UNIVERSITY OF THESSALY

SCHOOL OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

Diploma Thesis

DEVELOPMENT OF A MATHEMATICAL MODEL AND A SOURCE CODE FOR THE MULTI – DEPOT CAPACITATED VEHICLE ROUTING PROBLEM WITH HETEROGENEOUS FLEET

by

AFRODITI TEMOURTZIDOU

Submitted to fulfill part of the requirements

for the acquirement of the Diploma of Mechanical Engineer

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Afroditi Temourtzidou

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University of Thessaly, Department of Mechanical Engineering, 2016

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Summary

Transportation plays an important role in daily basis for thousands of companies and organisations engaged in the delivery and collection of goods or people. Especially, logistics based companies often use various types of vehicles and operate from more than one distribution center, usually referred to as depot. In this situation, determining the optimal routes in order to serve the customers is a significant problem, as it could lead not only to the reduction of the transportation cost, but also contribute to the environmental protection, through the reduction of CO_2 emissions.

In this study, we present, analyse and compare two optimisation algorithms for distributions of third party logistics companies, which have more than one depot and heterogeneous fleet of vehicles in order to serve customers with different types of demand. This problem is known as the Multi – Depot Capacitated Vehicle Routing Problem with Heterogeneous Fleet (MDHFCVRP). Simultaneously, we simulated the algorithm in C++ programming language by using the CPLEX Optimization Studio.

The proposed mathematical formulations are mixed integer – linear programming models. We estimate that the implementation of them by logistics based companies will decrease both their transportation cost and their carbon footprint.

Contents

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Acronyms

APJ	Asia – Pacific Journal of Operational Research
CIE	Computers & Industrial Engineering
COR	Computers & Operations Research
CVRP	Capacitated Vehicle Routing Problem
CP	Constraint Programming
DVRP	Dynamic Vehicle Routing Problem
EJOR	European Journal of Operational Research
EMA	Environmental Modeling and Assessment
ESA	Expert Systems with Applications
GenClust	Genetic Clustering
GRASP	Greedy Randomized Adaptive Search Procedure

HFVRP	Vehicle Routing Problem with Heterogeneous Fleet
IJL	International Journal of Logistics Research and Applications
INFOR	INFOR
INTER	Interfaces
JFE	Journal of Food Engineering
JMMA	Journal of Mathematical Modelling and Algorithms
JORS	Journal of the Operational Research Society
LPR	Location Routing Problem
m – TSP	Multiple Traveling Salesmen Problem
MDVRP	Multi – Depot Vehicle Routing Problem
MDFHCVRP	Multi – Depot Capacitated Vehicle Routing Problem with Heterogeneous Fleet
MILP	Mixed Integer Programming
MSOM	Manufacturing & Service Operations Management

NN	Nearest Neighbour
OME	Omega
OR	Operations Research
PVRP	Periodic Vehicle Routing Problem
SA	Simulated Annealing
SDVRP	Vehicle Routing Problem with Split Delivery
TB	Tabu Search
TS	Transportation Science
TSP	Traveling Salesman Problem
VNS	Variable Neighbourhood Search
VRP	Vehicle Routing Problem
VRPCS	Vehicle Routing Problem with Crew Scheduling
VRPPD	Vehicle Routing Problem with Pick – up and Delivery
VRPSD	Vehicle Routing Problem with Stochastic Demand

VRPSC	Vehicle Routing Problem with Stochastic Customers
VRPTW	Vehicle Routing Problem with Time Windows
VRP2002	The Vehicle Routing Problem, 2002
2L – VRP/ 3L – VRP	Vehicle Routing Problem with Two/ Three Dimensional Loading

Chapter 1 The Traveling Salesman Problem and the Vehicle Routing Problem

1.1 The Traveling Salesman Problem

1.1.1 Definition of the Problem

The Traveling Salesman Problem (TSP) is widely studied in Computer Science and Linear Programming. It is stated as, given a complete graph, G, with a set of vertices, V, a set of edges, E, and a cost, c_{ij}, associated with each edge in E. The value c_{ij} is the cost incurred when traversing from vertex $i \in V$ to vertex $j \in V$. Given this information, a solution to the TSP must return the Hamiltonian cycle of G with the minimum cost. A Hamiltonian cycle is a cycle that visits each node in a graph exactly once. This is referred to as a tour in TSP terms.

The essence of the traveling salesman problem is evident within many practical applications in real life. From a mail delivery person trying to figure out the most optimal route that will cover all of his/her daily stops, to a network architect trying to design the most efficient ring topology that will connect hundreds of computers. In all of these instances, the cost or distance between each location, whether it is a city, building or node in a network, is known. With this information, the fundamental goal is to find the optimal tour, which is to determine an order in which each location should be visited exactly once, and the total distance traveled, or cost incurred, is minimum. In the general TSP, there are no restrictions on the distance/cost values. (1)

1.1.2 History of the Problem

The origins of TSP range back to the 1800's, when the Irish mathematician Sir William Rowan Hamilton and the British mathematician Thomas Penyngton Kirkman treated the first mathematical problems related to it. However, the general form of the TSP appears to be first studied in the 1920's, when the mathematician Karl Menger brought it to the attention of his colleagues in Vienna (2). During the 1930's, the mathematical community of Princeton dealt with the problem, and in the 1940's, mathematician Merrill Meeks Flood, publicized the name, TSP, within the mathematical community (3). It was the year 1948 that Flood publicized the traveling salesman problem by presenting it at the RAND Corporation, which is a non-profit organization that is the focus of intellectual research and development within the United States (4).

The TSP soon became very popular due to its connection with the rising combinatorial problems of Linear Programming and its application in many tasks within people's daily lives. In 1950's Dantzig, Fulkerson, and Johnson presented a method for solving the TSP. They showed the effectiveness of their method by solving a 49-city instance (5).

1.1.3 Complexity

In the mid 1960's, it became evident that in general instance TSP is NP hard, which means that it could not be solved in polynomial time using Linear Programming techniques. In fact, it was proved that the TSP posed such computational complexity that any efforts to solve the problem would grow super polynomial with the problem size (6).

Due to its complexity, efficient approximation algorithms have been developed, which can be very useful in practice, such as «branch and bound», Lagrange relaxation and various numerical methods which provide a feasible solution very close to the optimal by minimizing the error. Since 19th century, many and more useful variations in real life applications of the TSP have been solved, in which certain restrictions are imposed.

1.1.4 Variations

Restrictions are set in the general TSP, either to make it easier to solve, or simply because such restrictions allow the problem to reflect certain and more realistic applications. The most popular variations of TSP are listed below (1).

- The Symmetric TSP: In this variation, all the edges have symmetric costs. This means that, for all nodes in the graph, the cost incurred, when traveling from node a to node b, is the same as cost incurred when traveling backwards (from node b to node a). On the other hand, the asymmetric TSP does not have such constraints. The general TSP is considered asymmetric. An input to the asymmetric TSP would be a directed graph.
- The Metric TSP: In this variation, all of the edge costs are symmetric and also satisfy the triangle inequality. The triangle inequality property means that for any three nodes a, b and c, the cost of going from node a directly to node c is always cheaper than going from node a to node c by passing through node b. In addition, the nodes are points in some space and the edge costs are determined by calculating the metric distance between them.
- The Euclidean TSP: In this variation, all of the nodes lie in the plane, which means it is symmetric and the triangle inequality is applied. The cost of each edge e, connecting nodes a and b, is defined by the Euclidean distance between the nodes a and b. In general, the plane can be d-dimensional, where $d > 1$.

1.1.5 Real Life Applications

There are many practical real life uses of the TSP. The most common of which are transportation routing problems.

The most popular application consists of finding a route that a salesman would follow in order to visit every geographical location in a specified list such that minimum total distance is traveled. Considering the case of a salesman traveling from door to door in a certain sub – division of houses, it would be very convenient if the salesperson could obtain a list of all the houses in that sub – division specified in the most optimal order to visit. Furthermore, considering the case of a salesperson that needs to visit hundreds of cities spread throughout an entire country, by knowing the optimal tour that will visit each city, days or even weeks of traveling time could potentially be saved. Moreover, considering a postal delivery person who goes to work in the morning with a truck full of parcels to deliver, in what order should those parcels be delivered in order to minimize the total distance traveled? For all these instances, the nodes in the graph would correspond to the geographical locations, and the distances would be metric values based on the lengths of the roads connecting the locations.

There are also other important practical uses of the TSP. Considering some of the machines in an assembly line, there are machines whose sole purposes are to drill various holes in a certain piece of material. The material may be a circuit board, the frame of a vehicle, or even a piece of wood to be used building a book shelf. The drill is repositioned by motors that slide along tracks such that the drill could move to any position within a certain area. It will take a certain amount of time to reposition the drill depending on the distance that drill needs to move. A solution to the TSP could be used to find the optimal order in which the holes should be drilled. In the case of an assembly line, saving several seconds to complete the process for each work – piece means producing much more work – pieces by the end of the day. To solve this problem using the TSP application, simply let the vertices represent the locations that the holes need to be drilled and let the edges be the distances between them.

Another application that a solution to the TSP can be applied to is electronic or mechanical connection placement. Considering the wiring of a circuit board, or the electrical wiring within a large building, or even the plumbing layout within a building, in many of these cases, the connections need to be laid out such that the components are all connected in a cycle. In the case of a circuit board, the connections are the wires and the components are the transistors, resistors, etc. Concerning the electrical setup of a building, the connections are the wires and the components are the switches, plugs, light fixtures, etc. Finally, regarding the plumbing layout of a building, the connections are the pipes and the components are the faucets and water taps. In all of these cases, the shortest Hamiltonian path should be found in order to save material and to optimize flow by reducing the length of the cycle. Connecting circuits or wiring electronic components so that the current has to travel the minimum possible distance will ultimately increase efficiency and overall performance.

The final application which will be analysed is the multiple salesmen model (m – TSP). The task of this application is to visit a set of cities, where each city has to be visited exactly once by any of the m salesmen. For each salesman j, who is being hires a fixed cost of d_j – the salary – should be paid. Each of the m salesmen must complete a subtour, and the combination of all the subtours that each of the m salesmen embark on must result in each city in set being visited exactly once. The salesmen start and end their sub tours at the same home base location. The object it to determine how many salesmen to hire and what the subtours should be done in order to minimize the total distance traveled and the cost of hiring the salesmen. This problem can be modeled as the original TSP by adding $(m - 1)$ additional vertices, denoted $-1, \ldots, -(m - 1)$ to the input graph. Now, the m salesmen, numbered 0 to $(m - 1)$, are represented by the source vertex and the new $(m - 1)$ vertices. Then, edges are added to connect these $(m - 1)$ new vertices to the rest of the vertices in the original graph. The costs of the newly added edges are determined by adding the costs d_j, for $0 \leq j \leq (m - 1)$, to the cost of existing edges in the graph (3).

1.2 The Vehicle Routing Problem

1.2.1 Definition of the Problem

The classical Vehicle Routing Problem (VRP) generalizes the Traveling Salesman problem and is one of the most popular problems in Combinatorial Optimization. It is defined on a complete undirected graph $G = (V, E)$, where $V = \{0, \ldots, n\}$ is a vertex set and $E = \{(i, j): i, j \in V, i < j\}$ is a set of edges. Each vertex $i \in V \setminus \{0\}$ represents a customer having a nonnegative demand q_i, while vertex 0 corresponds to the depot. Each edge $e \in E$ is associated with a travel cost c_{ij}. A fixed fleet of m identical vehicles, each of capacity Q, is available at the depot. Problem's objective is the determination of a set of at most m vehicle routes whose total travel cost is minimized such that:

- each customer is visited exactly once by one route,
- each route starts and ends at the depot,
- the total demand of the customers served by a route does not exceed the vehicle capacity Q, and
- the length of each route does not exceed a preset limit L.

The VRP arises in several forms because of the variety of constraints encountered in practice. The basic variation of VRP is the Capacitated VRP (CVRP) and is described as (7):

"A number of identical vehicles with a given capacity are located at a central depot. They are available for servicing a set of customer orders, (all deliveries, or, alternatively, all pickups). Each customer order has a specific location and size. Travel costs between all locations are given. The goal is to design a least cost set of routes for the vehicles in such a way that all customers are visited once and vehicle capacities are adhered to."

1.2.2 History of the problem

The Capacitated VRP was formally introduced in 1959 by Dantzig and Ramser. They proposed a simple matching – based heuristic for its solution and illustrated it on a toy – sized example. The following years saw the emergence of several heuristics based on a variety of principles including savings, geographical proximity, customer matchings, as well as intra – route and inter – route improvement steps. Perhaps the most famous heuristic of this category is the Clarke and Wright (1964) savings heuristic, which has resisted the test of time because of its speed, simplicity and reasonably good accuracy.

The development of exact algorithms for the VRP took off in 1981 with the publication of two papers by Christofides, Mingozzi and Toth in Networks. The first one proposed an algorithm based on dynamic programming with state – space relaxation whereas the second one proposed two mathematical formulations making use of q – paths and k – shortest spanning trees. A few years later, Laporte, Desrochers and Nobert proposed the first cutting plane approach for a VRP based on the solution of linear relaxation of an integer model. These seminal concepts have made their way into some of the more recent algorithms.

Since then, a variety of exact algorithms based on mathematical programming formulations have been proposed. Some formulations contain vehicle flow or commodity flow variables and are often solved by branch – and – cut method. The VRP can also be formulated as a set partitioning problem to which some valid inequalities are added. Some of the most successful implementations by Fukasawa (2006) and by Baldacci (2008) are based on this methodology.

The development of modern heuristics for the VRP really started in the 1990s with the advent of metaheuristics. It is fair to say that the study of the VRP has stimulated the growth and understanding of several metaheuristic concepts which are now known. The early research in this area was quite fragmented, with a notable bias towards tabu search-based approaches and some of the algorithms were over engineered, but some rationalization has started to take place in recent years. The best metaheuristics are those that simultaneously perform a wide and deep search of the solution space and can solve several variants of the problem. They generally either apply several operators, as in adaptive large neighbourhood search (Pisinger and Ropke 2007), or combine genetic search with local search, as in the hybrid genetic algorithm recently proposed by Vidal (2012) (8).

1.2.3 Complexity

The classical VRP is a generalization of the TSP (and especially m – TSP). Since TSP is proven to be NP – hard, the VRP also is NP – hard. Therefore, the computational effort required to solve the problem increases exponentially as the problem size increases.

1.2.4 Variations

As it is mentioned in section 1.2.1, the VRP arises in several forms because of the variety of constraints encountered in practice. The basic variation of VRP is the Capacitated VRP (CVRP). However, the real – life routing problems usually include much more complications which are not considered by the basic CVRP. Most of the complications are related to the following aspects (7).

- Planning horizon: In real life, routes are planed for a given planning horizon. This planning horizon can consist of multiple periods.

- Customer: In the basic VRP, each customer has a demand. In more complicated problems, the customers may have requirements on the service time and/ or the vehicle type. There could also be different types of services, e.g., pickup service, delivery service or pickup-and-delivery service. In addition, in some cases, customers are allowed to be visited multiple times by several vehicles instead of just once by one vehicle. Moreover, in some applications with multiple planning days, the customers have demands every day and they can store products for the following days if they have received more than they can consume. In this case, the distributor needs to make a routing plan according to the demands and inventories of the customers.
- Depot: There can be more than one depot in a large distribution network, which may serve different purposes, such as warehousing or crossdocking, to reduce the total cost in the supply chain.
- Vehicle: The vehicles used for distribution can have different capacities and sizes. There are usually a limited number of vehicles available in real – life planning. A vehicle may be used in multiple trips instead of a single trip in a routing plan. In the problem with multi depots, each vehicle may be associated to a base depot. The vehicle must start from and end at its base depot.
- Driver: In most of the real-life problems, distributors need to consider the drivers' working regulations, e.g., the working shift and the break rules.

- Objective: The objective function can be quite complex in practice. It may include the minimization of the total travel cost, the minimization of the difference between the longest and shortest route to balance the workload among drivers, the minimization of the number of vehicles to save the large overhead, and/ or the maximization of the number of served customers to improve the service level.
- Uncertainty: There can be uncertainties in the route planning. For example, the locations and/ or the demands of customers are unknown at the beginning but revealed over time when the vehicles have already been sent out to carry out tasks. In some occasions, the probability distribution of these uncertainties is available, whereas in other cases, it is not.
- Good packing: In some applications, customer demand is formed by a set of two – dimensional or three – dimensional weighted items. A feasible routing implies a feasible packing in the sense of geometrical layout.

These complications and even more real – life restrictions lead to different extensions of the CVRP. The variations of CVRP are:

- Multi – Depot VRP (MDVRP): When there is more than one depot from which the customers can be served, the problem is considered as MDVRP. The MDVRP requires the assignment of customers to depots. A fleet of vehicles is based at each depot. Each vehicle originates from one depot, services the customers assigned to that depot, and returns to the same depot. The objective is to minimize the total travel cost and the number of vehicles used in order to service all customers.

- VRP with Heterogeneous Fleet (HFVRP): When the available fleet of vehicles for distribution activities characterized by different capacities, types of cargo and/ or costs, the problem is considered as HFVRP. The objective is to minimize the total travel cost and the total cost of vehicles used (9).
- VRP with Time Windows (VRPTW): In VRPTW every customer is characterized by a specific time window $[a_i, b_i]$ within it should be serviced. A vehicle is allowed to arrive before a_i and wait until the customer becomes available. However, arrivals after b_i are prohibited. The objective is to minimize firstly the number of vehicles used and then the total distance traveled (10).
- VRP with Split Delivery (SDVRP): In SDVRP each customer can be served by more than one vehicle. This relaxation of the VRP facilitates the service of the customers especially when their size of orders is as big as the capacity of the vehicle. The objective is to minimize the vehicle fleet and the total distance travelled.
- VRP with Pick – up and Delivery (VRPPD): In VRPPD is taken into account the return of goods to the delivery vehicle. Hence, they have to fit into it. The objective is to minimize the vehicle fleet and the total distance travelled, with the restriction that the vehicle must have enough capacity for transporting the commodities to be delivered and those ones picked – up at customers for returning them to the depot.
- Periodic VRP (PVRP): In PVRP, there is a horizon of M days and a frequency for each customer stating how often within the M – day period each customer should

be visited. The objective is to minimize the total cost of all routes over the planning horizon (11).

- Dynamic VRP (DVRP): DVRP is the extension of the classical VRP in which the uncertainties of real life is taken into consideration. The objective is to minimize the total travel cost subject to constraints related to real life restrictions (7).
- VRP with Crew Scheduling (VRPCS): VRPCS is the combination of the vehicle routing and the crew scheduling problem. Although it is easier to study these problems separately, due to the dependence between them the combined problem may yield better schedules of the costly manpower and also may reduce the total cost significantly (7).
- VRP with Stochastic Customers (VRPSC): In VRPSC, each customer is present with probability p and absent with probability (1 – p). Two stages are made in order to get a solution. Firstly, a solution is determined before knowing the actual number of customers. In a second stage, a recourse or corrective action can be taken when the customers are determined, in order to define the optimal solution. The objective is to minimize the vehicles used and the total travel cost.
- VRP with Stochastic Demand (VRPSD): When the demand of each customer is a random variable, the problem is considered as VRPSD. As in VRPSC, the optimal solution is determined in two stages and the objective is to minimize the vehicle used and the total travel cost.

- VRP with Two/ Three Dimensional Loading (2L – VRP/ 3L – VRP): 2L – VRP/ 3L – VRP is the combination of the vehicle routing and the vehicles' loading problem. The objective is to find a partition of the customers into subsets, which are no more than the maximum number of available vehicles and, for each subset, a route starting and ending at the depot such that the total travel cost is minimized (12).

1.2.5 Real Life Applications and Literature Review

The VRP is rooted in a wide variety of real life applications worldwide. The key sectors of real life VRP applications are (13):

- oil, gas and fuel
- retail,
- waste collection and management
- mail and small package delivery, and
- food distribution.

1.2.5.1 Oil, gas and fuel applications

Concerning problems which describe applications related to the supply of oil, gas and fuel to houses, gas stations and companies, they present a number of specific features such as vehicles with capacitated compartments and sometimes the presence of flow meters to control the delivered quantity. The latter feature implies that sometimes the content of the same compartment can be used to satisfy the demands of several customers, whereas when there is no flow meter, the compartment must be completely emptied in a single customer tank. Cleaning operations may be needed between the loading of different products using the same compartment. These problems are generally solved over long – term planning horizons and incorporate mixed inventory and routing decisions (14).

Campbell et al. (15) worked with Praxair, an industrial gases company with about 60 production facilities and more than 10.000 customers across North America. The problem, modeled as an inventory – routing problem, was solved by means of a two – phase heuristic that first assigns delivery days to customers, and then creates vehicle routes. It was applied to instances in which facilities can have between 50 and 87 customers.

Chiang and Russell (16) integrated purchasing and routing decisions for a propane gas supply chain using set partitioning and tabu search techniques. They modeled the problem as a general multi – depot VRP with time windows in which a tanker starts from a depot, travels through a number of terminals (for pickups) and plants (for deliveries), and returns to the same depot. They reported results for the Illinois and Michigan dispatch areas. In the case of Michigan, they reduced the number of tankers used from 130 to 102 over a one-week planning period.

Avella et al. (17) studied the case of a company supplying three types of fuel to a set of gas stations located in an urban area. They considered that each tank in the delivery vehicles must be either completely full or completely empty. They generated all feasible routes having at most four clients and solved the resulting set partitioning problem by branch – and – price. They solved a one – week instance with 60 clients, and a fleet of six heterogeneous trucks was used to serve about 25 clients per day.

Cornillier et al. (18) studied a problem similar to that of Avella (17) but also considered the loading of tanker trucks divided into compartments, which is of primary importance since there are several small gas stations throughout Eastern Quebec, the area of application, and because trucks are not equipped with flow meters.

Song and Savelsbergh (19) worked with Praxair on another variant of the inventory – routing problem. They developed bounds on the volume delivered per mile, which was used to determine customer – plant assignment.

Ng et al. (20) designed a decision support system combining heuristic and optimal routing for a tanker assignment and routing problem for petroleum products in Hong Kong. They reported an increase in the volume delivered as well as better route designs.

Cornillier et al. (21) studied a richer petrol station replenishment problem with time windows and obtained a distance reduction of about 22% over the solution obtained by the company dispatcher. On a 42-station instance, they reduced the number of routes from 26 to 23.

Day et al. (22) studied the inventory replenishment of a company in Indiana, which distributes carbon dioxide to over 900 customer sites. They developed a heuristic capable of reducing driver labor cost by about 30%.

In Table 1, all the papers which are mentioned in this section are presented in summary.

Author	Year	Journal	Algorithm	Product/ Company/ Location	Estimated improvement
Campbell et al.	2002	VRP2002	Two-phase heuristic	Gases/ Praxair/ North America	
Chiang and Russell	2004	EJOR	Set partitioning and tabu search	Propane/ One of the largest USA distributor/ Illinois and Michigan	9.4% reduction in total cost and 21.5% in number of vehicles
Avella et al.	2004	EJOR	Set partitioning and branch-and-price	Fuel//	22-25% reduction in total cost
Song and Savelsbergh	2007	TS	Lower bounds	Gases/ Praxair/	
Cornillier et al.	2008	JORS	Matching and column generation	Fuel// Eastern Quebec, Canada	17.2% reduction in distance and 1.16% increase in quantity delivered

Ng et al.	2008	JORS	Heuristic and integer programming with multiple objectives		Better route design and increased volume delivered
Cornillier et al.	2009	COR	Heuristics based on arc and route preselection	Fuel// Eastern Quebec, Canada	22% reduction in distance
Day et al.	2009	OME	Three-phase heuristic	Carbon dioxide// Indiana	30% reduction in driver labor cost

Table 1: Summary of contributions about oil, gas and fuel distribution

1.2.5.2 Retail applications

Retail involves the sales of goods and the provision of services to end – users. In this section we list applications dealing with a number of final products, and in various sectors such as supermarkets and consultancy services. These applications generally involve time windows and loading constraints.

Prins (23) studied the case of a French furniture manufacturer and modeled it as a heterogeneous VRP in which each vehicle can perform several trips. To solve it, he adapted several well – known VRP algorithms and developed a tabu search algorithm. On a one –

week data set containing 775 stores, his results showed a reduction of 11.7% in distribution costs.

Poot et al. (24) described a savings – based heuristic implemented within Shortrec Distriplanner®, a commercial virtual reality system sold by ORTEC Consultants, a Dutch software provider. The authors dealt with several types of constraints such as consistently assigning the same customers to drivers, grouping customers that should be visited first (or last) in a route, and forbidding some product combinations. Results were reported for four anonymous companies.

Gaur and Fisher (25) solved a periodic inventory – routing problem for Albert Heijn, a supermarket chain in the Netherlands. They reported transportation savings of about 4% in the first year of implementation.

Gendreau et al. (26) studied the case of an Italian company manufacturing bedroom furniture. The problem was modeled as a capacitated VRP with three – dimensional loading constraints and was solved by tabu search. Solutions were obtained on five instances involving up to 64 customers, 181 products and four vehicles.

Kant et al. (27) reported the implementation of the ORTEC vehicle routing software for Coca-Cola. They considered a problem involving about 10,000 trucks daily, and reported an annual cost savings of about $45 million, as well as major improvements in customer service.

Belfiore and Yoshizaki (28) worked with a Brazilian retail group composed of 519 stores present in 11 Brazilian states. They modeled the problem as a heterogeneous VRP with time windows and split deliveries, and proposed a scatter search heuristic to solve it. Using one week of data, they reported a cost reduction of about 7.5% which could translate into a yearly saving of one million dollars.

Chang et al. (29) described a stochastic dynamic TSP with time windows which was applied to FamilyMart, the second-largest convenience store in Taiwan, with more than 1,500 sales points. They proposed an algorithm combining a shortest n-path algorithm with a convolution – propagation heuristic. They performed their experiments on a 12 – customer instance which was said to be representative of a typical route.

Wen et al. (30) solved a VRP with cross-docking for the Danish consultancy Transvision. In this application identical vehicles are used to transport orders from sup-pliers to customers through a cross-dock. They developed a tabu search heuristic embedded within an adaptive memory search to solve an instance containing up to 200 pairs of nodes. They obtained within a few minutes, solutions that were less than 5% away from optimality.

In Table 2, a summary of the papers mentioned in this section is presented.

Author	Year	Journal	Algorithm	Product/ Company/ Location	Estimated improvement
Prins	2002	JMMA	Construction, improvement and tabu search algorithm	Furniture// Nantes, France	Reduction in distribution time of 11.7%

Poot et al.	2002	JORS	ORTEC software based on savings and local search		
Gaur and Fisher	2004	OR	Mathematical programming and matching	Supermarket/ Albert Heijn/ the Netherlands	4% reduction in cost
Gendreau et al.	2006	TS	Tabu search	Bedroom furniture// Italy	
Kant et al.	2008	INTER	ORTEC software based on savings and local search	Soft drinks/ Coca-Cola/ USA	Annual cost saving of 45$ million
Belfiore and Yoshizaki	2009	EJOR	Scatter search	Supermarkets// Brazil	7.5% cost reduction
Chang et al.	2009	EJOR	Heuristic based on n – path algorithm and convolution-propagation	Convenience stores/ FamilyMart/ Taiwan	22% reduction in distance
Wen et al.	2009	JORS	Tabu search within an adaptive memory procedure	/ Transvision/ Denmark	30% reduction in driver labor cost

Table 2: Summary of contributions about retail applications

1.2.5.3 Waste collection and management

Waste collection is essential to the proper functioning of any collectivity.

Ghiani et al. (31) presented a survey of the strategic and tactical issues related to the application of operations research in solid waste management. A variant of the problem deals with hazardous waste management in which collection, transportation, treatment and disposal of hazardous materials are involved. These problems are characterized by loading and un-loading constraints, time windows, and inter-arrival time constraints at customer points.

Tung and Pinnoi (32) studied the waste collection of households and streets garbage cans in five districts of Hanoi. The service is provided by Urenco, a private company paid by the municipal government based on the volume collected. The authors reported a reduction of 4.6% in operating cost and showed that they could reduce their fleet size by 20% or, conversely, increase the volume of waste collected with the current fleet by 20%.

Shih and Chang (33) modeled the routing and scheduling of medical waste from a set of hospitals and clinics as a periodic VRP. The system, tested in central Taiwan, uses dynamic programming to partition customers into routes and a simple 2 – opt heuristic to improve each route individually. They solved an instance with 346 clinics over six days, with two or three routes scheduled per day with up to 47 visited clinics.

Baptista et al. (34) extended the algorithm of Christo des and Beasley (35) for the periodic VRP to the collection of recycling paper containers in Almada, Portugal. In this application, a single vehicle must perform a route in the morning and another in the afternoon to collect 59 containers. The problem was solved over a one – month horizon.

Still in Portugal, Teixeira et al. (36) studied an urban recyclable waste problem where three types of products (glass, paper and plastic/ metal) must be collected separately. They modeled the problem as a periodic VRP which was solved through a three – phase heuristic. Their algorithm yielded a distance reduction of about 29% over historic distances travelled.

A similar problem with different types of waste was studied by Nuortio et al. (37) in Eastern Finland. These authors developed a scheduler and an optimizer system based on a guided variable neighborhood thresholding metaheuristic and reported an average distance improvement of 12% and a reduction of 44% on a specific instance.

Sahoo et al. (38) worked with Waste Management Inc., a provider of waste-management services based in Houston, which services nearly 20 million residential customers and two million commercial customers throughout the Unites States and Canada. They developed a complete route – management system, deployed over 36 markets areas, and yielding 984 fewer routes and $18 million in savings after one year. In the long run, the number of routes was expected to be reduced by 10%.

Li et al. (39) developed a prototype decision support system (DSS) for the solid waste collection services in Porto Alegre, Brazil. They analyzed the impact of disruptions in trips and the strategy to use when unexpected events occur.

In the context of hazardous waste disposal, Alumur and Kara (40) proposed a model that determines where to open treatment centers, which technologies to employ, how to assign different types of hazardous waste to compatible treatment technologies, and how to route waste residues to disposal centers. The system was applied in the Central Anatolian region of Turkey.

Repoussis et al. (41) developed a complete DSS to manage waste lube oils collection and recycling operations for a multinational Greek company. They modeled this problem as an open VRP with time windows and solved it by means of a list – based threshold accepting metaheuristic. Unit cost reductions of up to 30% were achieved.

Coene et al. (42) studied the problem of a Belgian company collecting waste at slaughterhouses, butcher stores, and supermarkets. Waste products were divided into two categories – high – risk and low – risk – and different vehicles were used for each type. This led to two distinct periodic VRPs, one with 48 low – risk customers and three trucks over a planning period of one week, and one with 262 high – risk customers and three trucks over a two – week planning horizon. Since planning occurs over a time period of several days, the problem was solved as a periodic VRP using a two – phase heuristic in which customers are first assigned to days, and VRPs are solved for each day in the second phase.

Hauge et al. (43) dealt with the transportation of bulky waste containers. This roll – on/ roll – off routing problem arises in the collection of industrial waste. It was formulated as a generalized set partitioning problem and solved by means of a hybrid column generation and a tabu search procedure.

Hemmelmayr et al. (44) studied the problem of designing a collection system for general waste in Italy. They considered the bin configuration and sizing problem at each collection site, as well as the service frequency over a given horizon. They analyzed the resulting trade – offs between the bin investment cost and the routing cost. They proposed a hierarchical solution procedure in which the bin location problem was first solved and was followed by the solution of the VRP. They tested both a sequential and an integrated approach.

Battarra et al. (45) solved an urban garbage collection in Italy as a clustered VRP in which 456 large street bins are located at 385 collection points.

Aksen et al. (46) studied the case of a biodiesel production facility in Istanbul, which collects used vegetable oil from restaurants, catering companies and hotels. The resulting selective and periodic inventory-routing problem was solved by means of an adaptive large neighborhood search algorithm.

Huang and Lin (47) studied the problem of efficiently routing and scheduling collectors for municipal waste collection in Taiwan where it is required that residents personally bring their waste to collection vehicles. They proposed a bi-level optimization model that first selects collection points by solving a set covering problem and then solves a VRP with pickup delivery by means of an ant colony optimization heuristic. They used two instances from a subnetwork of Kaohsiung City in Taiwan, involving 262 and 611 nodes.

In Table 3, a summary of the papers mentioned in this section is presented.

Author	Year	Journal	Algorithm	Product/ Company/ Location	Estimated improvement
Tung and Pinnoi	2000	EJOR	Heuristic route construction and improvement	Street solid waste/ Urenco/ Hanoi, Vietnam	4.6% operating cost reduction
Shih and Chang	2001	EMA	Heuristic route construction and improvement	Medical waste// Tainan City, Taiwan	
Baptista et al.	2002	EJOR	Heuristic route construction and improvement	Recycling paper containers// Almada, Portugal	
Teixeira et al.	2004	EJOR	Three-phase heuristic	Glass, paper, plastic, metal// Portugal	29% reduction in distance
Sahoo et al.	2005	INTER	Iterative two-phase algorithm	Waste/ Waste Management Inc./ USA	984 fewer routes, saving $18 million

Nuortio et al.	2006	ESA	Guided variable neighborhood thresholding metaheuristic	Municipal waste/ Jätekukko Ltd/ Finland	12% distance reduction on average
Alumur and Kara	2007	COR	Mathematical model solved by CPLEX	Hazardous waste// Central Anatolia	22% reduction in distance
Li et al.	2007	COR	DSS with optimization	Municipal waste/ DMLU/ Porto Alegre, Brazil	30% reduction in driver labor cost
Repoussis et al.	2009	EJOR	DSS with hybrid metaheuristics	Lube oil// Greece	25% to 30% reduction in per unit cost
Coene et al.	2010	JORS	Two-phase mathematical based algorithm	Animal waste// Belgium	
Hauge et al.	2014	CIE	Hybrid column generation and tabu search	Industrial waste// Italy	
Hemmelmayr et al.	2013	TS	VNS and ILP	General waste// Italy	
Battarra et al.	2014	OR	BC & P	Garbage// Italy	

Aksen et. al	2014	EJOR	ALNS	Used oil// Instabul, Turkey
Huang and Lin	2015	OME	Set covering and ant colony	Municipal waste// Kaohsiung, Taiwan

Table 3: Summary of contributions about waste collection and management

1.2.5.4 Mail and small package delivery

Mail and package delivery is a very important industry. In this section, the reviews of real life applications range from mail delivery to the delivery of Internet orders, touching many variants of the classical VRP such as those involving time windows and pick – ups and deliveries.

Larsen et al. (48) modeled and solved the routing problem of an overnight mail service provider as an a priori dynamic TSP with time windows. The objective was to minimize the lateness of deliveries. They worked with United Parcel Service (UPS), using 10 days of data for each of four selected areas.

Hollis et al. (49) used a vehicle routing and crew scheduling algorithm based on set covering with column generation to solve the Melbourne metropolitan mail distribution at Australia Post. They worked with instances containing up to 339 locations and five depots and reported a potential cost saving of about 10%.

Cohn et al. (50) studied the load-matching and routing problem with equipment balancing for small package carriers. In this problem, all packages of a given commodity move through the same sequence of intermediate sorting facilities, and the commodities are grouped by common destination to fill trailers more efficiently. They used data from a regional subnetwork from UPS, with 263 nodes and more than 2,000 requests, and reported cost reductions of about 5%.

Groër et al. (51) solved a consistent VRP in a context where the objective is to plan the routes in order to have customers consistently visited by the same driver over time, so as to develop good working relationships. They solved an instance with 3,715 customers locations based on five weeks of real customer data provided by a company in the small package shipping industry.

Sungur et al. (52) studied a VRP with time windows in which customers appear probabilistically and have uncertain service times. They worked on two data sets provided by UPS having up to 5,178 potential customers and more than 25,000 service requests. They reported improvements of up to 20% over a weighted objective function value.

Pignac – Robitaille et al. (53) solved a pickup and delivery problem for a company specialized in transportation of biomedical samples in Quebec City. They worked on a data set containing 946 requests in which up to 30% of them were known one day in advance. By using the company's strategy which neglects this information, they reduced the number of routes from 54 to 50. Using information about known requests allowed cutting off one additional route and reducing the total distance by an additional 1.3%.

In Table 4, the papers presented in this section are summarized.

Author	Year	Journal	Algorithm	Product/ Company/ Location	Estimated improvement
Larsen et al.	2004	TS	Dynamic construction and improvement heuristics	Courier/UPS/ USA	
Hollis et al.	2006	EJOR	Set covering with column generation	Mail/ Australia Post/ Australia	Potential cost savings of 10%
Cohn et al.	2007	TS	Column generation and enumeration based heuristics	Courier/ UPS/ USA	Cost reduction of about 5%
Groër et al.	2009	MSOM	Record-to-record travel heuristic	Small packages//	
Sungur et al.	2010	TS	Insertion based and tabu search	Courier/ UPS/ USA	Up to 20% over a weighted objective function

Pignac – Robitaille et al.	2014	INFOR	Improvement heuristics	Medical samples// Quebec City, Canada

Table 4: Summary of contributions about mail and small package delivery

1.2.5.5 Food distribution

Food distribution has its own characteristics, constraints and challenges such as product quality, health and safety. The products often have a limited shelf – life, so that distribution operations must take into account temperature, humidity and time – in – transit considerations, as well as many other constraints related to products.

The review of Akkerman et al. (54) focuses on the challenges of food safety, quality and sustainability. These authors outline practical contributions related to strategic network design, tactical network planning and operational transportation planning.

Ahumada and Villalobos (55) studied the particular agricultural food (agri – food) supply chain and reviewed the main contribution in the specific field of production and distribution planning for agri – foods based on agricultural crops.

Tarantilis and Kiranoudis (56) dealt with the distribution of fresh milk for one of the largest dairy companies in Greece. The problem was formulated as a heterogeneous fixed fleet VRP and solved through a backtracking adaptive threshold accepting algorithm. The authors solved an instance containing 299 supermarkets located in Athens, with a heterogeneous fleet of 29 vehicles, reducing the total distance by 28% in comparison with the solution used by the company.

Cheong et al. (57) studied a soft drink distribution problem arising in several districts of Singapore. They reduced both the average and maximum number of vehicles needed over a 23-day period.

Tarantilis and Kiranoudis (58) modeled the distribution of fresh meat from depots to 174 butcher shops in Athens as an open multi – depot VRP and solved it by means of a threshold accepting – based metaheuristic. They reported reducing the total traveled distance by 17%.

Prindezis et al. (59) developed a solution system that was applied to the Greater Athens area for the benefit of Athens Central Food Market enterprises. The system, based on a tabu search metaheuristic, is used by nearly 150 Central Market enterprises for planning their daily routes.

Faulín (60) & (61) solved a logistics problem for Alimentos Congelados, S.A., a canning company located in Navarra, Spain. Heuristics were used for the initial solution which was improved by linear programming. Results obtained over 11 days showed a 4.6% average distance reduction.

Pamuk et al. (62) improved distribution operations for a major beer producer having about 4,000 customers in Ankara. They used a workload balancing and partitioning model to assign customers to workdays, followed by a simple nearest – neighbor routing heuristic.

Ruiz et al. (63) worked with Nanta S.A., a leading Iberian feed compounder in Spain, offering pig, poultry, ruminants, rabbits and other livestock feeding, and developed a complete DSS. They partitioned the customers into regions and created routes with few clients, generally less than six. They reported distance reductions ranging from 7% to 12% and cost reductions of 9% to 11%.

Faulín et al. (64) worked with the Frilac Company in Pamplona, northern Spain, which delivers frozen goods such as ice cream, vegetables, precooked dishes, seafood and meat. They developed a complete DSS with database and visualization capabilities based on a savings algorithm. They reported reductions of 13.5% in distance and 10.8% in cost seven months after the implementation.

Belenguer et al. (65) presented a computer program developed to design delivery routes for a medium-sized meat company in Valencia. They used seven days of data to plan the routes of a fleet of seven vehicles serving between 94 and 148 orders per day. They considerably reduced the total lateness and the routes lengths by 8.96%.

Ioannou (66) studied the supply chain of the Hellenic sugar industry in Greece. They handled the transportation part by means of the Map – Route system created by Ioannou et al. (67). This system was developed for a wholesaler and logistics service provider supplying packaged goods and beverages to supermarkets and retail outlets in the Central Athens area. Their objective was to minimize long-term average inventory and routing costs.

Prive et al. (68) studied the distribution of soft drinks and collection of recyclable containers for Distribution Jacques Dubois, a Quebec – based distributor in Canada. They considered vehicle routing costs and the revenue generated by the collection of recyclable containers for 164 customers ordering 125 different products over a one – week planning period. They reported a distance reduction of about 23% with respect to the manually designed routes of the company.

Cetinkaya et al. (69) improved the operations of Frito – Lay North America by modeling them as a large – scale, integrated multiproduct inventory lot – sizing and VRP. They solved the model using CPLEX by decomposing it into two sub – problems involving complementary inventory and routing components. They also used some classical TSP heuristics such as savings and cheapest insertions to improve the routes. Their results yield higher vehicle utilization and indicate that financial benefits could be achieved in inventory and delivery management.

Hu et al. (70) studied a food distribution problem for the Northern Grocery Company in Beijing. Routes were constructed over a circular transportation network, leading to special characteristics, which helped the generation process.

Battarra et al. (71) studied the distribution of three different types of foods to supermarkets (vegetables, fresh food and non-perishable), which were incompatible in the sense that they could not be delivered simultaneous in the same vehicle. The problem was modelled as a multi – trip VRP with time windows. Six days of data were used with an average of 422 customers per day.

Incompatibility constraints were also considered by Caramia and Guerriero (72) for a milk collection problem where some small farms are inaccessible by large trucks. Since farmers produce different milk types, they used multi – compartments trucks. They worked with ASSO. LA. C. which collects milk from 158 farmers in four towns in Calabria, in southern Italy. They were able to obtain a reduction of about 14.4% in the total distance traveled and they also increased the filling ratio of the tank trucks from 85% to 95%.

Zachariadis et al. (73) worked with a frozen food distribution company operating in Athens. This company uses 27 types of boxes and a homogeneous fleet of eight – pallet trucks. Thus, the problem was modelled as a pallet – picking VRP with three – dimensional rectangular boxes. The authors developed a tabu search algorithm in which pallet – packing is solved with a packing heuristic. Martínez and Amaya (74) worked with a home delivery service company that produces and delivers Spanish paella. The food is cooked in paella pans which are then delivered to the customers. This problem was solved as a multi – trip VRP with time windows and a loading component for these circulars items. On a set of 19 real instances they reported that their tabu search heuristic could reduce the total trip time by 25.5% on average.

Cattaruzza et al. (75) developed an iterated local search heuristic for the milk collection problem of Battarra et al. (71).

Lahrichi et al. (76) worked with the Fédération des producteurs de lait du Québec which is responsible for negotiating the transportation cost on behalf of the dairy producers' of the province of Quebec. They studied two examples having up to 226 farms, four depots and eight vehicles. When optimizing only the collection sequence, they reported small improvements of about 0.5% demonstrating that the current plan was very good. When they allowed the reassignment of farms and plants to vehicles, they obtained up to 4% in distance reduction, which corresponds to savings of a few hundred thousand dollars yearly.

Demir et al. (77) worked with Nabuurs B.V., a Netherlands – based logistics service provider specialized in refrigerated, frozen and ambient food products including beverages. They analyzed the shift from a single – depot planning to a centralized multi – depot planning process. They used a SHORTREC – based simulation model which includes many routing construction and improvement algorithms. They discussed the managerial implications as well as the implementation of the SHORTREC as a tactical planning tool.

Lahyani et al. (78) studied the olive oil collection process in Tunisia. Since olive oil comes in three different grades, it must be transported in multi – compartment vehicles. Cleaning operations may be needed if a compartment must be reused for a different oil grade. On a set of instances having up to seven producers and 39 requests they reported an average reduction of about 11.7% in the total distance traveled.

In Table 5, the contributions of this section are summarized.

Author	Year	Journal	Algorithm	Product/ Company/ Location	Estimated improvement
Tarantilis and Kiranoudis	2001	JFE	Backtracking adaptive threshold accepting	Milk//Athens	28% distance reduction
Cheong et al.	2002	APJ	Tree search, column generation over a set covering formulation	Soft drink// Singapore	Consistent reduction in the maximum number of vehicles required
Tarantilis and Kiranoudis	2002	JFE	List – based threshold accepting	Meat// Athens	17% distance reduction
Ioannou et al.	2002	JORS	DSS with GIS, look-ahead heuristic	Packaged goods and beverages// Athens	Lower number of routes and vehicles
Prindezis et al.	2003	JFE	Tabu search	Vegetables, fruits and meat/ Central Food Market/ Athens	
Faulín	2003	IJL	Heuristics and linear programming	Canning/ Alimentos Congelados S.A./ Spain	Average of 4.6% distance reduction

Ruiz et al.	2004	EJOR	B&B with Lingo	Animal food/ Nanta S.A./ Spain	Reduction of up to 11% in cost and 12% in distance
Pamuk et al.	2004	JORS	Workload balancing, partitioning and routing	Beer// Ankara	
Faulín et al.	2005	INTER	DSS based on savings and sweep algorithm	Frozen goods/ Frilac/ Pamplona, Spain	13.5% distance and 10.8% cost reduction
Belenguer et al.	2005	JFE	Constructive heuristic with tabu search improvement	Meat// Valencia, Spain	8.96% distance reduction
Ioannou	2005	JFE	DSS with GIS, look ahead heuristic	Sugar// Greece	About 25% in total transportation cost
Prive et al.	2006	JORS	Constructive and improvement	Soft drink/ Distribution J. Dubois/ Quebec, Canada	23% reduction in distance
Hu et al.	2009	COR	Route generation and selection	Packet meat/ Northern Grocery Co./ Beijing	

Cetinkaya et al.	2009	INTER	Mathematical decomposition and heuristics	Snacks/Frito – Lay/ North America	Average of 4.6% distance reduction
Battarra et al.	2009	COR	Heuristics with adaptive guidance mechanism	//	
Caramia and Guerriero	2010	INTER	Mathematical programming and local search multi – start	Milk/ ASSO. LA. C./ Italy	14.4% reduction in distance
Zachariadis et al.	2012	TS	Tabu search and packing heuristic	Frozen food// Athens	
Martnez and Amaya	2012	JORS	Insertion, tabu search and bin packing heuristics	Spanish paella//	25.5% in total trip time over a set of 19 instances
Cattaruzza et al.	2014	COR	Iterated local search	//	
Lahrichi et al.	2014	JORS	Generalized unified tabu search	Milk/ Fédération des producteurs de lait du Québec/ Canada	Lahrichi et al.

Demir et al.	2014	INTER	Construction and improvement	Refrigerated, frozen and ambient food products and beverages/ Nabuurs B.V./ the Netherlands	Demir et al.
Lahyani et al.	2015	OME	Branch – and – cut	Olive oil// Tunisia	11.7% distance reduction

Table 5: Summary of contributions about food distribution

Chapter 2 The Multi – Depot Capacitated Vehicle Routing Problem with Heterogeneous Fleet

2.1 Definition of the Problem

The Multi – Depot Capacitated Vehicle Routing Problem with Heterogeneous Fleet (MDHFCVRP) is the combination of the three following variations of VRP:

- Capacitated VRP (CVRP),
- Multi – Depot VRP (MDVRP), and
- VRP with Heterogeneous Fleet (HFVRP).

The objective of MDHFCVRP is to serve a given set of customers by minimizing the total distribution cost while taking into consideration the constraints of the three above mentioned problems simultaneously.

The fundamental constraint of the CVRP – the basic variation of VRP – is that the size of orders of all customers cannot exceed the capacity of the available on depot vehicles. In MDVRP, there is more than one depot with their fleet of vehicles from where the distributions carried out. The basic constraint in MDVRP is that all vehicles originate from one depot, service the customers assigned to that depot, and return to the same depot. In addition, the HFVRP sets that the available vehicles are not identical. However, they have different capacity, type of cargo and cost and these special characteristics should be described by appropriate constraints.

The MDHFCVRP is NP – hard, as its dimensions are also NP – hard problems, and it is difficult to obtain an optimal solution for large size problems within a reasonable amount of computing time. Therefore, time parameter constraints will be used in the simulation of the mathematical model of MDHFCVRP with CPEX solver.

2.2 Literature Review

Since 1980 more than one hundred of papers have studied the classical version of the MDVRP and the combination of it with other variants of VRP, some of them inspired from real – life applications. In this section, a literature review of the MDVRP, MDCVRP, MDHFVRP and MDHFCVRP is presented. Regarding the type of solution procedure applied – exact method, heuristic or meta–heuristic – this section is divided into three subsections.

2.2.1 Exact methods

The first papers presenting formal models or procedures to find optimal solution for the multi – depot vehicle routing problem are those of Laporte, Nobert, and Arpin (1984) who formulated the symmetric MDVRP as integer linear programs with three constraints. These authors then proposed a branch – and – bound algorithm using a LP relaxation (79).

The works of Kulkarni and Bhave (1985) (80), Laporte et al. (1988) (81) and Carpaneto, Dell'amico, Fischetti, and Toth (1989) (82) can also be considered as part of the pioneer works on exact methods for the MDVRP. The mathematical formulation proposed by Kulkarni and Bhave (1985) was later revised by Laporte (1989).

More recently, Baldacci and Mingozzi (2009) (83) proposed mathematical formulations for solving several classes of vehicle routing problems including the MDVRP.

Dondo, Mendez, and Cerdá (2003) (84) proposed a mixed – integer linear programming (MILP) model to minimize routing cost for MDHFVRP.

Cornillier, Boctor, and Renaud (2012) (85) presented a MILP model for the problem in which heterogeneous fleet of vehicles is available and with maximization of total net revenue as objective function, while maximum and minimum demands constraints are given.

Branch – and – cut algorithms were proposed by Benavent and Martínez (2013) (86) and Braekers, Caris, and Jenssens (2014) (87). Former authors focused also on studying the polyhedral structure of the problem which allowed the extension of their procedure to the location – routing problem (LRP), while latter authors considered a dial – a – ride problem with multiple depots.

The LRP with multiple depots was also studied by Contardo, Cordeau, and Gendron (2014) who proposed a cut – and column generation procedure for the capacitated case (88).

Other configurations of the MDVRP have been studied through exact algorithms. For instance, Contardo and Martinelli (2014) studied the capacitated MDVRP with route length constraints (89).

In Table 6, the papers of exact methods reviewed in this section are presented in summary.

Year	Authors	Variations		
		Multi – Depot	Heterogeneous Fleet	Capacitated
1984	Laporte et al. (1984)	✓		
1985	Kulkarni and Bhave (1985)	✓		✓
1988	Laporte et al. (1988)	✓		✓
1989	Laporte (1989)	✓		

1989	Carpaneto, Dell'amico, Fischetti, and Toth (1989)	✓		✓
2009	Baldacci and Mingozzi (2009)	✓		
2012	Cornillier, Boctor, and Renaud (2012)	✓	✓	
2013	Benavent and Martínez (2013)	✓		
2014	Braekers, Caris, and Jenssens (2014)	✓		
2014	Contardo, Cordeau, and Gendron (2014)	✓		✓

2014	Contardo and Martinelli (2014)	✓		✓

Table 6: Summary of MDVRP, MDCVRP and MDHFVRP with exact solution

2.2.2 Heuristics

Due to the NP – hardness of MDVRP, MDCVRP and MDHFVRP, several heuristic algorithms have been proposed in the literature.

The first works were published in the 1990's, in order to solve the capacitated version of MDVRP. Min, Current, and Schilling (1992) studied the version of the MDVRP with backhauling and proposed a heuristic procedure based on problem decomposition (90).

Salhi and Sari (1997) proposed the so-called "multi – level composite heuristic". This heuristic found as good solutions as those known at that time in the literature but using only 5 to 10% of their computing time. The heuristic was also tested on the problem with heterogeneous fleet (91).

Jin, Guo, Wang, and Lim (2004) modelled the MDVRP as a binary programming problem. Two solving methodologies were presented. The first one is a two – stage approach that decomposes and solves the problem into two independent subproblems. First the assignment problem is solved and then the routing problem. The second proposed approach treats both assignment and routing problems in an integrated manner. Their experimental results showed that the one – stage algorithm outperforms the other one (92).

The MDHFVRP, in which heterogeneous fleet of vehicles is considered have captured the attention of researchers since the work presented by Salhi and Sari (1997). Irnich (2000) proposed a set covering heuristic coupled with column generation and branch – and – price algorithm for cost minimization for the heterogeneous fleet and pickup and delivery MDVRP (93).

Tsirimpas, Tatarakis, Minis, and Kyriakidis (2007) considered the case of a single vehicle with limited capacity, multiple – products and multiple depot returns. Another characteristic of their problem is that the sequence of visits to customer is predefined. They developed a suitable dynamic programming algorithm for the determination of the optimal routing policy (94).

In Table 7, the papers reviewed in this section are presented in summary.

Year	Authors	Variations		
		Multi – Depot	Heterogeneous Fleet	Capacitated
1992	Min, Current, and Schilling (1992)	✓		✓

1997	Salhi and Sari (1997)	✓	✓	
2000	Irnich (2000)	✓	✓	
2004	Jin et al. (2004)	✓		
2007	Tsirimpas et al. (2007)	✓		✓

Table 7: Summary of MDVRP, MDCVRP and MDHFVRP with heuristics

2.2.3 Meta – heuristics

Meta – heuristic procedures have been employed by several researchers for efficiently solving the MDVRP and its variants.

The first meta – heuristic was proposed in the work of Renaud, Laporte et al. (1996) who studied the MDVRP with the constraints of vehicle capacities and maximum duration of routes (e.g. the time of a route cannot exceed the maximum working time of the vehicle). The objective to be optimized is the total operational cost (95). These authors proposed a tabu search algorithm for which the initial solution is built using the Improved Petal heuristic of Renaud, Boctor, and Laporte (1996b) (96).

Escobar, Linfati, Toth, and Baldoquin (in press) evaluated a granular tabu search algorithm to minimize the total sum of vehicle travelled distances (97).

The first genetic algorithms were proposed by Filipec, Skrlec, and Krajcar (1997) for the problem of minimizing total travel distance (98), by Salhi, Thangiah, and Rahman (1998) and by Skok, Skrlec, and Krajcar (2000), Skok, Skrlec, and Krajcar (2001) (99), (100) & (101).

Thangiah and Salhi (2001) proposed the use of genetic algorithms to define clusters of clients and then routes are found by solving a traveling salesman problem (TSP) using and insertion heuristic. This approach is called genetic clustering (GenClust) (102). Solutions are finally optimized using the post – optimization procedure of Salhi and Sari (1997).

Recently, Yücenur and Demirel (2011) proposed geometric shape based genetic clustering algorithm for the classical MDVRP. The procedure is compared with the nearest neighbour algorithm. Their experiments showed that their algorithm provides a better clustering performance in terms of the distance of each customer to each depot in clusters, in a considerably less computational time (103).

In the survey by Gendreau, Potvin, Bräumlaysy, Hasle, and Løkketangen (2008) focused on the application of meta – heuristics for solving various variants of the VRP, a short revision of the multi – depot problem is presented (104).

The most studied variant of the MDVRP it this which involves capacity constraints (MDCVRP). Among the meta – heuristics proposed in literature for MDCVRP, we can highlight the simulated annealing algorithms of Wu, Low, and Bai (2002) and Lim and Zhu (2006) and tabu search algorithms from Lim and Wang (2005), and Aras, Aksen, and Tekin (2011) (105).

Genetic Algorithms has been also proposed for MDCVRP, as illustrated in the works of Bae, Hwang, Cho, and Goan (2007) for heterogeneous fleet of vehicles (MDHFCVRP). The objective is the minimization of total route distance or cost (106).

Other meta-heuristics, such as GRASP for MDCVRP, are presented in the works of Villegas, Prins, Prodhon, Medaglia, and Velasco (2010), respectively minimizing route cost and distance.

Özyurt and Aksen (2007) solved the problem of depot location and vehicle routing using a hybrid approach based on lagrangian relaxation (LR) and tabu search (TS). These procedures improve the best solutions found for the set of instances proposed by Tüzün and Burke (1999) (105).

Moreover, the great amount of heuristics algorithms proposed for the problem variant with heterogeneous fleet (MDHFVRP) has been focused on the design of meta – heuristics algorithms. We can highlight the works of Jeon, Leep, and Shim (2007), who proposed a hybrid genetic algorithm that minimizes the total distance travelled (107).

Simulated annealing (SA) has been employed as well in MDHFVRP. Wu et al. coupled SA with tabu search (TS) to solve the heterogeneous fleet case with capacity constraints of the integrated location – routing problem. In their problem, location of depots, routes of vehicles and client assignment problems are solved simultaneously (108).

Crevier et al. (2007) considered a MDVRP in which there are intermediate depots along vehicles' routes where they may be replenished. This problem was inspired from a real – life application at the city of Montreal, Canada. A heuristic combining adaptive memory, tabu search and integer programming was proposed. The model allows the assignment of vehicles to routes that may begin and finish at the same depot or that connect two depots to increase the capacity of vehicles to deliver goods (109).

Zhen and Zhang (2009) considered a similar problem and proposed a heuristic combining the adaptive memory principle, a tabu search method for the solution of subproblems, and integer programming (110).

Concerning the performance of meta – heuristics, a good manner of improving their performance is to generate good initial solutions. Ho, Ho, Ji, and Lau (2008) proposed the use of the well – known Clarke & Wright Savings (C&WS) algorithm (Clarke & Wright, 1964) to generate initial solutions, as commonly used for other vehicle routing problems. Once the solution is generated, the nearest neighbour (NN) heuristic is employed to improve such solution. In comparison with the random generation of initial solutions, their experiments showed that this hybrid C&WS + NN approach produces better results regarding total delivery time (111).

Furthermore, Li and Liu (2008) considered the multi – depot open vehicle routing problem with replenishment during the execution of routes. They proposed a model and an ant colony optimization resolution procedure.

Other application of the ant colony optimization resolution procedure can be found in the works of Wang (2013) and Narasimha, Kivelevitch, Sharma and Kumar (2013). These last authors studied the MDVRP with minimization of the longest travel distance of a vehicle.

A realistic application of MDVRP found in vessel routing was studied by Hirsch, Schroeder, Maggiar, and Dolinskaya (2014). These authors proposed the implementation of various heuristics, including GRASP (Greedy Randomized Adaptive Search Procedure).

Vidal, Crainic, Gendreau, and Prins (2014) proposed a hybrid genetic algorithm with iterated local search and dynamic programming was presented for the classical MDVRP with unconstrained vehicle fleet.

Sitek, Wikarek, and Grzybowska (2014) presented a multi – agent system coupled with a mixed – integer linear programming (MILP) model and constraint programming (CP) for the multi – echelon capacitated vehicle routing problem (105).

In Table 8, all papers with meta – heuristics reviewed in this subsection are presented in summary.

Year	Authors	Variations		
		Multi – Depot	Heterogeneous Fleet	Capacitated
1996	Renaud, Laporte et al. (1996)	✓		✓
1997	Filipec et al. (1997)	✓		✓
1998	Salhi et al. (1998)	✓		
1999	Tüzün and Burke (1999)	✓		
2000	Skok, Skrlec, and Krajcar (2000)	✓		✓
2001	Skok, Skrlec, and Krajcar (2001)	✓		✓
2001	Thangiah and Salhi (2001)	✓		
2002	Wu et al. (2002)	✓	✓	✓

2005	Lim and Wang (2005)	✓		✓
2006	Lim and Zhu (2006)	✓		✓
2007	Crevier et al. (2007)	✓		
2007	Joen et al. (2007)	✓	✓	
2007	Özyurt and Aksen (2007)	✓		✓
2008	Gendreau, Potvin, Bräumlaysy, Hasle, and Løkketangen (2008)	✓		
2008	Ho et al. (2008)	✓		
2008	Li and Liu (2008)	✓		

2009	Zhen and Zhang (2009)	✓		
2010	Villegas et al. (2010)	✓		✓
2011	Aras et al. (2011)	✓		✓
2011	Yücenur and Demirel (2011)	✓		
2013	Narashima, Kivelevitch, Sharma and Kumar (2013)	✓		
2013	Wang (2013)	✓		✓
2014	Hirsch et al. (2014)	✓		
2014	Sitek et al. (2014)	✓		✓

2014	Vidal et al. (2014)	✓	✓
2014	Escobar et al. (in press)	✓	✓

Table 8: Summary of MDVRP, MDCVRP and MDHFVRP with meta – heuristics

Chapter 3 The Mathematical Models of Multi – Depot Capacitated Vehicle Routing Problem with Heterogeneous Fleet

3.1 Motivation and Basic Characteristics

Distribution and logistics based companies often use various types of vehicles and operate from more than one distribution centre, usually referred to as depot. Most of the time, the customers are not necessarily assigned to their nearest depots. The aim of the initial mathematical model of MDHFCVRP, which will be developed subsequently, is the minimization of the total distribution cost while satisfying the necessary constraints regarding the heterogeneity of the vehicles and, the size and type of the demand of customers. Afterwards and based on this model, a proposed model which accomplishes the same aim is presented.

The mathematical models will be also simulated by using CPLEX solver in a real – life instance of an existing Third Party Logistics company in Greece, which has two distribution centres in the prefecture of Attica.

3.2 Mathematical Formulations

3.2.1 Initial Model of MDHFCVRP

We developed a mathematical model of MDHFCVRP which is based on the proposed model of Said Sahli, Arif Imran and Niaz A. Wassan (2013) (112). This model is a mixed integer – linear programming model, as continuous and binary variables are used in the formulation, and is considered as a flow – based formulation. The objective of it is the minimisation of total distribution cost. More details will be defined in subsections 3.2.1.1 and 3.2.1.2, where the equations of the model and the explanation of them are cited respectively.

A four index binary variable which identifies the type of vehicle chosen to travel along a given arc and the originating depot is used. In addition, two index continuous variables which denote the total remaining load of the vehicle in kg and lit respectively are used.

Prior presenting the equations of the model, the parameters of it – indexes, data and sets – and the decision variables are defined.

Indexes

i, j: the nodes of network

k: the type of vehicle

d: the originating depot

Data

n: the number of customers

m: the number of depots

K: the number of vehicle types

w_i: the weight of demand of node i (defined in terms of kg), where $i = 1,\dots,(n+m)$

v_i: the volume of demand of node i (defined in terms of lit), where $i = 1,\dots,(n+m)$

p_i: the type of cargo. If $p_i = 1$, the cargo is considered as refrigerated. Otherwise, $p_i = 0$, the cargo is considered as dry.

QW_k: the capacity in kg of vehicle type k

QV_k: the capacity in lit of vehicle type k

F_k: the fixed cost in € of the vehicle of type k. Usually, the daily wage of the driver is considered as fixed cost.

a_k: the running cost in €/km of the vehicle of type k

D_{ij}: the distance in km between nodes i and j, where $i, j = 1,\dots,(n+m)$. Therefore, D_{ij} represents the distances from customer to customer and from depot to customer.

t_{ij}: the travel time between nodes i and j.

M_i: the total number of vehicles at depot i, where $i = [(n+1),\dots,(n+m)]$

M_{ik}: the number of vehicles of type k at depot i, where $i = [(n+1),\dots,(n+m)]$, $k = [1,\dots,K]$

mrt: the maximum time that each vehicle can daily travel.

Before proceeding to sets and decision variables a fundamental assumption of the formulation should be mentioned. The depots are the starting points of all vehicles and the place where they are loaded. Therefore, the depots have zero demand and this is denoted by $w_i = v_i = 0 \ \forall \ i = (n+1),\dots,(n+m)$.

Sets

$[1,\dots,(n+m)]$: the set of nodes

$[1,\dots,n]$: the set of customers

$[(n+1),\dots,(n+m)]$: the set of depots

$[1,\dots,K]$: the set of vehicle types

$[1,\dots,k_1-1]$: the set of dry cargo vehicles

$[k_1,\dots,K]$: the set of refrigerated cargo vehicles

Decision Variables

X_{ijkd}: binary variable. If $X_{ijkd} = 1$, then a vehicle of type k travelling along arc (i,j) and originating from depot d is selected, otherwise $X_{ijkd} = 0$. Where $k = 1,\ldots,K$, $i = j = 1,\ldots,(n+m)$ and $d = (n+1),\ldots,(n+m)$.

W_{ij}: non-negative continuous variable, which denotes the weight of load defined in terms of kg remaining in the vehicle before reaching node j while traveling along arc (i,j).

V_{ij}: non-negative continuous variable, which denotes the volume of load defined in terms of lit remaining in the vehicle before reaching node j while traveling along arc (i,j).

3.2.1.1 Initial Mathematical Model of MDHFCVRP

A mathematical model consists of the objective function and the constraints, which can be equalities and/ or inequalities.

$$Minimize\ z = \sum_{d=n+1}^{n+m} \sum_{k=1}^{K} \sum_{i=n+1}^{n+m} \sum_{j=1}^{n} F_k X_{ijkd} + \sum_{d=n+1}^{n+m} \sum_{k=1}^{K} \sum_{i=1}^{n+m} \sum_{j=1}^{n+m} a_k D_{ij} X_{ijkd} \qquad (3.2.1.1.1)$$

subject to

$$\sum_{d=n+1}^{n+m} \sum_{k=1}^{K} \sum_{i=1}^{n+m} X_{ijkd} = 1 \qquad \forall j = 1, \ldots, n \qquad (3.2.1.1.2)$$

$$\sum_{d=n+1}^{n+m}\sum_{k=1}^{K}\sum_{j=1}^{n+m} X_{ijkd} = 1 \qquad \forall\, i = 1, \dots, n \qquad (3.2.1.1.3)$$

$$\sum_{i=1}^{n+m} X_{ijkd} = \sum_{i=1}^{n+m} X_{jikd} \qquad \begin{array}{r} \forall\;\; k = 1, .. K; \\ j = 1, .., (n+m); \\ d = (n+1), \dots, (n+m) \end{array} \qquad (3.2.1.1.4)$$

$$\sum_{i=n+1}^{n+m}\sum_{j=1}^{n} W_{ij} = \sum_{j=1}^{n} w_j \qquad (3.2.1.1.5)$$

$$\sum_{i=n+1}^{n+m}\sum_{j=1}^{n} V_{ij} = \sum_{j=1}^{n} v_j \qquad (3.2.1.1.6)$$

$$\sum_{i=1}^{n+m} W_{ij} - \sum_{i=1}^{n+m} W_{ji} = w_j \qquad \forall\, j = 1, \dots, n \qquad (3.2.1.1.7)$$

$$\sum_{i=1}^{n+m} V_{ij} - \sum_{i=1}^{n+m} V_{ji} = v_j \qquad \forall\, j = 1, \dots, n \qquad (3.2.1.1.8)$$

$$W_{ij} \leq \sum_{d=n+1}^{n+m}\sum_{k=1}^{K} QW_k X_{ijkd} \qquad \begin{array}{r} \forall\, i = 1, \dots, (n+m); \\ j = 1, \dots, n \end{array} \qquad (3.2.1.1.9)$$

$$V_{ij} \leq \sum_{d=n+1}^{n+m} \sum_{k=1}^{K} QV_k X_{ijkd} \qquad \forall\, i = 1, \dots, (n+m); \quad j = 1, \dots, n \qquad (3.2.1.1.10)$$

$$\sum_{d=n+1}^{n+m} \sum_{k=k1}^{K} \sum_{j=1}^{n+m} X_{jikd} = p_i \qquad \forall\, i = 1, \dots, n \qquad (3.2.1.1.11)$$

$$\sum_{d=n+1}^{n+m} \sum_{k=1}^{k_1-1} \sum_{j=1}^{n+m} X_{jikd} = 1 - p_i \qquad \forall\, i = 1, \dots, n \qquad (3.2.1.1.12)$$

$$\sum_{k=1}^{K} \sum_{j=1}^{n} X_{ijki} \leq M_i \qquad \forall\, i = (n+1), \dots, (n+m) \qquad (3.2.1.1.13)$$

$$\sum_{j=1}^{n} X_{ijki} \leq M_{ik} \qquad \forall\, i = (n+1), \dots, (n+m); \quad k = 1, \dots, K \qquad (3.2.1.1.14)$$

$$\sum_{d=n+1}^{n+m} \sum_{i=1}^{n+m} \sum_{j=1}^{n+m} X_{ijkd}\, t_{ij} \leq mrt \qquad \forall\, k = 1, \dots, K \qquad (3.2.1.1.15)$$

$$\sum_{d=n+1}^{n+m} \sum_{j=1}^{n} \sum_{k=1}^{K} X_{ijkd} \geq 1 \qquad \forall\, i = n+1, \dots, n+m \qquad (3.2.1.1.16)$$

$$X_{d1ikd2} = 0 \qquad \forall\, i = 1, \dots, n;\; k = 1, \dots, K;\; d_1 \neq d_2 \qquad (3.2.1.1.17)$$

$$X_{id1kd2} = 0 \qquad \forall\, i = 1, \dots, n;\; k = 1, \dots, K;\; d_1 \neq d_2 \qquad (3.2.1.1.18)$$

$$X_{iikd} = 0 \qquad \forall\, i = 1, \dots, (n+m);\; k = 1, \dots, K;\; d = (n+1), \dots, (n+m) \qquad (3.2.1.1.19)$$

$$X_{ijkd2} = 0 \qquad \forall\, i, j = 1, \dots, n;\; k \in d_1 \qquad (3.2.1.1.20)$$

$$X_{ijkd1} = 0 \qquad \forall\, i, j = 1, \dots, n;\; k \in d_2 \qquad (3.2.1.1.21)$$

$$\begin{array}{lll} & \forall\, i,j = 1,\dots,(n+m); & \\ X_{ijkd} \in \{0,1\} & k = 1,\dots,K; & (3.2.1.1.22) \\ & d = (n+1),\dots,(n+m) & \end{array}$$

$$W_{ij} \geq 0 \qquad i,j = 1,\dots,(n+m) \qquad (3.2.1.1.23)$$

$$V_{ij} \geq 0 \qquad i,j = 1,\dots,(n+m) \qquad (3.2.1.1.24)$$

3.2.1.2 Explanation of the Objective Function and Constraints of Initial Model

The objective function (3.2.1.1.1) represents the total distribution cost, which consist of the fixed cost F_k and the running cost $a_k D_{ij}$. The daily wage of the driver is considered as the fixed cost of each vehicle. The coefficient a_k of the running cost is determined based on the consumption of each vehicle type and is defined in €/km. The objective of MDHFCVRP model is the minimization of total travel cost such that all customers are serviced.

Constraints (3.2.1.1.2) show that each customer is visited exactly once. Specifically, the transition from any node of the network – customer or depot – to a customer will be accomplished exactly once by a vehicle type k which has originating depot d. In mathematical terms, for every customer j, where j = 1,…,n, an equation is formed, which gives to exactly one X_{ijkd} the value 1, where i = 1,…,(m+n), k = 1,…,K and d = (n+1),…,(n+m) . Hence, there will be only one X_{ijkd} for every j = 1,…,n in the optimal solution.

Constraints (3.2.1.1.3) guarantee that from each customer exactly one departure is accomplished. Specifically, from every customer a transition to another customer or depot of the network by a vehicle type k, which has originating depot d, will happen. In mathematical terms, for every customer i, where i = 1,…,n, an equation is formed, which gives to exactly one X_{ijkd} the value 1, where j = 1,…,(m+n), k = 1,…,K and d = (n+1),…,(n+m). Thus, only one X_{ijkd} for every i = 1,…,n will be added in the optimal solution.

Constraints (3.2.1.1.4) ensure the flow conservation. These constraints ensure that when a vehicle type k with originating depot d enters a location, it must leave from this location. Analyzing the constraints (3.2.1.1.4), for every vehicle type k which has originating depot d and every node j (customer or depot), where k = 1,…,K, d = (n+1),…,(n+m) and j = 1,…,(n+m), the total sum of all X_{ijkd}, which indicates the transition from all nodes i to a node j, will be equal to the total sum of all X_{jikd}, which denotes the transition from a node j to all nodes i, where i = 1,…,(n+m). As it is obvious from the sets of i and j, this equation guarantees the flow conservation both for customers and depots. This leads to the fact that every vehicle which departs from its originating depot should return to it.

It should be mentioned that constraints (3.2.1.1.2), (3.2.1.1.3), and (3.2.1.1.4) are fundamental for the mathematical model, as they originate from TSP and determine the routing of vehicles.

Constraints (3.2.1.1.5) show that the weight of total load which leaves from all depots is exactly the total weight of customers' demand. Although decision variable W_{ij} indicates the weight of load in kg remaining in the vehicle before reaching node j while traveling along arc (i,j), in these constraints index i denotes only depots (i = (n+1),…,(n+m)) and j denotes only customers (j = 1,…,n), therefore the total remaining weight departing from all depots before reaching customers is equal to the total weight of customers' demand. Similarly, constraints (3.2.1.1.6) show that the volume of total load which leaves from all depots is exactly the total volume of customers' demand.

Constraints (3.2.1.1.7) guarantee that the weight of the remaining load after visiting each customer j, where j = 1,…,n, is exactly the weight of the remaining load before visiting this customer minus the weight of its demand. In the same way, constraints (3.2.1.1.8) guarantee that the volume of the remaining load after visiting each customer j, where j = 1,…,n, is exactly the volume of the remaining load before visiting this customer minus the volume of its demand.

Constraints (3.2.1.1.9) guarantee that the capacity defined in terms of kg of any vehicle type is not violated. Especially, for every transition from every node i to every customer j, where i = 1,…,(n+m) and j = 1,…, n, the weight of the remaining load before reaching customer j while traveling along arc (i,j) is not exceed the total capacity of all vehicle types k. Similarly, constraints (3.2.1.1.10) guarantee that the capacity defined in terms of lot of any vehicle type is not violated. Constraints (3.2.1.1.9) and (3.2.1.1.10) are significant for the model, as they connect decision variables X_{ijkd} with W_{ij} and V_{ij} respectively, and hence, the routing with capacity constraints is accomplished.

Constraints (3.2.1.1.11) and (3.2.1.1.12) connect the type of the vehicle k with the type of the demand of each customer. Concerning constraints (3.2.1.1.11), when a customer has a demand of type $p_i = 1$, which indicates refrigerated cargo, the appropriate type of vehicle should be used in order to satisfy its demand. On the other hand, constraints (3.2.1.1.12) ensure that when a customer has a demand of type $p_i = 0$, which indicates dry cargo, the appropriate type of vehicle should be used in order to satisfy its demand.

Constraints (3.2.1.1.13) guarantee that the number of vehicles, which leave from every depot i, where $i = (n+1),\dots,(n+m)$, in order to visit the customers of the network, should be less or equal to the total number of the vehicles, which are available at this depot.

On step ahead and regarding vehicle types, constraints (3.2.1.1.14) guarantee that the number of vehicles of type k, which leave from every depot i, where $i = (n+1),\dots,(n+m)$, in order to visit the customers of the network, should be less or equal to the total number of vehicles of type k, which are available at this depot.

Constraints (3.2.1.1.15) are time limit constraints and stem from the fact that a driver should not work more than 8 hours daily. Therefore, the travel time of each vehicle should not exceed the maximum allowed travel time, which is 8 hours per day.

Constraints (3.2.1.1.16) show that at least one vehicle type departing from each depot $i = (n+1),\dots,(n+m)$ should visit a customer. These constraints in combination with constraints (3.2.1.1.2), (3.2.1.1.3) and (3.2.1.1.4) ensure that there will not be subtours in the routing of vehicles and that all vehicles depart from and arrive to their originating depot.

Constraints (3.2.1.1.17) are assignment constraints and guarantee that a vehicle which leaves a depot cannot be linked to a different depot. In relation to constraints (3.2.1.1.17), constraints (3.2.1.1.18) guarantee that a vehicle which returns to a depot cannot be linked to a different depot.

Constraints (3.2.1.1.19) are considered as valid equalities for the mathematical formulation and they added in order to reduce the search space. These constraints impose that a vehicle type k, which has originating depot d, cannot leave a node i in order to visit the same node.

Constraints (3.2.1.1.20) and (3.2.1.1.21) show that specific type vehicles have specific originating depots. In the simulation of the model to a real – life instance, there will be 4 different vehicle types – k = 1, 2, 3, 4 – and types k = 1, 3 will originate from depot d_1, thus X_{ijkd2}=0, and types k = 2, 4 will originate from depot d_2, thus X_{ijkd1}=0 for every transition from customer i to j, where i = j = 1,…,n.

Finally, constraints (3.2.1.1.22), and (3.2.1.1.23) and (3.2.1.1.24) refer to the binary and non – negativity of the decision variables respectively.

3.2.2 Proposed Model of MDHFCVRP

The proposed mathematical model of MDHFCVRP is based on the initial mathematical model of MDHFCVRP of subsection 3.2.1. It is also a mixed integer – linear programming model, as continuous and binary variables are used in the formulation, and is considered as a flow – based formulation.

In the proposed model of MDHFCVRP, a three index binary variable X_{ijk} is used instead of the four index variable of the initial model X_{ijkd}. X_{ijk} identifies the type of vehicle chosen to travel along a given arc (i,j). Index d is eliminated from the binary decision variable of the formulation as its utility is the presentation of the originating depot and this is not essential for the model as it can be expressed through indexes i and j. In subsection 3.2.2.1, where the proposed mathematical model is presented this change becomes clear. Meanwhile, it should be mentioned that using a three index variable instead of a four index variable, a better solution value for the same time limit of simulation is expected, as they size of the mathematical model is reduced.

Concerning the two index continuous variables of the initial model, which denote the total remaining load of the vehicle in kg and lit respectively, they are used as they are, in the proposed model.

Prior presenting the equations of the proposed model, the parameters of it – indexes, data and sets – and the decision variables are defined.

Indexes

i, j: the nodes of network

k: the type of vehicle

Data

n: the number of customers

m: the number of depots

K: the number of vehicle types

w_i: the weight of demand of node i (defined in terms of kg), where i = 1,…,(n+m)

v_i: the volume of demand of node i (defined in terms of lit), where i = 1,…,(n+m)

p_i: the type of cargo. If $p_i = 1$, the cargo is considered as refrigerated. Otherwise, pi = 0, the cargo is considered as dry.

QW_k: the capacity in kg of vehicle type k

QV_k: the capacity in lot of vehicle type k

F_k: the fixed cost in € of the vehicle of type k. Usually, the daily wage of the driver is considered as fixed cost.

a_k: the running cost in €/km of the vehicle of type k

D_{ij}: the distance in km between nodes i and j, where i, j = 1,…,(n+m). Therefore, D_{ij} represents the distances from customer to customer and from depot to customer.

t_{ij}: the travel time between nodes i and j.

M_i: the total number of vehicles at depot i, where i = [(n+1),…,(n+m)]

M_{ik}: the number of vehicles of type k at depot i, where i = [(n+1),…,(n+m)], k = [1,…,K] mrt: the maximum time that each vehicle can daily travel.

Before proceeding to sets and decision variables the assumption of initial model is also valid for proposed model. Thus, depots are the starting points of all vehicles, where they are loaded, and have zero demand and this is denoted by $w_i = v_i = 0 \ \forall \ i = (n+1),\ldots,(n+m)$.

Sets

[1,…,(n+m)]: the set of nodes

[1,…,n]: the set of customers

[(n+1),…,(n+m)]: the set of depots

[1,…,K]: the set of vehicle types

[1,…,k_1-1]: the set of dry cargo vehicles

[k_1,…,K]: the set of refrigerated cargo vehicles

Decision Variables

X_{ijk}: binary variable. If $X_{ijk} = 1$, then a vehicle of type k travelling along arc (i,j) is selected, otherwise $X_{ijk} = 0$. Where $k = 1,\ldots,K$ and $i = j = 1,\ldots,(n+m)$.

W_{ij}: non-negative continuous variable, which denotes the weight of load defined in terms of kg remaining in the vehicle before reaching node j while traveling along arc (i,j).

V_{ij}: non-negative continuous variable, which denotes the volume of load defined in terms of lit remaining in the vehicle before reaching node j while traveling along arc (i,j).

3.2.2.1 Proposed Mathematical Model of MDHFCVRP

The objective functions and constraints of proposed mathematical model are presented in this subsection.

$$Minimize\ z = \sum_{k=1}^{K}\sum_{i=n+1}^{n+m}\sum_{j=1}^{n} F_k X_{ijk} + \sum_{k=1}^{K}\sum_{i=1}^{n+m}\sum_{j=1}^{n+m} a_k D_{ij} X_{ijk} \qquad (3.2.2.1.1)$$

subject to

$$\sum_{k=1}^{K}\sum_{i=1}^{n+m} X_{ijk} = 1 \qquad \forall\, j = 1, \dots, n \qquad (3.2.2.1.2)$$

$$\sum_{k=1}^{K}\sum_{j=1}^{n+m} X_{ijk} = 1 \qquad \forall\, i = 1, \dots, n \qquad (3.2.2.1.3)$$

$$\sum_{i=1}^{n+m} X_{ijk} = \sum_{i=1}^{n+m} X_{jik} \qquad \forall\ k = 1, .. K; \quad j = 1, .., (n+m); \qquad (3.2.2.1.4)$$

$$\sum_{i=n+1}^{n+m}\sum_{j=1}^{n} W_{ij} = \sum_{j=1}^{n} w_j \tag{3.2.2.1.5}$$

$$\sum_{i=n+1}^{n+m}\sum_{j=1}^{n} V_{ij} = \sum_{j=1}^{n} v_j \tag{3.2.2.1.6}$$

$$\sum_{i=1}^{n+m} W_{ij} - \sum_{i=1}^{n+m} W_{ji} = w_j \qquad \forall\, j = 1, \dots, n \tag{3.2.2.1.7}$$

$$\sum_{i=1}^{n+m} V_{ij} - \sum_{i=1}^{n+m} V_{ji} = v_j \qquad \forall\, j = 1, \dots, n \tag{3.2.2.1.8}$$

$$W_{ij} \leq \sum_{k=1}^{K} QW_k X_{ijk} \qquad \begin{array}{r} \forall\, i = 1, \dots, (n+m); \\ j = 1, \dots, n \end{array} \tag{3.2.2.1.9}$$

$$V_{ij} \leq \sum_{k=1}^{K} QV_k X_{ijk} \qquad \begin{array}{r} \forall\, i = 1, \dots, (n+m); \\ j = 1, \dots, n \end{array} \tag{3.2.2.1.10}$$

$$\sum_{k=k1}^{K}\sum_{j=1}^{n+m} X_{jik} = p_i \qquad \forall\, i = 1, \dots, n \tag{3.2.2.1.11}$$

$$\sum_{k=1}^{k_1-1}\sum_{j=1}^{n+m} X_{jik} = 1 - p_i \qquad \forall\, i = 1, \dots, n \tag{3.2.2.1.12}$$

$$\sum_{k=1}^{K}\sum_{j=1}^{n} X_{ijk} \le M_i \qquad \forall\, i = (n+1), \dots, (n+m) \qquad (3.2.2.1.13)$$

$$\sum_{j=1}^{n} X_{ijk} \le M_{ik} \qquad \begin{aligned} &\forall\, i = (n+1), \dots, (n+m);\\ &k = 1, \dots, K \end{aligned} \qquad (3.2.2.1.14)$$

$$\sum_{i=1}^{n+m}\sum_{j=1}^{n+m} X_{ijk}\, t_{ij} \le mrt \qquad \forall\, k = 1, \dots, K \qquad (3.2.2.1.15)$$

$$X_{iik} = 0 \qquad \begin{aligned} &\forall\, i = 1, \dots, (n+m);\\ &k = 1, \dots, K; \end{aligned} \qquad (3.2.2.1.16)$$

$$X_{ijk} = 0 \qquad \begin{aligned} &i = (n+m);\\ &\forall\, j = 1, \dots, n;\\ &k \in (n+1) \end{aligned} \qquad (3.2.2.1.17)$$

$$X_{ijk} = 0 \qquad \begin{aligned} &i = (n+1);\\ &\forall\, j = 1, \dots, n;\\ &k \in (n+m) \end{aligned} \qquad (3.2.2.1.18)$$

$$X_{ijk} \in \{0,1\} \qquad \begin{aligned} &\forall\, i,j = 1, \dots, (n+m);\\ &k = 1, \dots, K; \end{aligned} \qquad (3.2.2.1.19)$$

$$W_{ij} \geq 0 \qquad i,j = 1, \dots, (n+m) \qquad (3.2.2.1.20)$$

$$V_{ij} \geq 0 \qquad i,j = 1, \dots, (n+m) \qquad (3.2.2.1.21)$$

3.2.2.2 Explanation of the Objective Function and Constraints of Proposed Model

The objective function (3.2.2.1.1) represents the total distribution cost, which consist of the fixed cost F_k and the running cost a_kD_{ij}. The daily wage of the driver is considered as the fixed cost of each vehicle. The coefficient a_k of the running cost is determined based on the consumption of each vehicle type and is defined in €/km. The objective of MDHFCVRP model is the minimization of total travel cost such that all customers are serviced.

Constraints (3.2.2.1.2) show that each customer is visited exactly once. Specifically, the transition from any node of the network – customer or depot – to a customer will be accomplished exactly once by a vehicle type k. In mathematical terms, for every customer j, where j = 1,…,n, an equation is formed, which gives to exactly one X_{ijk} the value 1, where i = 1,…,(m+n) and k = 1,…,K. Hence, there will be only one X_{ijk} for every j = 1,…,n in the optimal solution.

Constraints (3.2.2.1.3) guarantee that from each customer exactly one departure is accomplished. Specifically, from every customer a transition to another customer or depot of the network by a vehicle type k will happen. In mathematical terms, for every customer i, where i = 1,…,n, an equation is formed, which gives to exactly one X_{ijk} the value 1, where j = 1,…,(m+n) and k = 1,…,K. Thus, only one X_{ijk} for every i = 1,…,n will be added in the optimal solution.

Constraints (3.2.2.1.4) ensure the flow conservation. These constraints ensure that when a vehicle type k with originating depot d enters a location, it must leave from this location. Analyzing these constraints, for every vehicle type k which has originating depot d and every node j (customer or depot), where k = 1,…,K and j = 1,…,(n+m), the total sum of all X_{ijk}, which indicates the transition from all nodes i to a node j, will be equal to the total sum of all X_{jik}, which denotes the transition from a node j to all nodes i, where i = 1,…,(n+m). As it is obvious from the sets of i and j, this equation guarantees the flow conservation both for customers and depots. This leads to the fact that every vehicle which departs from its originating depot should return to it.

It should be mentioned that constraints (3.2.2.1.2), (3.2.2.1.3) and (3.2.2.1.4) are also fundamental for the proposed mathematical model, as they originate from TSP and determine the routing of vehicles.

Constraints (3.2.2.1.5) show that the weight of total load which leaves from all depots is exactly the total weight of customers' demand. Although decision variable W_{ij} indicates the weight of load in kg remaining in the vehicle before reaching node j while traveling along arc (i,j), in these constraints index i denotes only depots (i = (n+1),…,(n+m)) and j denotes only customers (j = 1,…,n), therefore the total remaining weight departing from all depots before reaching customers is equal to the total weight of customers' demand. Similarly, constraints (3.2.2.1.6) show that the volume of total load which leaves from all depots is exactly the total volume of customers' demand.

Constraints (3.2.2.1.7) guarantee that the weight of the remaining load after visiting each customer j, where j = 1,…,n, is exactly the weight of the remaining load before visiting this customer minus the weight of its demand. In the same way, constraints (3.2.2.1.8) guarantee that the volume of the remaining load after visiting each customer j, where j = 1,…,n, is exactly the volume of the remaining load before visiting this customer minus the volume of its demand.

Constraints (3.2.2.1.9) guarantee that the capacity defined in terms of kg of any vehicle type is not violated. Especially, for every transition from every node i to every customer j, where i = 1,…,(n+m) and j = 1,…, n, the weight of the remaining load before reaching customer j while traveling along arc (i,j) is not exceed the total capacity of all vehicle types k. Similarly, constraints (3.2.2.1.10) guarantee that the capacity defined in terms of lot of any vehicle type is not violated. Constraints (3.2.2.1.9) and (3.2.2.1.10) are significant for the model, as they connect decision variables X_{ijk} with W_{ij} and V_{ij} respectively, and hence, the routing with capacity constraints is accomplished.

Constraints (3.2.2.1.11) and (3.2.2.1.12) connect the type of the vehicle k with the type of the demand of each customer. Concerning constraints (3.2.2.1.11), when a customer has a demand of type $p_i = 1$, which indicates refrigerated cargo, the appropriate type of vehicle should be used in order to satisfy its demand. On the other hand, constraints (3.2.2.1.12) ensure that when a customer has a demand of type $p_i = 0$, which indicates dry cargo, the appropriate type of vehicle should be used in order to satisfy its demand.

Constraints (3.2.2.1.13) guarantee that the number of vehicles, which leave from every depot i, where i = (n+1),…,(n+m), in order to visit the customers of the network, should be less or equal to the total number of the vehicles, which are available at this depot.

On step ahead and regarding vehicle types, constraints (3.2.2.1.14) guarantee that the number of vehicles of type k, which leave from every depot i, where i = (n+1),…,(n+m), in order to visit the customers of the network, should be less or equal to the total number of vehicles of type k, which are available at this depot.

Constraints (3.2.2.1.15) are time limit constraints and stem from the fact that a driver should not work more than 8 hours daily. Therefore, the travel time of each vehicle should not exceed the maximum allowed travel time, which is 8 hours per day.

Constraints (3.2.2.1.16) are considered as valid equalities for the proposed formulation and they added in order to reduce the search space. These constraints impose that a vehicle type k, cannot leave a node i in order to visit the same node.

Constraints (3.2.2.1.17) and (3.2.2.1.18) are assignment constraints, showing that specific types of vehicle belong to specific depots from where they start their routes. In the simulation of the model to a real – life instance, there will be 4 different vehicle types – k = 1, 2, 3, 4 – and types k = 1, 3 will originate from depot $d_1 = (n+1)$, thus $X_{ijk} = 0$ for k = 1, 3 for $i = d_2 =$

(n+m), and types k = 2, 4 will originate from depot d_2 = (n+m), thus $X_{ijk} = 0$ for k = 2, 4 for i = d_1 = (n+1), for every transition from depots to customers, where j = 1,…,n.

Finally, constraints (3.2.2.1.19), (3.2.2.1.20) and (3.2.2.1.21) refer to the binary and non – negativity of the decision variables respectively.

3.2.3 Differences between Initial and Proposed Model

The basic difference between two formulations concerns the binary decision variable which is used in order to define whether a vehicle of type k which travels along arc (i,j) is selected or not. In initial formulation the four index variable X_{ijkd} is used, but in proposed formulation the three index variable X_{ijk} is used. In the initial model, the four index variable ensures that a vehicle will return to its origination depot according to S. Salhi et al. (112). However, this could be accomplished even with the elimination of index d and by including in index i the set of depots in constraints (3.2.2.1.17) and (3.2.2.1.18), which are assignment constraints and show that specific type vehicles belong to specific depots. This fact will be proved in Chapters 4 and 5 where the results and the comparison of them are presented respectively.

Concerning the objective function of two models, their difference lies to the fact that index d is eliminated. Therefore, the summations which concern index d are written off from the objective function of proposed model, as they are unnecessary.

In addition and regarding constraints of two models, all constraints of initial model are changed due to the elimination of index d. In constraints (3.2.1.1.2), (3.2.1.1.3), (3.2.1.1.9), (3.2.1.1.10), (3.2.1.1.11), (3.2.1.1.12), (3.2.1.1.15) and (3.2.1.1.16) of initial model, the summations which concern index d are unnecessary and thus are eliminated. Constraints (3.2.1.1.17) and (3.2.1.1.18) of initial model, which guarantee the departure and the return of the vehicle types from and to their originating depot, are not included in the proposed model. This is accomplished through constraints (3.2.2.1.2), (3.2.2.1.3) and (3.2.2.1.4), which ensure the flow in the routing of each vehicle and because in indexes i and j the set of depots is included. Moreover, constraints (3.2.1.1.16) of initial model, which show that at least one vehicle departing from each depot should visit a customer, could be considered as additional and unnecessary, as this fact is accomplished through the constraints which ensure that the transition from any node of the network – customer or depot – to a customer and from each customer to any node will be accomplished exactly once by a vehicle k ((3.2.2.1.2), (3.2.2.1.3)) in combination with the assignment constraints ((3.2.2.1.17) and (3.2.2.1.18)). In constraints (3.2.2.1.17) and (3.2.2.1.18), which are assignment constraints and show that specific types of vehicles belong to specific depots, index i represents the depots due to the elimination of index d.

Finally, it should be highlighted that by eliminating index d a minimization to the size of the model is achieved. This fact is proved by adding counter structures in the source code of mathematical models. The number of decisions variables and the number of constraints' equations created in each model are presented in section 5.1.

Chapter 4 Application of the MDHFCVRP Mathematical Models

4.1 A Real – Life Instance

The initial and the proposed MDHFCVRP mathematical model were applied in a real – life instance of a Third Party Logistics company which has two distribution centers in the prefecture of Attica. The first depot is located in Aspropyrgos and the second one in Magoula. The instance concerns the one day horizon service of 55 customers which have known demands and they are located scattered in the prefecture of Attica. In Figure 1 the map with the locations of customers and depots is presented. The depots are displayed with the orange indicators and the customers are displayed with the blue stars.

For this instance, there are 4 different types of vehicle, regarding the capacity and the type of the cargo, and there is 1 vehicle of each type available, the different types of vehicles represented by index k, where k = 1, 2, 3, 4. The characteristics of each type are:

- k = 1: Small van for dry cargo

- k = 2: Medium duty truck for dry cargo
- k = 3: Refrigerated small van
- k = 4: Refrigerated medium duty truck

Vehicles k = 1, 3 are located in Aspropyrgos' depot (d_1) and k = 2, 4 are located in Magoula's depot (d_2).

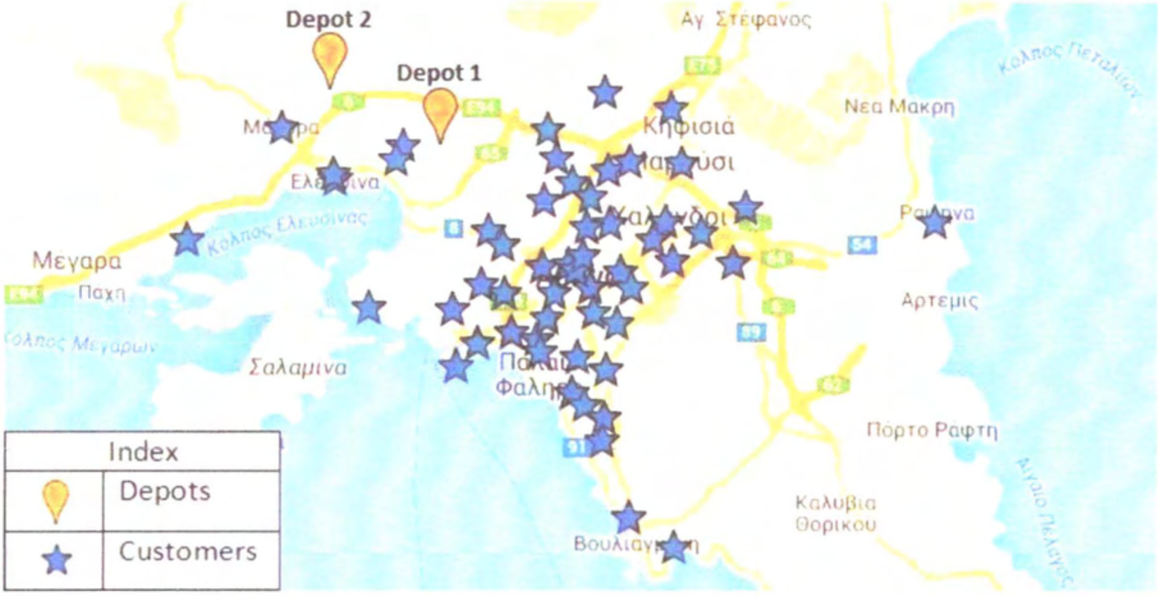

Figure 1: Map of customers and depots

4.2 Simulation and Results

4.2.1 Simulation

The mathematical models of MDHFCVRP were simulated in C++ programming language by using the CPLEX Optimization Studio. The source codes are available in Annex. Additionally, the essential data for the source code, such as demand of customers, characteristics of vehicles and, distances and traveling time between nodes of network are included in Annex.

The characteristics of the program and the software and hardware of the computer which were used for the simulation of models are:

Program edition

IBM CPLEX Optimization Studio 12.6

Windows edition

Windows 10 Pro

System

Processor: AMD FX(tm) – 8320 Eight – Core Processor 3.50 GHz

Installed memory (RAM): 8.00 GB

System type: 64 – bit Operating System, x64 – based processor

4.2.2 Results

The instance of service of 55 customers were tested both for Euclidean and real distances between the nodes of network for each model. The results of these tests will be compared in Chapter 5 Conclusions in order to come to conclusions regarding the difference between routings and consequently between total distribution cost.

Due to the size of the problem and for reasons of memory space and waiting time of simulation, a time limit of 900 seconds was set in the source code. This limitation leads to the fact that the solutions provided are not optimal, but feasible. In addition, this simulation has two process parts. The first part is a pre-calculation part, in which the simulation happens with the time limit of 900 sec and a feasible solution with an optimal gap are found. Then, in the second part the simulation happens again with the time limit of 900 sec beginning from the feasible solution which was estimated in the first part.

Moreover, it should be mentioned that in C++ the counting of all indexes begins from 0. Therefore, a shift back to the enumeration of nodes and vehicles is done (e.g. 1st customer is denoted with number 0 and, 1st and 2nd depot are denoted with number 55 and 56 respectively).

4.2.3 Results of Initial MDHFCVRP Model

4.2.3.1 Service of 55 Customers – Euclidean Distances

In the case of Euclidean distances, the final solution is feasible with value 305,6820 € with optimal gap 25,92%. The total calculation time was 1811,97 sec, from which the time of 909,73 sec is the calculation time of the first part and the time of 902,24 sec is the calculation time of second part.

Subsequently, the route of each vehicle and the display of it on map are given.

- Vehicle 0 – Small van for dry cargo with originating depot d_0=55

55 → 10 → 49 → 35 → 44 → 15 → 43 → 32 → 27 → 47 →0 → 55

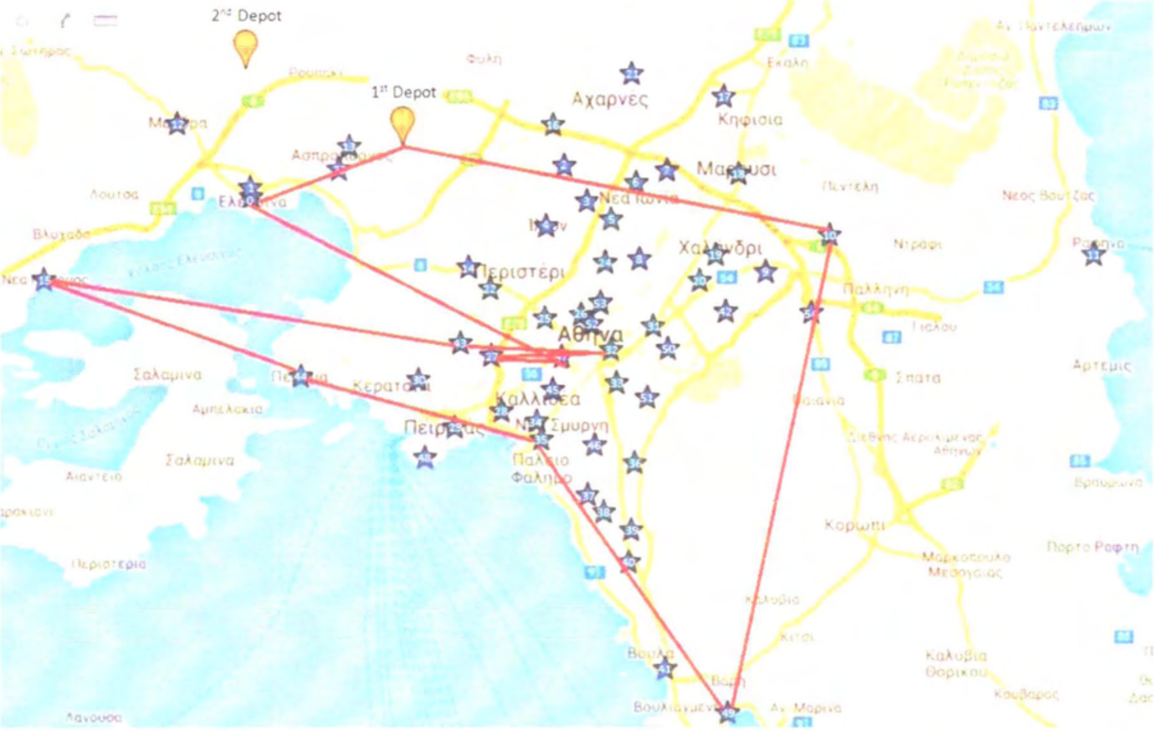

Figure 2: Routing of vehicle k = 0 with Euclidean distances

- Vehicle 1 – Medium duty truck for dry cargo with originating depot d_1=56

56 → 28 → 33 → 51 → 6 → 53 → 8 → 26 → 22 → 39 → 40 → 41 → 20 → 18 → 54 → 9 → 16 → 21 → 3 → 1 → 12 → 56

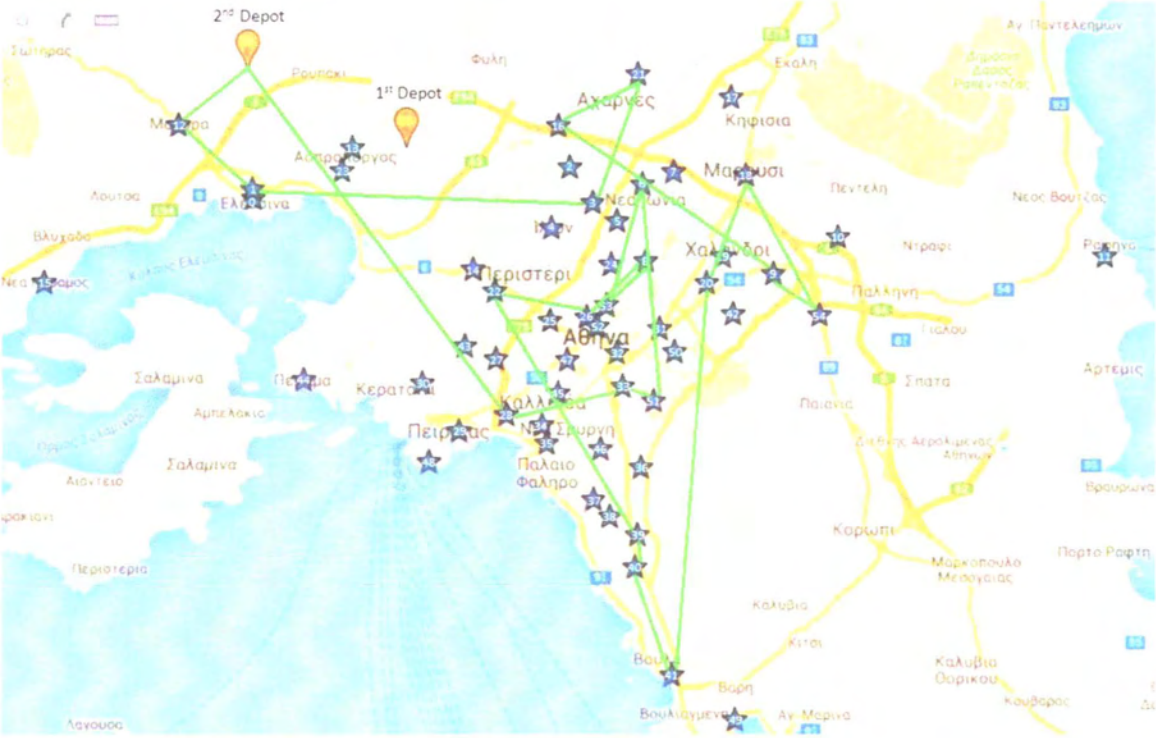

Figure 3: Routing of vehicle k = 1 with Euclidean distances

- Vehicle 2 – Refrigerated small van with originating depot $d_0=55$

  55 → 14 → 29 → 48 → 30 → 38 → 13 → 17 → 11 → 4 → 55

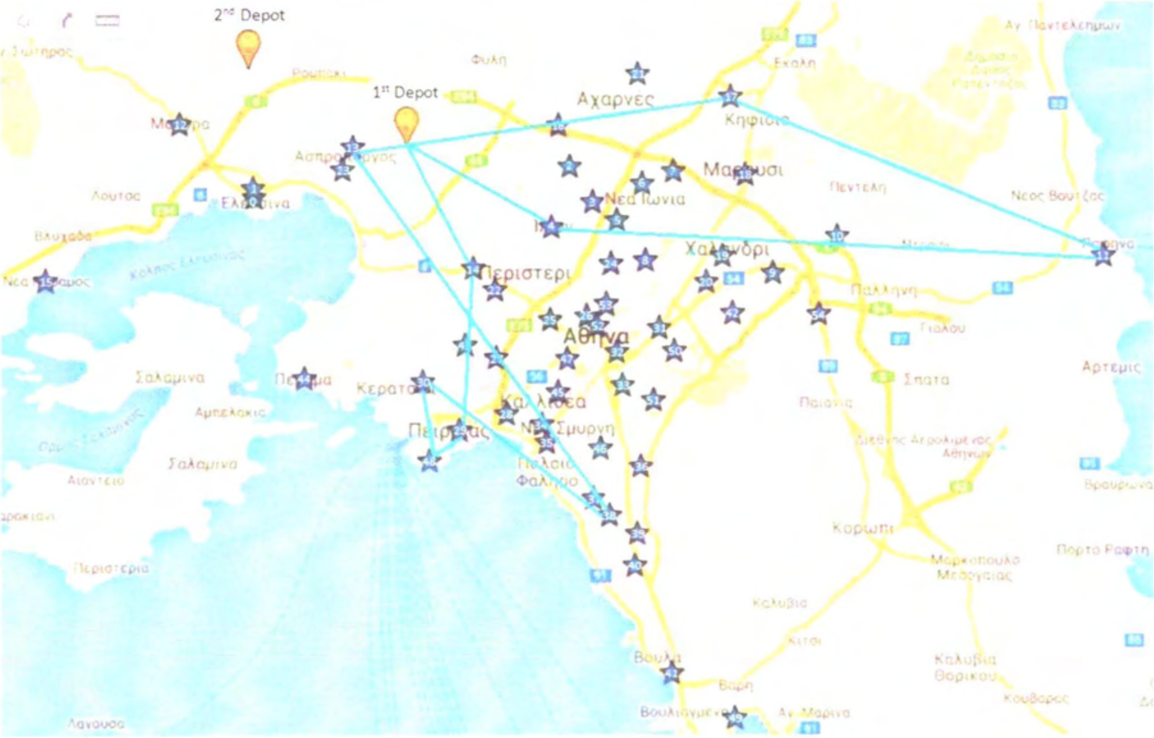

Figure 4: Routing of vehicle k = 2 with Euclidean distances

- Vehicle 3 – Refrigerated medium duty truck with originating depot $d_1=56$

  56 → 7 → 2 → 5 → 19 → 42 → 50 → 34 → 46 → 36 → 37 → 45 → 25 → 52 → 24 → 31 → 23 → 56

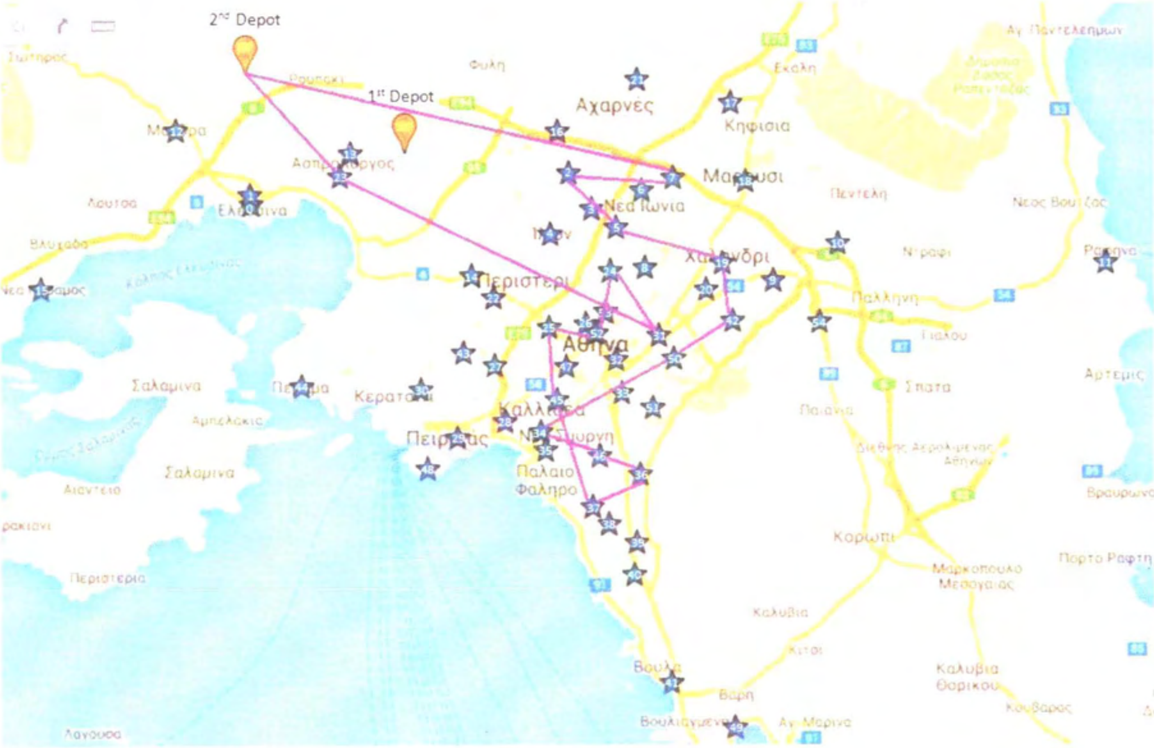

Figure 5: Routing of vehicle k = 3 with Euclidean distances

The remaining weight and volume in each transition are presented in the following table:

Transition	Remaining Weight (kg)	Remaining Volume (lit)
0 → 55	0	0
1 → 12	125,67	754,02
2 → 5	1602,37	9614,22
3 → 1	191,02	1146,12
4 → 55	0	0
5 → 19	1507,31	9043,86
6 → 53	1642,54	9855,24
7 → 2	1765,05	10590,3
8 → 26	1384,95	8309,7
9 → 16	591,39	3548,34
10 → 49	859,15	5154,9
11 → 4	107,45	644,7
12 → 56	0	0
13 → 17	392,24	2353,44
14 → 29	876,01	5256,06

15 → 43	432,26	2593,56
16 → 21	456,19	2737,14
17 → 11	246,24	1477,44
18 → 54	765,99	4595,94
19 → 42	1349,09	8094,54
20 → 18	918,15	5508,9
21 → 3	326,69	1960,14
22 → 39	1281,3	7687,8
23 → 56	0	0
24 → 31	260,5	1563
25 → 52	468,3	2809,8
26 → 22	1319,05	7914,3
27 → 47	234,02	1404,12
28 → 33	1860,47	11162,8
29 → 48	798,72	4792,32
30 → 38	642,89	3857,34
31 → 23	131,8	790,8

32 → 27	297,26	1783,56
33 → 51	1811,75	10870,5
34 → 46	1027,34	6164,04
35 → 44	672,891	4037,34
36 → 37	784,58	4707,48
37 → 45	656,82	3940,92
38 → 13	511,14	3066,84
39 → 40	1110,52	6663,12
40 → 41	1045,17	6271,02
41 → 20	982,49	5894,94
42 → 50	1263,86	7583,16
43 → 32	353,61	2121,66
44 → 15	575,6	3453,6
45 → 25	541,04	3246,24
46 → 36	898,64	5391,84
47 → 0	170,78	1024,68
48 → 30	708,67	4252,02

49 → 35	740,34	4442,04
50 → 34	1163,01	6978,06
51 → 6	1729,3	10375,8
52 → 24	389,28	2335,68
53 → 8	1523,74	9142,44
54 → 9	722,89	4337,34
55 → 10	967,45	5804,7
55 → 14	999,01	5994,06
56 → 7	1896,8	11380,8
56 → 28	1899,18	11395,1

Table 9: Remaining weight and volume in each transition concerning Euclidean distances

According to Table 9, vehicle $k = 0$ is loaded with 967,45 kg of dry cargo at its originating depot $d_0 = 55$, to serve the customers of its route. Also, the volume of the initial load is 5804,7 lit. Hence, the capacity of the vehicle is not exceeded both in terms of kg and lot, as its capacity is 1000 kg and 6000 lit respectively. In addition, vehicle $k = 1$ is loaded with 1899,18 kg of refrigerated cargo at its originating depot $d_1 = 56$ and the volume of the initial load is

11395,1 lit. Vehicle k = 2 is loaded with 999,01 kg and 5994,06 lit of dry cargo at $d_0 = 55$ and vehicle k=3 is loaded with 1896,8 kg and 11380,8 lit of refrigerated cargo at $d_1 = 56$. Additionally, neither of vehicles k = 1, k = 2 and k = 3 has not violation of its capacity both in terms of kg and lit.

4.2.3.2 Service of 55 Customers – Real Distances

In the case of real distances, the final solution is feasible with value 370,42 € with optimal gap 30,92%. The total calculation time was 1812,39 sec, from which the time of 911,23 sec is the calculation time of the first part and the time of 901,16 sec is the calculation time of second part.

Subsequently, the route of each vehicle and the display of it are presented.

- <u>Vehicle 0 – Small van for dry cargo with originating depot d_0=55</u>

55 → 16 → 18 → 54 → 51 → 39 → 40 → 35 → 44 → 15 → 55

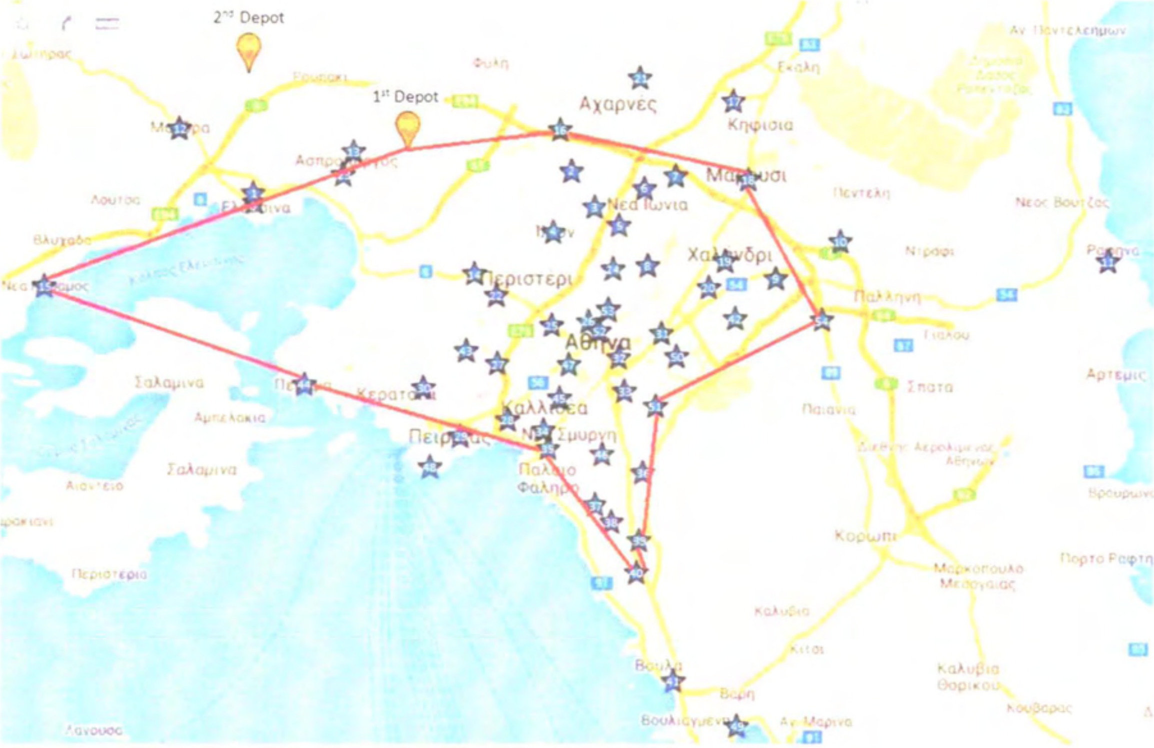

Figure 6: Routing of vehicle k = 0 with real distances

- <u>Vehicle 1 – Medium duty truck for dry cargo with originating depot d_1=56</u>

  56 → 12 → 0 → 1 → 43 → 6 → 9 → 26 → 28 → 41 → 49 → 10 → 20 → 32 → 33 → 27 → 8 → 21 → 3 → 47 → 53 → 22 → 56

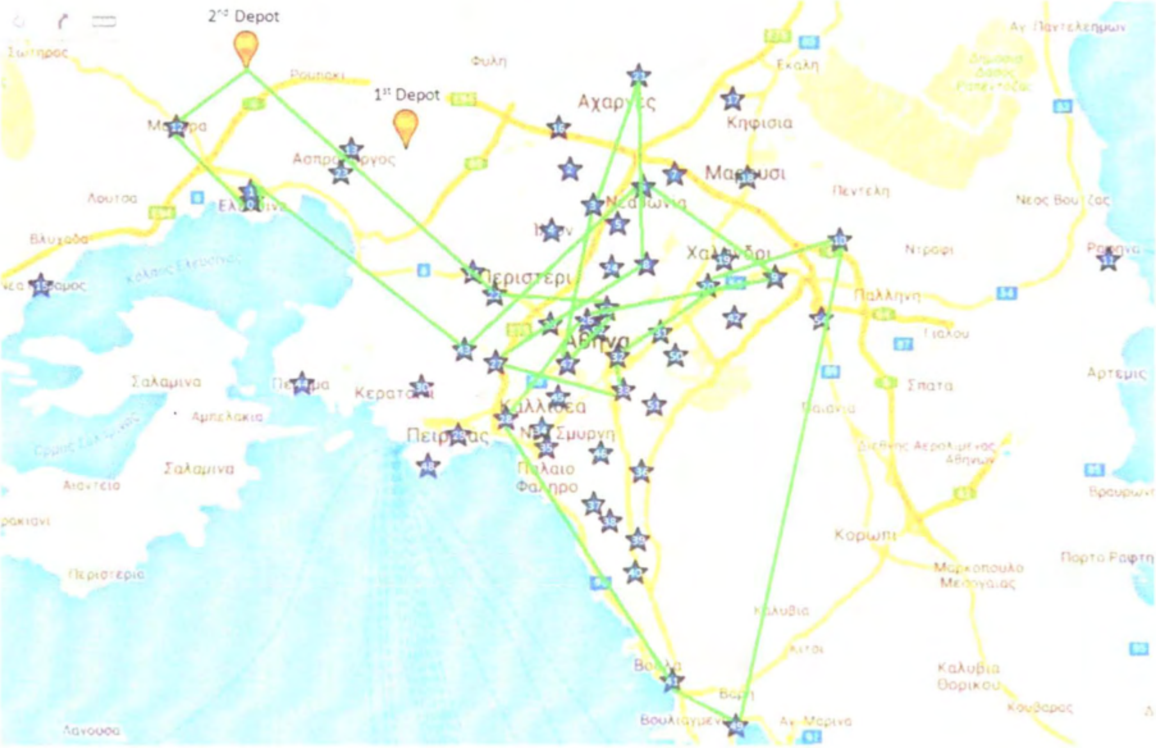

Figure 7: Routing of vehicle k = 1 with real distances

- Vehicle 2 – Refrigerated small van with originating depot d_0=55

55 → 7 → 11 → 42 → 36 → 24 → 4 → 48 → 30 → 23 → 55

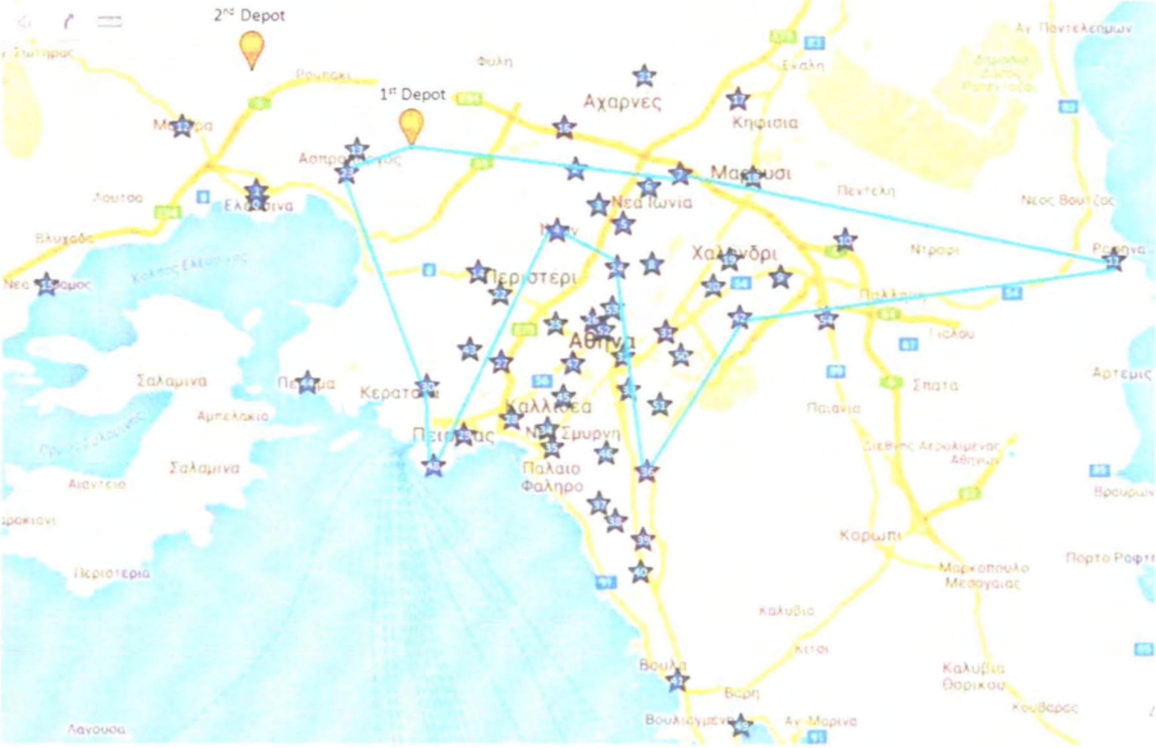

Figure 8: Routing of vehicle k = 2 with real distances

- Vehicle 3 – Refrigerated medium duty truck with originating depot d_1=56

56 → 13 → 17 → 52 → 14 → 25 → 45 → 34 → 50 → 19 → 5 → 29 → 37 → 38 → 46 → 31 → 2 → 56

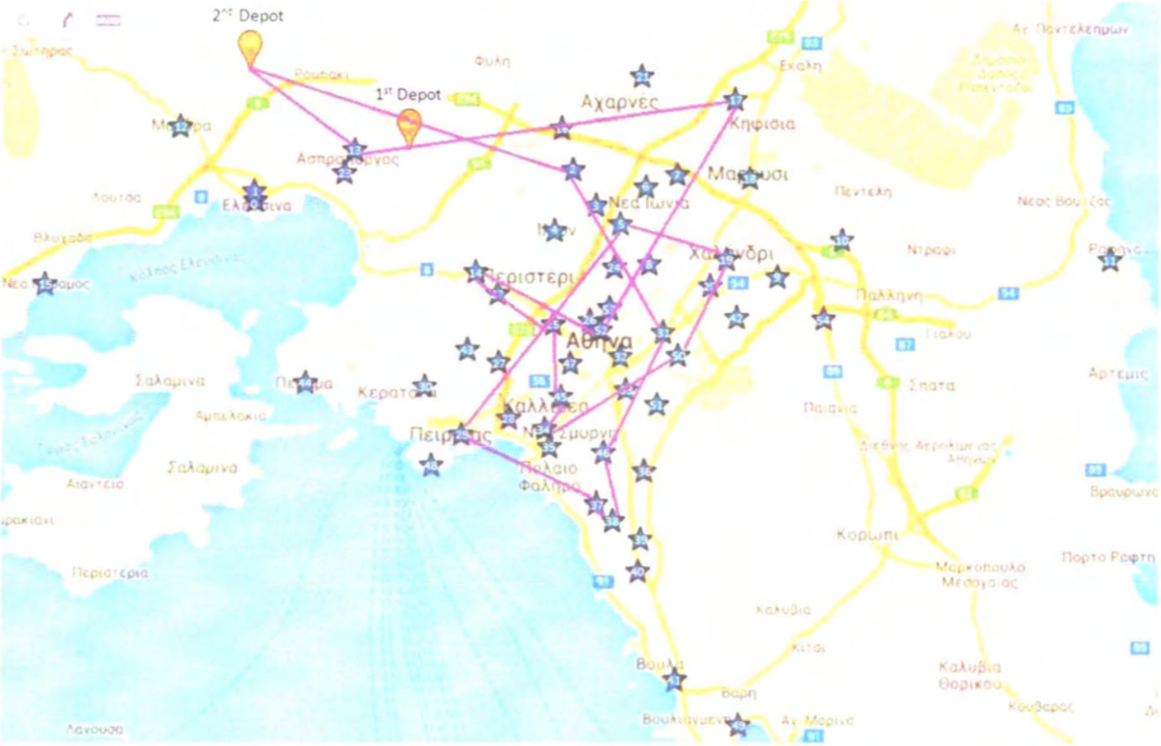

Figure 9: Routing of vehicle k = 3 with real distances

The remaining weight and volume in each transition are presented in the following table:

Transition	Remaining Weight (kg)	Remaining Volume (lit)
0 → 1	1613,06	9678,36

1 → 43	1547,71	9286,26
2 → 56	0	0
3 → 47	219,79	1318,74
4 → 48	287,63	1725,78
5 → 29	756,88	4541,28
6 → 9	1382,3	8293,8
7 → 11	861,94	5171,64
8 → 21	484,96	2909,76
9 → 26	1250,8	7504,8
10 → 20	856,4	5138,4
11 → 42	723,15	4338,9
12 → 0	1783,84	10703
13 → 17	1783,22	10699,3
14 → 25	1435,2	8611,2
15 → 55	0	0
16 → 18	821,92	4931,52
17 → 52	1673,22	9823,32

18 → 54	669,76	4018,56
19 → 5	851,94	5111,64
20 → 32	792,06	4752,36
21 → 3	355,46	2132,76
22 → 56	0	0
23 → 55	0	0
24 → 4	395,08	2370,48
25 → 45	1362,46	8174,76
26 → 28	1184,9	7109,4
27 → 8	623,75	3742,5
28 → 41	1146,19	6877,14
29 → 37	679,59	4077,54
30 → 23	131,8	790,8
31 → 2	162,68	976,08
32 → 33	735,71	4414,26
33 → 27	686,99	4121,94
34 → 50	1111,01	6666,06

35 → 44	240,63	1443,78
36 → 24	523,86	3143,16
37 → 38	551,83	3310,98
38 → 46	420,08	2520,48
39 → 40	373,43	2240,58
40 → 35	308,08	1848,48
41 → 49	1083,51	6501,06
42 → 36	637,92	3827,52
43 → 6	1469,06	8814,36
44 → 15	143,34	860,04
45 → 34	1246,68	7480,08
46 → 31	291,38	1748,28
47 → 53	156,55	939,3
48 → 30	197,58	1185,48
49 → 10	964,7	5788,2
50 → 19	1010,16	6060,96
51 → 39	544,21	3265,26

52 → 14	1558,2	9349,2
53 → 22	37,7505	226,501
54 → 51	626,66	3759,96
55 → 7	993,69	5962,14
55 → 16	957,12	5742,72
56 → 12	1909,51	11457,1
56 → 13	1902,12	11412,7

Table 10: Remaining weight and volume in each transition concerning real distances

According to Table 10, vehicle k = 0 is loaded with 957,12 kg of dry cargo at its originating depot d_0 = 55, to serve the customers of its route. Also, the volume of the initial load is 5742,72 lit. Hence, the capacity of the vehicle is not exceeded both in terms of kg and lot, as its capacity is 1000 kg and 6000 lit respectively. In addition, vehicle k = 1 is loaded with 1909,51 kg of refrigerated cargo at its originating depot d_1 = 56 and the volume of the initial load is 11457,1 lit. Vehicle k = 2 is loaded with 993,69 kg and 5962,14 lit of dry cargo at d_0 = 55 and vehicle k=3 is loaded with 1902,12 kg and 11412,7 lit of refrigerated cargo at d_1 = 56. Additionally, neither of vehicles k = 1, k = 2 and k = 3 has not violation of its capacity both in terms of kg and lit.

4.2.4 Results of Proposed MDHFCVRP Model

4.2.4.1 Service of 55 Customers – Euclidean Distances

In the case of Euclidean distances, the final solution is feasible with value 281,2358 € with optimal gap 19,81%. The total calculation time was 1807,56 sec, from which the time of 906,28 sec is the calculation time of the first part and the time of 901,28 sec is the calculation time of second part.

Subsequently, the route of each vehicle and the display of it on map are presented.

- Vehicle 0 – Small van for dry cargo with originating depot d_0=55

$55 \rightarrow 3 \rightarrow 47 \rightarrow 39 \rightarrow 40 \rightarrow 41 \rightarrow 49 \rightarrow 54 \rightarrow 18 \rightarrow 6 \rightarrow 55$

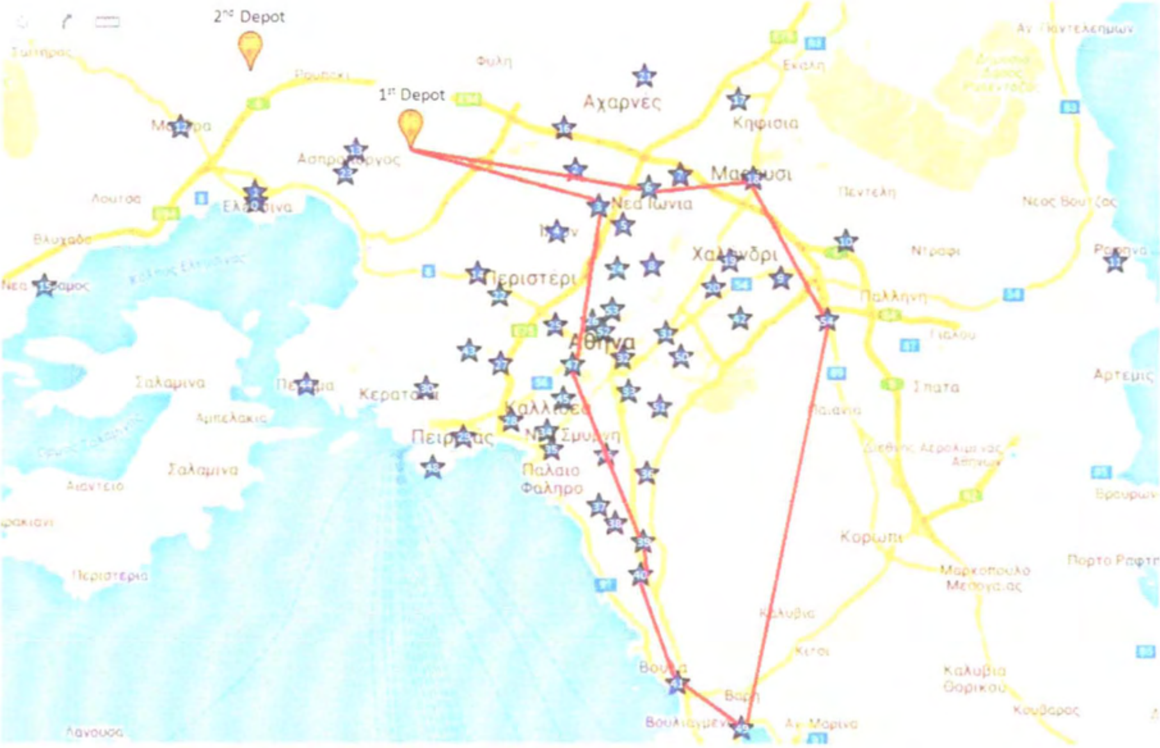

Figure 10: Routing of vehicle k = 0 with Euclidean distances

- Vehicle 1 – Medium duty truck for dry cargo with originating depot $d_1=56$

  56 → 0 → 1 → 43 → 28 → 27 → 22 → 16 → 21 → 32 → 26 → 33 → 8 → 10 → 9 → 20 → 53 → 51 → 35 → 44 → 15 → 12 → 56

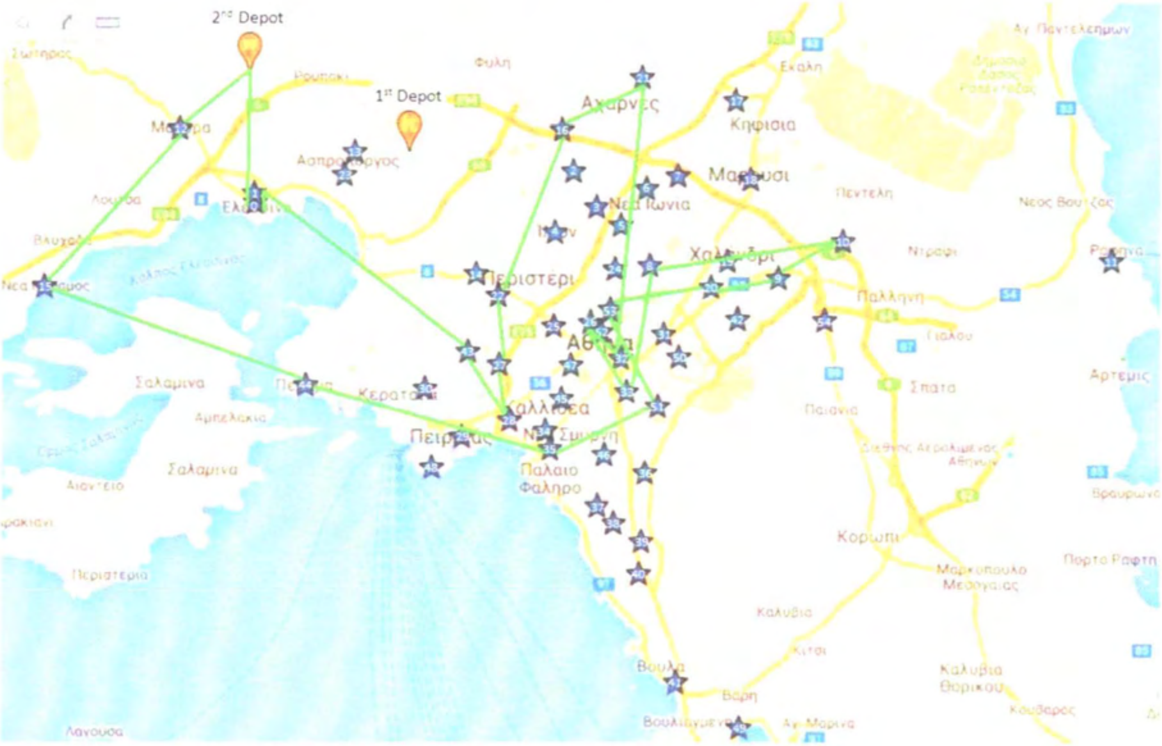

Figure 11: Routing of vehicle k = 1 with Euclidean distances

- <u>Vehicle 2 – Refrigerated small van with originating depot d_0=55</u>

  55 → 5 → 7 → 17 → 11 → 42 → 50 → 37 → 48 → 29 → 55

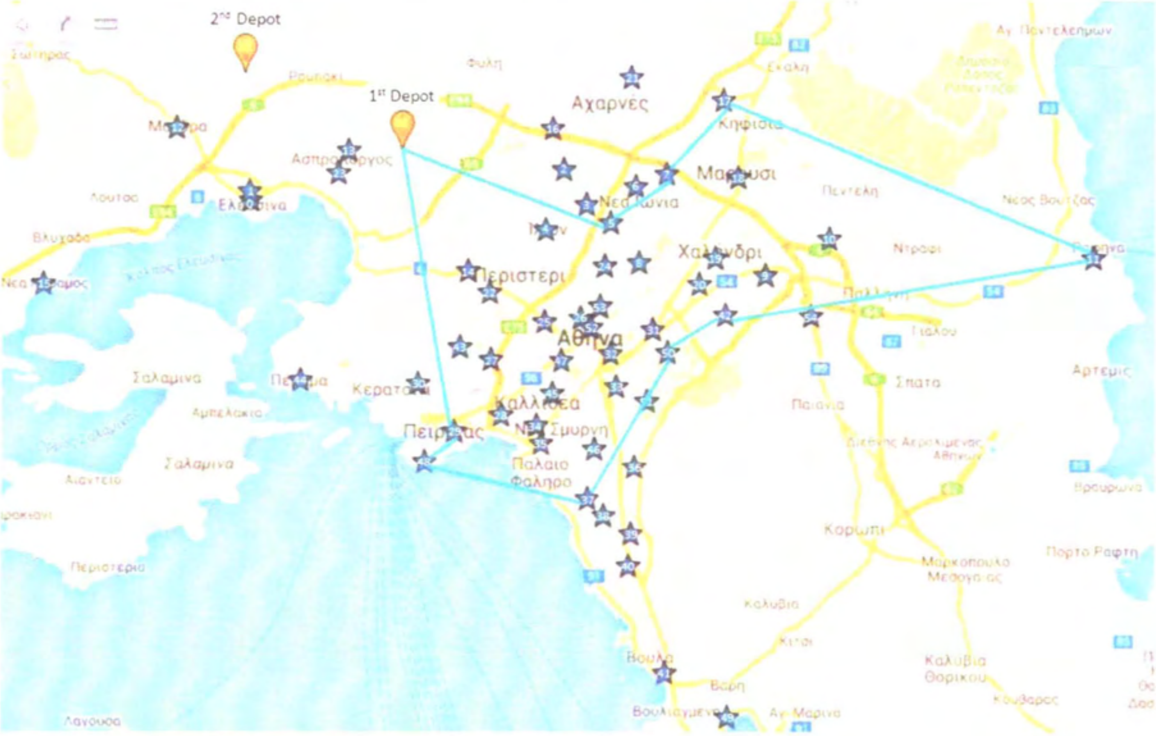

Figure 12: Routing of vehicle k = 2 with Euclidean distances

- Vehicle 3 – Refrigerated medium duty truck with originating depot d_1=56

  56 → 23 → 13 → 24 → 19 → 31 → 52 → 25 → 45 → 36 → 38 → 46 → 34 → 30 → 14 → 4 → 2 → 56

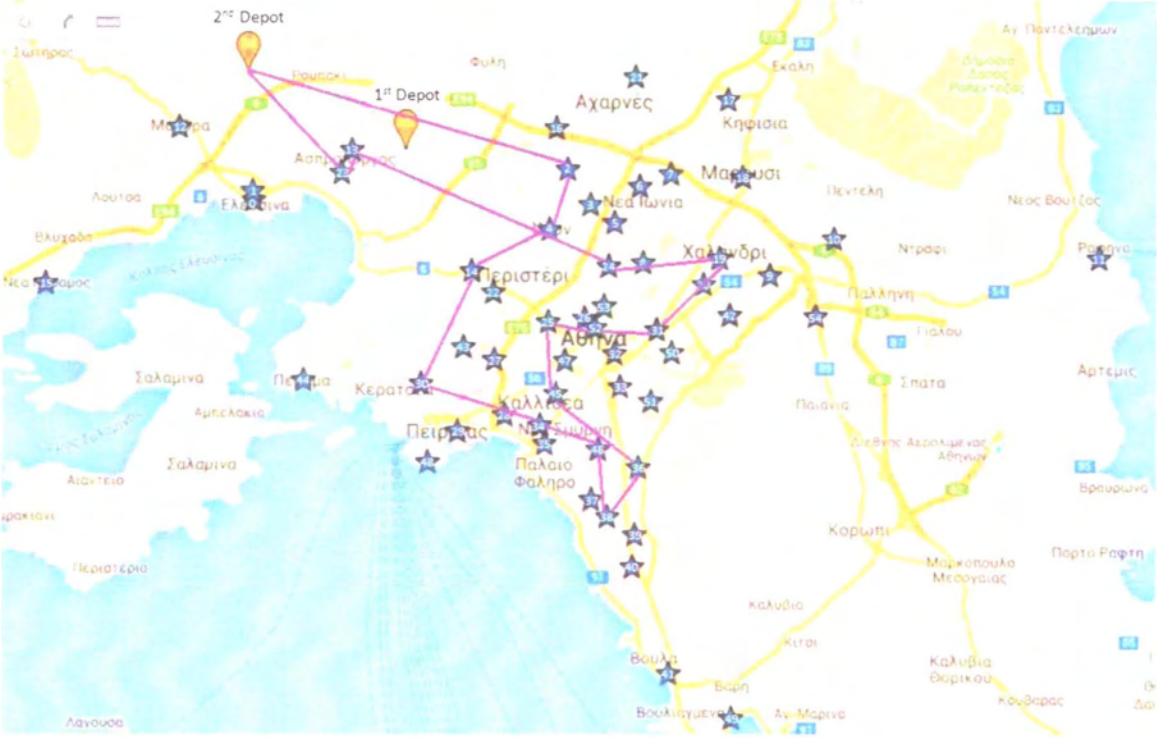

Figure 13: Routing of vehicle k = 3 with Euclidean distances

The remaining weight and volume in each transition are presented in the following table:

Transition	Remaining Weight (kg)	Remaining Volume (lit)
0 → 1	1797,3	10783,8

1 → 43	1731,95	10391,7
2 → 56	0	0
3 → 47	762,88	4577,28
4 → 2	162,68	976,08
5 → 7	897,72	5386,32
6 → 55	0	0
7 → 17	765,97	4595,82
8 → 10	939,14	5634,84
9 → 20	699,34	4196,04
10 → 9	830,84	4985,04
11 → 42	481,18	2887,08
12 → 56	0	0
13 → 24	1652,33	9913,98
14 → 4	270,13	1620,78
15 → 12	125,67	754,02
16 → 21	1378,4	8270,4
17 → 11	619,97	3719,82

18 → 6	86,76	520,56
19 → 31	1365,33	8191,98
20 → 53	635	3810
21 → 32	1248,9	7493,4
22 → 16	1513,6	9081,6
23 → 13	1771,23	10627,4
24 → 19	1523,55	9141,3
25 → 45	1084,87	6509,22
26 → 33	1126,65	6759,9
27 → 22	1551,35	9308,1
28 → 27	1614,59	9687,54
29 → 55	0	0
30 → 14	393,13	2358,78
31 → 52	1236,63	7419,78
32 → 26	1192,55	7155,3
33 → 8	1077,93	6467,58
34 → 30	458,91	2753,46

35 → 44	366,3	2197,8
36 → 38	855,03	5130,18
37 → 48	167,34	1004,04
38 → 46	723,28	4339,68
39 → 40	528,86	3173,16
40 → 41	463,51	2781,06
41 → 49	400,83	2404,98
42 → 50	395,95	2375,7
43 → 28	1653,3	9919,8
44 → 15	269,01	1614,06
45 → 36	969,09	5814,54
46 → 34	594,58	3567,48
47 → 39	699,64	4197,84
48 → 29	77,29	463,74
49 → 54	282,02	1692,12
50 → 37	295,1	1770,6
51 → 35	433,75	2602,5

52 → 25	1157,61	6945,66
53 → 51	516,2	3097,2
54 → 18	238,92	1433,52
55 → 3	898,55	5391,3
55 → 5	992,78	5956,68
56 → 0	1968,08	11808,5
56 → 23	1903,03	11418,2

Table 11: Remaining weight and volume in each transition concerning Euclidean distances

According to Table 11, vehicle $k = 0$ is loaded with 898,55 kg of dry cargo at its originating depot $d_0 = 55$, to serve the customers of its route. Also, the volume of the initial load is 5391,3 lit. Hence, the capacity of the vehicle is not exceeded both in terms of kg and lit, as its capacity is 1000 kg and 6000 lit respectively. In addition, vehicle $k = 1$ is loaded with 1968,08 kg of refrigerated cargo at its originating depot $d_1 = 56$ and the volume of the initial load is 11808,5 lit. Vehicle $k = 2$ is loaded with 992,78 kg and 5956,68 lit of dry cargo at $d_0 = 55$ and vehicle $k=3$ is loaded with 1903,03 kg and 11418,2 lit of refrigerated cargo at $d_1 = 56$. Additionally, neither of vehicles $k = 1$, $k = 2$ and $k = 3$ has not violation of its capacity both in terms of kg and lit.

4.2.4.2 Service of 55 Customers – Real Distances

In the case of real distances, the final solution is feasible with value 323,81 € with optimal gap 21,98%. The total calculation time was 1801,62, from which the time of 900,42 sec is the calculation time of the first part and the time of 901,20 sec is the calculation time of second part.

Subsequently, the route of each vehicle and the display of it on map are presented.

- Vehicle 0 – Small van for dry cargo with originating depot d_0=55

55 → 18 → 40 → 41 → 49 → 39 → 26 → 47 → 28 → 44 → 27 → 22 → 55

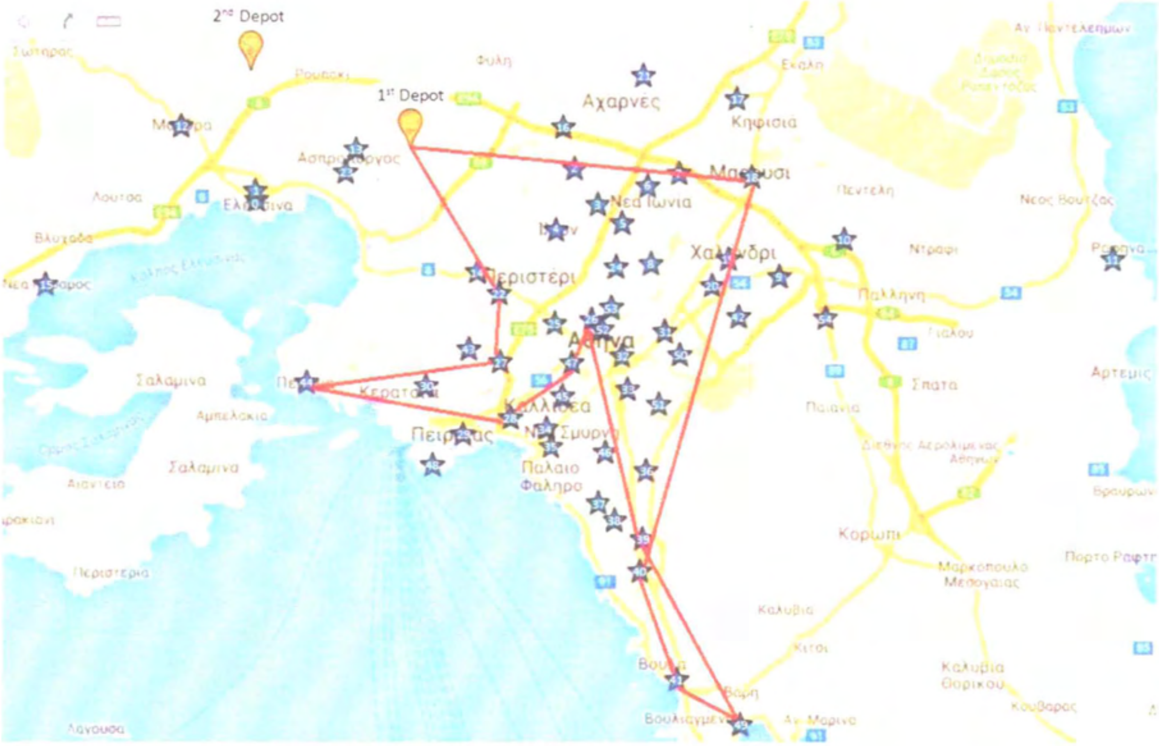

Figure 14: Routing of vehicle k = 0 with real distances

- Vehicle 1 – Medium duty truck for dry cargo with originating depot d_1=56

  56 → 12 → 0 → 1 → 53 → 32 → 51 → 33 → 35 → 43 → 8 → 10 → 54 → 9 → 20 → 6 → 3 → 21 → 16 → 15 → 56

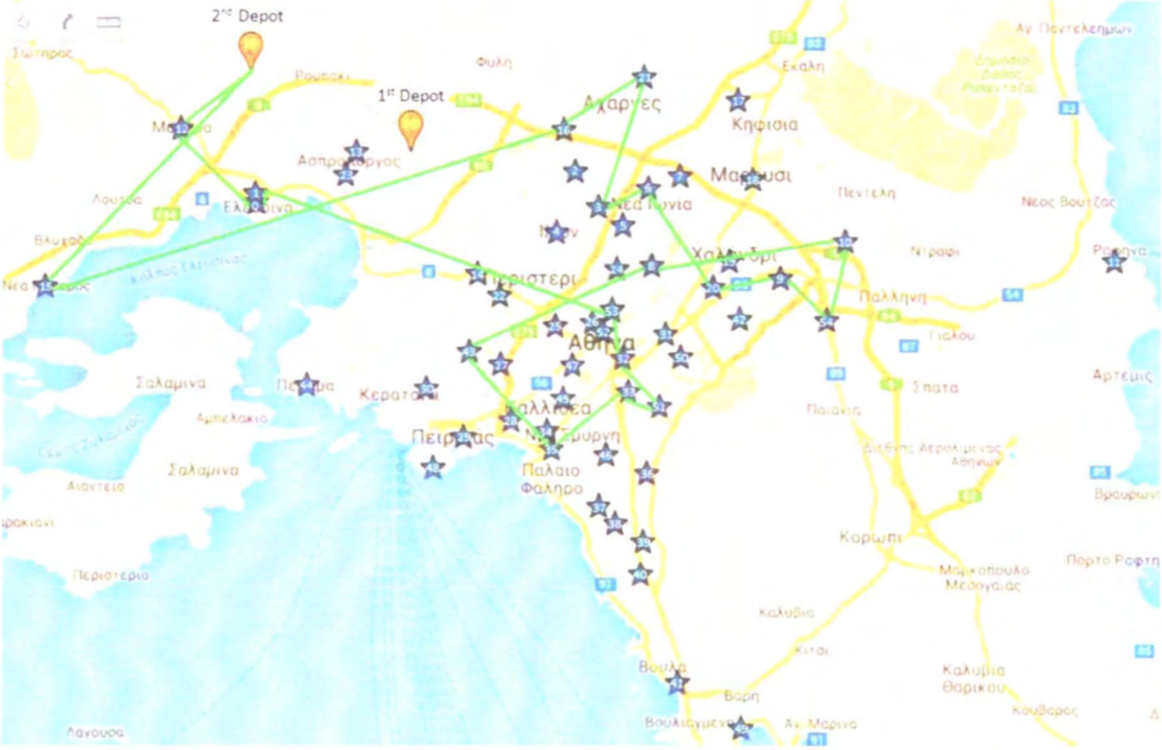

Figure 15: Routing of vehicle k = 1 with real distances

- Vehicle 2 – Refrigerated small van with originating depot d_0=55

$55 \rightarrow 17 \rightarrow 7 \rightarrow 11 \rightarrow 19 \rightarrow 42 \rightarrow 50 \rightarrow 38 \rightarrow 30 \rightarrow 55$

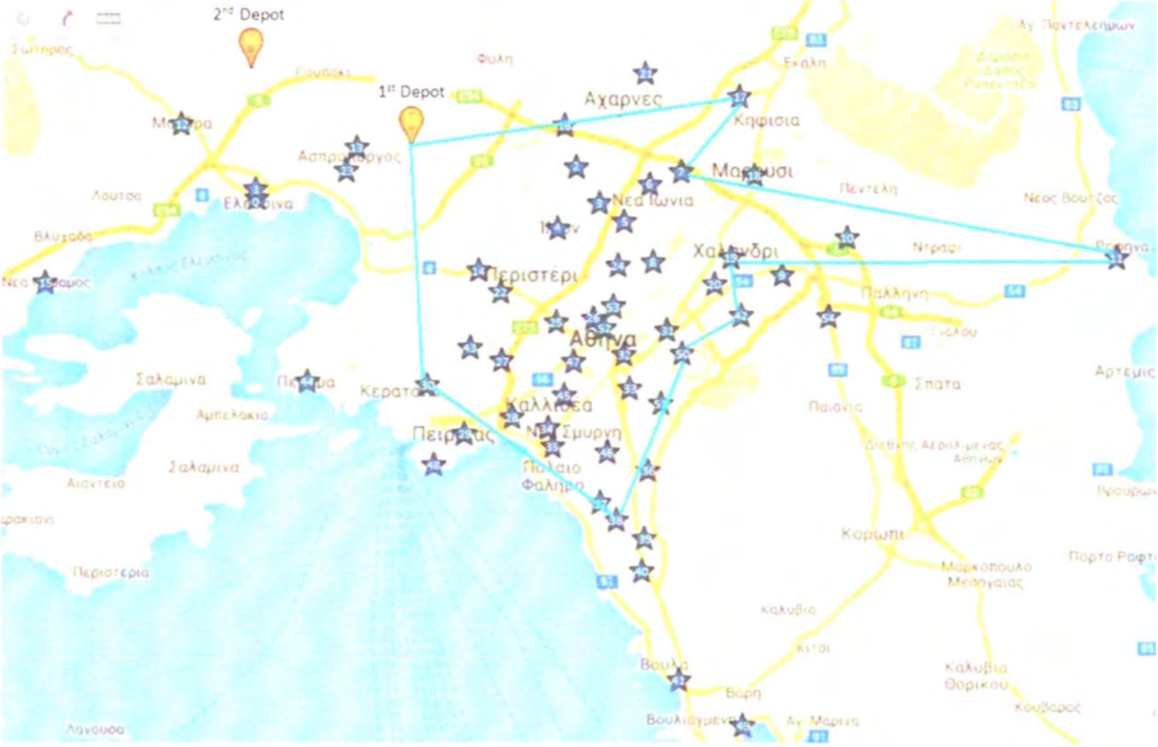

Figure 16: Routing of vehicle k = 2 with real distances

- Vehicle 3 – Refrigerated medium duty truck with originating depot d_1=56

56 → 14 → 25 → 52 → 34 → 45 → 46 → 36 → 37 → 29 → 48 → 31 → 24 → 5 → 4 → 2 → 23 → 13 →56

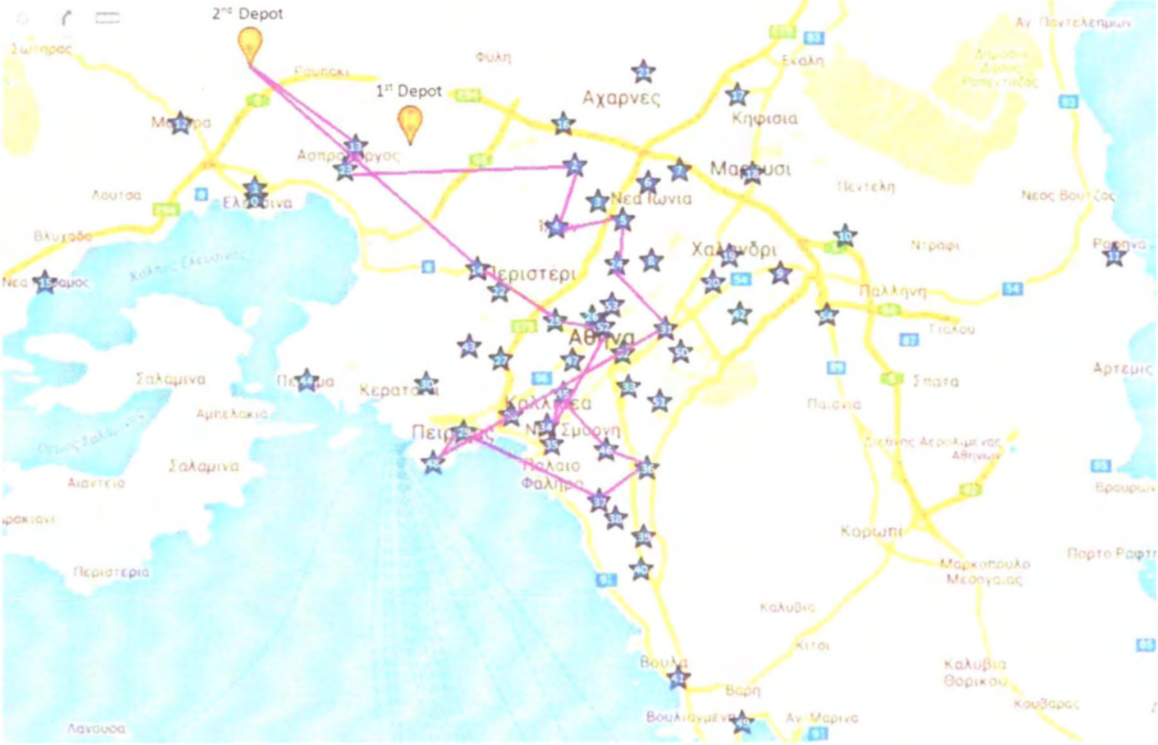

Figure 17: Routing of vehicle k = 3 with real distances

The remaining weight and volume in each transition are presented in the following table:

Transition	Remaining Weight (kg)	Remaining Volume (lit)
0 → 1	1634,27	9805,62
1 → 53	1568,92	9413,52
2 → 23	250,7	1504,2
3 → 21	408,04	2448,24
4 → 2	413,38	2480,28
5 → 4	520,83	3124,98
6 → 3	543,71	2448,24
7 → 11	680,62	4083,72
8 → 10	977,71	5866,26
9 → 20	694,81	4168,86
10 → 54	869,41	5216,46
11 → 19	541,83	3250,98
12 → 0	1805,05	10830,3
13 → 56	0	0
14 → 25	1814,44	10886,6

15 → 56	0	0
16 → 15	143,34	860,04
17 → 7	812,37	4874,22
18 → 40	783,75	4702,5
19 → 42	383,61	2301,66
20 → 6	630,47	3782,82
21 → 16	278,54	1671,24
22 → 55	0	0
23 → 13	118,9	713,4
24 → 5	615,89	3695,34
25 → 52	1741,7	10450,2
26 → 47	300,23	1801,38
27 → 22	37,75	226,5
28 → 44	198,28	1189,68
29 → 48	963,42	5780,52
30 → 55	0	0
31 → 24	744,67	4468,02

32 → 51	1393,77	8362,62
33 → 35	1262,6	7575,6
34 → 45	1527,01	9162,06
35 → 43	1195,15	7170,9
36 → 37	1168,47	7010,82
37 → 29	1040,71	6244,26
38 → 30	65,78	394,68
39 → 26	366,13	2196,78
40 → 41	718,4	4310,4
41 → 49	655,72	3934,32
42 → 50	298,38	1790,28
43 → 8	1116,5	6699
44 → 27	100,99	605,94
45 → 46	1411,23	8467,38
46 → 36	1282,53	7695,18
47 → 28	236,99	1421,94
48 → 31	873,37	5240,22

49 → 39	536,91	3221,46
50 → 38	197,53	1185,18
51 → 33	1311,32	8700,72
52 → 34	1662,68	9976,08
53 → 32	1450,12	8700,72
54 → 9	826,31	4957,86
55 → 17	958,37	5750,22
55 → 18	935,91	5615,46
56 → 12	1930,72	11584,3
56 → 14	1937,44	11624,6

Table 12: Remaining weight and volume in each transition concerning real distances

According to Table 12, vehicle k = 0 is loaded with 935,91 kg of dry cargo at its originating depot d_0 = 55, to serve the customers of its route. Also, the volume of the initial load is 5615,46 lit. Hence, the capacity of the vehicle is not exceeded both in terms of kg and lit, as its capacity is 1000 kg and 6000 lit respectively. In addition, vehicle k = 1 is loaded with 1930,72 kg of refrigerated cargo at its originating depot d_1 = 56 and the volume of the initial load is 11584,3 lit. Vehicle k = 2 is loaded with 958,37 kg and 5750,22 lit of dry cargo at $d_0 = 55$ and vehicle k=3 is loaded with 1937,44 kg and 11624,6 lit of refrigerated cargo at $d_1 = 56$. Additionally, neither of vehicles k = 1, k = 2 and k = 3 has not violation of its capacity both in terms of kg and lit.

In order to verify that the total remaining weight and volume of load before visiting each customer is enough to satisfy its demand, the case of routing of vehicle k = 2 with Euclidean distances of proposed model is presented in Table 13:

Routing	55 →	5	→	7	→	17	→	11	→	42	→	50	→	37	→	48	→	29	→ 55
Customer's demand (kg)	0	95,06		131,75		146		138,79		85,23		100,85		127,76		90,05		77,29	
Remaining weight (kg)	992,78		897,72		765,97		619,97		481,18		395,95		295,1		167,34		77,29		0

Table 13: Verification of the service of customers with vehicle k = 2

Subtracting the demand of each customer presented in "Customer's demand (kg)" row from the remaining weight before reaching him/her presented in "Remaining weight (kg)" row, we estimate the remaining weight of each next transition. The cross check results and the results of simulations are the same for all cases of all instances.

Chapter 5 Conclusions

5.1 Comparison of the results

Comparing the results provided by CPLEX for initial and proposed model in Euclidean and real distances, we come to the conclusion that for the same time limit – 900 sec – the proposed model provides better values in the objective function both for Euclidean and real distances as it converges faster in the optimal solution. With the term of "better solution", we define the solution with the minimum total distribution cost and with smaller optimal gap. This is due to the elimination of index d, which leads to a model with a smaller size. Thus, in proposed model, the number of decision variables X created is fewer in proposed model than in initial model. In addition, the number of constraints' equations created in proposed model is fewer than initial model. In Tables 14 and 15, the difference in the solution value and in optimal gap between two models is presented respectively. In addition, in Tables 16 and 17 the difference between the size of two models regarding the number of decision variables X and the number of constraints' equations created is presented respectively.

Solution value in objective function

	Initial model	Proposed model	Percentage of difference
Euclidean Distances	305,6820 €	281,2358 €	7,99%
Real Distances	370,42 €	323,81 €	12,58%

Table 14: Percentage of difference between solution values in objective function

Optimal gap

	Initial model	Proposed model	Difference
Euclidean Distances	25,92%	19,81%	6,11%
Real Distances	30,92%	21,98%	8,94%

Table 15: Difference between percentages of optimal gap

Concerning the decision variables X created, in proposed model 50% less decision variables are being created than in the initial model. This is shown in Table 16.

	Initial model	Proposed model	Percentage of difference
Number of decision variables X created	25992	12996	50%

Table 16: Percentage of difference of the number of decision variables created

Constraints (3.2.1.1.16), (3.2.1.1.17), (3.2.1.1.18) of initial model are not included in proposed model, as they considered unnecessary as it is mentioned in subsection 3.2.2.2, where the objective function and the constraints of proposed mathematical model are explained. In addition, constraints (3.2.1.1.20) and (3.2.1.1.21) of initial model are replaced with (3.2.2.1.17) and (3.2.2.1.18) of proposed model. Initial model has 20510 equations in total and proposed model has 13218 less equations, thus 7292 in total. In Table 17 the percentage difference of the number of constraints' equations created is presented.

Number of constraints' equations created

Constraints	Initial model	Proposed model	Percentage of difference
(3.2.1.1.2) → (3.2.2.1.2)	55	55	0%

(3.2.1.1.3) → (3.2.2.1.3)	55	55	0%
(3.2.1.1.4) → (3.2.2.1.4)	456	228	50%
(3.2.1.1.5) → (3.2.2.1.5)	1	1	0%
(3.2.1.1.6) → (3.2.2.1.6)	1	1	0%
(3.2.1.1.7) → (3.2.2.1.7)	55	55	0%
(3.2.1.1.8) → (3.2.2.1.8)	55	55	0%
(3.2.1.1.9) → (3.2.2.1.9)	3135	3135	0%
(3.2.1.1.10) → (3.2.2.1.10)	3135	3135	0%
(3.2.1.1.11) → (3.2.2.1.11)	55	55	0%
(3.2.1.1.12) → (3.2.2.1.12)	55	55	0%
(3.2.1.1.13) → (3.2.2.1.13)	2	2	0%
(3.2.1.1.14) → (3.2.2.1.14)	8	8	0%
(3.2.1.1.15) → (3.2.2.1.15)	4	4	0%
(3.2.1.1.16)	2	-	100%

(3.2.1.1.17)	440	-	100%
(3.2.1.1.18)	440	-	50%
(3.2.1.1.19) → (3.2.2.1.16)	456	228	50%
(3.2.1.1.20 & 21) → (3.2.2.1.17 & 18)	12100	220	98,18%

Table 17: Percentage difference of the number of constraints' equations created

Furthermore, we converted the solutions of Euclidean distances to real distances. The proposed model provides better solution value than the initial model. The percentage difference of the converted solution of initial and proposed model is presented in Table 18.

	Initial model	Proposed model	Percentage of difference
Value of converted solutions (€)	372,65	348,42	6,502%

Table 18: Difference of converted solutions

5.2 Evaluation of Proposed MDHFCVRP Model

The proposed MDHFCVRP model can be a useful tool for logistics based companies and/ or enterprises, which perform on their own the distribution of their products, and have more than one distribution center. The utility of this model in real – life applications based on the fact of heterogeneity of vehicles' capacity and type of cargo. Thus, customers with different size and type of demand can be served at a minimum distribution cost. Moreover, the model can be applied in cases with one distribution center by setting only one value in the set of depots.

Although applications with large number of customers increase the complexity and the solution time, setting time limit to the source code of the model, regarding the allowed waiting time and the computational ability of our operating system, the solutions provided are not optimal but feasible and better than a random routing.

In a future research, more constraints based on real – life restrictions can be added to this model in order to extent it. A basic constraint that could be added concerns the fact that many customers have specific time windows in which their service should be accomplished. This leads to the fact that a vehicle might arrive earlier than the earliest permitted time of service and thus should wait until the customer is available for service, but is prohibited to arrive later than the latest permitted time of service.

Annex

In this section of the thesis, the source code of the initial and proposed model and all necessary data needed to simulate these models in CPLEX are given. However, due to the large size of models and data, all this information has been saved in a CD. This CD is provided to the members of the examination committee with the hard copy of the thesis.

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