UNIVERSITY OF THESSALY
SCHOOL OF ENGINEERING

DEPARTMENT OF
MECHANICAL ENGINEERING

Analysis and Evaluation of Pricing Mechanisms in Markets with Non-Convexities with Reference to Electricity Markets

Panagiotis Andrianesis

Volos, 2016

ΠΑΝΕΠΙΣΤΗΜΙΟ ΘΕΣΣΑΛΙΑΣ

ΠΟΛΥΤΕΧΝΙΚΗ ΣΧΟΛΗ

ΤΜΗΜΑ ΜΗΧΑΝΟΛΟΓΩΝ ΜΗΧΑΝΙΚΩΝ

Διδακτορική Διατριβή

ΑΝΑΛΥΣΗ ΚΑΙ ΑΞΙΟΛΟΓΗΣΗ ΜΗΧΑΝΙΣΜΩΝ ΤΙΜΟΛΟΓΗΣΗΣ ΣΕ ΑΓΟΡΕΣ ΜΕ ΜΗ-ΚΥΡΤΟΤΗΤΕΣ ΜΕ ΑΝΑΦΟΡΑ ΣΤΙΣ ΑΓΟΡΕΣ ΗΛΕΚΤΡΙΚΗΣ ΕΝΕΡΓΕΙΑΣ

υπό

ΠΑΝΑΓΙΩΤΗ ΑΝΔΡΙΑΝΕΣΗ

Μ.Δ.Ε. "Σύγχρονες Μέθοδοι Σχεδιασμού και Ανάλυσης στη Βιομηχανία",
Τμήμα Μηχανολόγων Μηχανικών, Πανεπιστήμιο Θεσσαλίας, 2011

Διπλωματούχου Ηλεκτρολόγου Μηχανικού και Μηχανικού Υπολογιστών
Εθνικού Μετσόβιου Πολυτεχνείου, 2010

Πτυχιούχου Οικονομικών Επιστημών
Εθνικού και Καποδιστριακού Πανεπιστημίου Αθηνών, 2004

Αποφοίτου Στρατιωτικής Σχολής Ευελπίδων, 2001

Υπεβλήθη για την εκπλήρωση μέρους των
Απαιτήσεων για την απόκτηση του
Διδακτορικού Διπλώματος

2016

Η έγκριση της διδακτορικής διατριβής από το Τμήμα Μηχανολόγων Μηχανικών της Πολυτεχνικής Σχολής του Πανεπιστημίου Θεσσαλίας δεν υποδηλώνει αποδοχή των απόψεων του συγγραφέα (Ν. 5343/32 αρ. 202 παρ. 2).

Εγκρίθηκε από τα Μέλη της Επταμελούς Εξεταστικής Επιτροπής:

Πρώτος Εξεταστής (Επιβλέπων)	Δρ. Γεώργιος Λυμπερόπουλος, Καθηγητής, Τμήμα Μηχανολόγων Μηχανικών, Πανεπιστήμιο Θεσσαλίας
Δεύτερος Εξεταστής	Δρ. Αθανάσιος Ζηλιασκόπουλος, Καθηγητής, Τμήμα Μηχανολόγων Μηχανικών, Πανεπιστήμιο Θεσσαλίας
Τρίτος Εξεταστής	Δρ. Νικόλαος Ανδρίτσος, Καθηγητής, Τμήμα Μηχανολόγων Μηχανικών, Πανεπιστήμιο Θεσσαλίας
Τέταρτος Εξεταστής	Δρ. Γεώργιος Κοζανίδης, Επίκουρος Καθηγητής, Τμήμα Μηχανολόγων Μηχανικών, Πανεπιστήμιο Θεσσαλίας
Πέμπτος Εξεταστής	Δρ. Αναστάσιος Μπακιρτζής, Καθηγητής, Τμήμα Ηλεκτρολόγων Μηχανικών και Μηχανικών Υπολογιστών, Αριστοτέλειο Πανεπιστήμιο Θεσσαλονίκης
Έκτος Εξεταστής	Δρ. Μιχάλης Καραμανής, Καθηγητής, Department of Mechanical Engineering, Boston University, USA
Έβδομος Εξεταστής	Dr. Antonio J. Conejo, Professor, Department of Electrical and Computer Engineering, The Ohio State University, USA

ΑΝΑΛΥΣΗ ΚΑΙ ΑΞΙΟΛΟΓΗΣΗ ΜΗΧΑΝΙΣΜΩΝ ΤΙΜΟΛΟΓΗΣΗΣ ΣΕ ΑΓΟΡΕΣ ΜΕ ΜΗ-ΚΥΡΤΟΤΗΤΕΣ ΜΕ ΑΝΑΦΟΡΑ ΣΤΙΣ ΑΓΟΡΕΣ ΗΛΕΚΤΡΙΚΗΣ ΕΝΕΡΓΕΙΑΣ

ΠΑΝΑΓΙΩΤΗΣ ΑΝΔΡΙΑΝΕΣΗΣ
Πανεπιστήμιο Θεσσαλίας, Τμήμα Μηχανολόγων Μηχανικών, 2016

Επιβλέπων Καθηγητής: Δρ. Γεώργιος Λυμπερόπουλος, Καθηγητής

Περίληψη

Στην παρούσα διδακτορική διατριβή, εξετάζεται το πρόβλημα της τιμολόγησης και της υποβολής προσφορών σε αγορές με μη κυρτότητες. Το πρόβλημα αυτό, το οποίο βρίσκεται στη διεπαφή της οικονομικής επιστήμης, της επιχειρησιακής έρευνας και του σχεδιασμού των αγορών, έχει πρόσφατα προσελκύσει εκ νέου το ενδιαφέρον στο πλαίσιο της εξέλιξης των αγορών ηλεκτρικής ενέργειας και κυρίως των προ-ημερήσιων χονδρεμπορικών αγορών, στις οποίες το κόστος ένταξης και κατανομής των μονάδων καθώς και οι περιορισμοί στην παραγωγική δυναμικότητα δημιουργούν μη κυρτότητες.

Η παρούσα διατριβή αποτελείται από δύο μέρη. Σε κάθε μέρος, επιχειρείται η εμβάθυνση στις δύο κύριες περιοχές ενδιαφέροντος, την τιμολόγηση και την υποβολή προσφορών, με τη χρήση αναλυτικών μεθόδων αλλά και αριθμητικών συγκρίσεων και αξιολογήσεων.

Στο πρώτο μέρος εξετάζεται μία αγορά στην οποία προμηθευτές με μη συμμετρικές δυναμικότητες και μη συμμετρικά οριακά και σταθερά κόστη ανταγωνίζονται για την ικανοποίηση ντετερμινιστικής και ανελαστικής ζήτησης ενός προϊόντος, σε μία περίοδο. Οι προμηθευτές υποβάλλουν προσφορές για τα κόστη τους σε μία δημοπρασία στην οποία καθορίζεται η βέλτιστη κατανομή και οι σχετικές πληρωμές, όπως κατά κανόνα γίνεται στις απελευθερωμένες αγορές ηλεκτρικής ενέργειας.

Στο πλαίσιο της κλασικής θεωρίας για την τιμολόγηση στο οριακό κόστος, η μη κυρτότητα του συνολικού κόστους είναι δυνατό να προκαλέσει ζημίες σε ορισμένους προμηθευτές, οι οποίοι δεν κατορθώνουν να ανακτήσουν τα σταθερά τους κόστη από τις

πληρωμές για το προϊόν. Για την αντιμετώπιση του προβλήματος αυτού, έχουν προταθεί στη βιβλιογραφία διάφορα σχήματα τιμολόγησης τα οποία αυξάνουν την τιμή πάνω από το οριακό κόστος ή/και παρέχουν πρόσθετες πληρωμές στους προμηθευτές. Στη διατριβή παρατίθεται μία αναλυτική επισκόπηση των σχημάτων αυτών και προτείνεται ένα νέο σχήμα για μια αγορά με δύο προμηθευτές. Για κάθε ένα από τα σχήματα εξάγονται αναλυτικοί τύποι για την τιμή, τις πρόσθετες πληρωμές και τα κέρδη των προμηθευτών, βάσει των οποίων καθίσταται δυνατή η μεταξύ τους σύγκριση.

Η ανωτέρω ανάλυση συμπληρώνεται με αριθμητικά αποτελέσματα και επεκτείνεται για την περίπτωση περισσοτέρων των δύο (πολλαπλών) προμηθευτών, καθώς επίσης και για την περίπτωση ελαστικής ζήτησης. Για την περίπτωση πολλαπλών προμηθευτών παρατίθενται επίσης αριθμητικά αποτελέσματα που συμπληρώνουν υφιστάμενα στη σχετική βιβλιογραφία. Τέλος, τα αριθμητικά αποτελέσματα ολοκληρώνονται με ένα παράδειγμα μιας πραγματικής αγοράς ηλεκτρικής ενέργειας, βασισμένο στην Ελληνική χονδρεμπορική προ-ημερήσια αγορά.

Στο δεύτερο μέρος μελετάται η συμπεριφορά υποβολής προσφορών των συμμετεχόντων σε αγορές με μη κυρτότητες για διάφορα σχήματα (ή μηχανισμούς) τιμολόγησης στο οριακό κόστος, που παρέχουν διαφορετικού τύπου πρόσθετες πληρωμές στους συμμετέχοντες μετά την εκκαθάριση της αγοράς, ώστε να καλύψουν πιθανές ζημίες με βάση τα κόστη τους ή τις προσφορές τους.

Εξετάζονται επίσης οι επιδράσεις που έχει κάθε σχήμα στα κίνητρα των συμμετεχόντων. Αρχικά, εντοπίζονται και σχολιάζονται αποτελέσματα τιμών ισορροπίας σε ένα δυοπώλιο με μη κυρτά κόστη, συμπληρώνοντας και επεκτείνοντας τα λιγοστά αποτελέσματα που υπάρχουν στη σχετική βιβλιογραφία. Το απλό παράδειγμα του δυοπωλίου είναι χρήσιμο για την αναγνώριση των αποτελεσμάτων ισορροπίας και την ανακάλυψη σημαντικών ιδιοτήτων των υπό εξέταση μηχανισμών.

Επίσης, διερευνάται η συμπεριφορά υποβολής προσφορών των συμμετεχόντων για διάφορους μηχανισμούς ανάκτησης κόστους, οι οποίοι διατηρούν την τιμολόγηση στο οριακό κόστος, και αποζημιώνουν τους συμμετέχοντες που εμφανίζουν ζημίες, για πιο πολύπλοκες μορφές αγορών. Τέτοιες αγορές είναι οι πραγματικές αγορές ηλεκτρικής ενέργειας, όπου η ανεύρεση σημείων ισορροπίας με αναλυτικές μεθόδους είναι πρακτικώς αδύνατη. Οι υπό εξέταση μηχανισμοί, ορισμένοι από τους οποίους είναι παραλλαγές ή γενικεύσεις των μηχανισμών που εξετάσθηκαν στην περίπτωση του δυοπωλίου, διαφέρουν ως προς τον τύπο και το μέγεθος των πρόσθετων πληρωμών.

Για την αξιολόγηση της στρατηγικής υποβολής προσφορών των μονάδων παραγωγής, υπό οιονδήποτε μηχανισμό, προτείνεται μία μεθοδολογία που χρησιμοποιεί έναν επαναληπτικό αριθμητικό αλγόριθμο που αποσκοπεί στην εξεύρεση των συνδυασμένων βέλτιστων στρατηγικών υποβολής προσφορών των μονάδων οι οποίες επιζητούν τη μεγιστοποίηση του κέρδους τους. Η προτεινόμενη μεθοδολογία εφαρμόζεται σε ένα μοντέλο που αντιπροσωπεύει τη συνδυασμένη προ-ημερήσια Ελληνική αγορά ηλεκτρικής ενέργειας και εφεδρειών. Τα αποτελέσματα παρέχουν χρήσιμα συμπεράσματα για την αξιολόγηση της επίδοσης και των παρεχόμενων κινήτρων των υπό εξέταση μηχανισμών.

UNIVERSITY OF THESSALY

SCHOOL OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

Dissertation

ANALYSIS AND EVALUATION OF PRICING MECHANISMS IN MARKETS WITH NON-CONVEXITIES WITH REFERENCE TO ELECTRICITY MARKETS

by

PANAGIOTIS ANDRIANESIS

M.Sc. in Mechanical Engineering,
University of Thessaly, 2011

Diploma in Electrical & Computer Engineering,
National Technical University of Athens, 2010

B.Sc. in Economics,
National and Kapodistrian University of Athens, 2004

B.Sc., Hellenic Military Academy, 2001

Submitted in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

2016

Approved by

First Reader (Supervisor)	Dr. George Liberopoulos, Professor, Department of Mechanical Engineering, University of Thessaly
Second Reader	Dr. Athanasios Ziliaskopoulos, Professor, Department of Mechanical Engineering, University of Thessaly
Third Reader	Dr. Nikos Andritsos, Professor, Department of Mechanical Engineering, University of Thessaly
Fourth Reader	Dr. George Kozanidis, Assistant Professor, Department of Mechanical Engineering, University of Thessaly
Fifth Reader	Dr. Anastasios Bakirtzis, Professor, School of Electrical and Computer Engineering, Aristotle University of Thessaloniki
Sixth Reader	Dr. Michael Caramanis, Professor, Department of Mechanical Engineering, Boston University, USA
Seventh Reader	Dr. Antonio J. Conejo, Professor, Department of Electrical and Computer Engineering, The Ohio State University, USA

Acknowledgements

First and foremost, I would like to express my sincerest gratitude to my advisor Professor George Liberopoulos. This thesis could not have been completed without his guidance and support. George taught me that research is about hard work and that it is quality that matters. He was always there to discuss my anxieties, and he encouraged me to enter in new fields and endeavors. I am greatly indebted for providing me the opportunity to participate in many conferences, where I had the chance to meet and exchange ideas with many interesting people, and broaden my perspective in an area that was new to me. I am also deeply indebted for the time he devoted all these years, the great effort he put in our work, the thorough and tireless "torturing" of our manuscripts, and the heartful understanding he showed for my other professional activities. When I embarked on this journey, I did not expect that I would have enjoyed it so much, and George played the greatest role in this. I could never have imagined having a better advisor and mentor. Special mention goes to his wife Alexandra, and their lovely daughters, Anna and Elli, along with apologies for keeping George busy many late evenings and weekends in Volos and Filothei.

Apart from my advisor, I would like to thank the members of my thesis committee: Professors Athanasios Ziliaskopoulos, Nikolaos Andritsos, George Kozanidis, Anastasios Bakirtzis, Antonio Conejo, and Michael Caramanis for their comments and the time they devoted in this process. In particular, I would like to thank Professor Ziliaskopoulos for teaching me the art of optimization during my post-graduate studies, and for generously sharing his worldwide experience in the classroom; Professor Andritsos for the interest he showed in our work and especially during some late hours at my advisor's office; Professor Kozanidis for his valuable help and participation in our work, the very thorough reading of the thesis, the confidence and trust he showed in me, and the valuable discussions we had and still have; Professor Bakirtzis for the so kind advice he provided during my doctoral studies, as if I were one of his students; Professor Conejo, who always offered his advice, despite his very busy schedule, and especially during our last published article, and who pointed out interesting directions for further research.

But mostly, I would like to express my gratitude to Professor Caramanis, as he was the first that introduced me to the area of electricity markets. Michael happens to be the advisor of my advisor, and it is no exaggeration to say that this thesis would not have been started if it was not for Michael. I will always remember the long discussions we had in his house, and his patience in explaining the basics and the trends in the area. I am very grateful for the invitations to attend RAE discussions and watch the process of formulating market rules in the Greek electricity market. But above all, I would like to thank him for being so kind and warm, and offering his advice whenever needed.

I would also like to thank Professor Pandelis Biskas, who provided great assistance during my doctoral studies, and devoted much of his time in long discussions on various topics of my research. Pandelis, his talented wife Maria, little Sofia and Nikos have become family for me and my wife.

Special mention goes to Dr. Alex Papalexopoulos, who generously shared his international experience on electricity market design, and provided me the opportunity to gain hands-on experience in this exciting field. Despite his busy consulting career, Alex still has the bug for academic work, and I would like to thank him for being a co-author in many of our works.

I would also like to thank Professor Eddie Anderson, area editor of Operations Research, for the valuable comments he provided in our last published article, which greatly improved the quality of our work.

I also feel the need to express my gratitude to Professor George Fikioris, my advisor at NTUA. Our relationship only started after I finished my diploma thesis, and since then George has become a dear friend, and has always offered his advice during my doctoral studies.

I would also like to thank Professor Dimitris Pandelis, my university colleagues Olympia Hatzikonstantinou, Andreas Gavranis, and Thanasis Lois, for their friendship and discussions we had all these years. Also, Professors George Saharidis and Panos Pardalos for their warm advice and the wonderful conference they organized last year in Crete.

I should also mention the people that I met during my doctoral studies, with whom we exchanged interesting ideas, and who helped me better understand issues in electricity markets and power systems: Kostis Sakellaris, Miltos Aslanoglou, Kostis Perrakis, Andreas Vlachos, Fotis Kalantzis, Kostas Kanellopoulos, from RAE; Yiannis Blanas, Dimitris Michos, George Fotopoulos from ADMIE; Kostas Baslis, Christos Simoglou from Aristotle University of Thessaloniki; Constantinos Varnavas from EAC and the Cyprus TSO, Andreas Poullikkas from EAC and CERA; Yann Rebours from EdF; Professor Mohammad Reza Hesamzadeh from KTH, Stefanos Delikaraoglou from DTU. Special thanks also to Diamantis Marinakis from ABB, and my classmates from NTUA Petros Aristeidou and Alexandros Andronikakis, for the fruitful discussions, their advice and friendship.

Last but not least, I would like to thank my family, my parents Evangelos, and Eleni, my brother Kostas, and my wife Olena, for their love and support. Words cannot express my gratitude, so I would like to dedicate this thesis to them.

Panagiotis Andrianesis

Abstract

This thesis addresses the problem of pricing and bidding in markets with non-convexities. This problem, which lies at the interface of economics, operations research, and market design, has attracted renewed attention in the context of electricity markets, most notably day-ahead wholesale electricity markets where unit commitment costs and capacity constraints give rise to non-convexities. The thesis is structured in two parts. Each part aims at furthering our understanding in each of the two areas — pricing and bidding — by employing exact analysis and numerical comparisons and evaluation.

In the first part, we start by considering a market in which suppliers with asymmetric capacities and asymmetric marginal and fixed costs compete to satisfy a deterministic and inelastic demand of a commodity in a single period. The suppliers bid their costs to an auctioneer who determines the optimal allocation and the resulting payments, a typical situation in deregulated electricity markets. Under classical marginal-cost pricing, the non-convexity of the total cost may result in losses for some suppliers, as they may fail to recover their fixed cost through commodity payments only. To address this problem, various pricing schemes that lift the price above marginal cost and/or provide side-payments (uplifts) have been proposed in the literature. We review several of these schemes, also proposing a new variant, in a two-supplier setting. We derive closed-form expressions for the price, uplifts, and profits that each scheme generates, which enable us to analytically compare these schemes along these three dimensions. Our analysis is complemented with numerical results. We extend some of our analytical comparisons to the case of more than two suppliers and discuss and add to extant numerical comparisons for this case. Finally,

we close the numerical investigation by considering an actual market setting, based on the Greek wholesale electricity market.

In the second part, we study the bidding behavior of market participants in markets with non-convexities under different pricing schemes (or mechanisms) that use marginal pricing and provide side-payments to the participants after the market is cleared (ex post) to recover any losses they may have based on their costs or bids. We also explore the implications that each scheme has on the incentives of the participants. Initially, we identify and discuss equilibrium outcomes for a stylized duopoly with non-convex costs, complementing and extending the scarce results that exist in the related literature. The simple stylized duopoly is useful for identifying equilibrium outcomes and revealing key properties of the considered mechanisms. We also explore the bidding behavior of participants under different recovery mechanisms that maintain marginal pricing and reimburse the participants that exhibit losses, for more complicated market designs, such as actual electricity markets, where finding Nash equilibria becomes practically infeasible. The mechanisms that we consider differ in the type and amount of recovery payments; some are variants or generalizations of the mechanisms considered in the context of the duopoly model. To evaluate the bidding strategy behavior of the participating units under each mechanism, we propose a methodology that employs an iterative numerical algorithm aimed at finding the joint optimal bidding strategies of the profit-maximizing units. We apply this methodology and evaluate the performance and incentive compatibility properties of the considered mechanisms on a test case model representing the Greek joint energy/reserve day-ahead electricity market. The results allow us to gain insights and draw useful conclusions on the performance and incentive compatibility properties of the recovery mechanisms.

Abbreviations

AC	Average Cost
AGC	Automatic Generation Control
APX	Amsterdam Power Exchange
ASP	Ancillary Services Price
CC	Commitment Cost
CCGT	Combined Cycle Gas Turbine
CH	Convex Hull
DAS	Day-Ahead Scheduling
DS	Dispatch Scheduling
EFORD	Equivalent Demand Forced Outage Rate
EPEC	Equilibrium Problem with Equilibrium Constraints
EU	European Union
EXPIP	Ex Post Imbalance Pricing
FCR	Fixed Cost Recovery
FERC	Federal Energy Regulatory Commission
GU	Generalized Uplift
HTSO	Hellenic Transmission System Operator
IEEE	Institute of Electrical and Electronics Engineers
IP	Integer Programming
IPTO	Independent Power Transmission Operator
IS	Imbalances Settlement
ISO	Independent System Operator
JPMVEC	JP Morgan Ventures Energy Corp.
LP	Linear Programming

LPR	Loss-Related Profits Recovery
LR	Lagrangean Relaxation
MILP	Mixed Integer Linear Programming
MINLP	Mixed Integer Non-Linear Programming
mIP	modified Integer Programming
MIP	Mixed Integer Programming
MO	Market Operator
MU	Minimum Uplift
MZU	Minimum Zero-Sum Uplift
OCGT	Open Cycle Gas Turbine
PCR	Price Coupling of Regions
PD	Primal-Dual
PJM	Pennsylvania, New-Jersey, Maryland
RAE	Regulatory Authority for Energy
rcIP+	regulated cap IP+
RES	Renewable Energy Sources
RTD	Real-Time Dispatch
SIMP	System Imbalance Marginal Price
SLR	Semi-Lagrangean Relaxation
SMP	System Marginal Price
TPs	Total Payments
TSO	Transmission System Operator
U.S.	United States
VC	Variable Cost
WASMP	Weighted Average System Marginal Price
ZIMP	Zonal Imbalance Marginal Price
ZMP	Zonal Marginal Price

Contents

Acknowledgements **xiii**

Abstract **xvi**

Abbreviations **xix**

1 Introduction **1**

1.1 Background and Motivation 1

1.2 Literature Review 7

1.2.1 Pricing in Markets with Non-Convexities 7

1.2.2 Bidding in Markets with Non-Convexities 14

1.3 Contribution of the Thesis 18

1.3.1 Contribution to Pricing in Markets with Non-Convexities . . . 18

1.3.2 Contribution to Bidding in Markets with Non-Convexities . . 20

1.4 Structure of the Thesis 24

I Pricing in Markets with Non-Convexities **25**

2 Exact Analysis of Pricing Mechanisms **26**

2.1 Introduction 26

2.2 Two-Supplier Model 27

2.3 Uniform Pricing plus External Uplifts 31

2.3.1 Integer Programming (IP) Pricing 31

2.3.2 Modified IP (mIP) Pricing 32

2.3.3 Convex-Hull (CH) Pricing . . . 34
2.4 Zero-Sum Uplift Pricing . . . 35
2.4.1 Generalized Uplift (GU) Pricing . . . 35
2.4.2 Minimum Zero-Sum Uplift (MZU) Pricing: A Proposed Variant of the IP Scheme . . . 38
2.5 Revenue-Adequate Pricing . . . 40
2.5.1 Average-Cost (AC) Pricing . . . 40
2.5.2 Semi-Lagrangean Relaxation (SLR) Pricing . . . 41
2.5.3 Primal-Dual (PD) Pricing . . . 43
2.6 Comparison of Pricing Schemes . . . 46
2.7 Discussion of Trade-Offs . . . 54
2.8 Extensions . . . 56
2.8.1 Multiple Suppliers . . . 56
2.8.2 Price-Elastic Demand . . . 57
2.9 Conclusions . . . 59

3 Numerical Results for Pricing Mechanisms **61**
3.1 Introduction . . . 61
3.2 2-Supplier Case . . . 62
3.3 Scarf's Example . . . 77
3.3.1 Model of Modified Scarf's Example . . . 78
3.3.2 Pricing Approaches . . . 79
3.3.3 Numerical Results and Discussion . . . 83
3.4 Greek Electricity Market . . . 86
3.4.1 The Greek Electricity Market Framework . . . 87
3.4.2 The DAS Model . . . 90
3.4.3 Input Data and Test Cases . . . 97
3.4.4 Numerical Results and Discussion . . . 101
3.5 Conclusions . . . 103

II Bidding in Markets with Non-Convexities under Marginal Cost Pricing **106**

4 Equilibrium Outcomes in a Duopoly **107**

4.1 Introduction . . . 107

4.2 Duopoly Model Setting . . . 109

4.3 Cost-Based Recovery Mechanisms . . . 110

4.3.1 Model Assumptions . . . 110

4.3.2 FCR Mechanism . . . 112

4.3.3 LPR Mechanism . . . 115

4.3.4 Design Issues . . . 120

4.4 Bid-Based Recovery Mechanisms . . . 123

4.4.1 Model Assumptions . . . 123

4.4.2 Bid/Cost Recovery Mechanisms . . . 125

4.4.3 Equilibrium Analysis . . . 126

4.4.4 Discussion . . . 132

4.5 Conclusions . . . 135

5 Numerical Evaluation in Electricity Markets **137**

5.1 Introduction . . . 137

5.2 Joint Energy/Reserve Day-Ahead Market Problem . . . 139

5.3 Proposed Recovery Mechanisms . . . 141

5.3.1 Cost-Based Side-Payments . . . 144

5.3.2 Bid-Based Side-Payments . . . 146

5.4 Numerical Evaluation Methodology . . . 149

5.5 Implementation . . . 152

5.5.1 Test Case Data . . . 153

5.5.2 Assumptions . . . 155

5.5.3 Performance Measures . . . 157

5.5.4 Computational Issues . . . 158

5.6 Numerical Results . . . 159

5.6.1 Average Aggregate Results for All Mechanisms . . . 159

5.6.2 A Closer Look at the Prevailing Mechanisms 161
5.7 Sensitivity Analysis and Extensions 165
5.7.1 Load Sensitivity Analysis (Low-Demand Scenario) 166
5.7.2 Bidding Strategy Assumptions Sensitivity Analysis 167
5.7.3 Bidding Under the Cost 170
5.7.4 Further Sensitivity Analysis 171
5.7.5 Agent-Based Simulation 173
5.8 Conclusions and Issues for Further Research 175

6 Thesis Summary **180**

A Proofs of Chapter 2 **186**
A.1 Proof of Proposition 2.1 186
A.2 Proof of Proposition 2.3 187
A.3 Proof of Proposition 2.5 189
A.4 Proof of Proposition 2.8 193
A.5 Proof of Proposition 2.9 195
A.6 Proof of Proposition 2.10 202
A.7 Proof of Proposition 2.11 203
A.8 Proof of Proposition 2.12 205

B Nomenclature for the DAS Problem **210**

C Proofs of Chapter 4 **215**
C.1 Proof of Proposition 4.1 215
C.2 Proof of Proposition 4.3 220

D List of Thesis Publications **237**

Bibliography **241**

List of Tables

1.1 Summary of existing results on Nash equilibria for a symmetric duopoly with non-convex costs. 17

2.1 Concise summary of results for the high-demand case. 60

3.1 Summary of numerical illustrations for the two-supplier case. 63
3.2 Modified Scarf example. 78
3.3 Conventional generation units (as of December 2011). 98
3.4 Annual aggregate results. 101

4.1 Equilibrium outcomes for the FCR mechanism. 113
4.2 Equilibrium outcomes for the LPR mechanism. 116
4.3 Equilibria involving one supplier bidding at the price cap for the high-demand case under the rcIP+ mechanism. 132

5.1 Recovery mechanisms. 153
5.2 Unit's data (DAS input). 154
5.3 Energy demand and reserve requirements. 154
5.4 Recovery mechanisms average aggregate results. 160
5.5 Type-M units' profits under different recovery mechanisms. 162
5.6 Type-M units' bids under different recovery mechanisms. 163
5.7 Energy demand and reserve requirements (low demand scenario). . . 166
5.8 Average aggregate results (low demand scenario). 167
5.9 Average aggregate results (sensitivity analysis on bidding strategy assumptions). 168

5.10 Average aggregate results (bidding up to 30% under cost). 170
5.11 Average aggregate results (peak load pricing). 172

A.1 Price, uplifts and profits generated by the GU pricing scheme for the high-demand case. 193

D.1 Relation of thesis publications to main chapters. 240

List of Figures

2.1 Total cost versus quantity for suppliers i and I, for the three possible cases of parameters b_n, f_n, $n = 1, 2$, and k_1. 28

2.2 Optimal dispatch quantities versus demand for the three possible cases of capacity and cost parameters. 30

2.3 Price versus $b_i + f_i/k_i$ for cases: (a) low demand; (b) high demand. . 47

2.4 Price graphs for cases (a), (b), and (c) of Figure 2.2, for low demand $[b' = b_I + f_I/k_i - (b_I - b_i)(k_i - d)/k_i]$. 49

2.5 Price graphs for cases (a), (b), and (c) of Figure 2.2, for high demand. 51

2.6 Profits versus $b_i k_i + f_i$ for the high-demand case for suppliers: (a) i; (b) I. 52

2.7 Profit graphs for cases (a), (b), and (c) of Figure 2.2, for high demand. 53

2.8 Intersection of supply (S) and demand (D) functions. 58

3.1 Quantities, price, and profits versus demand for Case (a) of Figure 2.2, for $b_1 = 5, b_2 = 4, f_1 = 5, f_2 = 4, k_1 = 7, k_2 = 10$ (instance 1 of Table 3.1). 64

3.2 Quantities, price, and profits versus demand for Case (a) of Figure 2.2, for $b_1 = 5, b_2 = 4, f_1 = 5, f_2 = 9, k_1 = 7, k_2 = 10$ (instance 2 of Table 3.1). 65

3.3 Quantities, price, and profits versus demand for Case (a) of Figure 2.2, for $b_1 = 5, b_2 = 4, f_1 = 5, f_2 = 14, k_1 = 7, k_2 = 10$ (instance 3 of Table 3.1). 66

3.4 Quantities, price, and profits versus demand for Case (a) of Figure 2.2, for $b_1 = 5, b_2 = 4, f_1 = 5, f_2 = 20, k_1 = 7, k_2 = 10$ (instance 4 of Table 3.1). 67
3.5 Quantities, price, and profits versus demand for Case (a) of Figure 2.2, for $b_1 = 5, b_2 = 4, f_1 = 16, f_2 = 20, k_1 = 7, k_2 = 10$ (instance 5 of Table 3.1). 68
3.6 Quantities, price, and profits versus demand for Case (b) of Figure 2.2, for $b_1 = 4, b_2 = 5, f_1 = 9, f_2 = 5, k_1 = 7, k_2 = 10$ (instance 6 of Table 3.1). 72
3.7 Quantities, price, and profits versus demand for Case (b) of Figure 2.2, for $b_1 = 4, b_2 = 5, f_1 = 16, f_2 = 5, k_1 = 7, k_2 = 10$ (instance 7 of Table 3.1). 73
3.8 Quantities, price, and profits versus demand for Case (c) of Figure 2.2, for $b_1 = 4, b_2 = 5, f_1 = 1, f_2 = 5, k_1 = 7, k_2 = 10$ (instance 8 of Table 3.1). 75
3.9 Quantities, price, and profits versus demand for Case (c) of Figure 2.2, for $b_1 = 4, b_2 = 5, f_1 = 6, f_2 = 5, k_1 = 7, k_2 = 10$ (instance 9 of Table 3.1). 76
3.10 Prices versus demand for different pricing mechanisms (granularity of demand 0.5 units). 83
3.11 Cost increase under the PD allocation compared to the optimal MILP solution. 84
3.12 Prices under the different revenue-adequate approaches (granularity of demand 0.05 units). 85
3.13 Prices under the different revenue-adequate approaches for a modified 2-unit example (granularity of demand 0.01 units). 87

4.1 Total costs and equilibrium prices for the low demand case under the IP+ mechanism. 129

5.1 Net profits for alternative mechanism designs. 148
5.2 Frequency of bidding over the cap for mechanism B.2. 164

5.3 Bidding pattern for mechanism B.2, with $\beta = 3$. 165
5.4 Uplifts for different regulating parameter values. 172
5.5 Bidding pattern for mechanism B.2, with $\beta = 3$ using exponential smoothing $\alpha = 0.2$. 175

A.1 Values of q_i^{PD}, λ^{PD}, and the suppliers' profits for the five possible regions of the q_i' vs. q_i'' space and the conditions that define each region. 200
A.2 (a) Price vs. $b_i + f_i/k_i$ and (b) profits vs. $b_i k_i + f_i$ for suppliers i and I, for the PD scheme for three representative instances of the problem parameters, for the case $d > k_2$. 201
A.3 Price vs. $b_i + f_i/k_i$ for the GU scheme for cases: (a) $3k_i/2 < d \leq 2k_i$; (b) $k_i \leq d < 3k_i/2$. 207
A.4 Profits vs. $b_i k_i + f_i$ for the GU scheme for cases: (a) $3k_i/2 < d \leq 2k_i$; (b) $k_i \leq d < 3k_i/2$. 208

Chapter 1

Introduction

In this chapter, we introduce the main topic of this thesis which is the analysis and evaluation of pricing mechanisms in markets with non-convexities with special reference to electricity markets. In Section 1.1, we present some background information and the motivation behind this work. In Section 1.2, we review the literature on pricing mechanisms and bidding strategies in markets with non-convexities with an emphasis on works that are most closely related to this thesis. In Section 1.3, we present the main contribution of this thesis. Finally, in Section 1.4, we provide the structure of the remainder of this thesis.

1.1 Background and Motivation

The process of liberalization and deregulation in various electricity markets worldwide has led to significantly different institutions and market designs. Despite their differences, most electricity market designs are based on the principle of separating the competitive functions of generation and retail from the monopolistic functions of transmission and distribution. This separation has led to the establishment of a wholesale electricity market and a retail electricity market.

The role of the wholesale market is to allow trading between generators and retailers, possibly through other financial intermediaries. Most wholesale market designs consist of two or more (often overlapping) sequential markets with varying horizon lengths. Usually, there is a market for trading long-term capacity assurance contracts that are typically sold a year or more in advance. Short-term delivery of electricity is traded in the day-ahead market, in which large volumes of electricity are transacted for the day ahead. In many market designs, there is even an hour-ahead market.

All the above markets are forward, financial markets in the sense that the physical commitment of units, although based on the transactions in these markets, is indicative rather than binding, and suppliers need not own a generating unit to participate in the market. The only true physical market is the real-time market, where all trades correspond to actual electricity flows. The real-time market is therefore also the only true spot market, in which electricity is traded for immediate delivery, although the term "spot market" is often used to include all short-term markets, namely, the day-ahead, hour-ahead and real-time markets [1].

The main issue in market design is the choice of market architecture. One of the key features of market architecture is the degree of centralization. In bilateral markets, sellers and buyers trade privately with each other, possibly through a middleman. Such markets are completely decentralized and are well-suited for long-term electricity contracts. Balancing supply and demand in real time, on the other hand, requires extremely fast, coordinated dispatching, which can be best accomplished through a tightly centralized design.

The choice between a centralized and a decentralized scheme is less clear at the level of the day-ahead market. Totally decentralized bilateral markets are known to address poorly the unit commitment problem, i.e., the problem of deciding which generating units to operate. Starting up the wrong units, as a result of a somewhat random bilateral matching of supply and demand, may lead to inefficiency and reduced reliability. In addition, bilateral markets are less transparent and may have higher transaction costs. Although many day-ahead markets, mainly in Europe, rely largely on over-the-counter bilateral trading, with prices unique to each transaction, most day-ahead markets are centralized to some extent and degree, using auctions run by a system operator to establish uniform clearing prices. We distinguish between two trends in auction-based day-ahead market designs that are run centrally by a system operator: *pools* and *exchanges*.

Pools are the natural descendants of the standard procedures used by vertically integrated power utilities to solve the centralized, security-constrained unit commitment and economic dispatch problem. They require the participating generating units to bid all their costs and dispatch constraints to the system operator; hence, they are considered multi-part auctions. The system operator uses these bids to optimally solve the unit commitment problem subject to various system constraints. The market-clearing prices are derived as shadow prices on system constraints and reflect the marginal cost of producing electricity [2]. Pools are used in most U.S. markets, which have been moving towards a standard market design during the last decade. The main advantage of pools is that they integrate all aspects of system operations, including energy, transmission and ancillary services, and could thus lead to higher efficiency and reliability levels.

Some of concerns about the pool approach focus on incentive issues. Because generating units submit bids rather than their true costs and constraints, they may find it profitable to misstate their parameters. As a consequence, the resulting unit commitment, which is efficient in terms of the submitted bids, may be inefficient when the true parameters are used. Another source of concern is that the centralized unit commitment problem requires substantial computational effort to be solved, because of its complexity. The solution methodologies employed to solve it — Lagrangean Relaxation (LR) and Mixed Integer Programming (MIP) methods — have been shown to lead to equity and incentive problems, because often in practice they have to be terminated before reaching complete optimality due to time-limitations [3]. The issue of complexity and tight administration also raises concerns of perception by the market participants. Pools have a long history in the operation of large power systems, but are an uncommon form of market trading. Traders tend to prefer market-clearing prices that they can easily derive from their bids, rather than shadow prices on system constraints, which are indirectly linked to their bids. Also, pools may be perceived by some traders to be too constrained by regulatory policies and inflexible procedural rules. Once a generator subscribes to a pool's basic contract, the daily participation in the pool is mandatory and the unit must adhere to the system operator's day-ahead unit commitments and schedules, presumed availability for reserves.

Exchanges are simple market-clearing mechanisms for energy. They require the participating generating units to bid their energy costs only; hence, they are considered single-part auctions. The market-clearing price in an exchange balances supply and demand for energy, leaving to subsequent markets the determination of transmission adjustments and usage charges and reserve commitments for ancillary services. Exchanges are used in most European markets in addition to over-the-counter trading in the day-ahead market. The advantage of exchanges is that they are simple and less binding for the traders, as unit commitment decisions are often left to the bidders. For this reason, they are often described as decentralized market designs, even though they typically establish uniform clearing prices through a central exchange operator. The main concern in electricity auctions is that, as in the case of bilateral trading, they may lead to efficiency losses by not properly coordinating commitment and dispatch decisions.

Regardless of the market design – exchange or pool – that is employed in the day-ahead market, the generation units incur commitment costs, such as start-up and minimum-load costs, besides the marginal costs for generating electricity. These costs, which we refer to as "fixed costs," along with certain technical constraints, such as the minimum output requirements, make the total costs non-convex, rendering marginal costs less than average variable costs.

In the case of pools, as was mentioned above, the generation units submit multi-part bids for both their marginal and fixed costs; however, under marginal pricing, which is the prevailing pricing basis in pools, the clearing price reflects only the marginal cost for electricity. As a result, the energy payments that generation units receive may not be enough to cover their total costs, a problem often classified as a type of "missing-money" problem. To overcome this drawback, which is also described by the terms "lumpiness" [4] and "non-convexities" [5], most centralized pools give out additional "make-whole" payments to the generation units in order to compensate for any losses. The guarantee of offer revenue sufficiency is a crucial market rule in the U.S. energy and reserves markets with multi-part supply offers. The rule generally states that for all generation service offers by a particular generating unit that are accepted in a spot auction, the total daily market revenue must at least equal the offer requirements. If this is not achieved, the supplier is eligible for a "make-whole" payment that is recovered as an uplift charge to all load. This payment allows for an efficient dispatch where no generator is able to increase its profits by deviating from the accepted supply schedule. Without such payment generators may be tempted to deviate from their schedules, potentially causing reliability problems in real time, as has been experienced in California [6]. In this thesis, we use the term "recovery mechanism" to refer to market rules that define and provide side-payments to the generation units in order to compensate them for potential losses and possibly allow for some positive profits.

In the case of exchanges, the generation units must internalize all their costs including commitment costs in their energy bids. The drawback is that these bids, and consequently the electricity prices, do not reflect the marginal cost for electricity. The lack of a recovery mechanism that gives out side or make-whole payments may drive the generation units to bid well above their marginal costs. As a result, the clearing price that affects the total volume of electricity transacted in the day-ahead market may be well above the marginal cost for electricity.

Although our motivation originates from the area of electricity markets, it is true that many other markets involve non-convexities in the form of economies of scale, start-up and/or shut-down costs, avoidable costs, indivisibilities, minimum supply requirements, etc. It has been widely noted that in the presence of such non-convexities there may be no linear prices that support market equilibrium. In much of the post-war economic theory, the issue of pricing in markets with non-convexities has been dealt with by "convexifying" these markets, following the work of Starr [7] and applying central results of convex economic theory, arguing that these results are good approximations at least for "large" markets where non-convexities are "small."

For cases where non-convexities can be modeled using discrete variables, there have been some early approaches to define dual prices or price functions for Integer Programming (IP) or Mixed Integer Linear Programming (MILP) problems, following the work of Gomory and Baumol [8] on a possible economic interpretation of Gomory's pioneering cutting plane algorithm for solving general IP problems. Wolsey [9] examined the economic implications of this theory and showed that in the IP case we need to use price functions instead of prices in order to identify interpretable and computable duals.

In the 1990s, Scarf [10, 11] revived the discussion on the connection between economic theory and mathematical programming. He pointed out that in markets with standard convexity assumptions, Simplex is an effective device for discovering equilibrium prices from the underlying Linear Programming (LP) problem. If the optimal solution of the LP problem has been determined and the market is in equilibrium, he argued, then a necessary and sufficient condition for introducing a new activity in the market is that this activity is profitable at the old equilibrium price. As such prices may not exist in the presence of non-convexities, this pricing test fails in this case. In view of this failure, Scarf suggested a “neighboring system” as the discrete approximation to the marginal rate of substitution revealed by linear prices.

More recently, the problem of finding interpretable prices/quantities in markets with non-convexities has attracted renewed interest due to the deregulation of the electricity sector worldwide. Several pricing schemes and mechanisms have been proposed to address this problem. We review the most important schemes in the following section.

Finally, we note that the effect of pricing on bidding behavior has substantial practical implications. In a recent case that attracted substantial public attention [12], JP Morgan Ventures Energy Corp. (JPMVEC) carried out a manipulative bidding strategy in California’s day-ahead electricity market that resulted in tens of millions of dollars in overpayments from the grid operator. The strategy, which exploited the make-whole (bid/cost recovery) mechanism, was to:

(i) offer a negative bid for electricity to ensure commitment,

(ii) receive commodity payments at the prevailing market price, and

(iii) qualify for a bid cost recovery payment on the minimum load cost up to twice its actual value.

Following complaints to the Federal Energy Regulatory Commission (FERC), an investigation commenced and was eventually settled with JPMVEC agreeing to pay a total of $410 million in penalties [13].

1.2 Literature Review

In this section, we review the literature related to this thesis. In Subsection 1.2.1, we present the works on pricing in markets with non-convexities. We further discuss these works in Chapter 2, where we present analytical results and comparisons of various pricing approaches. This discussion is continued in Chapter 3, where we provide numerical results illustrating the various pricing approaches. In Subsection 1.2.2, we present the relevant works on bidding in markets with non-convexities. The literature on bidding strategies is further addressed in Chapter 4, where we identify equilibrium outcomes for a stylized duopoly, and in Chapter 5, where we employ a numerical methodology to investigate strategic bidding behavior in the context of electricity markets.

1.2.1 Pricing in Markets with Non-Convexities

Pricing under non-convex market designs has been widely studied. The standard practice for addressing the "lumpiness" (or "missing money") problem in non-convex electricity markets has been to maintain uniform marginal-cost pricing for energy, and provide side-payments to the committed suppliers that would otherwise lose money, in order to make them whole. These side-payments, or "uplifts," as they are often called, may be significant, in which case they may modify the suppliers' incentives by converting the payment scheme towards pay-as-bid. To address this issue, several alternative uniform pricing schemes have been proposed in the last decade (e.g., O'Neill *et al.* [5], Bjørndal and Jörnsten [14, 15], Hogan and Ring [16], Gribik *et al.* [17], Motto and Galiana [18], Galiana *et al.* [19], Van Vyve [20], Araoz and Jörnsten [21], Ruiz *et al.* [22], Gabriel *et al.* [23]). For the most part, these schemes are based on raising the commodity price above marginal cost to increase commodity payments and consequently reduce or even eliminate uplifts. Note that in this thesis we only treat uniform pricing; we refer to the work of Cramton and Stoft [24] for a discussion on the advantages of uniform-price auctions over pay-as-bid pricing in electricity markets, and to an early work of Hobbs *et al.* [25] in which the application of a Vickrey-Clarke-Groves auction has been shown to have several undesirable properties.

There are also a number of works that treat aspects of pricing in markets with non-convexities; e.g., the works of Motto and Galiana [26, 27] who study the coordination problem in energy-only markets with non-convexities, establishing a minimum input requirement without considering monetary uplifts, as well as of Muratore [28] that addresses the issue of non-convexities proposing a peak-load pricing scheme that can recover fixed costs in a yearly period, and of Toczylowski and Zoltowska [29] for a multi-period pool-based electricity auction.

In Europe, the feasible region of the optimization problem that underlies the market coupling problem between European day-ahead electricity exchanges is non-convex, due to the presence of binary decision variables. The common practice is to avoid compensation payments and to implement strict linear prices. This major difference renders methods that are based on compensation payments inapplicable [30]. Madani and Van Vyve [31] consider the case of the non-convex, uniform-price Pan-European day-ahead electricity market "PCR" (Price Coupling of Regions), with non-convexities arising from so-called complex and block orders. This work also extends the results of O'Neill *et al.* [32], who propose a multi-part, discriminatory pricing mechanism in power exchange markets, such as in the Amsterdam Power Exchange (APX) Day-Ahead market or the Nord Pool Elspot market, that allow non-convex, "fill-or-kill" block bids by market participants. We do not deal with such market designs in this thesis, as our main focus is on the non-convex market design used in the U.S. markets [33].

In what follows, we summarize the main schemes proposed in the context of electricity markets, including a variant proposed in this thesis. We divide the schemes into three categories depending on whether the uplifts that they provide are external, internal, or zero. We also refer to an example introduced by Scarf [11] that has been used as a benchmark in the related literature.

Uniform Pricing plus External Uplifts

The following schemes provide commodity payments based on uniform pricing, plus additional uplifts that would normally be passed on to the buyers.

Integer Programming (IP) (O'Neill *et al.* [5]) This scheme mathematically formalizes the standard approach of marginal-cost pricing with make-whole uplifts. It is based on reformulating the original MILP problem as an LP by replacing the integral constraints with constraints that fix the integer variables at their optimal values, solving the LP, and using the dual variables to price the traded commodity and the integral activities causing the non-convexities. IP pricing results in zero profits for all suppliers. A variant of IP pricing used in practice allows profitable suppliers to keep their profits. We refer to this variant as **IP+** pricing.

Modified IP (mIP) (Bjørndal and Jörnsten [14, 15]) This scheme modifies the IP scheme to generate more stable prices. It adds extra constraints to O'Neill *et al.*'s reformulated LP that fix certain continuous variables at their optimal values, as needed. These variables are selected so that if the reformulated LP is viewed as a Benders sub-problem in which the complicating variables are held fixed at their optimal values, the Benders cut that is generated when solving this sub-problem is a supporting valid inequality.

Minimum Uplift (MU) — Convex Hull (CH) (Hogan and Ring [16], Gribik *et al.* [17]) This scheme increases the price above marginal cost and seeks the minimum total uplift for compensating the self-interested suppliers. The price and uplifts are determined by approximating the cumulative non-convex cost of the original MILP problem with its convex hull, solving the resulting LP problem, and using the dual variables to price the commodity and the integral activities. Zhang *et al.* [34] extend the CH pricing to include the no-load costs of the generating units. Another extension of the CH pricing algorithm for integrated energy and reserve markets is provided by Wang *et al.* [35, 36].

Zero-Sum Uplift Pricing

The following schemes consider uplifts as internal zero-sum transfers between the suppliers.

Generalized Uplift (GU) (Motto and Galiana [18], Galiana *et al.* [19]) This scheme increases the price above marginal cost and provides additional minimized, multi-part, positive or negative, zero-sum uplifts to the self-interested suppliers. The price and uplifts are determined by solving a quadratic programming problem that seeks to minimize the norm of the uplift components. Bouffard and Galiana [37] extend this scheme to time-dynamic markets.

Minimum Zero-Sum Uplift (MZU) (proposed in this thesis) This scheme increases the price above marginal cost and transfers all the additional commodity payments that the profitable (under marginal-cost pricing) suppliers receive as a result of the price increase, to the unprofitable suppliers in the form of internal zero-sum uplifts, to make them whole at the smallest possible price.

Revenue-Adequate Pricing

The following schemes generate high-enough prices to ensure that the suppliers cover their costs, without the need for additional uplifts.

Average Cost (AC) This scheme seeks the smallest revenue-adequate price under the optimal allocation. This price is the maximum average cost of the suppliers. Van Vyve [20] proposes a zero-sum uplift pricing scheme that aims to minimize the maximum contribution to the financing of the uplifts, in a model where both suppliers and buyers place bids. That scheme is equivalent to AC pricing, when the demand is inelastic.

Semi-Lagrangean Relaxation (SLR) (Araoz and Jörnsten [21]) This scheme seeks the smallest revenue-adequate price for the self-interested suppliers. This price is determined by formulating an SLR of the original MILP problem by semi-relaxing the linear market-clearing equality constraint, and solving its dual.

Primal-Dual (PD) (Ruiz *et al.* [22]) This scheme seeks an efficient revenue-adequate price. This price is determined by relaxing the integrality constraints of the MILP problem so that it becomes a (primal) LP, deriving its dual, formulating a new LP that seeks to minimize the duality gap of the primal and dual LPs, subject to both primal and dual constraints, and adding back the integrality constraints along with additional non-linear revenue-adequate constraints. Recently, Abbaspourtorbati *et al.* [38] extend this scheme to a stochastic non-convex market-clearing model. PD is also somewhat related to an approach for solving Discretely Constrained Mixed Linear Complementarity Problems, recently proposed in [23], which is outside the scope of this thesis.

Comparisons of Pricing Schemes

There are strong arguments for keeping the marginal price as the market signal, and designing mechanisms that aim at keeping the uplifts low. Marginal prices are simple and accurate economic signals that are easily understood by the market participants. As long as the "distortion" caused by the uplifts is small, keeping the marginal price as the economic signal is an attractive option for market designers. In the context of electricity markets in particular, proposals for changing marginal pricing rules are often mistrusted by the market participants and prove to be a source of friction. On the other hand, there are counter-arguments that support schemes that require no external payments and hence impose no uplifts. By avoiding uplifts, these schemes eliminate the cost allocation problem —i.e, allocating the cost that these uplifts impose, which is often another source of friction especially if the magnitude of these uplifts becomes large. A common benchmark that has been used for evaluating various pricing schemes that deal with non-convexities is an example introduced by Scarf [11].

Scarf's example considers a market with two types of units (suppliers), called "smokestack" and "high-tech," where smokestack has higher capacity and higher fixed and marginal costs than high-tech. O'Neill *et al.* [5] show how to compute IP prices for a range of values of demand for Scarf's example, when a finite number of units of each type is available.

Hogan and Ring [16] modify this example by adding a third unit type, called "med-tech," with lower capacity than the other two types, and a minimum output, to capture a common feature in electricity markets. The new type has zero fixed cost and a marginal cost which is higher than the average cost at full capacity of the other two types. For this example, they demonstrate that CH prices are less volatile than IP prices and that the breakup of payments into commodity and uplifts payments is different under IP and CH. Bjørndal and Jörnsten [14] add the mIP scheme to the comparison, and demonstrate that mIP prices are piecewise constant and increasing in demand and the resulting uplifts are higher but less volatile than are CH uplifts. Araoz and Jörnsten [21] add SLR but assume that med-tech units have no minimum output requirement. They show that SLR prices are higher and less volatile than IP prices and note that they are also higher than CH prices. Finally, Ruiz *et al.* [22] evaluate PD against IP, mIP and CH, for Hogan and Ring's example and observe that PD prices are close to CH prices.

Ruiz *et al.* [22] also evaluate PD for a more realistic electricity market case study (based on the IEEE reliability test system 1996 [39]) for which they observe instances of inefficient dispatching. For a similar case study, Wang *et al.* [35, 36] compare the outcomes of energy-only and energy-reserve co-optimized markets and observe that it remains unclear whether marginal-cost pricing or convex-hull pricing leads to higher total payment.

Unit Commitment Problem

The main reference of pricing under non-convexities in electricity markets is the unit commitment and generation scheduling problem, which is a very important and particularly challenging problem in the electrical power industry. In general, this problem aims at minimizing the system operational costs of the generation units by providing an optimal schedule of power production for each unit so that the demand for electricity is met. The generators must operate within certain technical limits; however, the operational constraints, such as the ramp and the minimum up and down time constraints, in addition to the scale of the problem, make large unit commitment problems particularly challenging to solve.

Even though the unit commitment model originated in the monopolistic era, it has been easily extended to produce generation schedules in competitive market environments [40, 41]. The problem remains particularly important today, after the deregulation of the power industry, where in many wholesale electricity markets the Independent System Operator (ISO) has to determine day-ahead schedules for generation units based on a centralized unit commitment.

The unit commitment problem is generally modeled as a large-scale non-convex problem, and various approaches have been developed to solve it. These approaches have ranged from highly complex and theoretically complicated methods to simple rule-of thumb methods. A thorough bibliographical survey of the unit commitment problem up to the early 2000s that lists mathematical formulations and area approaches based on more than 150 published articles is provided in [42]. Reviews on the methodologies that were employed to solve the unit commitment and generation scheduling problem are also listed in [43].

In the real world, the time required to solve the unit commitment models has introduced a hard practical limitation that has challenged the size and scope of the problem's formulations. Up until recently, the LR algorithm was the only practical means of solving an ISO-scale unit commitment problem and was used by most ISOs in the U.S. However, recent advances in software development has made branch-and bound based techniques for solving MILP an attractive alternative. According to [44], the first MILP unit commitment formulation was described in [45]. That formulation, which uses three sets of binary variables to model the on/off status of generation units, has been extensively used ever since. Recent advances in computing capabilities and optimization algorithms now make the solution of MILP formulations tractable, often with optimality gaps smaller than those of LR algorithms. This development has led most ISOs to adopt this approach. Furthermore, strengthening the basic unit commitment formulation is expected to have a positive effect in solving more advanced models, and a number of recent works have addressed this issue (e.g. Carrión and Arroyo [46], Ostrowski *et al.* [44], Morales *et al.* [47]).

On the downside, Sioshansi *et al.* [48] demonstrate, using actual market data from an ISO, that both LR and MILP solutions suffer from the equity issues that were first identified by [3] for the LR case. Because different commitments that are similar in terms of total system costs can result in different surpluses to individual units, this drawback is inevitable unless the ISO unit commitment problems can be solved to complete optimality within the available timeframe, which is beyond current computational capabilities. From a fairness perspective, this is important in competitive markets because two near-optimal solutions can produce considerably different payments to generator owners.

1.2.2 Bidding in Markets with Non-Convexities

Although several different market designs have been proposed to address non-convexities, when it comes to fully evaluating a pricing scheme, it is necessary to explore its implications on the incentives and thus the likely bidding behavior of the market participants. Such an evaluation involves finding the optimal bidding strategy of each market participant and evaluating the incentive compatibility associated with these designs. From a market design point of view, competitive bidding in uniform-price auction markets, i.e., bidding above marginal cost, should be expected because this guides the market toward long-run efficient outcomes [49].

In what follows we briefly review the most relevant works on strategic bidding in the context of electricity markets and on equilibria characterization for stylized models.

Strategic Bidding in Electricity Markets

Strategic bidding behavior has been studied extensively, mainly in markets with standard convexity assumptions. Ventosa *et al.* [50] classify electricity market models into three major streams: optimization, equilibrium, and simulation.

Optimization models take the view of a single market participant that tries to maximize his profit as a price-taker (e.g., Conejo *et al.* [51], De Ladurantaye *et al.* [52], Simoglou *et al.* [53], Díaz *et al.* [54], Baringo and Conejo [55], the latter employing robust optimization) or price-maker (e.g., Anderson and Philpott [56], de la Torre *et al.* [57], Bakirtzis *et al.* [58], Pousinho *et al.* [59]).

In equilibrium models, each participant tries to maximize his profit taking into account the other participants' strategies. This is a game-theoretic approach to the strategic bidding problem aiming at identifying market equilibria. Hobbs *et al.* [60]

use an iterative scheme (diagonalization) to compute a market equilibrium, where in each iteration, each market participant solves a profit maximization problem, assuming that the other participants' bids remain fixed at the values of the previous iteration. De la Torre *et al.* [61] consider multi-period Nash equilibria and apply an iterative procedure to identify behavior patterns of the generation units; the units decide on the quantity to bid, while the price is determined by price quota curves. The problem formulation assumes a simple pricing rule according to which the market-clearing price is the price of the last accepted production bid, without considering reserves or any recovery mechanism. A similar iterative procedure is also employed by Haghighat *et al.* [62], in an attempt to find Nash equilibria in joint energy/reserve markets, under a pay-as-bid pricing scheme, without considering the non-convexities of the unit commitment problem. Barroso *et al.* [63] present an MILP solution approach for finding Nash equilibria in strategic bidding in short-term energy-only electricity markets with equilibrium constraints. Hasan *et al.* [64] and Hasan and Galiana [65, 66] address the issue of Nash equilibria for an energy-only electricity market, without taking into account unit commitment and associated costs, technical minimum, and inter-temporal constraints. Cournot models (in which firms compete in quantity strategies) and supply function equilibrium models (in which firms compete in offer curve strategies), as defined by Klemperer and Meyer [67], are special cases of this stream (e.g., Green and Newbery [68], Hobbs *et al.* [60], Anderson and Philpott [69], Anderson and Xu [70]).

Finally, simulation models are used when the problem is too complex to be tackled with formal equilibrium models, in which each participant tries to maximize his profit taking into account the other participants' strategies (e.g., Otero-Novas *et al.* [71], Krause *et al.* [72], Weidlich and Veit [73], Guerci *et al.* [74]).

Equilibria Characterization

There also exist a few works that set out to analytically characterize equilibria for simple stylized representations of market auctions under convexity assumptions (e.g.,von Fehr and Harbord [75], Fabra *et al.* [76] that characterize Nash equilibria for duopoly models). More specifically, Fabra *et al.* [76] characterized the bidding behavior and market outcomes in uniform and discriminatory electricity auctions, in a basic duopoly model — similar to the one described by von Fehr and Harbord [75] — with asymmetric marginal costs and capacities.

The literature on models with non-convexities, on the other hand, is scarce. We distinguish the works of Sioshansi and Nicholson [77], Wang *et al.* [78], and Wang [79], that derive Nash equilibria for a duopoly model, under IP+ and CH pricing and under different assumptions on model parameters (symmetric/asymmetric costs and capacities) and bidding format (single-part/two-part bidding). Sioshansi and Nicholson [77] debate on centrally committed vs. self committed markets and characterize Nash equilibria for a stylized single-period symmetric duopoly. In the former design, the generators submit two part offers (energy and start-up) and the recovery of their bids is guaranteed with make-whole payments (hence, the setting is identical to IP+ pricing).

Table 1.1 summarizes the results of these works, which to our knowledge are the only ones that derive Nash equilibria for uniform-price auctions, under IP+ and CH pricing, in a duopoly setting for the symmetric-capacity case. The works differ in their assumptions about the costs and bidding format. Symbols c_n and s_n denote the suppliers' actual marginal and fixed costs, respectively, and $b^{\max}$ and $f^{\max}$ denote the caps on the bid costs b_n and f_n, $n = 1, 2$, respectively. Low-demand refers to a level of demand equal to or lower than the capacity of a single supplier, and high-demand refers to a level of demand which is greater than the capacity of a single supplier (hence both suppliers need to be committed to satisfy this level of demand).

Table 1.1 indicates that in the low-demand case, pure-strategy Nash equilibria exist for the considered schemes. In fact, the equilibrium strategies for IP+ and CH are identical. Specifically, there is one Bertrand-type equilibrium, in which the supplier with the highest actual total cost at d, say R, bids his actual costs, and the other supplier, say r, just underbids R (subject to the caps). Hence, in case of excess capacity, competition drives suppliers to bid at or close to their actual costs.

In the high-demand case, IP+ admits only a mixed-strategy equilibrium, in which the suppliers bid their fixed cost at the cap, i.e., $f_n = f^{\max}$, and mix their marginal-cost bid b_n over $[0, b^{\max}]$. If they have to submit truthful bids for their fixed costs, they

Table 1.1: Summary of existing results on Nash equilibria for a symmetric duopoly with non-convex costs.

	Model assumptions		Bidding assumptions		Nash equilibrium strategy		
Reference	**Capacities**	**Costs**	**Marginal-cost bids (b_n)**	**Fixed-cost bids (f_n)**	**Pricing**	**Low-demand**	**High-demand**
[77]	symmetric	symmetric	$[0, b^{\max}]$	$[0, f^{\max}]$	IP+	pure	mixed
[79], ch. 6.3	symmetric	asymmetric	$[0, b^{\max}]$	$[0, f^{\max}]$	IP+	pure	mixed
					CH	pure	pure
[78] (IP+ only); [79], ch. 5	symmetric	asymmetric	$[0, b^{\max}]$	Truthful (s_n)	IP+	pure	mixed
					CH	pure	pure

mix b_n over the same support $[b^{\min}, b^{\max}]$, where $b^{\min} > \max(c_1, c_2)$. Under CH, a pure-strategy equilibrium exists, in which the supplier with the highest actual fixed cost bids at the marginal-cost cap. A secondary pure-strategy equilibrium may also exist, under certain conditions. The strategic bidding behavior under both schemes makes the actual cost information inaccessible to the auctioneer, and may consequently lead to inefficient allocation in terms of actual total cost. Specifically, under both schemes, the supplier with the smallest actual marginal cost may bid aggressively high, sacrificing market share. Moreover, the CH price may exceed the bids, which means that $b^{\max}$ fails to cap the CH price. Also, CH provides more freedom to game, unless more strict regulatory measures, such as a lower $b^{\max}$ value, are imposed.

Wang [79], chapter 6.2 extends the analysis for the IP+ and CH schemes to the asymmetric-capacity case. He shows that in the medium-demand region, there exists one and possibly a second pure-strategy equilibrium, for both schemes. In the first equilibrium, supplier 1 aggressively reduces his bid to offset his capacity disadvantage, forcing supplier 2 to give up part of his market share and profit and raise his bid up to the cap. In the second equilibrium, supplier 2 undercuts supplier 1's bid profitably and assumes full share of the market. The resulting profits in both equilibria are limited for both suppliers. Although the equilibrium strategies are the same in both IP+ and CH, the resulting prices and profits differ. Notably, under CH, supplier 1 can manipulate the "opportunity-cost" uplift payment by deliberately underbidding his marginal cost, even if he is not committed.

1.3 Contribution of the Thesis

The contribution of this thesis is mainly in the areas of pricing and bidding in markets with non-convexities that we reviewed in Section 1.2. In Subsection 1.3.1 that follows, we summarize the contribution in pricing, and in Subsection 1.3.2, we summarize the contribution in bidding.

1.3.1 Contribution to Pricing in Markets with Non-Convexities

The development of the pricing schemes mentioned in Subsection 1.2.1 — namely, IP+, mIP, CH, GU, MZU, AC, SLR, PD — suggests that the issue of pricing in markets with non-convexities remains to this day an open challenge with significant practical implications. Although most of these schemes have been well motivated and described, there are limited results on the prices and uplifts that they generate. Moreover, the connection between different schemes has not been thoroughly studied, and existing comparisons are restricted to observations based on limited numerical experimentation, for the most part, on Scarf's [11] benchmark example. It is thus difficult to draw general conclusions.

This thesis contributes to furthering our understanding on the behavior of different mechanisms and on the connections between them by employing the following:

(i) Exact analysis of pricing schemes in markets with non-convex costs;

(ii) Numerical comparison of pricing schemes in markets with non-convexities.

Exact Analysis of Pricing Schemes in Markets with Non-Convex Costs

We critically review the pricing schemes listed in Section 1.2.1, including the MZU variant which we propose, by considering a basic model of two suppliers with asymmetric capacities and asymmetric marginal and fixed costs, who compete to satisfy a deterministic and inelastic demand of a commodity in a single period. The suppliers simultaneously bid their costs to an auctioneer, who determines the optimal allocation and the resulting payments.

In contrast to the extant literature, we derive closed-form expressions for the price, uplifts, and profits for each scheme in the context of this model, and we use these expressions to compare these schemes along these three dimensions. Our analysis identifies trade-offs between the market outcome characteristics that are weighed differently by each scheme.

We also extend some of our analytical comparisons to the case of more than two suppliers and discuss the case of price-elastic demand.

Numerical Comparison of Pricing Schemes in Markets with Non-Convexities

Based on the closed-form expressions that we derive for each pricing scheme, we numerically explore and compare the quantities, prices, and profits as a function of the demand. We present several graphs for the two-supplier case for various sets of parameters, and we comment on their similarities and differences.

Furthermore, we numerically evaluate several pricing mechanisms using Scarf's example [11] as it was modified in [16]. Given that comparisons between the IP+, mIP, CH, and PD pricing schemes already exist, we restrict our attention to mechanisms that do not provide external uplifts, i.e., GU, MZU, AC, SLR, and PD.

Lastly, we consider the impact of recovery mechanisms on actual markets, using the Greek wholesale electricity market as a test case of a real market with non-convexities. For this market, we evaluate the aggregate (annual) impact of the recovery mechanism that has been implemented against a standard bid/cost recovery mechanism.

1.3.2 Contribution to Bidding in Markets with Non-Convexities

Fabra *et al.* [76] characterize the bidding behavior and market outcomes in uniform and discriminatory electricity auctions in a basic duopoly model with asymmetric marginal costs and capacities. However, their analysis does not consider fixed costs or any other non-convexities.

In the works of Sioshansi and Nicholson [77], Wang *et al.* [78], and Wang [79], equilibrium outcomes are identified for the IP+ and CH pricing schemes for a symmetric-capacity duopoly where the participants incur fixed costs. In these works, a regulator can only control the price cap to influence the market outcome, which is particularly limiting from a market design point of view.

In general, the models that have been employed in the literature to study strategic bidding behavior either limit the players' bidding options or suppress important market structure features, such as discontinuities in the cost structure and inter-temporal effects. The literature on the evaluation of the impact of recovery mechanisms in actual electricity markets, and the incentives that different mechanisms induce to the market participants is underdeveloped.

In this thesis, rather than modifying the objective function of the unit commitment problem or the clearing prices, we do not directly interfere with the day-ahead market design and solution, so we let the commodity prices be equal to the shadow prices of the respective market-clearing constraints. Instead, we introduce simple rules for recovery payments that will allow the generation units to have positive profits. These payments are settled after the day-ahead market is cleared; hence, they depend on the market outcome. The advantage of this approach is that the dispatching and pricing of the commodities is still subject to the existing and well-established day-ahead market rules for co-optimizing energy and ancillary services. This approach can be particularly attractive to regulators because proposals that change the pricing rules (e.g., the payment cost minimization based clearing format that some claim reduces the amount of payments [80, 81, 82]) are often misguided, misunderstood or mistrusted by the market participants and prove to be a source of friction. Hence, keeping the widely accepted marginal pricing scheme for the procured commodities is particularly important for the market.

This thesis contributes to furthering our understanding on the bidding behavior of participants in markets with non-convexities by employing the following:

(i) Exact analysis to identify equilibrium outcomes in electricity auctions under cost-based and bid-based recovery mechanisms;

(ii) Numerical evaluation of recovery mechanisms in electricity markets.

Exact Analysis to Identify Equilibrium Outcomes in Electricity Auctions

Following on the work of Fabra *et al.* [76], we introduce a fixed cost component in the electricity auction that they consider and study recovery mechanisms that ensure that the units will not exhibit losses. The form of such mechanisms is a challenge in mechanism design. To be in line with practice, we introduce side-payments in the decentralized auction model, so that the market participants will no longer have to internalize their fixed costs in their offers. Instead, the market will ensure that the participants will not incur losses through a recovery mechanism that will provide appropriate side-payments. We are interested in designing and evaluating such recovery mechanisms, exploring the bidding behavior of the market participants and identifying equilibrium outcomes. We are also interested in providing useful insights for a regulator to "control" the market outcome, through the adjustment of certain design parameters. We choose to study an auction format with energy-only bids, so that we will not lose the main advantage of simplicity in the market rules. This design offers the possibility of analytically finding Nash equilibria, that characterize the bidding behavior of the generation units. As pointed out by Maskin [83], since our problem lies in the area of mechanism design, the principal theoretical and practical drawbacks of Nash equilibrium as a solution concept are far less troublesome than in most other applications of game theory; hence, this solution concept can be justified. Also, the duopoly model facilitates the discussion on particular design issues related to the parameters that a regulator can control in a real market.

At first, one could argue that since the non-convexity problem arises from the fixed costs, this problem could be eliminated by fully compensating the players for their fixed costs, whenever they occur, thus allowing them to compete with their bids reflecting their marginal cost. We show that this actually results in the design of [76], and we present the outcomes for a slightly more general case where the marginal costs of the two players are non-zero (in [76] it is assumed that the lowest marginal cost is zero).

An alternative option is to compensate the players for their losses with what we call a recovery mechanism with "loss-related profits" that actually compensates and allows for a positive profit that is proportional to the losses. The idea is that a supplier that exhibits losses has an incentive to maximize these losses, and hence has an incentive for low prices (the lower the price the lower the market revenues and hence the higher the losses). We show that the paradoxical behavior of this mechanism, i.e., having increasing profits with decreasing prices, has some particularly nice and interesting properties and that it can be designed to outperform the fixed cost recovery mechanism in terms of equilibrium prices and total payments.

In the electricity auction model that we consider, we use a two-step approach, where in the first step, the allocation is made according to the energy bids, and in the second step, the recovery mechanism that guarantees that the generation units will not incur losses is cost-based and not bid-based. In the first step, we use the standard market design in which the auctioneer solves a stylized "unit commitment" problem, taking into account both marginal and fixed costs to determine the commitment and dispatch of the generation units. This yields a rather non-trivial electricity auction that provides the market with an economic signal that is consistent with marginal pricing but also involves side-payments due to the non-convexities.

In the second step, we consider a stylized Bertrand-type capacity-constrained duopoly, where we add a recovery mechanism that "recovers" (compensates) potentially incurred losses and occasionally allows for some positive profits. An example of such a mechanism is the IP+ pricing scheme, which is referred to as standard bid/cost recovery, and which unconditionally allows for make-whole payments based on the as-bid costs. Note that the equilibrium outcomes for this mechanism can be found in [77, 78] for the symmetric capacity case. Here, we extend these outcomes to the asymmetric capacity case. We also introduce a modified version of the IP+ mechanism in which the make-whole payments are provided under the condition that the offered bids are within a certain regulated margin from the actual marginal costs. This mechanism is referred to as bid/cost recovery with regulated cap. We identify and discuss the equilibrium outcomes for this mechanism and show that it can be designed so that it outperforms the standard/bid cost recovery mechanism.

Numerical Evaluation of Recovery Mechanisms in Electricity Markets

It is evident that for more complicated market designs than the simple stylized duopoly models — e.g., actual electricity markets — analytically finding Nash equilibria becomes practically infeasible. Nevertheless, attempting to numerically find Nash equilibria by some iterative scheme may reveal insightful patterns of bidding behavior under the specific market rules, even if this scheme does not converge to a solution.

We study four alternative recovery mechanism designs that address the issue of non-convexities in joint energy/reserve, unit commitment-based day-ahead electricity markets. The first design lets the suppliers that incur losses keep a fixed percentage of their actual variable costs (a variant of this mechanism has been used in the Greek electricity market). The second lets them keep a fixed percentage of their losses based on their actual costs. The other two designs are the standard bid/cost recovery and the bid/cost recovery with regulated cap.

We propose a methodology for evaluating the bidding strategy behavior of the participating units under each mechanism. We evaluate the proposed numerical methodology on a simplified test case model and derive insights on the performance and incentive compatibility of the recovery mechanisms under consideration. We also perform sensitivity analysis and discuss certain extensions to explore the accuracy and extendability of our results.

1.4 Structure of the Thesis

The remainder of this thesis is structured in two parts as follows:

Part I addresses pricing mechanisms and consists of Chapters 2 and 3. In Chapter 2, we present the exact analysis of several pricing schemes in markets with non-convex costs for a two-supplier model and its extensions. In Chapter 3, we provide numerical results for the two-supplier case, Scarf's modified example, and an actual market setting.

Part II addresses bidding in markets with non-convexities, assuming marginal cost pricing, and consists of Chapters 4 and 5. In Chapter 4, we identify and discuss equilibrium outcomes for a duopoly model under cost-based and bid-based recovery mechanisms. In Chapter 5, we provide a numerical methodology for the evaluation of the performance and incentive compatibility of recovery mechanisms in actual electricity markets.

Finally, Chapter 6 provides a brief summary of our main findings.

Most of the proofs and supplementary materials of this thesis are put in appendices. More specifically, in Appendix A, we provide proofs and other supplementary material for the main propositions of Chapter 2. In Appendix B, we list the nomenclature for the Day-Ahead Scheduling (DAS) problem employed in Chapters 3 and 5. In Appendix C, we provide proofs for the propositions of Chapter 4. Finally, in Appendix D, we list the main publications that have resulted from the work of this thesis to date.

Part I

Pricing in Markets with Non-Convexities

Chapter 2

Exact Analysis of Pricing Mechanisms in Markets with Non-Convex Costs

2.1 Introduction

In this chapter, we review several existing pricing schemes in markets with non-convexities, also proposing a variant of one of these schemes, by considering a basic model of two suppliers with asymmetric capacities and asymmetric marginal and fixed costs, who compete to satisfy a deterministic and inelastic demand of a commodity in a single period. The suppliers simultaneously bid their costs to an auctioneer, who determines the optimal allocation and the resulting payments. In contrast to the extant literature, we derive closed-form expressions for the price, uplifts, and profits in the context of this model for each scheme presented in Section 1.2.1, and we use these expressions to compare these schemes along these three dimensions. We extend some of our analytical comparisons to the case of more than two suppliers, and we also present an extension to price-elastic demand.

The remainder of this chapter is organized as follows. In Section 2.2, we present the two-supplier model, and in Sections 2.3–2.5, we analyze the alternative pricing schemes for this model, namely uniform pricing plus external uplifts, zero-sum uplift pricing, and revenue-adequate pricing. In Section 2.6, we compare the prices and profits that these schemes generate. In Section 2.7, we discuss the tradeoffs between various market outcome characteristics underlying the scheme differences. The discussions on multiple suppliers and price-elastic demand are presented in Section 2.8. Finally, we draw conclusions in Section 2.9. The proofs and other supplementary material are included in Appendix A.

2.2 Two-Supplier Model

We consider a model of two suppliers with asymmetric marginal and fixed costs and asymmetric capacities k_n, $n = 1, 2$, where $k_1 \le k_2$, without loss of generality. The suppliers compete to satisfy a deterministic inelastic demand d in a single period. We assume that $0 < d \le k_1 + k_2$. The suppliers simultaneously submit bids b_n and f_n, $n = 1, 2$, for their marginal and fixed costs, respectively, to an auctioneer, who must determine the allocation and payments to the suppliers. Throughout this chapter, we will be using the following definition:

$$k = \begin{cases} k_1, & \text{if } b_2 > b_1 + f_1/k_1 \ ; \\ k_2, & \text{otherwise.} \end{cases} \tag{2.1}$$

Because of the non-convexity in the total bid costs caused by the fixed costs, there is no unique definition of the least costly supplier. Throughout this chapter, we will be using different sets of indices to distinguish the suppliers in terms of their bid costs. For ease of presentation, henceforth, we will omit the term "bid" when we refer to the costs/profits. The different sets of indices are:

- i (I): index of supplier with smallest (largest) marginal cost, i.e., $b_i \le b_I$;
- $r(d)$ ($R(d)$): index of supplier with smallest (largest) total cost at demand level d, for $0 < d \le k_1$, i.e., $b_{r(d)}d + f_{r(d)} \le b_{R(d)}d + f_{R(d)}$, $0 < d \le k_1$;

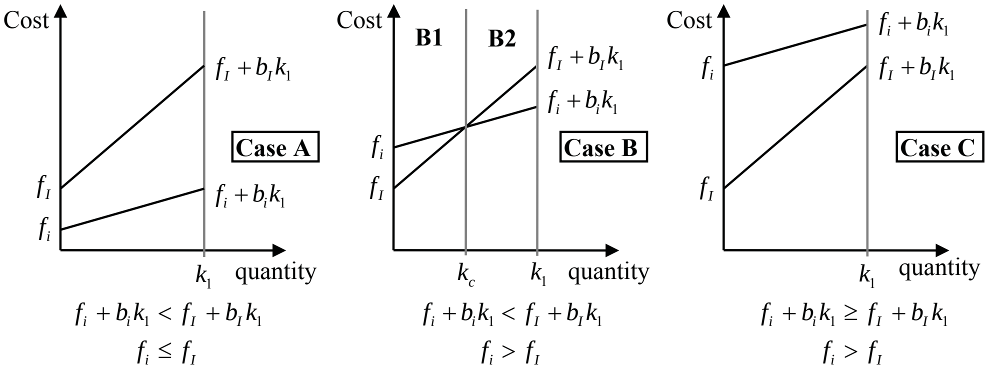

Figure 2.1: Total cost versus quantity for suppliers i and I, for the three possible cases of parameters b_n, f_n, $n = 1, 2$, and k_1.

$r'(d)$ $(R'(d))$: index defined as follows: $r'(d) = r(d)$ and $R'(d) = R(d)$, if $d \leq k_1$; $r'(d) = 2$ and $R'(d) = 1$, if $k_1 < d \leq k$;

j (J): index of supplier with smallest (largest) average cost at full capacity, i.e., $b_j + f_j/k_j \leq b_J + f_J/k_J$.

Depending on the values of parameters b_n, f_n, $n = 1, 2$, and k_1, there are three cases to consider, shown in Figure 2.1.

Given bids b_n, f_n, $n = 1, 2$, the auctioneer determines the optimal allocation, expressed by decision variables z_n (binary) and q_n (continuous), $n = 1, 2$, representing the suppliers' commitment and dispatch quantities, respectively, by solving the following MILP problem:

$$\underset{q_n, z_n, n=1,2}{\text{Minimize}} \; L_{\text{MILP}} = \sum_{n=1,2} (b_n q_n + f_n z_n), \tag{2.2}$$

subject to:

$$\sum_{n=1,2} q_n = d, \tag{2.3}$$

$$q_n \leq k_n z_n, \qquad n = 1, 2, \tag{2.4}$$

$$q_n \geq 0, \qquad n = 1, 2, \tag{2.5}$$

$$z_n \in \{0, 1\}, \qquad n = 1, 2. \tag{2.6}$$

Objective function (2.2) expresses the total cost, which is non-convex. Equality (2.3) is the market-clearing constraint, and inequalities (2.4)–(2.5) express the capacity constraints.

Given a feasible solution of problem (2.2)–(2.6), a uniform commodity price, denoted by λ, and additional side-payments (uplifts), denoted by σ_n, the profit of supplier n, denoted by π_n, is given by:

$$\pi_n = \lambda q_n - (b_n q_n + f_n z_n) + \sigma_n, \qquad n = 1, 2. \tag{2.7}$$

Proposition 2.1 gives the optimal solution of problem (2.2)–(2.6), denoted by $z_n^{\text{MILP}}, q_n^{\text{MILP}}, n = 1, 2$.

Proposition 2.1. *The optimal solution of the MILP problem (2.2)–(2.6) is as follows:*

(i) *If $d \le k$, then $z_{r'(d)}^{\text{MILP}} = 1$, $z_{R'(d)}^{\text{MILP}} = 0$, $q_{r'(d)}^{\text{MILP}} = d$, and $q_{R'(d)}^{\text{MILP}} = 0$.*

(ii) *If $d > k$, then $z_i^{\text{MILP}} = z_I^{\text{MILP}} = 1$, $q_i^{\text{MILP}} = k_i$, and $q_I^{\text{MILP}} = d - k_i$.*

Proof. The proof is found in Appendix A (Section A.1). □

When the suppliers have asymmetric capacities, three cases may arise. Figure 2.2 shows the optimal dispatch quantities versus d for these cases. In each case, there are three regions of interest where d may lie: lowest ($0 < d \le k_1$), middle ($k_1 < d \le k_2$), and highest ($k_2 < d \le k_1 + k_2$). Proposition 2.1 implies that, as far as the optimal allocation is concerned, the three regions effectively map onto two regions: a “low-demand” and a “high-demand” region, where the latter is shown as shaded in Figure 2.2. The border between these regions, denoted by k, is given by (2.1). In the low-demand region ($0 < d \le k$ or $d \le k$ for short), only one supplier, namely $r'(d)$, is dispatched. In the high-demand region ($k < d \le k_1 + k_2$ or $d > k$ for short), both suppliers are dispatched: supplier i at full capacity k_i, and supplier I at the residual demand $d - k_i$. The lowest value of q_I^{MILP} in the high-demand region is attained at $d \to k^+$, given by the following corollary.

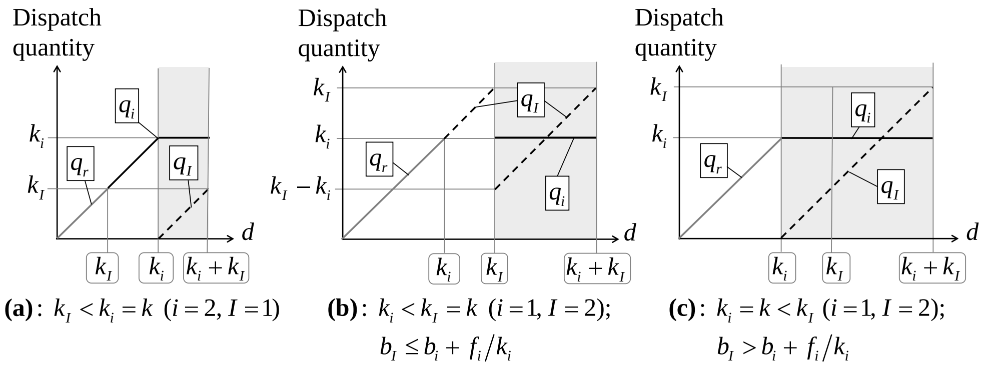

Figure 2.2: Optimal dispatch quantities versus demand for the three possible cases of capacity and cost parameters.

Corollary 2.1. *If $d > k$, then the following holds:*

(i) *If $k = k_i$, then $\lim_{d \to k^+} q_I^{\text{MILP}} = 0$ (Figure 2.2(a),(c)).*

(ii) *If $k = k_I$, then $\lim_{d \to k^+} q_I^{\text{MILP}} = k_I - k_i > 0$ (Figure 2.2(b)).*

Problem (2.2)–(2.6) is a simple MILP. The issue that we address in this chapter is not how to solve it, but how to price the commodity, given that marginal-cost pricing fails to cover the suppliers' fixed costs. Specifically, the marginal price, denoted by λ^{MILP}, and the suppliers' profits, if they are paid λ^{MILP} for the commodity and receive no other payments, denoted by π_n^{MILP}, are given by the following corollary.

Corollary 2.2. *Under marginal-cost pricing and no additional side-payments:*

(i) *If $d \le k$, then $\lambda^{\text{MILP}} = b_{r'(d)}$. The resulting profit is $\pi_{r'(d)}^{\text{MILP}} = -f_{r'(d)}$.*

(ii) *If $d > k$, then $\lambda^{\text{MILP}} = b_I$. The resulting profits are $\pi_i^{\text{MILP}} = b_I k_i - (b_i k_i + f_i)$ and $\pi_I^{\text{MILP}} = -f_I$.*

Corollary 2.2 implies that in the low-demand case, the marginal supplier is $r'(d)$, and in the high-demand case, it is I. It also implies that under marginal-cost pricing, at least the marginal supplier has a negative profit. Next, we analyze several alternative pricing schemes that address this "missing money" problem.

2.3 Uniform Pricing plus External Uplifts

In this section, we present pricing schemes that provide commodity payments based on uniform pricing, plus additional uplifts, which would normally be passed on to the buyers.

2.3.1 Integer Programming (IP) Pricing

O'Neill *et al.* [5] introduced a pricing scheme that uses uniform marginal-cost pricing for the commodity, and discriminatory pricing for the integral activities causing the non-convexities. This scheme, which was referred to as "IP-pricing" by [16], is based on:

(i) reformulating the original MILP problem as an LP, by replacing the integer constraints with constraints that set the integer variables equal to their optimal values,

(ii) solving the LP, and

(iii) using the dual variables to price the commodity and the integral activities.

In the context of our two-supplier model, the reformulated LP is obtained from the original MILP (2.2)–(2.6) after replacing the integer constraint (2.6) with the constraints $z_n = z_n^{\text{MILP}}$ and $z_n \geq 0$, $n = 1, 2$. We refer to the reformulated problem as the "IP" problem (even though it is an LP), because it is used to generate the "IP prices." The IP problem is presented below.

$$\underset{q_n, z_n, n=1,2}{\text{Minimize}} \; L_{\text{IP}} = \sum_{n=1,2} (b_n q_n + f_n z_n), \tag{2.8}$$

subject to:

$$\sum_{n=1,2} q_n = d, \tag{2.9}$$

$$q_n \leq k_n z_n, \qquad n = 1, 2, \tag{2.10}$$

$$z_n = z_n^{\text{MILP}}, \qquad n = 1, 2, \tag{2.11}$$

$$q_n \geq 0, \qquad n = 1, 2, \tag{2.12}$$

$$z_n \geq 0, \qquad n = 1, 2. \tag{2.13}$$

Clearly, the IP problem has the same optimal solution as the MILP problem. Once the IP problem is solved, supplier n receives a commodity payment $\lambda^{\mathrm{IP}} q_n^{\mathrm{IP}}$ for producing q_n^{IP} units, plus an uplift, denoted by σ_n^{IP}, equal to $\nu_n^{\mathrm{IP}} z_n^{\mathrm{IP}}$, for being committed, where λ^{IP} and ν_n^{IP} are the dual variables of the market-clearing constraint (2.9) and the new constraint (2.11), respectively. It is straightforward to show that $\lambda^{\mathrm{IP}} = \lambda^{\mathrm{MILP}}$ and $\sigma_n^{\mathrm{IP}} = -\pi_n^{\mathrm{MILP}}$, rendering the profits $\pi_n^{\mathrm{IP}} = 0$, $n = 1, 2$. This means that under IP pricing, a supplier who is unprofitable (profitable) under marginal-cost pricing, receives a positive (negative) uplift to end up with a zero profit.

The zero-profit condition implied by the IP scheme is meant to ensure the long-run equilibrium in a market with infinite potential suppliers. In short-run auctions (e.g., daily power markets), where entry cannot occur instantaneously, because the number of suppliers is fixed, O'Neill *et al.* suggested that the zero-profit condition can be removed by ignoring negative uplifts, thus allowing the suppliers to keep their profits, if positive. Under this variant, which we refer to as "IP+," the uplifts and profits of the suppliers are simply given by the following proposition, where we use the notation $(x)^+ \equiv \max(0, x)$.

Proposition 2.2. *Under IP+ pricing,* $\lambda^{\mathrm{IP+}} = \lambda^{\mathrm{IP}} = \lambda^{\mathrm{MILP}}$, $\sigma_n^{\mathrm{IP+}} = (\sigma_n^{\mathrm{IP}})^+ = (-\pi_n^{\mathrm{MILP}})^+$ *and* $\pi_n^{\mathrm{IP+}} = (\pi_n^{\mathrm{MILP}})^+$, $n = 1, 2$.

The proof is straightforward and hence omitted.

Essentially, the only difference between IP and IP+ is in the high-demand case, where, under IP+ pricing, the infra-marginal supplier i is allowed to keep his profit, $b_I k_i - (b_i k_i + f_i)$, if positive.

2.3.2 Modified IP (mIP) Pricing

Bjørndal and Jörnsten [14, 15] noted that the prices generated by the IP scheme can be volatile, and proposed a "modified IP" (mIP) scheme that aims to produce more stable prices. In our two-supplier model, the mIP scheme turns out to be almost exactly the same as the IP+ scheme. Namely, total payments are the same but the mIP scheme generates prices that are piecewise constant and nondecreasing in d. More specifically, the relationship between the mIP and IP+ prices is $\lambda^{\mathrm{mIP}}|_d = \min_{d' \geq d}\{\lambda^{\mathrm{IP+}}|_{d'}\}$.

The mIP scheme is based on the idea of viewing the IP problem as a Benders sub-problem in which the complicating variables (i.e., the variables that are held fixed at some trial values in Benders decomposition to generate an easy to solve convex sub-problem in the remaining variables) are held fixed at their optimal values. If the Benders cut that is generated when solving this sub-problem is a valid inequality (i.e., an inequality which, when added to the relaxed original MILP problem, does not cut off any feasible solution) for some but not all values of d, then the resulting prices are volatile. To reduce this volatility, additional variables must join the complicating variables (for values of d where the Benders cut is not a valid inequality) by adding to the IP problem extra constraints which fix these variables at their optimal values, making the resulting Benders cut a supporting valid inequality. We refer to the resulting problem as the mIP problem.

For our two-supplier model, the commodity price λ^{mIP} and the resulting uplifts σ_n^{mIP} and profits π_n^{mIP} of the committed suppliers under the mIP scheme are given by the following proposition.

Proposition 2.3. *Under mIP pricing:*

(i) *If $d \leq k_1$, then $\lambda^{\mathrm{mIP}} = \min(b_{r(k_1)}, b_2)$, $\sigma_{r(d)}^{\mathrm{mIP}} = f_{r(d)} + (b_{r(d)} - \lambda^{\mathrm{mIP}})d$, and $\pi_{r(d)}^{\mathrm{mIP}} = \pi_{r(d)}^{\mathrm{IP+}} = 0$.*

(ii) *If $d > k_1$, then $\lambda^{\mathrm{mIP}} = \lambda^{\mathrm{IP+}} (= \lambda^{\mathrm{MILP}})$, $\sigma_n^{\mathrm{mIP}} = \sigma_n^{\mathrm{IP+}}$, and $\pi_n^{\mathrm{mIP}} = \pi_n^{\mathrm{IP+}}$, $n = 1, 2$.*

Proof. The proof is found in Appendix A (Section A.2). □

Proposition 2.3 implies that the mIP and IP+ schemes differ only in cases C (when $i = 2$) and B1 of Figure 2.1, when $d \leq k_1$. In these two cases, $\lambda^{\mathrm{mIP}} = \min(b_{r(k_1)}, b_2) = b_i$, whereas $\lambda^{\mathrm{IP+}} = b_{r(d)} = b_I$; in all other cases, $\lambda^{\mathrm{mIP}} = \lambda^{\mathrm{IP+}}$. Even in these two cases, however, under both schemes, the marginal supplier $r(d)$ receives the same total payments which bring him to zero profit. The difference between the IP+ and mIP schemes therefore is merely in the way that these payments are divided into commodity and uplift payments. Namely, under IP+, the marginal supplier I receives a commodity payment $b_I d$ and an uplift payment f_I, whereas under mIP, he receives $b_i d$ and $f_I + (b_I - b_i)d$, respectively. A closer look reveals that cases C (when $i = 2$) and B are the only instances where the minimum total cost is non-convex in d for $d \leq k$. In both cases, the IP+ scheme generates a piecewise constant price which is decreasing in d for part of or all the low-demand region, reflecting the non-convexity. More specifically, in case C (when $i = 2$), $\lambda^{\mathrm{IP+}} = b_I$ for $d \leq k_1$ and $\lambda^{\mathrm{IP+}} = b_i$ for $k_1 < d \leq k = k_2$. Similarly, in case B, $\lambda^{\mathrm{IP+}} = b_I$, for $d \leq k_c$, and $\lambda^{\mathrm{IP+}} = b_i$, for $k_1 < d \leq k_c$. The mIP scheme, on the other hand, generates a constant price $\lambda^{\mathrm{mIP}} = b_i$ that avoids the non-convexity.

2.3.3 Convex-Hull (CH) Pricing

The IP+ and mIP schemes may lead to large uplift requirements, and thus modify the suppliers' incentives by converting the payment scheme toward pay-as-bid. To address this issue, Hogan and Ring [16] proposed the concept of "Minimum Uplift" (MU) pricing which is based on the idea of paying each supplier the smallest uplift that would make him indifferent between:

(i) accepting the optimal solution and receiving the uplift, and

(ii) choosing the best self-scheduling option in the absence of any uplift.

This uplift is equal to the potential extra profit that the supplier would make if he were allowed to self-schedule instead of accepting the optimal solution. For each commodity price, there is an uplift that renders the supplier indifferent. The MU price is the price that minimizes the total uplift payments.

Gribik *et al.* [17] refined the MU pricing concept into the "Convex-Hull" (CH) scheme, which actually generates the minimum uplifts. CH is based on:

(i) approximating the cumulative non-convex cost of the original MILP problem with its convex hull, thus eliminating the integer variables,

(ii) solving the resulting LP, and

(iii) using the dual variables of the LP to price the commodity and the integral activities.

For our two-supplier model, the following proposition lists expressions for the price λ^{CH} and resulting profits π_n^{CH} under CH pricing.

Proposition 2.4. *Under CH pricing:*

(i) *If* $d \leq k_j$, *then* $\lambda^{\text{CH}} = b_j + f_j/k_j$, *and* $\pi_{r'(d)}^{\text{CH}} = 0$.

(ii) *If* $d > k_j$, *then* $\lambda^{\text{CH}} = b_J + f_J/k_J$, $\pi_j^{\text{CH}} = (\lambda^{\text{CH}} - b_j)k_j - f_j$, *and* $\pi_J^{\text{CH}} = 0$.

The proof is straightforward and hence omitted.

Proposition 2.4 implies that if $k_1 = k_j < d \leq k = k_2$ (cases (a) and (b) of Figure 2.2), then uncommitted supplier 1 makes a profit of $[b_2 + f_2/k_2 - (b_1 + f_1/k_1)]k_1$.

2.4 Zero-Sum Uplift Pricing

In this section, we look at two schemes that consider uplifts as internal zero-sum transfers between the suppliers.

2.4.1 Generalized Uplift (GU) Pricing

Motto and Galiana [18] and Galiana *et al.* [19] proposed a "Generalized Uplift" (GU) pricing scheme which is based on the concepts of:

(i) compensating suppliers that earn less under centralized scheduling than under self-scheduling,

(ii) setting the commodity price and the uplifts so that the suppliers would choose to adopt the optimal MILP solution, if they were allowed to self-schedule,

(iii) restricting the uplifts to internal zero-sum transfers between the suppliers, and

(iv) sharing the cost of compensating the total loss of profit between the suppliers and the buyers.

Under GU, supplier n is asked to provide extra multi-part payments, $\Delta b_n q_n + \Delta f_n z_n$, where Δf_n and Δb_n are positive or negative scalars (uplift parameters) that are added to his fixed and marginal costs, respectively. These payments represent internal, zero-sum transfers between the suppliers. The auctioneer solves a modified version of the MILP problem (2.2)–(2.6), in which the objective function is expressed in terms of the modified costs $f_n + \Delta f_n$ and $b_n + \Delta b_n$. Solving the modified MILP problem generates the optimal quantities q_n^{GU}, z_n^{GU}, $n = 1, 2$, and price λ^{GU}, which is equal to the modified marginal cost.

Motto and Galiana showed that there exist uplift parameters Δf_n and Δb_n, $n = 1, 2$, such that the resulting modified MILP problem:

(i) is strongly dualizable (i.e., has no duality gap) through decomposition by each supplier,

(ii) produces the same optimal solution as the original MILP problem (2.2)–(2.6), and

(iii) produces an optimal price λ^{GU} which guarantees that each supplier would choose to adopt the optimal solution if he were allowed to self-schedule.

To find parameters Δf_n and Δb_n, $n = 1, 2$, that exhibit the above properties, they showed that it suffices to solve the following mathematical programming problem:

$$\underset{\lambda, \Delta b_n, \Delta f_n, n=1,2}{\text{Minimize}} \; L_{\mathrm{GU}} = \sum_{n=1,2} (\Delta b_n q_n^{\mathrm{MILP}})^2 + (\Delta f_n z_n^{\mathrm{MILP}})^2, \tag{2.14}$$

subject to:

$$\lambda \geq b_n + \Delta b_n, \qquad \text{if } \; q_n^{\mathrm{MILP}} = k_n, \; n = 1, 2, \tag{2.15}$$

$$\lambda = b_n + \Delta b_n, \qquad \text{if } \; 0 < q_n^{\mathrm{MILP}} < k_n, \; n = 1, 2, \tag{2.16}$$

$$\lambda \leq b_n + \Delta b_n, \qquad \text{if } \; q_n^{\mathrm{MILP}} = 0, \; n = 1, 2, \tag{2.17}$$

$$(1 - z_n^{\mathrm{MILP}}) \Delta f_n = 0, \qquad n = 1, 2, \tag{2.18}$$

$$[\lambda - (b_n + \Delta b_n)] q_n^{\mathrm{MILP}} - (f_n + \Delta f_n) z_n^{\mathrm{MILP}} \geq 0, \qquad n = 1, 2, \tag{2.19}$$

$$\sum_{n=1,2} (\Delta b_n q_n^{\text{MILP}} + \Delta f_n z_n^{\text{MILP}}) = 0. \tag{2.20}$$

Constraints (2.15)–(2.17) ensure that the price is appropriately defined for the modified MILP problem. Constraints (2.18)–(2.20) ensure the following:

(i) if supplier n is not committed, then $\Delta f_n = 0$,

(ii) the suppliers incur no losses, and

(iii) the extra GU payments are internal zero-sum transfers between the suppliers, respectively.

To find unique values λ, Δb_n and Δf_n, $n = 1, 2$, Motto and Galiana [18] suggested minimizing the norm of the payment components; quadratic function (2.14) is the norm definition that they used in their paradigm.

For our two-supplier model, we can obtain analytical expressions for the price and uplift parameters that solve the quadratic programming problem (2.14)–(2.20). These expressions, along with those for the resulting profits of the committed suppliers, denoted by π_n^{GU}, are given by the following proposition, where the extra GU payments (uplifts) received by each supplier, σ_n^{GU}, are defined as

$$\sigma_n^{\text{GU}} = -(\Delta b_n q_n^{\text{MILP}} + \Delta f_n z_n^{\text{MILP}}), \qquad n = 1, 2. \tag{2.21}$$

These uplifts sum to zero, by (2.20).

Proposition 2.5. *Under GU pricing:*

(i) *If $d \leq k$, then $\lambda^{\text{GU}} = b_{r'(d)} + f_{r'(d)}/d$ and $\sigma_{r'(d)}^{\text{GU}} = 0$. The resulting profit is $\pi_{r'(d)}^{\text{GU}} = 0$.*

(ii) *If $d > k$, then:*

(a) *$\lambda^{\text{GU}} = b_I + \Delta b_I^{\text{GU}}$, where $\Delta b_I^{\text{GU}} = \max(\Delta b_I^{(1)}, \Delta b_I^{(2)}, \Delta b_I^{(3)})$, with $\Delta b_I^{(1)} = f_I/[3(d - k_i)]$, $\Delta b_I^{(2)} = (f_i + b_i k_i - b_I k_i + f_I)/d$, and $\Delta b_I^{(3)} = (f_i + b_i k_i - b_I k_i)(2d + k_i)/(4d^2 - 4k_i d + 3k_i^2)$.*

(b) *The uplift parameters* Δf_I^{GU} *are:* $-f_I$, $-f_I$, *and* $\Delta b_I^{(3)}(d-k_i)(2d-3k_i)/(2d+k_i)$, *for* $\Delta b_I^{\mathrm{GU}} = \Delta b_I^{(1)}$, $\Delta b_I^{(2)}$, *and* $\Delta b_I^{(3)}$, *respectively.*

(c) *The profits are: 1)* $\pi_i^{\mathrm{GU}} > 0$, $\pi_I^{\mathrm{GU}} = 0$, *2)* $\pi_i^{\mathrm{GU}} = 0$, $\pi_I^{\mathrm{GU}} = 0$, *and 3)* $\pi_i^{\mathrm{GU}} = 0$, $\pi_I^{\mathrm{GU}} > 0$, *for* $\Delta b_I^{\mathrm{GU}} = \Delta b_I^{(1)}, \Delta b_I^{(2)}$, *and* $\Delta b_I^{(3)}$, *respectively.*

Proof. The proof is found in Appendix A (Section A.3), along with the exact conditions under which Δb_I^{GU} assumes each of the three possible values given in (ii)(a), and the exact expressions for the profits in the high-demand case. □

Proposition 2.5 states that under GU pricing, in the low-demand case, the suppliers are paid the smallest average cost at d. In the high-demand case, the price is the marginal cost b_I uplifted by the maximum of three quantities that depend on the problem parameters (in the proof, it is shown that $\Delta b_I^{\mathrm{GU}} \geq 0$). Note that if $k = k_i$, the denominator of $\Delta b_I^{(1)}$, $d - k_i$, which is equal to q_I^{MILP}, goes to zero as $d \to k^+$ (see Corollary 2.1). In this case, $\lim_{d\to k^+} \Delta b_I^{\mathrm{GU}} = \lim_{d\to k^+} \Delta b_I^{(1)} = \infty$, implying that $\lim_{d\to k^+} \lambda^{\mathrm{GU}} = \infty$ and $\lim_{d\to k^+} \pi_i^{\mathrm{GU}} = \infty$. This is an adverse property of the GU scheme which stems from the fact that the objective of GU is restricted to minimizing the norm of the uplift components expressed by (2.14). To be more specific, nothing prohibits Δb_n from becoming excessively large when q_n^{MILP} is infinitesimally small (as is the case with q_I^{MILP} when $k = k_i$ and $d \to k^+$), as long as their product, appearing in (2.14), remains small. The problem is that if $\Delta b_n \to \infty$ (with n being the marginal supplier), then $\lambda^{\mathrm{GU}} \to \infty$ too, by (2.16).

2.4.2 Minimum Zero-Sum Uplift (MZU) Pricing: A Proposed Variant of the IP Scheme

The GU scheme leads to complicated prices and uplifts, even for the simple two-supplier model. This is partly due to the structure of objective function (2.14) which separates uplifts into two components and considers a quadratic form for each component. We propose an alternative scheme, called "Minimum Zero-sum Uplift" (MZU), which focuses on the total uplifts that each supplier receives/pays rather than on the individual components.

MZU is based on the idea of maintaining the optimal MILP solution and increasing the price beyond λ^{MILP}, so that all suppliers who would incur losses under marginal-cost pricing eventually break even; at the same time, profitable suppliers keep their profits under marginal-cost pricing but are not allowed to gain any more profits. This can be achieved if the extra commodity payments that they receive as a result of the price increase are transferred as side-payments to the unprofitable suppliers, on top of the extra commodity payments that the latter suppliers also receive as a result of the price increase. The smallest price at which all unprofitable suppliers break even, denoted by λ^{MZU}, is such that the total additional payments that they receive, namely $(\lambda^{\mathrm{MZU}} - \lambda^{\mathrm{MILP}})d$, are just enough (hence the term "minimum zero-sum") to cover their losses.

For our two-supplier model, the commodity price λ^{MZU} and the resulting uplifts σ_n^{MZU} and profits π_n^{MZU}, of each committed supplier, are given by the following proposition.

Proposition 2.6. *Under MZU pricing:*

(i) *If $d \leq k$, then $\lambda^{\mathrm{MZU}} = b_{r'(d)} + f_{r'(d)}/d$ and $\sigma_{r'(d)}^{\mathrm{MZU}} = 0$.*

(ii) *If $d > k$, then $\lambda^{\mathrm{MZU}} = b_I + f_I/d + (b_i + f_i/k_i - b_I)^+ k_i/d$, $\sigma_I^{\mathrm{MZU}} = f_I k_i/d - (b_i k_i + f_i - b_I k_i)^+ (d - k_i)/d$, and $\sigma_i^{\mathrm{MZU}} = -\sigma_I^{\mathrm{MZU}}$.*

(iii) *In both the low- and high-demand cases, the resulting profits are $\pi_n^{\mathrm{MZU}} = \pi_n^{\mathrm{IP+}}$, $n = 1, 2$.*

The proof is straightforward and hence omitted.

Proposition 2.6 implies that in the low-demand case, $\lambda^{\mathrm{MZU}} = \lambda^{\mathrm{GU}}$. In the high-demand case, λ^{MZU} is equal to the average cost of supplier I at level d, if supplier i is profitable at the marginal cost b_I; otherwise, it is higher than this value. Proposition 2.6 also states that the suppliers have the same profits under MZU and IP+; therefore, MZU can be considered as a variant of IP+. The difference between the two schemes is in the way that total payments are divided into commodity and uplift components. Specifically, under IP+, suppliers are paid the marginal cost for the commodity and external uplifts $\sigma_i^{\mathrm{IP+}} = (b_i k_i + f_i - b_I k_i)^+$ and $\sigma_I^{\mathrm{IP+}} = f_I$. Under MZU, suppliers are paid a higher price than the marginal cost and receive lower zero-sum positive/negative uplifts. Essentially, the IP+ price expresses the cost of producing an additional unit of the commodity, whereas the MZU price represents the average cost of buying an additional unit, taking into account the fixed costs; it therefore provides a more accurate price signal to buyers.

2.5 Revenue-Adequate Pricing

Revenue-adequate pricing refers to schemes that generate high-enough prices to ensure that the suppliers cover their costs, without the need for additional uplifts.

2.5.1 Average-Cost (AC) Pricing

The simplest revenue-adequate scheme is "Average Cost" (AC) pricing. AC seeks the smallest price which guarantees that no supplier incurs losses under the optimal allocation. For our two-supplier model, this is stated as follows:

$$\underset{\lambda}{\text{Minimize}}\ \lambda, \quad \text{subject to:}\ \ \lambda q_n^{\text{MILP}} \geq b_n q_n^{\text{MILP}} + f_n z_n^{\text{MILP}}, \qquad n = 1, 2. \tag{2.22}$$

Clearly, the solution of the above LP is the maximum average cost of the committed suppliers. Specifically, the AC price, denoted by λ^{AC}, and the resulting profits of the committed suppliers, denoted by π_n^{AC}, are given by the following proposition:

Proposition 2.7. *Under AC pricing:*

(i) *If $d \leq k$, then $\lambda^{\text{AC}} = b_{r'(d)} + f_{r'(d)}/d$ and $\pi_{r'(d)}^{\text{AC}} = 0$.*

(ii) *If $d > k$, then $\lambda^{\text{AC}} = \max[b_i + f_i/k_i, b_I + f_I/(d-k_i)]$, $\pi_i^{\text{AC}} = [b_I k_i + f_I k_i/(d - k_i) - (b_i k_i + f_i)]^+$ and $\pi_I^{\text{AC}} = \{b_i k_i + f_i - [b_I k_i + f_I k_i/(d-k_i)]\}^+(d-k_i)/k_i$.*

The proof is straightforward and hence omitted.

In words, in the low-demand case, the committed supplier $r'(d)$ is paid his average cost which brings him to zero profit. In the high-demand case, the supplier with the highest average cost sets the price but makes no profit; the other supplier makes a profit equal to the difference of the total costs. Note that if $k = k_i$, then $q_I^{\mathrm{MILP}} = d - k_i \to 0$ as $d \to k^+$ (see Corollary 2.1). In this case, $\lim_{d\to k^+} \lambda^{\mathrm{AC}} = \lim_{d\to k^+} \pi_i^{\mathrm{AC}} = \infty$, indicating that AC has the same adverse property as GU. Namely, if the marginal supplier's quantity is extremely small, an extremely large price is required to cover his losses.

Van Vyve [20] proposed a zero-sum uplift scheme that aims to minimize the maximum contribution to the financing of the uplifts, in a model where both suppliers and buyers place bids. Notably, if the demand is inelastic, that scheme is equivalent to standard AC pricing with no uplifts, as is the case in our model.

Recently, two new pricing schemes that generate revenue-adequate prices appeared in the literature. In the remainder of this section, we analyze both schemes for our two-supplier model.

2.5.2 Semi-Lagrangean Relaxation (SLR) Pricing

Araoz and Jörnsten [21] proposed a "Semi-Lagrangean Relaxation" (SLR) approach to compute a uniform price that produces the same solution as the original MILP problem while ensuring that no supplier incurs losses. SLR was introduced in [84] and the closely related work by [85]. It is based on:

(i) formulating an SLR of the original MILP problem by semi-relaxing the linear equality constraints of interest using standard Lagrange multipliers, but keeping weaker inequality constraints in their place, and

(ii) solving the dual problem.

In the context of our two-supplier model, the SLR of the MILP problem (2.2)–(2.6) is as follows:

$$\underset{q_n, z_n, n=1,2}{\text{Minimize}} \; L_{\mathrm{SLR}} = \sum_{n=1,2} (b_n q_n + f_n z_n) + \lambda \left(d - \sum_{n=1,2} q_n \right), \tag{2.23}$$

subject to:

$$\sum_{n=1,2} q_n \leq d, \tag{2.24}$$

$$q_n \leq k_n z_n, \qquad n = 1, 2, \tag{2.25}$$

$$q_n \geq 0, \qquad n = 1, 2, \tag{2.26}$$

$$z_n \in \{0, 1\}, \qquad n = 1, 2. \tag{2.27}$$

Note that the market-clearing equality constraint (2.3) of the original MILP has been relaxed into inequality (2.24). At the same time, a Lagrange multiplier λ has been introduced in objective function (2.23) to penalize the amount of the demand not served. Letting $L^*_{\mathrm{SLR}}(\lambda)$ denote the minimum value of objective function (2.23) for a given λ, the SLR approach consists of solving the dual problem, maximize$_\lambda$ $L^*_{\mathrm{SLR}}(\lambda)$.

Beltran *et al.* [84] showed that the SLR dual function ($L^*_{\mathrm{SLR}}(\lambda)$ in our model) is concave and non-differentiable in λ. They also showed that the SLR approach has no duality gap, i.e., produces the same optimal value as the MILP problem. To see this in our two-supplier model, note that an excessively large value of λ would drive $\sum q_n$ to exceed d, in order to minimize the objective function (2.23). As constraint (2.24) prohibits this, $\sum q_n$ would be set equal to d, thus meeting the market-clearing equality (2.3) in the original MILP problem and forcing the term $\lambda(d - \sum q_n)$ in (2.23) to zero. The question then is, what is the smallest uniform price λ that maximizes $L^*_{\mathrm{SLR}}(\lambda)$ and, if used in the relaxed problem (2.23)–(2.27), produces the optimal solution of the MILP problem (2.2)–(2.6), while guaranteeing that no supplier incurs losses. This problem can be stated as $\lambda^{\mathrm{SLR}} = \arg\min_\lambda \{\max L^*_{\mathrm{SLR}}(\lambda)\}$.

To find λ^{SLR}, Araoz and Jörnsten suggested an iterative algorithm that increases λ in each iteration and solves the relaxed problem (2.23)–(2.27) until objective function (2.23) reaches the optimal value of the objective function of the MILP problem. For our two-supplier model, we can obtain analytical expressions for λ^{SLR} and the resulting profits π_n^{SLR}. These expressions are given by the following proposition.

Proposition 2.8. *Under SLR pricing:*

(i) *If $d \leq k_1$, then $\lambda^{\mathrm{SLR}} = b_{r(d)} + f_{r(d)}/d$. The resulting profit is $\pi^{\mathrm{SLR}}_{r(d)} = 0$.*

(ii) *If $k_1 < d \leq k$, then $\lambda^{\mathrm{SLR}} = b_2 + f_2/d + [b_2 + f_2/d - (b_1 + f_1/k_1)]^+ k_1/(d - k_1)$. The resulting profit is $\pi_2^{\mathrm{SLR}} = [b_2 k_1 + f_2 k_1/d - (b_1 k_1 + f_1)]^+ d/(d - k_1)$.*

(iii) *If* $d > k$*, then* $\lambda^{\text{SLR}} = b_I + f_I/(d-k_i) + \{b_i + f_i/k_i - [b_I + (f_I/k_i)(d-k_I)^+/(d-k_i)]\}^+ k_i/(d-k_I)$*. The resulting profits are* $\pi_i = b_I k_i + f_I k_i/(d-k_i) - (b_i k_i + f_i) + \{b_i k_i + f_i - [b_I k_i + f_I(d-k_I)^+/(d-k_i)]\}^+ k_i/(d-k_I)$ *and* $\pi_I = \{b_i k_i + f_i - [b_I k_i + f_I(d-k_I)^+/(d-k_i)]\}^+ (d-k_i)/(d-k_I)$.

Proof. The proof is found in Appendix A (Section A.4). □

Proposition 2.8 states that in the lowest-demand case (where $r'(d) = r(d)$), the SLR price is equal to the marginal price $b_{r'(d)}$ plus an increment of $f_{r'(d)}/d$ which is necessary to bring supplier $r'(d)$ to zero losses. If $k_1 < d \leq k$, in which case $r'(d) = 2$, it may happen that at this increased price the optimal SLR solution is to dispatch supplier 1 at k_1 and not commit supplier 2. This will occur if the difference in SLR cost yielded by the MILP solution and this solution is positive. In this case, an extra price increment equal to this difference over $d-k_1$ is needed to cover the difference and pay for the extra $d - k_1$ units; supplier 2 will reap this difference and make a profit. In fact, this is the only situation within all pricing schemes where the committed supplier can make a profit in the low-demand case. Note that if $b_2 k_1 + f_2 > b_1 k_1 + f_1$, then $\lim_{d \to k_1^+} \lambda^{\text{SLR}} = \lim_{d \to k_1^+} \pi_2^{\text{SLR}} = \infty$, indicating that SLR has the same adverse property as GU and AC.

In the high-demand case, the price and profits have a similar interpretation. In this case too, if $k = k_i$, then $d \to k^+$ implies $q_I^{\text{MILP}} = d - k_i \to 0$ (see Corollary 2.1); hence, $\lim_{d \to k^+} \lambda^{\text{SLR}} = \lim_{d \to k^+} \pi_i^{\text{SLR}} = \infty$, indicating again that SLR has the same adverse property as GU and AC.

2.5.3 Primal-Dual (PD) Pricing

Recently, Ruiz *et al.* [22] proposed a so-called "Primal-Dual" (PD) approach for deriving efficient uniform revenue-adequate prices. This approach consists of:

(i) relaxing the integrality constraints of the MILP problem so that it becomes a (primal) LP,

(ii) deriving the dual LP associated with the primal LP,

(iii) formulating a new LP problem that seeks to minimize the duality gap of the primal and dual LPs, subject to both primal and dual constraints, and

(iv) adding the integrality constraints back to the problem as well as additional non-linear constraints to ensure that no participant incurs losses.

In the context of our two-supplier model, the resulting Mixed Integer Non-Linear Programming (MINLP) problem — referred to as "PD" — can be written as

$$\underset{\lambda, q_n, z_n, \mu_n, \nu_n, n=1,2}{\text{Minimize}} \quad L_{\text{PD}} = \sum_{n=1,2} (b_n q_n + f_n z_n) - \lambda d + \sum_{n=1,2} \nu_n, \tag{2.28}$$

subject to:

$$\sum_{n=1,2} q_n = d, \tag{2.29}$$

$$q_n \leq k_n z_n, \qquad n = 1, 2, \tag{2.30}$$

$$\lambda - \mu_n \leq b_n, \qquad n = 1, 2, \tag{2.31}$$

$$\nu_n \geq k_n \mu_n - f_n, \qquad n = 1, 2, \tag{2.32}$$

$$\lambda q_n \geq b_n q_n + f_n z_n, \qquad n = 1, 2, \tag{2.33}$$

$$q_n, \mu_n, \nu_n \geq 0, \qquad n = 1, 2, \tag{2.34}$$

$$z_n \in \{0, 1\}, \qquad n = 1, 2. \tag{2.35}$$

Constraints (2.29)–(2.30) are the same as (2.3)–(2.4) in the original MILP, and (2.31)–(2.32) are the constraints of the relaxed dual LP. Decision variables λ and μ_n, $n = 1, 2$, are the dual variables of constraints (2.3)–(2.4) in the relaxed primal LP, and ν_n, $n = 1, 2$, is the dual variable of constraint $z_n \leq 1$, $n = 1, 2$, which replaces (2.6) in the relaxed primal LP. Finally, (2.33) ensures that no supplier incurs losses. Note that the first summation in (2.28) is identical to objective function (2.2) in the original MILP; the remaining terms originate from the objective function of the relaxed dual LP maximization problem. Solving the PD problem (2.28)–(2.35) yields the optimal quantities z_n^{PD} and q_n^{PD}, $n = 1, 2$, and price λ^{PD}. The following proposition gives analytical expressions for these quantities, and the resulting profits π_n^{PD}.

Proposition 2.9. *Under PD pricing, there exists $k^{\mathrm{PD}}: \ k \leq k^{\mathrm{PD}} \leq k_2$, such that:*

(i) *If $d \leq k^{\mathrm{PD}}$, then $q^{\mathrm{PD}}_{r'(d)} = d$ and $\lambda^{\mathrm{PD}} = b_{r'(d)} + f_{r'(d)}/d$. The resulting profit is $\pi^{\mathrm{PD}}_{r'(d)} = 0$.*

(ii) *If $d > k^{\mathrm{PD}}$, then:*

(a) *$q_i^{\mathrm{PD}} = \min[\max(q_i', q_i'', d - k_I), k_i]$, $q_I^{\mathrm{PD}} = d - q_i^{\mathrm{PD}}$, where q_i' is the point of intersection of the average cost functions $b_i + f_i/q_i$ and $b_I + f_I/(d - q_i)$, and q_i'' is the minimizer of $(b_i - b_I)q_i + (k_i + k_I - d)[b_I + f_I/(d - q_i)]$.*

(b) *$\lambda^{\mathrm{PD}} = \max(\lambda_i, \lambda_I)$, where $\lambda_i = b_i + f_i/q_i^{\mathrm{PD}}$ and $\lambda_I = b_I + f_I/(d - q_i^{\mathrm{PD}})$.*

(c) *The profits are: 1) $\pi_i^{\mathrm{PD}} = 0$, $\pi_I^{\mathrm{PD}} > 0$, if $q_i^{\mathrm{PD}} = k_i$ and $\lambda^{\mathrm{PD}} = \lambda_i$, 2) $\pi_i^{\mathrm{PD}} = \pi_I^{\mathrm{PD}} = 0$, if $q_i^{\mathrm{PD}} = q_i'$ and $\lambda^{\mathrm{PD}} = \lambda_i = \lambda_I$, 3) $\pi_i^{\mathrm{PD}} > 0$, $\pi_I^{\mathrm{PD}} = 0$, if $q_i^{\mathrm{PD}} \in \{q_i'', d - k_I, k_i\}$ and $\lambda^{\mathrm{PD}} = \lambda_I$.*

Proof. The proof is found in Appendix A (Section A.5), along with expressions for k^{PD}, q_i', q_i'', conditions for the different possible values of q_i^{PD}, expressions for the respective profits, and representative graphs of the price and profits. □

Proposition 2.9 implies that under PD pricing, the demand space is divided into a low- and a high-demand region, as far as the optimal allocation is concerned. The border between these regions is denoted by k^{PD}, where $k \leq k^{\mathrm{PD}} \leq k_2$. This means that if $k < d \leq k^{\mathrm{PD}}$, the optimal PD allocation differs from the optimal (cost efficient) MILP allocation. Even if $d > k^{\mathrm{PD}}$, however, the optimal PD allocation may still deviate from the optimal MILP allocation. Specifically, the proof of Proposition 2.9 shows that when $d > k^{\mathrm{PD}}$, the effective goal of PD is to minimize the total marginal cost plus the foregone revenues of the unused capacity. The decision variables to achieve this goal are λ and q_n, $n = 1, 2$. The optimal price λ^{PD}, being the smallest revenue-adequate price, is the maximum average cost of the suppliers, and hence is a function of the quantities q_n. Therefore, (2.28) reduces to a function of q_n, $n = 1, 2$, only, namely, $\sum_{n=1,2} b_n q_n + \max_{n=1,2}\{b_n + f_n/q_n\}(\sum_{n=1,2} k_n - d)$. Effectively, PD seeks to reallocate the demand in order to minimize this function. Unlike all other schemes, PD trades cost efficiency for price efficiency, as long as this tradeoff reduces the value of the objective function. The following proposition provides the conditions under which the PD allocation is cost efficient.

Proposition 2.10. *The necessary and sufficient conditions under which the PD allocation is cost efficient are:*

(i) $d \leq k$, *or*

(ii) $k < d \leq k_2$ *and* $b_2+f_2/(d-k_1) \geq b_1+\max\{f_1/k_1+[f_2/(d-k_1)](k_1+k_2)/d, [f_2/(d-k_1)]k_2/(d-k_1)\}$, *or*

(iii) $d > k_2$ *and* (a) $b_I + f_I/(d-k_i) \leq b_i + f_i/k_i$ or (b) $b_I - b_i \geq [(\sum_{n=1,2} k_n - d)/(d-k_i)]f_I/(d-k_i)$.

Proof. The proof is found in Appendix A (Section A.6). □

To understand the logic behind the above conditions, consider one of them, say (iii). This condition implies that in the highest-demand case, where both suppliers are needed to cover the demand, if the average cost of supplier i dispatched at full capacity is greater than the respective cost of supplier I dispatched at the residual demand (condition (iii)(a)), then there is no incentive for a less efficient solution, as this would increase both the price and cost. If the opposite is true, then there is an incentive to reallocate some of supplier i's quantity to supplier I, as this would lower the price. However, this reallocation incurs a cost increase of $b_I - b_i$ per unit, which under condition (iii)(b), outweighs the benefit from the price decrease.

Finally, it can be shown that the PD scheme produces prices and profits that are always bounded, as a result of its ability to deviate from the optimal allocation, unlike GU, AC, and SLR, which may produce unbounded prices and profits, as was seen earlier.

2.6 Comparison of Pricing Schemes

In this section, we use the results from the preceding sections to compare the price and profits generated by the considered schemes. We omit the IP scheme because

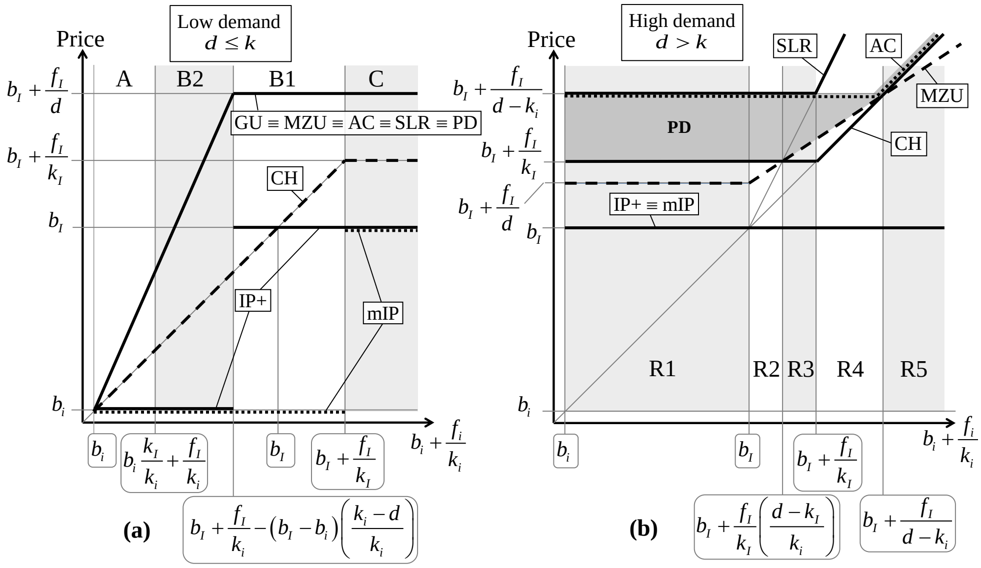

Figure 2.3: Price versus $b_i + f_i/k_i$ for cases: (a) low demand; (b) high demand.

it results in zero profits for both suppliers, but we include its extensions/variants, namely IP+, mIP, and MZU.

Figure 2.3 shows graphs of price versus $b_i + f_i/k_i$ for different schemes. All expressions are given in terms of the asymmetric capacities, but the graphs are first drawn assuming symmetric capacities, i.e., assuming $k_i = k_I$. The full set of graphs for all asymmetric-capacity cases follows, in Figures 2.4 and 2.5.

As can be seen from Figure 2.3(a), in the low-demand case, $b_i + f_i/k_i$ may belong to one of four regions corresponding to cases A, B2, B1, and C in Figure 2.1. The highest price is generated by GU, MZU, AC, SLR, and PD and equals the smallest average cost at d; it is therefore decreasing in d. The lowest price is generated by mIP and is piecewise constant and nondecreasing in d (as can be deduced from Figure 2.4). The IP+ and CH prices are between the highest and lowest prices, and their relative ordering depends on the region. The CH price is the smallest (largest) average cost at full capacity if $d \le k_j$ ($k_j < d \le k$); hence, it is piecewise constant and nondecreasing in d. The IP+ price is the marginal cost of the supplier with the smallest average cost at d and is piecewise constant and possibly decreasing in d. Finally, recall that in the low-demand case, the committed supplier has zero profit under all schemes, except SLR when $k_1 < d \leq k = k_2$ and $b_2 + f_2/d > b_1 + f_1/k_1$ (see Proposition 2.8(ii)). Also recall that CH is the only scheme where the uncommitted supplier has positive profit when $k_1 = k_j < d \leq k = k_2$.

Figure 2.3(b) shows price graphs for the high-demand case, for all schemes except GU. GU is examined separately, because its increased complexity gives rise to two different graphs, depending on the value of d. As can be seen, $b_i + f_i/k_i$ may belong to one of five regions, denoted by R1–R5, where R1–R3 correspond to cases A and B of Figure 2.1, and R4 and R5 correspond to case C. The darkly shaded area indicates the region that contains λ^{PD} and is defined by the following proposition.

Proposition 2.11. *If $d > k$, then λ^{PD} and π_n^{PD}, $n = 1, 2$, are bounded as follows:*

(i) *If $b_i + f_i/k_i \geq b_I + f_I/(d - k_i)$, then $\lambda^{\text{PD}} = \lambda^{\text{CH}} = \lambda^{\text{AC}}$, $\pi_i^{\text{PD}} = 0$, and $\pi_I^{\text{PD}} = \pi_I^{\text{AC}}$.*

(ii) *If $b_i + f_i/k_i < b_I + f_I/(d - k_i)$, then $\max(\lambda^{\text{CH}}, \lambda^{\text{MZU}}) \leq \lambda^{\text{PD}} \leq \lambda^{\text{AC}}$, $\pi_i^{\text{PD}} \leq \pi_i^{\text{AC}}$, and $\pi_I^{\text{PD}} = 0$.*

Proof. The proof is found in Appendix A (Section A.7). Indicative price graphs for the PD scheme are shown in Figure A.2(a) in Appendix A. □

Figure 2.3(b) shows that the highest price is generated by SLR, followed by AC, followed by PD. The lowest price is generated by mIP and IP+. The CH and MZU prices are in between, and their relative ordering depends on the region.

Figure 2.4 shows the price graphs for the asymmetric capacity (cases (a), (b), and (c) of Figure 2.2) low-demand case.

We distinguish between cases $k = k_1$ (graphs (i)–(v)) and $k = k_2$ (graph (vi)); the latter is valid only for cases (a) and (b). Note that for $d \leq k_1$, the price is shown versus $b_i + f_i/k_i$, whereas for $k_1 < d \leq k_2 = k$, it is shown versus $b_2 + f_2/k_2$. Case (a) for $d \leq k_1$ has two sub-cases, denoted by (1) and (2) (graphs (i) and (ii)); cases (b) and (c) have three sub-cases, denoted by (1)–(3) (graphs (iii), (iv), and (v)). The difference between these sub-cases is the relative position of point $b_I + f_I/k_i - (b_I - b_i)(k_i - d)/k_i$

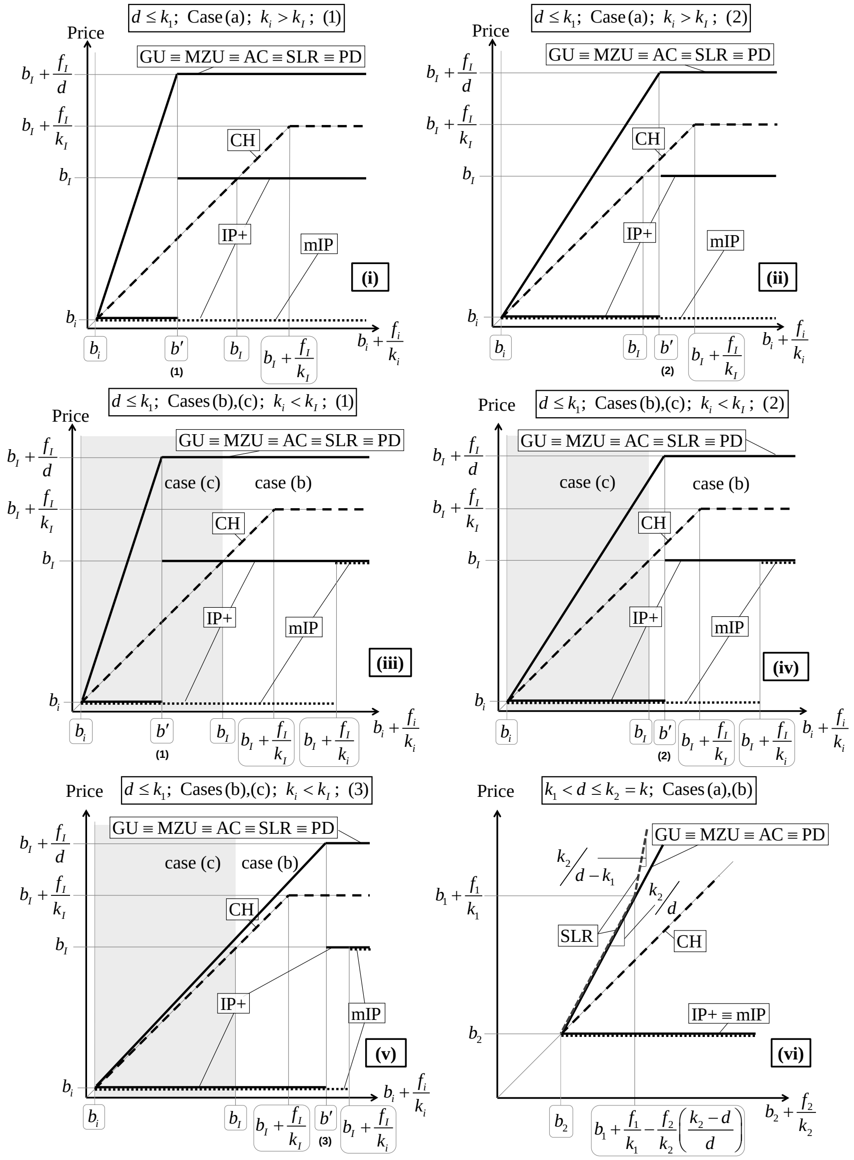

Figure 2.4: Price graphs for cases (a), (b), and (c) of Figure 2.2, for low demand $[b' = b_I + f_I/k_i - (b_I - b_i)(k_i - d)/k_i]$.

which is denoted by b'. Note that case (c) is defined for $b_i + f_i/k_i < b_I$ and case (b) for $b_i + f_i/k_i \geq b_I$.

For $d \leq k_1$, the relative ordering of the prices are practically the same as in the symmetric-capacity case shown in Figure 2.3(a). The highest price is generated by GU, MZU, AC, SLR and PD; the lowest by mIP; IP+ and CH are in between. Note that CH is lower than IP+ only in a region shown in sub-case (1) (graphs (i) and (iii)). Sub-cases (2) and (3) differ with respect to the region where IP+ equals mIP. For $k_1 < d \leq k_2 = k$, the difference with respect to the symmetric case is that the SLR price may be strictly higher than the GU, MZU, AC and PD prices (graph (vi)). IP+ equals mIP, and CH is higher than IP+.

Figure 2.5 shows the price graphs for the asymmetric capacity high-demand case. It can be seen that the highest price is generated by SLR, followed by AC; mIP and IP+ generate the lowest prices, and CH and MZU are in between. PD is further discussed in Appendix A (Figure A.2).

Figure 2.6 shows graphs of the suppliers' profits versus $b_i k_i + f_i$ in the high-demand case, again assuming symmetric capacities. Graphs for all asymmetric-capacity cases follow next.

From Figure 2.6(a), the highest profit of supplier i is generated by SLR, followed by AC, followed by CH, followed by mIP, IP+, and MZU. The darkly shaded area indicates the region that contains π_i^{PD}, defined by Proposition 2.11. Indicative profit graphs for the PD scheme are shown in Appendix A (Figure A.2(b)). From Figure 2.6(b), the profit of supplier I generated by CH and SLR is greater than that generated by AC and PD. Figure 2.7 shows cases where the CH and SLR profits diverge. The profit of supplier I generated by mIP, IP+, and MZU is always zero.

Figure 2.7 shows the profits versus $b_i k_i + f_i$ graphs, for high demand, for cases (a), (b), and (c) of Figure 2.2.

The remarks for supplier i are similar to those in the symmetric-capacity case. For supplier I the profit of CH and SLR is greater than that generated by AC and PD; the profits generated by mIP, IP+, and MZU are always zero. The main difference with the symmetric-capacity case is that SLR generates higher profits than CH in case (a) and vice versa in cases (b) and (c).

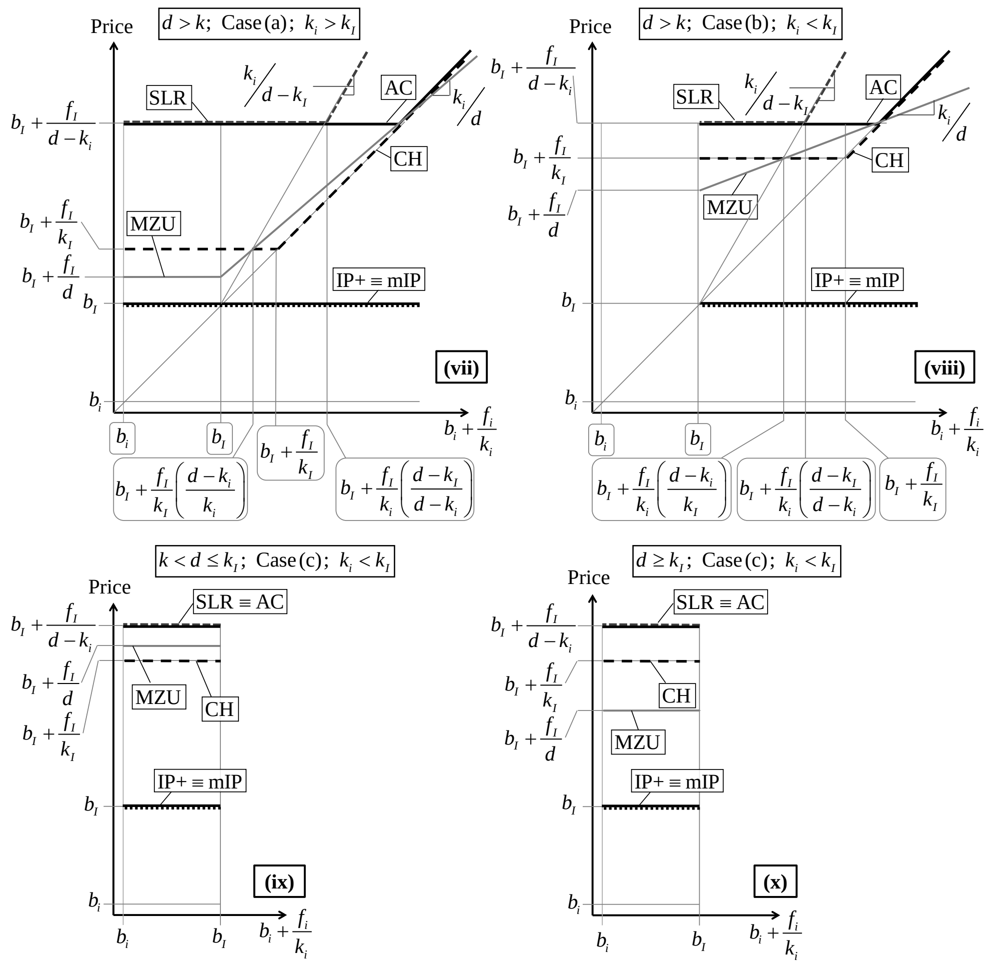

Figure 2.5: Price graphs for cases (a), (b), and (c) of Figure 2.2, for high demand.

The following proposition provides bounds on the GU price and profits with respect to other schemes.

Proposition 2.12. *If $d > k$, then λ^{GU} and π_n^{GU}, $n = 1, 2$, are bounded as follows:*

(i) *If $3k_i/2 < d \leq k_i + k_I$, then $\lambda^{\mathrm{IP+}} = \lambda^{\mathrm{mIP}} < \lambda^{\mathrm{GU}} \leq \lambda^{\mathrm{MZU}}$ and $\pi_i^{\mathrm{GU}} \leq \pi_i^{\mathrm{IP+}} =$*

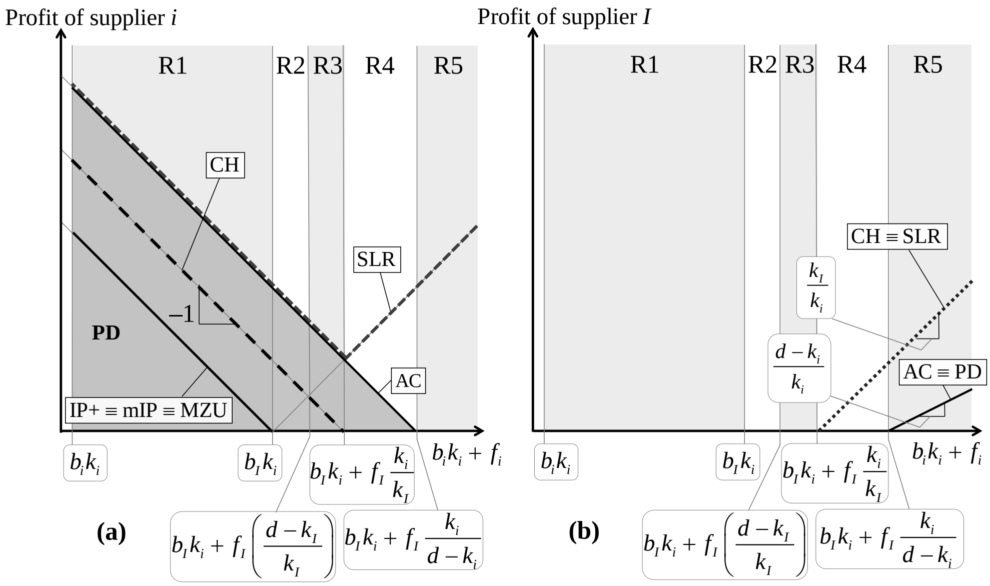

Figure 2.6: Profits versus $b_ik_i + f_i$ for the high-demand case for suppliers: (a) i; (b) I.

$$\pi_i^{\mathrm{mIP}} = \pi_i^{\mathrm{MZU}}.$$

(ii) *If* $d = 3k_i/2$*, then* $\lambda^{\mathrm{GU}} = \lambda^{\mathrm{MZU}}$ *and* $\pi_i^{\mathrm{GU}} = \pi_i^{\mathrm{MZU}} = \pi_i^{\mathrm{IP+}} = \pi_i^{\mathrm{mIP}}$.

(iii) *If* $k < d < 3k_i/2$*, then* $\lambda^{\mathrm{MZU}} \leq \lambda^{\mathrm{GU}} < \lambda^{\mathrm{AC}}$, $\pi_i^{\mathrm{mIP}} = \pi_i^{\mathrm{IP+}} = \pi_i^{\mathrm{MZU}} < \pi_i^{\mathrm{GU}} < \pi_i^{\mathrm{AC}}$ *and* $\pi_I^{\mathrm{GU}} < \pi_I^{\mathrm{AC}}$.

Proof. The proof is found in Appendix A (Section A.8), along with graphs and tighter, more detailed bounds on the GU price and profits. □

Note that when $d > 3k_i/2$, GU generates a lower profit for supplier i than does IP+ even though the GU price is higher than the IP+ price. This is because under IP+, supplier i is allowed to keep all his profit, whereas under GU, he transfers part of his profit to I.

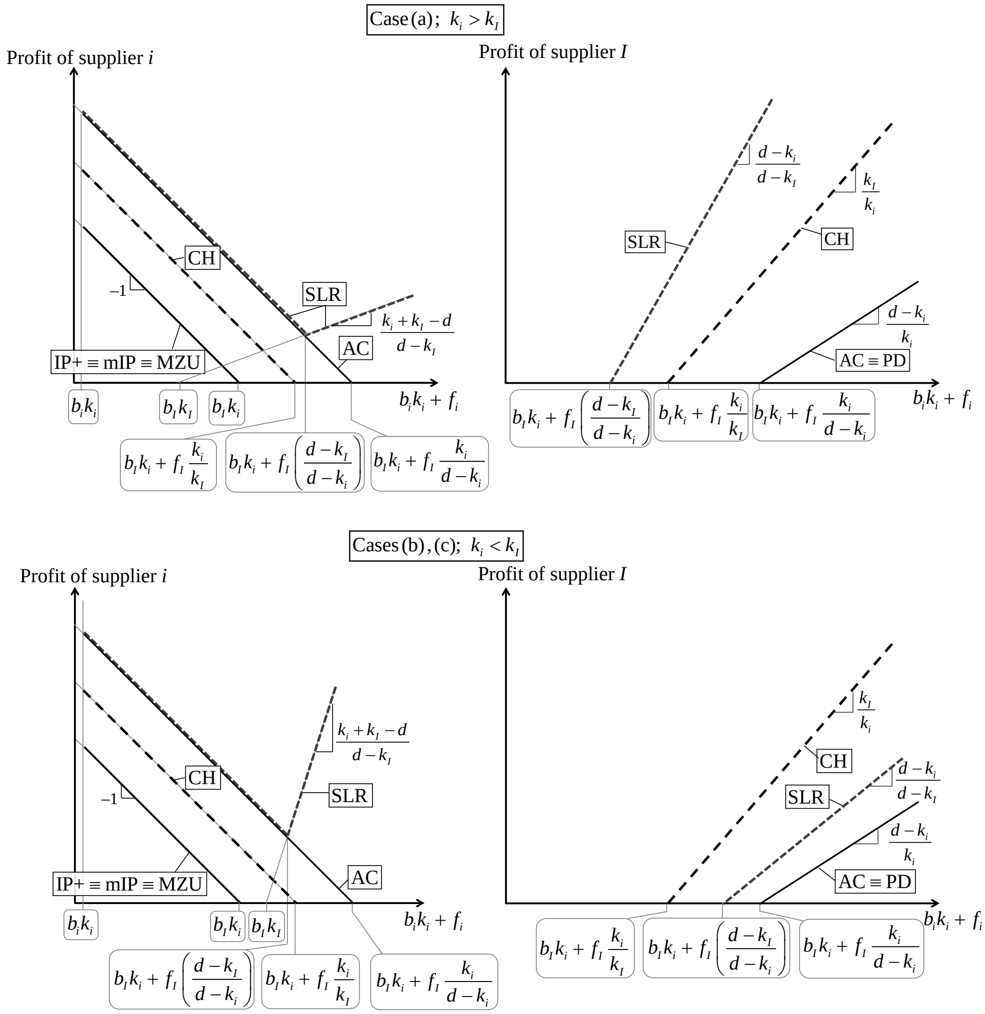

Figure 2.7: Profit graphs for cases (a), (b), and (c) of Figure 2.2, for high demand.

Regarding the effect of d on the price, note that the IP+, mIP, and CH prices are constant in d, whereas the GU, MZU, AC, and SLR prices are decreasing in d. The PD price also depends on d but this dependence is not necessarily monotonic, as can be shown.

It is important to note that, as far as the ordering of the schemes with respect to price and profits is concerned, the graphs for the symmetric-capacity case, shown in Figures 2.3 and 2.6, are indicative for the asymmetric-capacity case too, shown in Figures 2.4, 2.5, and 2.7.

2.7 Discussion of Trade-Offs between Market Outcome Characteristics

The divergence in prices and profits generated by the considered schemes, which is more evident in the high-demand region as shown in Section 2.6, suggests that there are trade-offs between market outcome characteristics that are weighed differently by each scheme. These trade-offs are discussed next.

IP+ formalizes the standard approach for dealing with non-convexities, notably in electricity markets. It uses uniform marginal-cost pricing and make-whole uplifts. IP+ may generate volatile prices when the optimal total cost is non-convex, because the IP+ price reflects this cost. The mIP scheme reduces this volatility by avoiding this non-convexity, generating prices that are nondecreasing in d. The trade-off is that the mIP price may be below marginal cost, in which case the make-whole uplifts are even higher than under IP+. The profits under mIP, however, remain the same as under IP+.

CH raises the price above marginal cost to minimize the external uplifts and resulting payment discrimination. This creates an opportunity for the marginal supplier to increase his profit by choosing to dispatch at full capacity. To cover the resulting opportunity cost, the CH price may end up being higher than the bare minimum to make the supplier whole. As a result, a supplier that incurs losses under marginal-cost pricing may make considerable profits under CH (e.g., supplier I in regions R4–R5 of Figure 2.6(b)). In addition, raising the price to cover the opportunity cost of one supplier increases the profit of another supplier, who may already be profitable under marginal-cost pricing. On the positive side, the CH price is piecewise constant and nondecreasing in d, and hence is stable.

SLR goes a step further and completely eliminates uplifts. The trade-off is that the SLR price and profits can be unbounded when the quantity of the marginal supplier tends to zero. Also, similarly to the CH price, the SLR price may be higher than the bare minimum to cover the losses, as is the case in regions R4–R5 of Figure 2.3(b).

PD also eliminates uplifts by transferring part of the quantity of the infra-marginal supplier (along with the associated payments) to the marginal supplier, as long as the value of the PD objective function is reduced. This transfer effectively constitutes a cross-subsidy between suppliers. The PD price and profits can be significantly lower than those generated by SLR, at the cost of a less efficient allocation. If such a transfer cannot reduce the value of the PD objective function, then PD yields the optimal allocation, and the resulting price and profits are identical to those generated by AC.

GU considers uplifts as internal zero-sum transfers between suppliers and aims to minimize the sum of the uplift norms, while ensuring allocation efficiency. The resulting prices and uplifts are complicated and depend on the uplift norm definition. The trade-off for focusing solely on minimizing the uplifts is that the price and profits can be excessively high, even unbounded when the quantity of the marginal supplier tends to zero, as in the case of AC and SLR. This adverse property could be mitigated if the fixed cost of the marginal supplier were reduced, softening the non-convexity, or if the quantity were subject to a minimum capacity constraint, as is often the case in electricity generation units.

MZU also considers uplifts as internal zero-sum transfers between suppliers, but is simpler than GU. Using these transfers, MZU increases the price above marginal cost and reduces the uplifts without generating excess profits for the suppliers. The trade-off is that the resulting price is decreasing in d, as is also the case with GU, AC, and SLR, as well as PD, in certain cases. The zero-sum uplifts condition is reminiscent of the zero-profit condition in IP. The difference is that under IP no supplier is allowed to make positive profits, whereas under MZU no supplier is allowed to earn more than under IP+.

We close with a few comments on the policy implications of the trade-offs discussed above. Designing pricing schemes in markets with non-convexities is a challenging multi-criteria decision problem with significant implications for market competition and regulation. The weights of the criteria depend on the maturity and prospects of the market, the number, market share and power of the players, the technology level driving fixed and marginal costs, and other factors. None of the considered schemes seems to dominate with respect to all criteria. If simplicity and transparency of the pricing rule is important, IP+, CH, AC, and MZU prevail. If the containment of profits to reasonable levels is sought, IP+, mIP, and MZU dominate. If the price should reflect the average cost of buying the commodity, schemes with no external uplifts prevail. If allocation efficiency is crucial, PD falls behind. If price stability and monotonicity is desired, mIP and CH generate piecewise constant, nondecreasing prices in d. If limiting the discriminatory uplifts is deemed an important driver for inciting truthful bidding, the revenue-adequate schemes are preferred.

2.8 Extensions

In this section, we extend our analysis for the cases of multiple suppliers (Subsection 2.8.1) and the case of price-elastic demand (Subsection 2.8.2).

2.8.1 The Case with Multiple Suppliers

The market model that we analyzed thus far assumes two suppliers. A question that arises naturally is, can we extend any of the conclusions to a larger number of suppliers. In this case, the optimal allocation, determined by the solution of the resulting MILP problem, does not have a simple structure. Still, however, given the optimal solution, we can compute the prices generated by the simpler schemes quite easily.

Specifically, suppose there are N suppliers with capacities k_n and marginal and fixed costs b_n and f_n, $n = 1, \ldots, N$. Let z_n^*, q_n^*, $n = 1, \ldots, N$, be the optimal MILP solution and $\lambda(d)$ be the price as a function of d, where $0 < d \leq \sum_{n=1}^{N} k_n$.

The IP+ and mIP prices are $\lambda^{\text{IP+}}(d) = b_m$, where $m = \arg\max_{n: z_n^*=1}\{b_n\}$, and $\lambda^{\text{mIP}}(d) = \min_{d': d' \geq d}\{\lambda^{\text{IP+}}(d')\}$.

To obtain the CH price, let (n) denote the supplier with the n^{th} smallest average cost at full capacity; then, $\lambda^{\text{CH}}(d) = b_{(n)} + f_{(n)}/k_{(n)}$, for $\sum_{i=1}^{n-1} k_{(i)} < d \leq \sum_{i=1}^{n} k_{(i)}$.

The MZU and AC prices are:

$$\begin{aligned}
\lambda^{\text{MZU}}(d) &= b_m + f_m/d + \textstyle\sum_{n:z_n^*=1, n\neq m} [f_n + (b_n - b_m)k_n]^+/d, \\
\lambda^{\text{AC}}(d) &= \max_{n:z_n^*=1}\{b_n + f_n/q_n^*\} = \\
&= \max[b_m + f_m/(d - \textstyle\sum_{n:z_n^*=1, n\neq m} k_n),\ \max_{n:z_n^*=1, n\neq m}\{b_n + f_n/k_n\}].
\end{aligned}$$

The above prices satisfy: $\lambda^{\text{mIP}}(d) \leq \{\lambda^{\text{CH}}(d), \lambda^{\text{IP+}}(d) \leq \lambda^{\text{MZU}}(d)\} \leq \lambda^{\text{AC}}(d)$.

GU, SLR, and PD are too complex to yield any manageable expressions.

Finally, we should note that, beyond the above cases, one must rely on numerical comparisons, which to date have been based for the most part on a benchmark example introduced in [11]. We treat this example in Chapter 3.

2.8.2 Extension to Price-Elastic Demand

For the schemes that we analyzed in this chapter, we developed exact expressions for the commodity price and uplifts paid to the suppliers as a function of demand d. The price at which the commodity is sold to the buyers (selling price) can be computed as the total payments to the suppliers (sum of commodity payments plus uplifts) averaged over d, assuming that uplifts are passed on to the buyers. If the uplifts are zero (AC, SLR, PD) or zero-sum internal transfers (GU, MZU), then the selling price coincides with the commodity price paid to the suppliers. If the uplifts are external (IP+, mIP, CH), then the selling price is greater than the price paid to the suppliers.

The selling price as a function of d constitutes a *supply function*. To determine the shape of this function, recall that in the low-demand region, the committed supplier $r'(d)$ has zero profit under all schemes (except for SLR when $k_1 < d \leq k = k_2$ and $b_2 + f_2/d > b_1 + f_1/k_1$). This means that the total payment to the committed supplier is equal to his total cost, which further implies that the selling price equals the average cost $b_{r'(d)} + f_{r'(d)}/d$; hence in the low-demand region, the supply function is decreasing in d. It can be shown that for the aforementioned special case of SLR, as well as the special case of CH in which the uncommitted supplier has positive profit ($k_j = k_1 < d \leq k$), the supply function is piecewise decreasing in d with an upward jump at k_1. In the high-demand region, it can be shown that the selling price

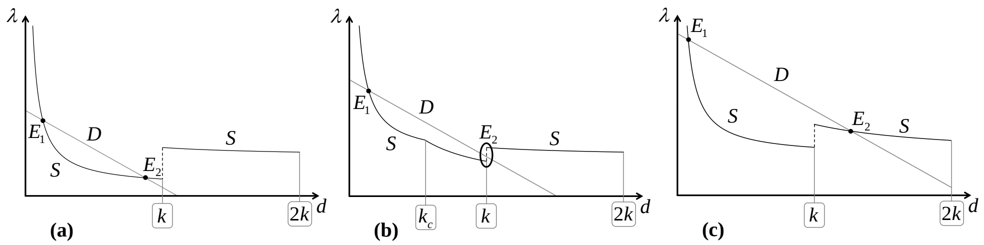

Figure 2.8: Intersection of supply (S) and demand (D) functions.

is also decreasing in d under all schemes (except for AC, where it may be partially constant, and for PD, where it may be partially constant or increasing). Finally, at the boundary between the low- and high-demand regions, k, the selling price exhibits an upward jump, reflecting the commitment of an additional supplier in the high-demand region.

Now, suppose that the demand is a smooth, downward-sloping, bounded function of price λ. Standard economic theory implies that the equilibrium price and quantity is given by the intersection of the supply and demand functions. As both functions are downward sloping (one monotonically and the other with an upward jump), they may have several intersections. To illustrate the types of situations that may arise, Figure 2.8 shows three indicative instances of supply and demand functions. In all cases, the demand function is linear, with the same negative slope but increasing intercept. Also, in all cases, the suppliers' capacities and costs are the same, except for f_i, which is increasing as we move from case (a) to (b) to (c).

In all cases, there are two intersections of the supply and demand functions, denoted by E_1 and E_2. In cases (a) and (c) both intersections have clearly-defined prices and quantities. In case (b), E_2 does not have a clearly-defined price, as the demand function crosses the supply function at its discontinuity (upward jump at k). In all cases, E_1 is in the low-demand region, whereas E_2 can be in the low-demand region, high-demand region, or at their boundary, depending on the case.

The fact that there are two equilibrium price-quantity outcomes in the presence of elastic demand would be seen as a weakness. A closer look, however, reveals that in all cases, E_1 is an unstable equilibrium, because the supply function crosses the demand function from above it to below it. This implies that the market can only be at E_1 if it starts at E_1, and any disruption from E_1 will lead the market away from E_1. In cases (a) and (b), E_2 is a stable equilibrium, because the supply function crosses the demand function from below it to above it. Therefore, for all practical purposes, the market would be attracted towards E_2. In case (b), where E_2 does not have a clearly-defined price, a special rule could be applied. For instance, as the buyers are willing to pay more for quantity k than the selling price at k, the clearing price could be simply set equal to the price that the buyers are willing to pay for k.

In all three cases, there are two intersections of the supply and demand functions. It is easy to imagine situations with more than two intersections, especially if the demand is highly elastic. The important point here is that our analytical results in Sections 2.3–2.5 enable us to compute and characterize the equilibria for any pricing scheme and any demand function.

2.9 Conclusions

Pricing in markets with non-convexities is a challenging interdisciplinary problem which has attracted renewed interest in the context of deregulated electricity markets. To address this problem, various pricing schemes have been proposed in recent years, but the connection between them has not been thoroughly studied. The two-supplier model that we analyzed, despite its simplicity, proved to be a useful test bed for evaluating and comparing in exact terms several of these schemes for markets with non-convex costs. This part of the analysis was based on closed-form expressions rather than on numerical comparisons.

Our comparison shows that the mIP scheme generates the same profits as IP+ but with lower and less volatile prices and higher uplifts. CH and MZU generally generate lower uplifts and higher prices than IP+. In the case of CH, the uplifts are external; hence, the profits are higher. Under MZU, the profits remain unchanged, as the uplifts are internal zero-sum payments between the suppliers. GU also provides internal zero-sum payments, but at prices and profits which can be much higher than

Table 2.1: Concise summary of results for the high-demand case.

Feature	**IP+**	**mIP**	**CH**	**GU**	**MZU**	**AC**	**PD**	**SLR**
Allocation efficiency	✓	✓	✓	✓	✓	✓		✓
Uplifts	external	external	external	internal	internal	0	0	0
Price	Low	Low	Med	Med-High	Med	High	Med-High	High
Price as a function of d	Const	Const	Const	↓	↓	↓	↓↑	↓
Profit of infra-marginal supplier	Low	Low	Med	Low-High	Low	High	Low-High	High
Profit of marginal supplier	0	0	High	Low	0	Med	Med	High

their MZU counterparts and are potentially unbounded. AC and SLR completely eliminate uplifts, but the resulting prices and profits can be substantial and also potentially unbounded. Finally, PD also eliminates uplifts at a possibly lower price than AC and SLR, trading off price efficiency for cost efficiency.

Table 2.1 summarizes the main results for the more involved high-demand case. As we argue in Subsection 2.8.2, our results, which were developed for inelastic demand, allow us to compute the prices and quantities for price-elastic demand as well. We also extended some of our analytical comparisons to the case of more than two suppliers.

Chapter 3

Numerical Results for Pricing Mechanisms in Markets with Non-Convexities

3.1 Introduction

In Chapter 2, we derived closed-form expressions for the prices, uplifts, and profits generated by different pricing mechanisms. We used these expressions to compare these mechanisms along these three dimensions for all possible ranges of the suppliers' fixed and variable costs and capacities, distinguishing between the low- and high-demand cases. We also commented on the sign of the slope of the supply function (price versus demand) for each mechanism, without any further elaboration. In this chapter, we numerically explore and compare the behavior of the quantities, prices and profits as a function of the demand for the different mechanisms.

Furthermore, we numerically evaluate several pricing mechanisms using Scarf's example [11] as it was modified in [16]. This example refers to a slightly more complex market model than the two-supplier model investigated in Chapter 2 and has become a common benchmark test-bed for evaluating different pricing schemes in markets with non-convexities. Given that comparisons between the IP+, mIP, CH, and PD pricing schemes already exist, with the comparisons involving the PD scheme being restricted to prices (refer to graphs in [22]), here we restrict our attention to mechanisms that do not provide external uplifts, i.e., GU, MZU, AC, SLR, and PD.

At the end of this chapter, we also consider an actual market setting, based on the Greek wholesale electricity market. Our aim is to present the impact of recovery mechanisms on actual markets. We evaluate a recovery mechanism that is implemented in the Greek market, against a standard bid/cost recovery mechanism. Our objective is not to assess potential bidding behavior, but to illustrate the aggregate (annual) impact of a recovery mechanism on a real test case.

The remainder of this chapter is structured as follows. In Section 3.2, we present illustrative examples of the various pricing mechanisms for the 2-supplier model introduced in Chapter 2. In Section 3.3, we use Scarf's example to evaluate pricing mechanisms that do not provide external uplifts. In Section 3.4, we consider the Greek wholesale electricity market setting, and we obtain numerical results for two alternative recovery mechanisms. Lastly, we conclude in Section 3.5.

3.2 Numerical Illustration for the Two-Supplier Case

In this section, we provide graphs that illustrate the behavior of the allocated quantities, prices and profits versus demand for different pricing mechanisms. For ease of exposition, we distinguish between the three different cases of Figure 2.2 in Chapter 2 where for each case we consider several problem instances. In Table 3.1, we summarize the sets of parameters that we considered for each instance, and the relevant figure in this chapter.

Graphs for Case (a) of Figure 2.2

In Case (a), the low-demand is up to $k = k_i = k_2$.

In Figure 3.1, we show the quantities, the price, and the profits for supplier $i = 2$ versus demand graphs, for $b_1 = 5, b_2 = 4, f_1 = 5, f_2 = 4, k_1 = 7, k_2 = 10$ (instance 1). The profits of supplier $I = 1$ are zero.

Table 3.1: Summary of numerical illustrations for the two-supplier case.

Instance	Cases of Figure 2.2	Parameters b_1	b_2	f_1	f_2	k_1	k_2	Figure
1	Case (a)	5	4	5	4	7	10	Figure 3.1
2		5	4	5	9	7	10	Figure 3.2
3		5	4	5	14	7	10	Figure 3.3
4		5	4	5	20	7	10	Figure 3.4
5		5	4	16	20	7	10	Figure 3.5
6	Case (b)	4	5	9	5	7	10	Figure 3.6
7		4	5	16	5	7	10	Figure 3.7
8	Case (c)	4	5	1	5	7	10	Figure 3.8
9		4	5	6	5	7	10	Figure 3.9

The PD allocation is not efficient for the entire high-demand case. For $10 < d \leq 13.923$, the outcome is not cost efficient, and the quantities are found in area Q4 of Figure A.1, whereas they are found in area Q2 for $d > 13.923$ (cost efficient). The outcome verifies Proposition 2.10, since the PD allocation is cost efficient for $d \leq k$, condition (i), and for $d > 13.923$, where condition (iii)(b) holds.

For the low-demand case, we observe that the GU, MZU, AC, SLR, and PD prices are equal and decreasing in d. They are higher than the CH price, which in turn is higher than the mIP and IP+ prices (the latter two prices are equal). A comparison with the graphs provided in Figures 2.4 and 2.5 in Chapter 2, shows that the values of the parameters chosen in Figure 3.1 refer to Sub-Cases (i), (ii), with $b_i + f_i/k_i = b_2 + f_2/k_2 = 4.4 < b_I = b_1 = 5$, b' ranging from 4.5 to 5.2 (although this is indifferent since $b_i + f_i/k_i < b_I$). We also have $b_2 + f_2/k_2 = 4.4 < b_1 + f_1/k_1 - f_2(k_2 - d)/(k_2 d)$ in Sub-Case (vi) for $k_1 < d \leq k_2$.

For the high-demand case, we verify the adverse property of GU, AC, and SLR as $d \to k^+$. We also see that AC and SLR prices are equal and higher than all the

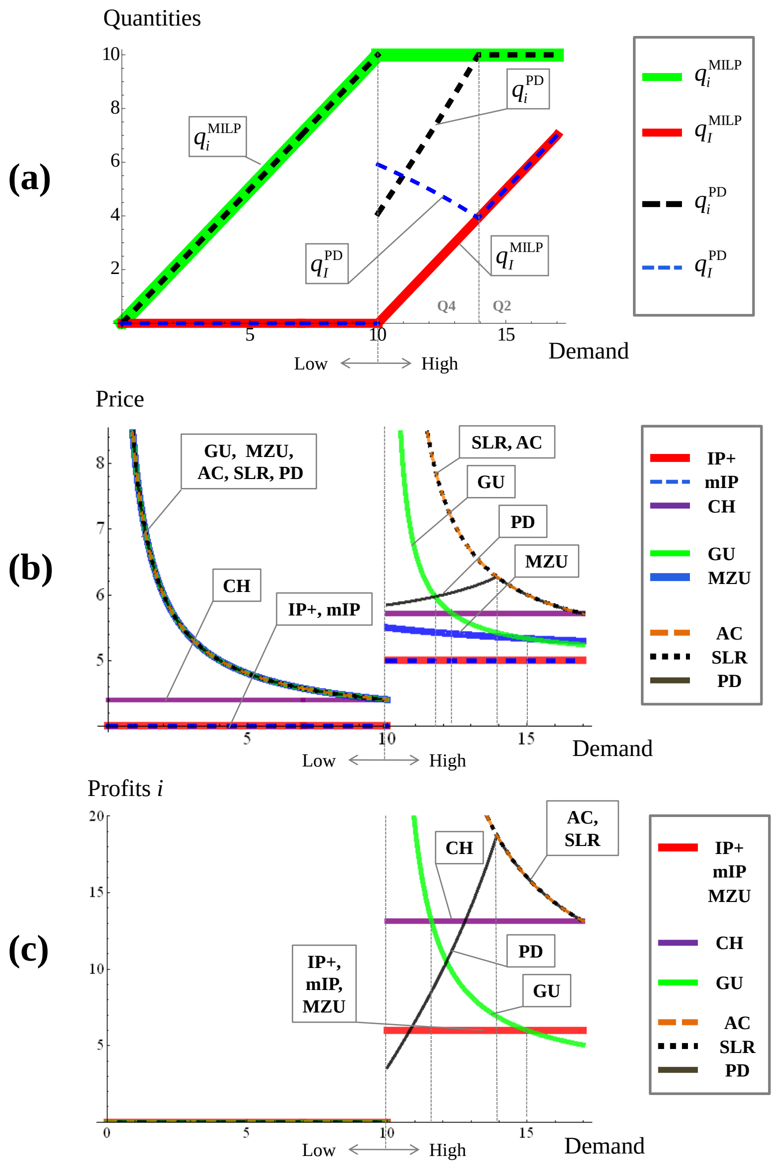

Figure 3.1: Quantities, price, and profits versus demand for Case (a) of Figure 2.2, for $b_1 = 5, b_2 = 4, f_1 = 5, f_2 = 4, k_1 = 7, k_2 = 10$ (instance 1 of Table 3.1).

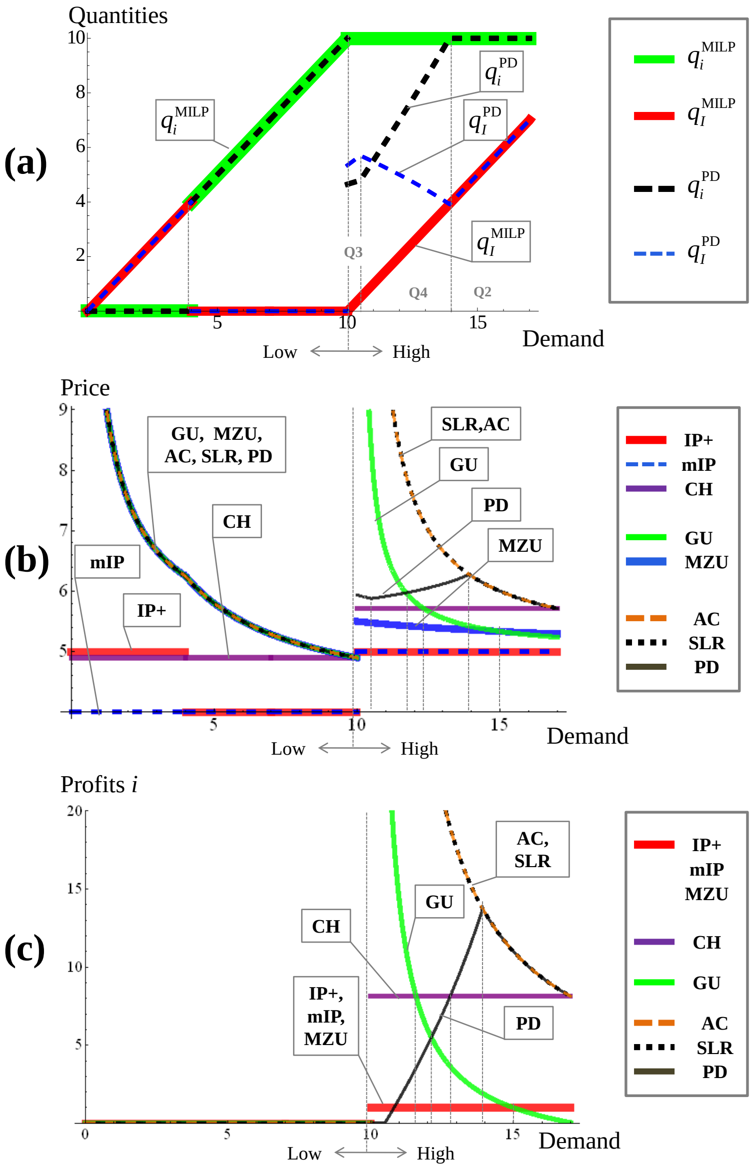

Figure 3.2: Quantities, price, and profits versus demand for Case (a) of Figure 2.2, for $b_1 = 5, b_2 = 4, f_1 = 5, f_2 = 9, k_1 = 7, k_2 = 10$ (instance 2 of Table 3.1).

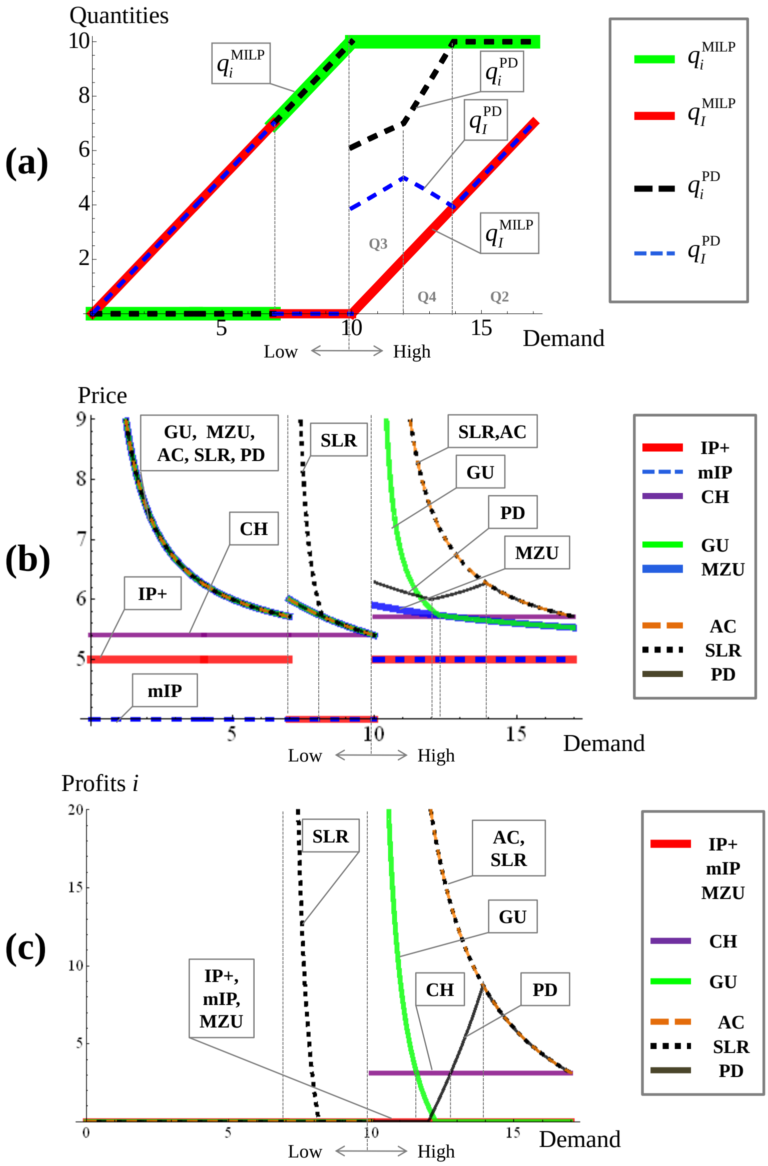

Figure 3.3: Quantities, price, and profits versus demand for Case (a) of Figure 2.2, for $b_1 = 5, b_2 = 4, f_1 = 5, f_2 = 14, k_1 = 7, k_2 = 10$ (instance 3 of Table 3.1).

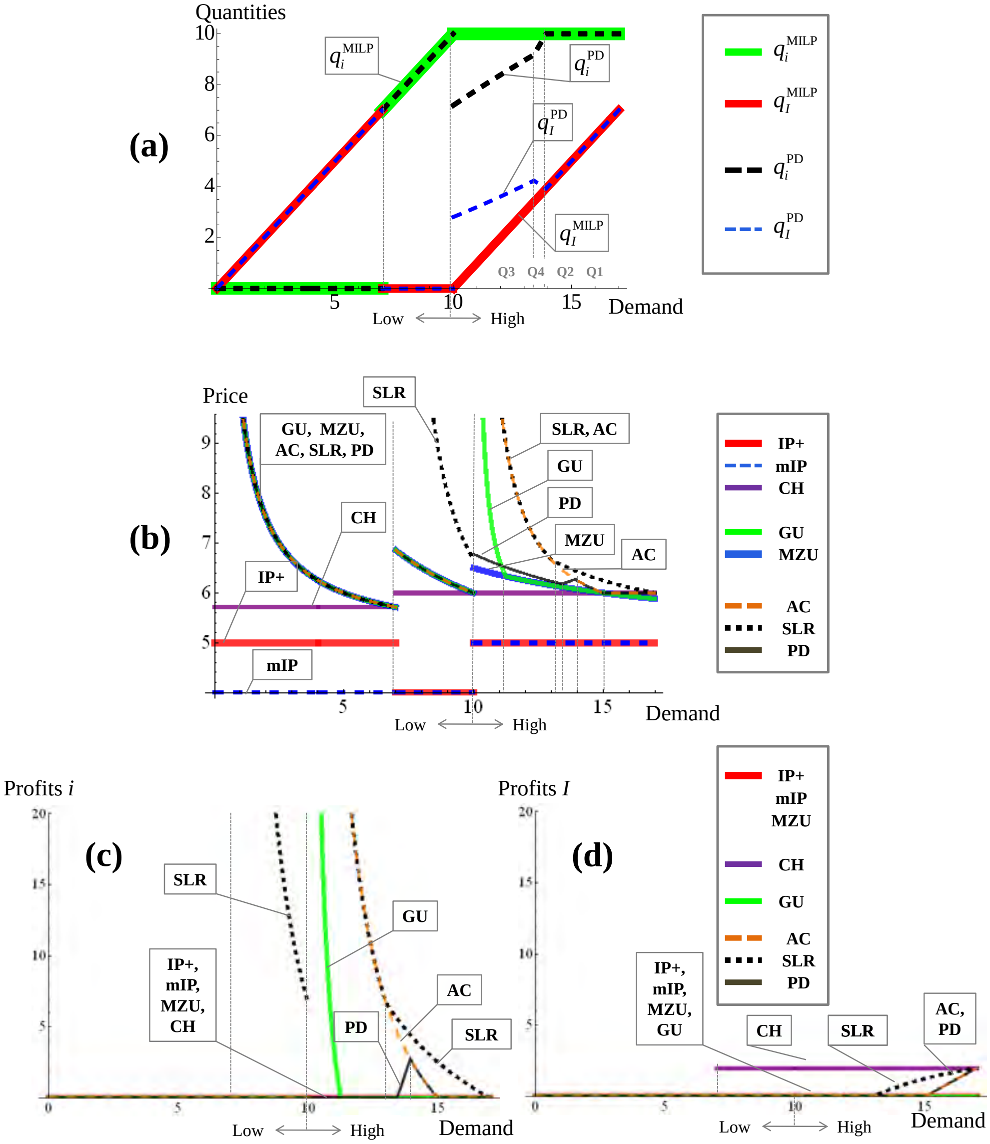

Figure 3.4: Quantities, price, and profits versus demand for Case (a) of Figure 2.2, for $b_1 = 5, b_2 = 4, f_1 = 5, f_2 = 20, k_1 = 7, k_2 = 10$ (instance 4 of Table 3.1).

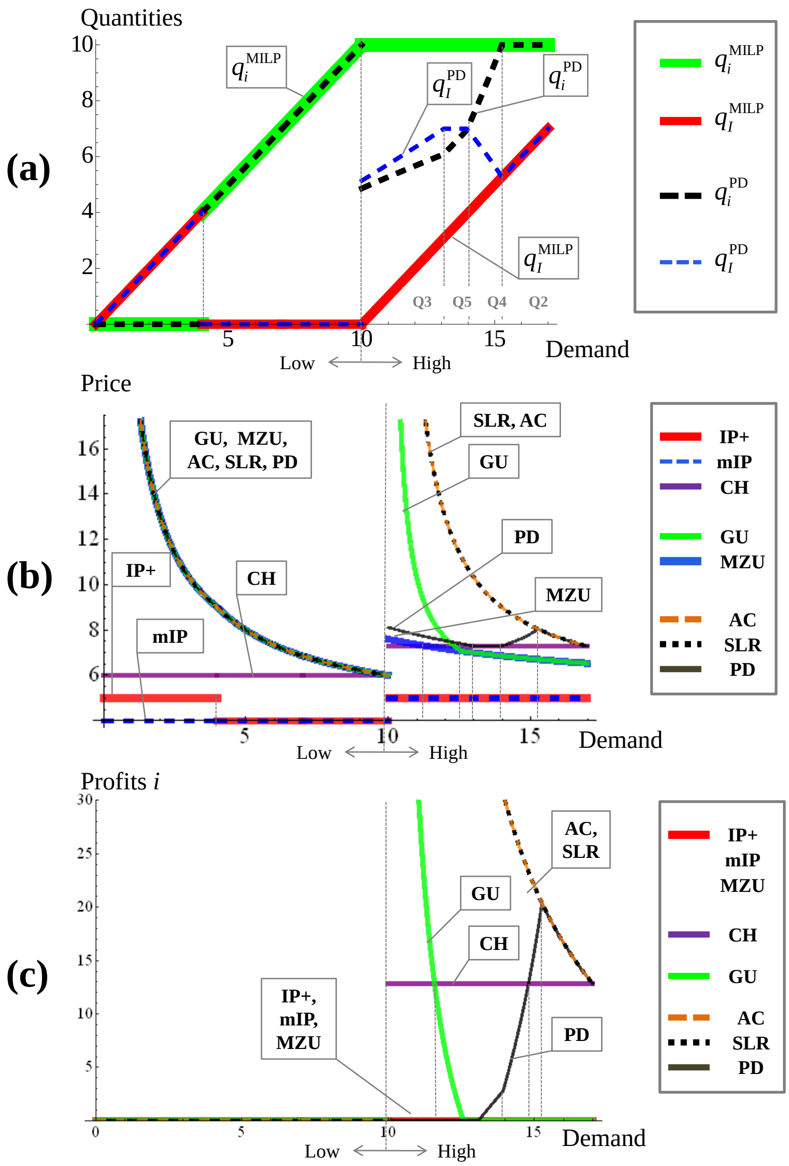

Figure 3.5: Quantities, price, and profits versus demand for Case (a) of Figure 2.2, for $b_1 = 5, b_2 = 4, f_1 = 16, f_2 = 20, k_1 = 7, k_2 = 10$ (instance 5 of Table 3.1).

others. The GU price is higher than the PD price for $d < 11.71$, higher than the CH price for $d < 10 + 7/3$ (i.e., if $b_I + f_I/[3(d - k_i)] > b_I + f_I/k_I$, i.e., $3(d - k_2) < k_1$, or $3(d - 10) < 7$, which yields $d < 10 + 7/3$), and higher than the MZU price for $d < 15$ (we note that for the parameters of Figure 3.1 we have $3k_i/2 = 15$). The PD price meets the AC price at $d = 13.923$, where the PD allocation becomes efficient. The CH price is higher than the MZU price, which is higher than the IP+ and mIP prices. The results are consistent with Figure 2.5(vii) which shows the SLR, AC, MZU, CH, mIP, and IP+ prices. The GU price is shown in Figure A.4.

For the profits of supplier $i = 2$, we have $b_i k_i + f_i = 44 < b_I k_i = 50$. The SLR and AC profits are equal to each other and are the highest. The GU profits follow for $d = 11.591$, where they cross CH. The PD profits intersect the IP+ profits (equal to the mIP and MZU profits), then the GU profits, and meet the AC and SLR profits at $d = 13.923$. The GU profits intersect the MZU profits at $d = 15$.

Next, we set $f_2 = 9$ (instance 2). The resulting quantities, the price, and the profits for supplier $i = 2$ are shown in Figure 3.2. The profits for supplier $I = 1$ are zero.

For the low-demand case, we observe that the GU, MZU, AC, SLR, and PD prices are equal and decreasing in d. They are higher than the CH, mIP and IP+ prices. A comparison with the graphs in Figures 2.4 and 2.5 of Chapter 2 shows that the values of the parameters chosen refer to Sub-Cases (i), (ii), with $b_i + f_i/k_i = b_2 + f_2/k_2 = 4.9 < b_I = b_1 = 5$, b' ranging from 4.5 to 5.2. For $b' > 4.9$, i.e., for $d < 4$, the IP price is higher than the CH and mIP prices, whereas for $b' < 4.9$, i.e., for $d > 4$, the CH price is higher than the IP+ and mIP prices (the latter being equal for $d > 4$). We also observe that $k_c = 4$, (refer to Figure 2.1, Case B). We also have $b_2 + f_2/k_2 = 4.9 < b_1 + f_1/k_1 - f_2(k_2 - d)/(k_2 d)$ in Sub-Case (vi) for $k_1 < d \leq k_2$.

For the high-demand case, the main difference is that PD now "crosses" regions Q3, Q4, and Q2 in Figure A.1 in Appendix A.

Next, we set $f_2 = 14$ (instance 3). The resulting quantities, the price, and the profits for supplier $i = 2$ are shown in Figure 3.3. The profits for supplier $I = 1$ are zero.

For the low-demand case, we observe that the GU, MZU, AC, SLR, and PD prices are equal and decreasing in d for $d \leq k_1$. They are higher than the CH, mIP and IP+ prices. A comparison with the graphs in Figures 2.4 and 2.5 shows that the values of the parameters chosen refer to Sub-Cases (i), (ii), with $b_i + f_i/k_i = b_2 + f_2/k_2 = 5.4 > b_I = b_1 = 5$, b' ranging from 4.5 to 5.2, hence $b' < 5.4$. For $d \leq 7$, the IP price is higher than the mIP price. For $k_1 < d \leq k_2$, we have $b_2 + f_2/k_2 = 5.4 > b_1 + f_1/k_1 - f_2(k_2 - d)/(k_2 d)$ for $d \leq 8.167$, and we observe that the SLR price is higher than the GU, MZU, AC, and PD prices in Sub-Case (vi).

For the high-demand case, the MZU price is higher than the CH price for $d < 12.3$, and vice versa for $d > 12.3$. The GU and CH prices still intersect at $d = 10 + 7/3$. The GU price is higher than the MZU price for $d <= 12.272$ and equal to it for $d > 12.272$. PD still "crosses" regions Q3, Q4, and Q2. The GU profits become zero at $d = 12.272$.

Finally, we set $f_2 = 20$ (instance 4). The resulting quantities, the price, and the profits for both suppliers are shown in Figure 3.4.

For the low-demand case, we have similar observations with Figure 3.3. A comparison with the graphs provided in Chapter 2, Figures 2.4 and 2.5 shows that the values of the parameters chosen refer to Sub-Cases (i), (ii), with $b_i + f_i/k_i = b_2 + f_2/k_2 = 6 > b_I = b_1 = 5$, b' ranging from 4.5 to 5.2, hence $b' < 6$. For $d \leq 7$, the IP price is higher than the mIP price. For $k_1 < d \leq k_2$, we have $b_2 + f_2/k_2 = 6 > b_1 + f_1/k_1 - f_2(k_2 - d)/(k_2 d)$, and we observe that the SLR price is higher than the GU, MZU, AC, and PD prices in Sub-Case (vi).

For the high-demand case, the MZU price is higher than the CH price for $d < 12.3$, and vice versa for $d > 12.3$. The GU and CH prices still intersect at $d = 10 + 7/3$. The GU price is higher than the MZU price for $d <= 11.25$ and equal for $d > 11.25$. The PD "crosses" regions Q3, Q4, Q2, Q1. Lastly, for $d > 13$, we have $b_I + f_I(d - k_I)/(k_i(d - k_i)) < 6 = b_i + f_i/k_i$, and the SLR price is higher than the AC price.

For the profits of supplier i, for $d > 15$, only SLR yields positive profits. For $13 < d < 15$, the AC profits are positive and lower than the SLR profit. The PD profits reach the AC profits at $d = 13.923$. For $d < 13$ the SLR profits equal the AC profits. The CH and MZU profits are zero. GU yields positive profits for $d < 11.25$, lower than SLR, AC profits. For the profits of supplier I, the CH profits are higher than the SLR profits (positive for $d > 13$), and higher than the AC profits, which equal the PD profits (positive for $d > 15$).

We then modify parameter f_1 to $f_1 = 16$ (instance 5). The resulting quantities, the price, and the profits for supplier $i = 2$ are shown in Figure 3.5. The profits for supplier $I = 1$ are zero.

For the low-demand case, we have $k_c = 4$. The GU, MZU, AC, SLR, and PD prices are equal and decreasing in d, followed by the CH price, followed by the IP+ price, followed by the mIP price. The mIP price is equal to the IP+ price for $d > 4$. b' ranges from 5.6 to 6.3, with $b' = 6$ for $d = 4$. Also, still $b_i + f_i/k_i = b_2 + f_2/k_2 = 6 > b_I = b_1 = 5$.

For the high-demand case, PD "crosses" regions Q3, Q5, Q4, Q2. The SLR price equals the AC price. The MZU price intersects the CH price at $d = 11.375$. The GU price is higher than the MZU price for $d <= 12.58$ and equal for $d > 12.58$.

For the profits of supplier i, we have $b_i k_i + f_i = 60 < b_I k_i + f_I k_i/k_I = 50 + 160/7$. The AC and SLR profits are equal and highest, followed by the CH profits. The MZU profits are zero. The GU profits intersect the CH profits at $d = 11.591$. The GU profits become zero at $d = 12.581$. The PD profits reach the AC profits at $d = 15.266$.

Graphs for Case (b) of Figure 2.2

In Case (b), low-demand is up to $k = k_I = k_2 = 10$.

We first consider the following set of parameters: $b_1 = 4, b_2 = 5, f_1 = 9, f_2 = 5, k_1 = 7, k_2 = 10$ (instance 6). Note that in Case (b), we have $b_I = 5 \leq b_i + f_i/k_i = 4 + 9/7$.

The results for the quantities, the price, and the profits for both suppliers are shown in Figure 3.6.

For the low-demand case $d \leq 10$, the SLR, AC, PD, MZU, GU prices are equal and decreasing in d, with the SLR price being the highest for $k_1 < d \leq k_2$, since $b_2 + f_2/k_2 = 5.5 > b_1 + f_1/k_1 - f_2(k_2 - d)/(k_2 d)$. They are followed by the CH, IP+, and mIP prices. We note that $k_c = 4$.

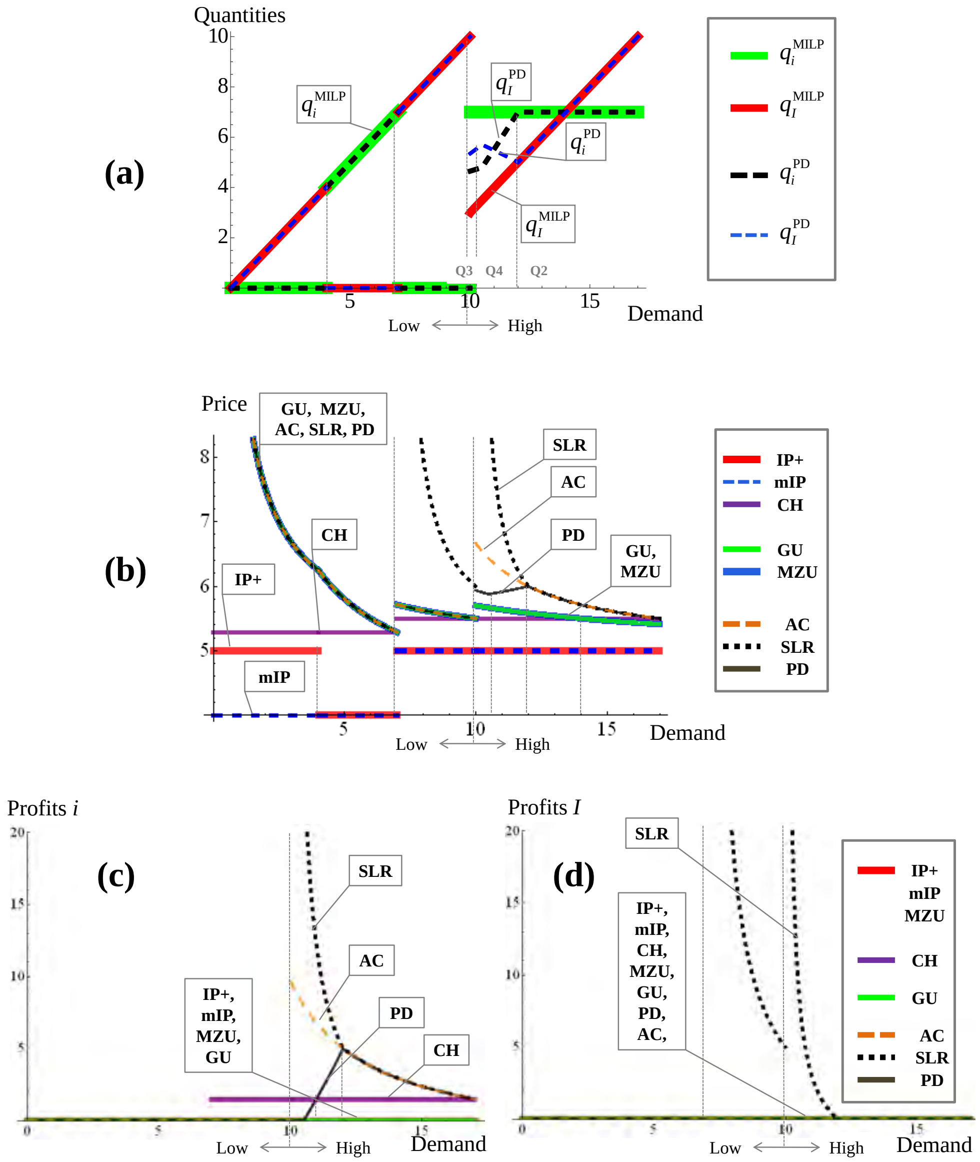

Figure 3.6: Quantities, price, and profits versus demand for Case (b) of Figure 2.2, for $b_1 = 4, b_2 = 5, f_1 = 9, f_2 = 5, k_1 = 7, k_2 = 10$ (instance 6 of Table 3.1).

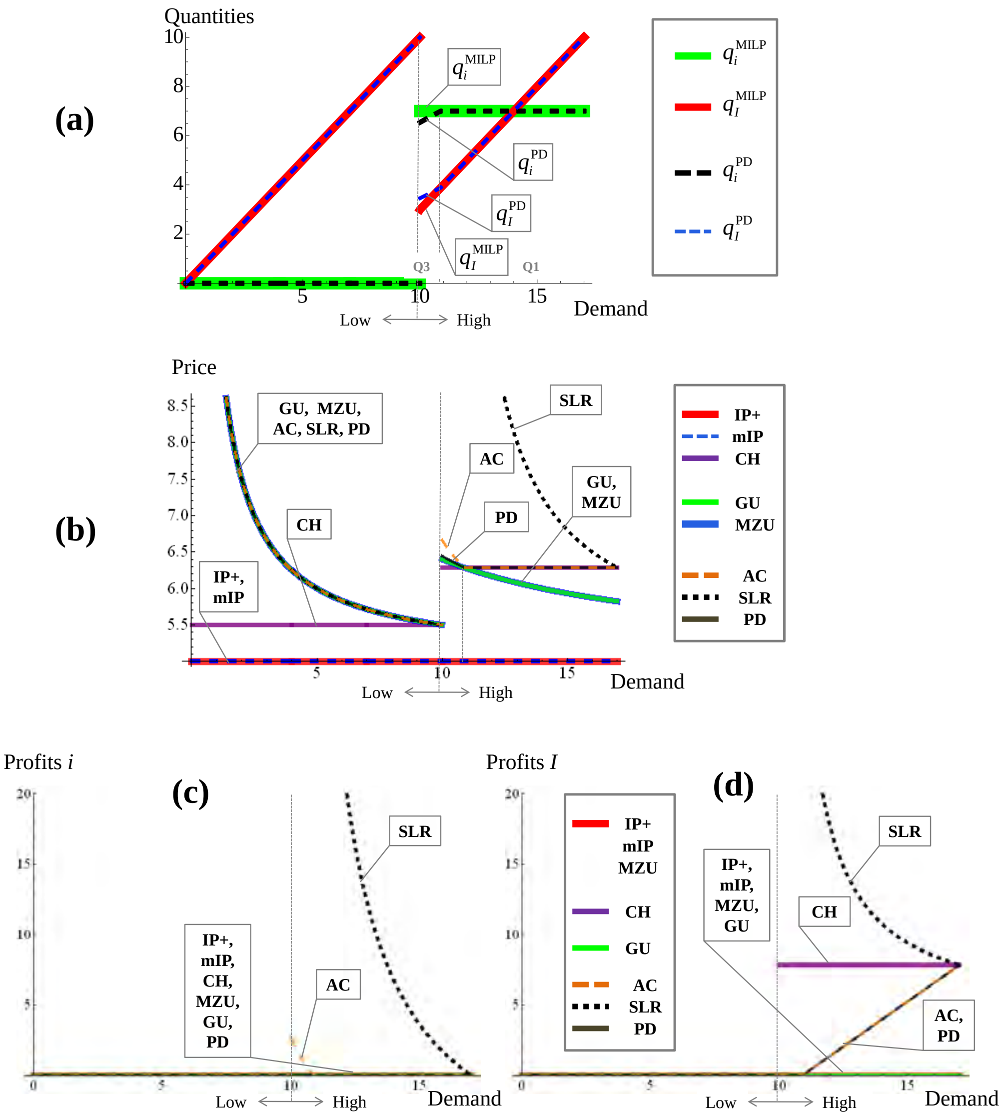

Figure 3.7: Quantities, price, and profits versus demand for Case (b) of Figure 2.2, for $b_1 = 4, b_2 = 5, f_1 = 16, f_2 = 5, k_1 = 7, k_2 = 10$ (instance 7 of Table 3.1).

For the high-demand case, the SLR price is the highest for $d < 12$. For $d \geq 12$, the SLR price is equal to the AC and PD prices. The MZU price intersects the CH price at $d = 14$. The MZU price equals the GU price. PD "crosses" Q3, Q4, Q2.

For the profits of supplier i, for $d < 12$ SLR yields the highest profits, followed by AC. The SLR profits reach the AC profits at $d = 12$. The CH profits follow, and MZU yields zero profits. The PD profits reach the AC profits at $d = 12$. GU yields zero profits. For the profits of supplier I, only SLR yields positive profits for $d < 12$.

We then modify parameter f_1 to $f_1 = 16$ (instance 7). The resulting quantities, the price, and the profits for both suppliers are shown in Figure 3.7.

For the low-demand case $d \leq 10$, the SLR, AC, PD, MZU, GU price are equal and decreasing in d, followed by the CH price, followed by the IP+ and mIP prices.

For the high-demand case, the SLR price is the highest for $d < 12$. For $d \geq 12$, the SLR price is equal to the AC and PD prices. The MZU price intersects the CH price at $d = 10.89$. The MZU price equals the GU price. PD crosses Q3, Q1.

For the profits of supplier i, SLR yields positive profits, followed by AC (positive for $d < 10.89$). For the profits of supplier I, SLR yields positive profits, followed by CH, followed by AC and PD.

Graphs for Case (c) of Figure 2.2

In Case (c), low-demand is up to $k = k_I = k_2 = 10$.

We first consider the following set of parameters: $b_1 = 4, b_2 = 5, f_1 = 1, f_2 = 5, k_1 = 7, k_2 = 10$ (instance 8). Note that in Case (c), we have $b_I = 5 > b_i + f_i/k_i = 4 + 1/7$.

The results for the quantities, the price, and the profits for supplier i are shown in Figure 3.8. The profits of supplier I are zero.

For the low-demand case $d \leq 7$, the SLR, AC, PD, MZU, GU prices are equal and decreasing in d. They are followed by the the CH, IP+, and mIP prices. Note that $k^{\mathrm{PD}} = 9.27$ and that for $7 < d < 9.27$ we have only one supplier (supplier 2) committed under the PD scheme.

For the high-demand case, the SLR price is equal to the AC price and is the highest. The MZU price intersects the CH price at $d = 10$ (refer to Figure 2.5). PD

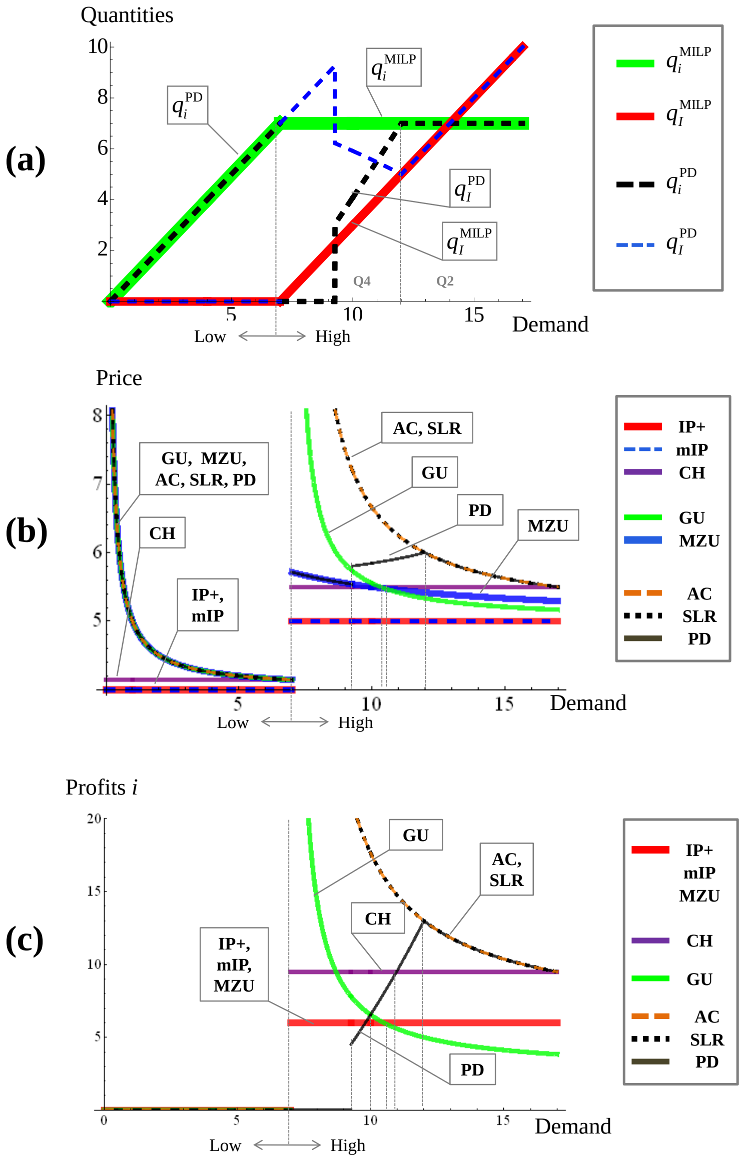

Figure 3.8: Quantities, price, and profits versus demand for Case (c) of Figure 2.2, for $b_1 = 4, b_2 = 5, f_1 = 1, f_2 = 5, k_1 = 7, k_2 = 10$ (instance 8 of Table 3.1).

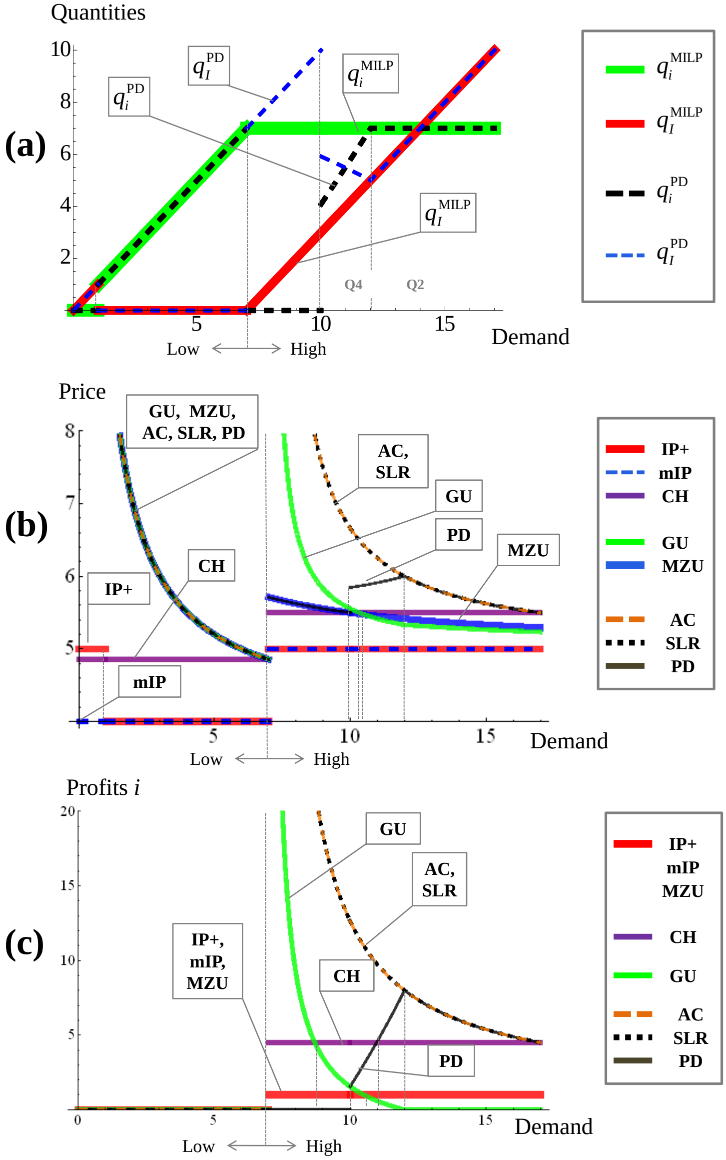

Figure 3.9: Quantities, price, and profits versus demand for Case (c) of Figure 2.2, for $b_1 = 4, b_2 = 5, f_1 = 6, f_2 = 5, k_1 = 7, k_2 = 10$ (instance 9 of Table 3.1).

"crosses" Q4 and Q2, being efficient at $d = 12$. The GU price intersects the CH price at $d = 10.333$, and the MZU price at $d = 10.5$.

For the profits of supplier i, we note that AC and SLR are equal and highest, followed by CH, followed by MZU that are equal to IP+ and mIP. The GU profits are higher than CH for $d < 8.707$ and higher than MZU for $d < 10.5$. PD profits are zero for $d < 9.27$ and reach AC, SLR at $d = 12$.

We then modify parameter f_1 to $f_1 = 6$ (instance 9). The resulting quantities, the price, and the profits for supplier i are shown in Figure 3.9. The profits of supplier I are zero.

For the low-demand case, the difference with the previous case is the ordering of CH, IP+ and mIP prices. We note that $k_c = 1$, $k^{\mathrm{PD}} = 10$, and that for $7 < d \leq 10$, we have only one supplier (supplier 2) committed under the PD scheme.

For the high-demand case, the AC and SLR prices are still equal and highest. PD crosses Q5, Q4, Q2, becoming efficient at $d = 12$. The MZU price crosses CH at $d = 10$. The GU price intersects the CH price at $d = 10.333$ and the MZU price at $d = 10.5$.

For the profits of supplier i, we note that AC and SLR are equal and highest, followed by CH, followed by MZU that are equal to IP+ and mIP. The GU profits are higher than the CH profits for $d < 8.707$ and higher than the MZU profits for $d < 10.5$. The GU profits reach zero at $d = 12$. The PD profits are zero for $d < 10$ and reach the AC, SLR profits at $d = 12$.

3.3 Numerical Illustration for Scarf's Example

In this section, we consider a commonly used benchmark test-bed for evaluating different pricing schemes that deal with non-convexities. The test-bed is based on an example introduced by Scarf [11], as modified in [16]. In Subsection 3.3.1, we present the characteristics of the example and the basic MILP formulation. In Subsection 3.3.2, we present the pricing approaches under consideration, which either resort to "augmented pricing" and additional internal uplifts in the form of zero-sum transfers between the suppliers, or are pure revenue-adequate in that the prices that they

Table 3.2: Modified Scarf example.

Unit	**SmokeStack**	**HighTech**	**MedTech**
Capacity	16	7	6
Minimum Output	0	0	2
Startup Cost	53	30	0
Marginal Cost	3	2	7
Number of Units	6	5	5

generate guarantee that no supplier incurs losses, without the need for additional external/internal uplifts. We list and discuss the results in Subsection 3.3.3.

3.3.1 Model of Modified Scarf's Example

The modification of Scarf's example introduced in [16] considers three types of generating units (Smokestack, HighTech, and MedTech). The characteristics of the units are shown in Table 3.2.

Demand is assumed to be deterministic and inelastic, with values up to 161 units corresponding to the sum of the capacities of all generating units.

The example above is more general than the two-supplier model described in Chapter 2 because it considers more than two types of suppliers where each type comes in a finite number of units and may be subject to a minimum output constraint. For clarity, we provide the model with n (multiple) suppliers that are obliged to produce above their minimum output, denoted by m_n. The nomenclature is the same as that in Chapter 2. Given bids b_n, f_n (for the marginal and fixed cost, respectively), the auctioneer determines the optimal allocation, expressed by decision variables z_n (binary) and q_n (continuous), representing the suppliers' commitment and dispatch quantities, respectively, by solving the following MILP problem:

$$\underset{q_n, z_n}{\text{Minimize}} \; L_{\text{MILP}} = \sum_n (b_n q_n + f_n z_n), \tag{3.1}$$

subject to:

$$\sum_n q_n = d, \tag{3.2}$$

$$q_n \leq k_n z_n, \qquad \forall n, \tag{3.3}$$

$$q_n \geq m_n z_n, \qquad \forall n, \tag{3.4}$$

$$q_n \geq 0, \qquad \forall n, \tag{3.5}$$

$$z_n \in \{0, 1\}, \qquad \forall n. \tag{3.6}$$

Problem (3.1)–(3.6) is characterized by non-convexities due to the presence of the fixed costs and the minimum output requirements. In what follows, we refer to the optimal solution of the MILP problem, as z_n^{MILP} and q_n^{MILP} for the optimal commitment and dispatch quantities, respectively. We also denote by λ^{MILP} the marginal cost price, which is equal to the dual variable associated with constraint (3.2), if the commitment variables are fixed to their optimal value so that problem (3.1)–(3.6) is transformed to an LP problem.

3.3.2 Pricing Approaches

In what follows, we present the formulations of the approaches that do not provide external uplifts, namely:

1. Generalized Uplift (GU);
2. Minimum Zero-Sum Uplift (MZU);
3. Average Cost (AC);
4. Semi-Lagrangean Relaxation (SLR);
5. Primal-Dual (PD).

GU Pricing

Under this scheme, scalars Δb_n^{GU} and Δf_n^{GU} are defined for supplier n and are added to his marginal and fixed costs, respectively. The supplier then receives positive or negative side-payments "uplifts" σ_n^{GU}, given as follows:

$$\sigma_n^{\mathrm{GU}} = \Delta b_n^{\mathrm{GU}} q_n^{\mathrm{MILP}} + \Delta f_n^{\mathrm{GU}} z_n^{\mathrm{MILP}} \tag{3.7}$$

These payments represent internal, zero-sum monetary transfers between the suppliers. The GU formulation is as follows:

$$\underset{\lambda^{\mathrm{GU}},\Delta b_n^{\mathrm{GU}},\Delta f_n^{\mathrm{GU}}}{\text{Minimize}} \quad L_{\mathrm{GU}} = \sum_n (\Delta b_n^{\mathrm{GU}} q_n^{\mathrm{MILP}})^2 + (\Delta f_n^{\mathrm{GU}} z_n^{\mathrm{MILP}})^2, \tag{3.8}$$

subject to:

$$\lambda^{\mathrm{GU}} \geq b_n + \Delta b_n^{\mathrm{GU}}, \qquad \text{if} \;\; q_n^{\mathrm{MILP}} = k_n, \; \forall n, \tag{3.9}$$

$$\lambda^{\mathrm{GU}} = b_n + \Delta b_n^{\mathrm{GU}}, \qquad \text{if} \;\; m_n z_n^{\mathrm{MILP}} < q_n^{\mathrm{MILP}} < k_n, \; \forall n, \tag{3.10}$$

$$\lambda^{\mathrm{GU}} \leq b_n + \Delta b_n^{\mathrm{GU}}, \qquad \text{if} \;\; q_n^{\mathrm{MILP}} = m_n z_n^{\mathrm{MILP}}, \; \forall n, \tag{3.11}$$

$$(1 - z_n^{\mathrm{MILP}}) \Delta f_n^{\mathrm{GU}} = 0, \qquad \forall n, \tag{3.12}$$

$$[\lambda^{\mathrm{GU}} - (b_n + \Delta b_n^{\mathrm{GU}})] q_n^{\mathrm{MILP}} - (f_n + \Delta f_n^{\mathrm{GU}}) z_n^{\mathrm{MILP}} \geq 0, \qquad \forall n, \tag{3.13}$$

$$\sum_n (\Delta b_n^{\mathrm{GU}} q_n^{\mathrm{MILP}} + \Delta f_n^{\mathrm{GU}} z_n^{\mathrm{MILP}}) = 0. \tag{3.14}$$

The profits of supplier n, denoted by π_n, are given by

$$\pi_n^{\mathrm{GU}} = [\lambda^{\mathrm{GU}} - (b_n + \Delta b_n^{\mathrm{GU}})] q_n^{\mathrm{MILP}} - (f_n + \Delta f_n^{\mathrm{GU}}) z_n^{\mathrm{MILP}}. \tag{3.15}$$

MZU Pricing

The MZU price is given as follows:

$$\lambda^{\mathrm{MZU}} = \lambda^{\mathrm{MILP}} + \frac{\sum_n (-\tilde{\pi}_n^{\mathrm{MILP}})^+}{d}, \tag{3.16}$$

where $\tilde{\pi}_n^{\mathrm{MILP}}$ the gross profits for λ^{MILP} and no uplifts, i.e., $\tilde{\pi}_n^{\mathrm{MILP}} = (\lambda^{\mathrm{MILP}} - b_n) q_n^{\mathrm{MILP}} - z_n^{\mathrm{MILP}} f_n$, and the notation $(x)^+ = \max\{0, x\}$.

The profits of supplier n are given by

$$\pi_n^{\mathrm{MZU}} = (\tilde{\pi}_n^{\mathrm{MILP}})^{+}. \tag{3.17}$$

Note that $\pi_n^{\mathrm{MZU}} = \pi_n^{\mathrm{IP+}} = \pi_n^{\mathrm{mIP}}$.

AC Pricing

Under this simple scheme, the price is set at the maximum average cost of the committed supplier.

$$\lambda^{\mathrm{AC}} = \max_{n:z_n^{\mathrm{MILP}}=1} \left\{ b_n + \frac{f_n}{q_n^{\mathrm{MILP}}} \right\} \tag{3.18}$$

The profits of supplier n are given by

$$\pi_n^{\mathrm{AC}} = (\lambda^{\mathrm{AC}} - b_n) q_n^{\mathrm{MILP}} - f_n z_n^{\mathrm{MILP}}. \tag{3.19}$$

SLR Pricing

The formulation of the SLR problem is presented below.

$$\underset{q_n, z_n}{\text{Minimize}} \;\; L_{\mathrm{SLR}} = \sum_n (b_n q_n + f_n z_n) + \lambda^{\mathrm{SLR}} \left(d - \sum_n q_n \right), \tag{3.20}$$

subject to:

$$\sum_n q_n \leq d, \tag{3.21}$$

and primal constraints (3.3)–(3.6).

In fact, the SLR approach consists of solving the dual problem maximize$_{\lambda}^{\mathrm{SLR}}$ $L_{\mathrm{SLR}}^{*}(\lambda^{\mathrm{SLR}})$ with $L_{\mathrm{SLR}}^{*}(\lambda)$ denoting the optimal value (minimum cost) which is proven to be equal to the optimal MILP solution of problem (3.1)–(3.6). The profits of supplier n are given by

$$\pi_n^{\mathrm{SLR}} = (\lambda^{\mathrm{SLR}} - b_n) q_n^{\mathrm{MILP}} - f_n z_n^{\mathrm{MILP}}. \tag{3.22}$$

PD Pricing

This approach produces a Mixed Integer Non-Linear Programming (MINLP) problem, whose formulation is presented below. We first consider the LP relaxation of problem (3.1)–(3.6), i.e., we replace (3.6) by the following constraint:

$$0 \leq z_n \leq 1 \qquad \forall n. \tag{3.23}$$

Assuming dual variables λ, μ_n, ν_n, ξ_n, associated with constraints (3.2), (3.3), (3.4), and (3.23), respectively, the dual problem is written as follows:

$$\underset{\lambda^{\mathrm{PD}},\mu_n,\nu_n,\xi_n}{\text{Maximize}} \left\{ \lambda^{\mathrm{PD}} d - \sum_n \xi_i \right\}, \tag{3.24}$$

subject to:

$$\lambda^{\mathrm{PD}} - \mu_n + \nu_n \leq b_n \qquad \forall n, \tag{3.25}$$

$$k_n \mu_n - m_n \nu_n - \xi_n \leq f_n \qquad \forall n, \tag{3.26}$$

$$\lambda^{\mathrm{PD}} \in \Re, \tag{3.27}$$

$$\mu_n, \nu_n, \xi_n \geq 0 \qquad \forall n. \tag{3.28}$$

The PD problem formulation is as follows:

$$\underset{z_n^{\mathrm{PD}},q_n^{\mathrm{PD}},\lambda^{\mathrm{PD}},\mu_n,\nu_n,\xi_n}{\text{Minimize}} \sum_n (b_n q_n^{\mathrm{PD}} + f_n z_n^{\mathrm{PD}}) - \lambda^{\mathrm{PD}} d + \sum_n \xi_n, \tag{3.29}$$

subject to:

Primal Constraints (3.2)–(3.6),
Dual Constraints (3.25)–(3.28), and

$$\lambda^{\mathrm{PD}} q_n - b_n q_n^{\mathrm{PD}} - f_n z_n^{\mathrm{PD}} \geq 0 \qquad \forall n. \tag{3.30}$$

The profits of supplier n are given by

$$\pi_n^{\mathrm{PD}} = (\lambda^{\mathrm{PD}} - b_n) q_n^{\mathrm{PD}} - f_n z_n^{\mathrm{PD}}. \tag{3.31}$$

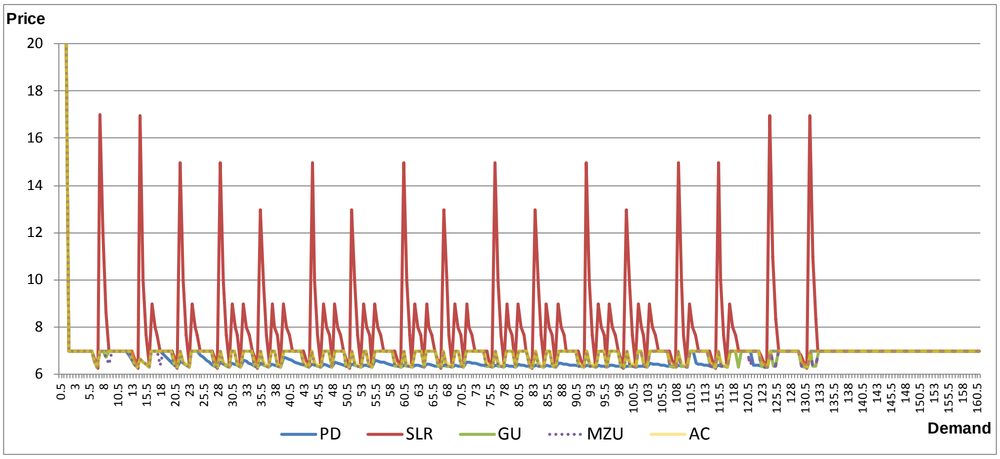

Figure 3.10: Prices versus demand for different pricing mechanisms (granularity of demand 0.5 units).

Note that under this scheme the commitment and dispatch variables z_n^{PD}, q_n^{PD} may differ from the ones of the original MILP problem z_n^{MILP}, q_n^{MILP}.

3.3.3 Numerical Results and Discussion

We modeled the pricing approaches using GAMS 24.1.2 [86] and solved the SLR, GU, MZU, and AC schemes with the CPLEX 12.5.1 solver and the PD scheme with AlphaECP, on an Intel Core i5 at 2.67GHz, with 6GB RAM.

Figure 3.10 shows the price versus the demand graph for the aforementioned pricing schemes for a demand granularity of 0.5 units. Note that all schemes except PD actually use the optimal MILP solution. PD is the only scheme that allows for different allocations.

Figure 3.11 shows the percentage increase of the total cost under PD compared to the optimal (minimum) total cost.

Remark 3.1. *Under all pricing schemes, prices are not monotonically increasing in demand.*

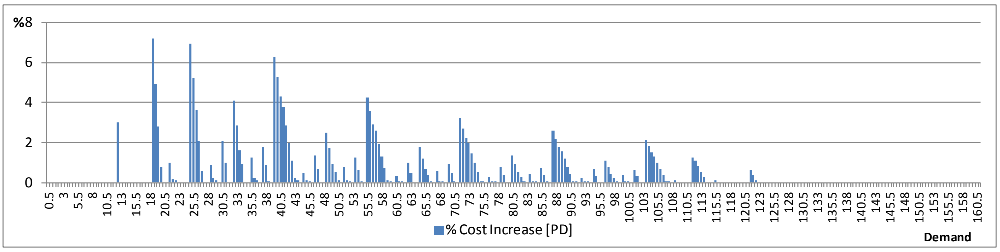

Figure 3.11: Cost increase under the PD allocation compared to the optimal MILP solution.

This is the main effect of the non-convexities. A remedy for this effect would be to consider convexified prices, as for example in the CH approach [17]. However, this would introduce external uplifts, to counter potential losses. Also, mIP prices are monotonically increasing (non-decreasing, to be more precise), but they may generate large external uplifts and prices that are below marginal cost.

Remark 3.2. *The PD scheme may result in inefficient commitment and dispatch quantities.*

This observation is straightforward from Figure 3.11, which shows that the PD scheme may result in a cost increase up to about 7% for the considered example. This effect is due to the fact that the PD scheme exchanges price for cost efficiency. Since this scheme does not introduce uplifts, the price should be high enough to cover the average cost at the dispatched quantity. The PD scheme may re-allocate the quantities, so that the average costs are actually lower than the ones of the optimal MILP allocation.

Remark 3.3. *The SLR scheme exhibits price spikes.*

In [21], it is shown that the SLR prices obtained yield competitive prices that are high enough to make the market participants willing to generate the amounts of electricity scheduled by the system operator. To achieve this, the SLR scheme may result in prices that are higher than the ones required to cover the losses. For this

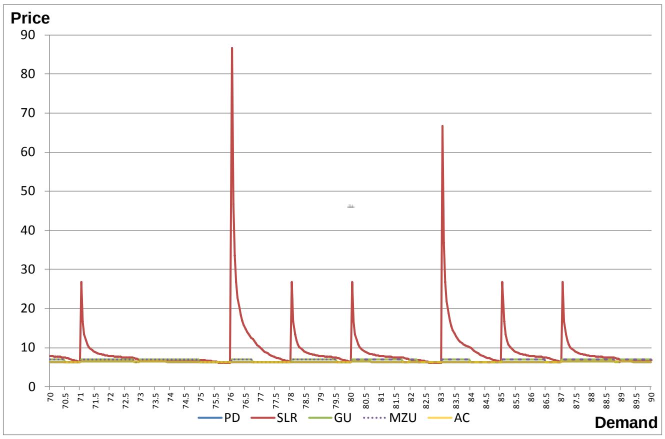

Figure 3.12: Prices under the different revenue-adequate approaches (granularity of demand 0.05 units).

reason, the SLR prices can be significantly higher than the AC prices, as seen in Figure 3.10.

In addition, it can be shown that the SLR price spikes may be excessively high when the allocated capacity to a committed unit is low. This is shown in Figure 3.12 that depicts the prices for demand levels between 70 and 90 with a granularity equal to 0.05.

Remark 3.4. *The prices of the PD and MZU schemes are comparable.*

The MZU scheme allows for internal transfers between the suppliers, and the uplifts are zero-sum. Hence, the profitable suppliers may transfer part of their revenues to the non-profitable ones, which in general keeps prices low. The PD scheme is discussed next.

Remark 3.5. *The PD scheme may yield lower prices than the MZU price, exchanging price for cost efficiency.*

In all cases where the PD price is lower than the MZU price, we observe that the dispatching is less efficient (positive percentage in Figure 3.11) than the optimal one.

This is the tradeoff for seeking price efficiency. A special case is the demand level 47.5, where there are multiple solutions (3 SmokeStack units with cost 301.5 versus 1 smokestack, 4 high tech and 1 med tech also with cost 301.5). The MZU price is equal to 7, whereas the PD price is equal to 6.347. This is due to the fact that the PD dispatch allocates 3 smokestack units but with equal quantities (15.833 each) which result in zero profits.

Interestingly, the AC prices seem to also be comparable to the PD and MZU prices. This is mainly due to the particularity of the example that MedTech has zero fixed cost, and hence can set the price. Note also that the average costs at full capacity range between 6.2857 and 7. As demand increases, the optimal allocation includes SmokeStack and HighTech at quantities that are close to the their capacity, complemented by MedTech units that have constant average cost, which explains the small variation in AC prices. However, this is not likely to be always the case.

To verify the above, we consider a smaller example with one SmokeStack and one HighTech unit, where we reverse the marginal costs, i.e., we assume that the Smokestack unit has a marginal cost equal to 2 and the HighTech unit has a marginal cost equal to 3. The maximum total capacity is now 23. In Figure 3.13, we present the price versus demand curve for this modified 2-unit example.

We observe that the AC and GU price exhibit price spikes when the demand is above 16, where the MILP solution allocates a very small quantity to the HighTech unit. Also, note that the price of PD is higher than the price of MZU even though the PD allocation deviates from the optimal. Nevertheless, for higher demand levels, the PD allocation is cost-efficient and the PD price coincides with the AC price. In this example, the MZU price is the lowest one.

3.4 Case Study: Greek Electricity Market

This section considers the Greek wholesale electricity market design that is a typical market with non-convexities. We explore the impact of a recovery mechanism that has been implemented in practice and compare it to an alternative bid/cost mechanism on a yearly basis. We also perform sensitivity analysis with respect to the hydro

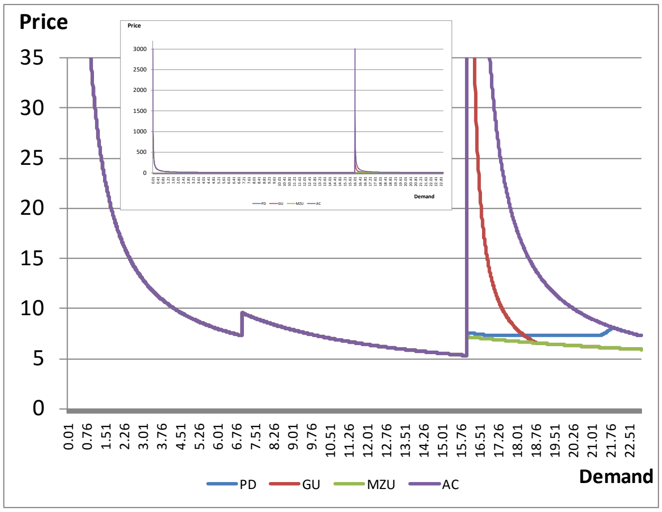

Figure 3.13: Prices under the different revenue-adequate approaches for a modified 2-unit example (granularity of demand 0.01 units).

production and the carbon price.

The remainder of this section is organized as follows. We present an overview of the Greek wholesale electricity market framework in Subsection 3.4.1. The basis of our analysis is the Day-Ahead Scheduling problem, which we list in Subsection 3.4.2. In Subsection 3.4.3, we provide the input data and the test cases that are used for the yearly simulations. We discuss the numerical results in Subsection 3.4.4.

3.4.1 The Greek Electricity Market Framework

The liberalization of national electricity markets in Europe, which was established by European Directive 96/92/EC, led to fundamental changes in the organization and operation of the electricity markets within the EU member states. In Greece, the liberalized electricity market is supervised by the Regulatory Authority for Energy (RAE) [87]. The entity that performed all the market operations was the Hellenic Transmission System Operator (HTSO) [88]. In February 2012, the HTSO was split into the Independent Power Transmission Operator (IPTO) [89], which performs the duties of system operation, maintenance and development, and the Hellenic Electricity Market Operator [90], which runs the day-ahead market. We shall refer to these entities as the TSO (Transmission System Operator) and the MO (Market Operator), respectively.

The Greek electricity pool market framework consists of a Wholesale Energy and Ancillary Services Market, which uses the following processes:

1. Day-Ahead Scheduling (DAS);
2. Dispatch Scheduling (DS);
3. Real Time Dispatch (RTD) operation;
4. Imbalances Settlement (IS).

Day-Ahead Scheduling

In DAS, the producers submit energy offers for all the available capacity of their generating units, and the load representatives submit load declarations for their customers. The energy offers are piece-wise constant step functions of price versus capacity with up to ten segments (blocks), where successive prices are strictly non-decreasing. Imports to Greece are treated as generation, and exports as demand. The injections of the renewable energy sources (RES) and the mandatory injections of hydro units are always scheduled, since they participate with non-priced offers, whereas all the other hydro units participate with priced offers. The generation units also submit reserve offers for primary and secondary reserve in the form of single price-versus-quantity pairs. Energy and ancillary services bid caps are established in order to prevent excessive price spikes, in case the available capacity is insufficient to meet the demand. The price cap for the energy offers is currently set to 150 €/MWh and for the reserve offers (currently, primary and secondary) is set to 10 €/MW. The system load in DAS is actually equal to the sum of non-priced load declarations of the load representatives for their local customers.

The day-ahead market clearing is performed simultaneously for all Dispatch Periods in the Dispatch Day. It is based on the co-optimization of the energy offers

(Energy Market), the reserve offers (Ancillary Services Market), and the commitment costs, subject to the energy balance equation, reserve requirements constraints, unit constraints (technical minimum/maximum, ramp rate, etc.), and system inter-zonal constraints (North-South corridor limits). It is noteworthy that co-optimization of energy and reserves is mostly used in U.S. day-ahead markets (e.g. New York, PJM), whereas most European markets clear only energy in the day-ahead markets.

The DAS solution produces a 24-hour production schedule for each generation unit, for each Dispatch Period of the Dispatch Day. In case the inter-zonal constraint is binding during a Dispatch Period, a Zonal Marginal Price (ZMP) is produced for each operational zone (North, South) for that Dispatch Period, and producers are paid at this ZMP. Load representatives, however, always pay at a uniform System Marginal Price (SMP), which is computed as a weighted average of the ZMPs of the operational zones, where the weights are the zonal generation quantities. Therefore, when the transmission constraint is not activated, as is currently the case due to the reduced load brought by the recent economic recession, ZMPs and SMP are equal. Imports are paid the day-ahead ZMP at the relevant interconnection, and exports are charged the day-ahead SMP. Dispatchable load (pumps) is charged the day-ahead SMP for its scheduled consumption.

The ancillary services are procured optimally in the day-ahead market, simultaneously with the energy market clearing. The day-ahead procedure produces hourly Ancillary Services Prices (ASPs) and hourly ancillary services cleared quantities.

Dispatch Scheduling

In the time period between DAS and RTD (the operational timescale), the TSO receives re-declarations from producers whenever there has been a change in the availability of their units, and responds to other changes in the system, such as variation in demand or modifications to interconnection flows. The TSO executes the DS procedure periodically and as needed, and adjusts unit commitment, scheduling, and ancillary services quantities, in response to the above changes and to enforce inter-zonal constraints. It uses ancillary services (principally primary, secondary and tertiary reserve) according to the day-ahead schedule, as modified by DS, and in real time, to keep the system in balance and respond to any contingencies.

Real-Time Dispatch

The generating units are subject to optimal re-dispatch in real time to meet actual system demand. Unlike other market designs worldwide (e.g. real-time balancing market in PJM and New York or intraday markets in Spain), Greece lacks a real-time market, and RTD uses the bids of the day-ahead market. The RTD procedure is executed every 5 minutes and produces an economic dispatch for the next 5-min time interval without performing any unit commitment; the unit commitment status is inherited from the DS.

Imbalances Settlement

The TSO determines hourly ex post Zonal Imbalance Marginal Prices (ZIMPs) and a weighted average hourly System Imbalance Marginal Price (SIMP) by executing the Ex Post Imbalance Pricing (EXPIP) procedure after the Dispatch Day. This procedure is similar to the DAS procedure but with actual system demand and actual unit commitment status and availability. The deviations of the generators are divided into "instructed" and "uninstructed" deviations. The "instructed" deviations are the real-time deviations of the actual production of a generating unit from the scheduled production by the DAS, due to a Dispatch Instruction by the TSO. Positive instructed deviations are paid the relevant ZIMP, whereas positive uninstructed deviations are not paid. Negative instructed deviations are charged as bid, whereas negative uninstructed deviations are charged the relevant ZIMP. Load deviations are settled at the SIMP. The TSO does not calculate any Ancillary Services Imbalance Price; the ancillary services quantities that are provided in real time are paid at the relevant DAS ASP.

3.4.2 The DAS Model

In the herein presented DAS model, we consider all available generation units, namely, thermal and hydro plants, imports, RES injections, exports, and pumping stations.

For the purposes of our analysis, we shall assume imports, exports and pumping as parameters of the optimization problem. Reserves include primary, secondary up and down, and tertiary reserve.

The producers submit energy offers for each hour of the following day as a step-wise function of price-quantity pairs, with successive prices being strictly non-decreasing. For simplicity, we assume a single price bid for energy. The producers also submit reserve bids as price-quantity pairs, as well as their commitment costs. Energy and reserves bid caps are established in order to prevent excessive price spikes, in case the available capacity is insufficient to meet the demand. The price cap for the energy offers is currently set at 150 €/MWh and for the reserve offers (primary and secondary) at 10 €/MW.

The technical characteristics of the generation units that constitute the constraints of the DAS problem include the technical minimum/maximum output, the AGC minimum/maximum, the maximum reserve availability, the minimum up/down times, and the ramp up/down limits, although the latter are rarely binding, and their impact on the annual results is negligible.

DAS can be formulated as a MIP problem. In what follows, we list the objective function, and the constraints. The nomenclature appears in Appendix B.

Objective Function

$$\text{Minimize} \left\{ \begin{array}{c} \text{Generation Cost} + \text{Ancillary Services Cost} + \\ \text{Commitment Cost} - \text{Load Revenues} + \text{Penalty Cost} \end{array} \right\}. \tag{3.32}$$

Cost Components:

$$\text{Generation Cost} = \sum_{u,l,t} b^{\mathrm{G}}_{u,l,t} \cdot q^{\mathrm{G}}_{u,l,t} + \sum_{i,l,t} b^{\mathrm{Imp}}_{i,l,t} \cdot q^{\mathrm{Imp}}_{i,l,t}, \tag{3.33}$$

$$\text{Ancillary Services Cost} = \sum_{u,t} \left\{ b^{\mathrm{PR}}_{u,t} \cdot q^{\mathrm{PR}}_{u,t} + b^{\mathrm{SRR}}_{u,t} \left(q^{\mathrm{SRU}}_{u,t} + q^{\mathrm{SRD}}_{u,t} \right) \right\}, \tag{3.34}$$

$$\text{Commitment Cost} = \sum_{u,t} f^{\mathrm{SD}}_{u} \cdot z^{\mathrm{SD}}_{u,t}, \tag{3.35}$$

$$\text{Load Revenues} = \sum_{o,l,t} b^{\text{LD}}_{o,l,t} \cdot q^{\text{LD}}_{o,l,t} + \sum_{i,l,t} b^{\text{Exp}}_{i,l,t} \cdot q^{\text{Exp}}_{i,l,t}, \tag{3.36}$$

$$\text{Penalty Cost} = \sum_{t} \left\{ \begin{array}{c} P^{\text{G}} \left(q_t^{\text{G, def}} + q_t^{\text{G, sur}} \right) + P^{\text{PR}} \cdot q_t^{\text{PR, def}} + \\ P^{\text{SR}} \left(q_t^{\text{SRU, def}} + q_t^{\text{SRD, sur}} \right) + P^{\text{TR}} \cdot q_t^{\text{TR, def}} \end{array} \right\}. \tag{3.37}$$

The objective, as defined in (3.32), is to minimize the as-bid cost for energy and ancillary services, plus the commitment cost, minus the load revenues. Equations (3.33)–(3.37) define the cost components of the objective function. The generation cost in (3.33) includes the as-bid cost of the generation units plus the cost of imports. The ancillary services cost includes the cost for primary and secondary reserve. The commitment cost in (3.35) includes only the shutdown cost. The shutdown cost is considered to be equal to the warm start-up cost, so as to deter the DAS program, which concerns a rather short horizon (24h) relatively to the time and effort it takes to start up some units, from reaching a solution in which it easily shuts down those units. A discussion on a longer DAS horizon is found in [91]. The load revenues includes the load declarations plus the exports. The penalty cost in (3.37) is an additional term imposed in the objective function to deal with problem infeasibilities.

Constraints

Energy Balance:

$$q_{u,t}^{\text{G, Total}} + \sum_{i} q_{i,t}^{\text{Imp, Total}} + q_t^{\text{G, def}} - q_t^{\text{G, sur}} = d_t^{\text{G}} + \sum_{o,t} q_{o,t}^{\text{LD, Total}} + \sum_{i} q_{i,t}^{\text{Exp, Total}} \qquad \forall t. \tag{3.38}$$

Ancillary Services Requirements:

$$\sum_{u,t} q_{u,t}^{\text{PR}} + q_t^{\text{PR, def}} \geq d_t^{\text{PR}} \qquad \forall t, \tag{3.39}$$

$$\sum_{u,t} q_{u,t}^{\text{SRU}} + q_t^{\text{SRU, def}} \geq d_t^{\text{SRU}} \qquad \forall t, \tag{3.40}$$

$$\sum_{u,t} q_{u,t}^{\text{SRD}} + q_t^{\text{SRD, def}} \geq d_t^{\text{SRD}} \qquad \forall t, \tag{3.41}$$

$$\sum_{u,t} q_{u,t}^{\text{TR}} + q_t^{\text{TR, def}} \geq d_t^{\text{TR}} \qquad \forall t. \tag{3.42}$$

Block Limits:

$$q_{u,l,t}^{\text{G}} \leq \bar{k}_{u,l}^{\text{G}} \qquad \forall u, l, t, \tag{3.43}$$

$$q_{o,l,t}^{\text{LD}} \leq \bar{k}_{o,l}^{\text{LD}} \qquad \forall o, l, t, \tag{3.44}$$

$$q_{i,l,t}^{\text{Imp}} \leq \bar{k}_{i,l}^{\text{Imp}} \qquad \forall i, l, t, \tag{3.45}$$

$$q_{i,l,t}^{\text{Exp}} \leq \bar{k}_{i,l}^{\text{Exp}} \qquad \forall i, l, t. \tag{3.46}$$

Technical Minimum:

$$q_{u,t}^{\text{G, Total}} - q_{u,t}^{\text{SRD}} \geq (z_{u,t}^{\text{St}} - z_{u,t}^{\text{AGC}})\underline{k}_u^{\text{G}} + z_{u,t}^{\text{AGC}} \cdot \underline{k}_u^{\text{AGC}} \qquad \forall u, t. \tag{3.47}$$

Technical Maximum:

$$q_{u,t}^{\text{G, Total}} + q_{u,t}^{\text{PR}} + q_{u,t}^{\text{SRU}} + q_{u,t}^{\text{TR}} \leq (z_{u,t}^{\text{St}} - z_{u,t}^{\text{AGC}})\bar{k}_u^{\text{G}} + z_{u,t}^{\text{AGC}} \cdot \bar{k}_u^{\text{AGC}} \qquad \forall u, t. \tag{3.48}$$

Reserves Availability:

$$q_{u,t}^{\text{PR}} \leq z_{u,t}^{\text{St}} \cdot \bar{k}_u^{\text{PR}} \qquad \forall u, t, \tag{3.49}$$

$$q_{u,t}^{\text{SRU}} + q_{u,t}^{\text{SRD}} \leq z_{u,t}^{\text{AGC}} \cdot \bar{k}_u^{\text{SRR}} \qquad \forall u, t. \tag{3.50}$$

Net Transfer Capacity:

$$\sum_{i \in I_x} q_{i,t}^{\text{Imp, Total}} - \sum_{i \in I_x} q_{i,t}^{\text{Exp, Total}} \leq k_{x,t}^{\text{NTC}} \qquad \forall x, t, \tag{3.51}$$

$$\sum_{i \in I_x} q_{i,t}^{\text{Exp, Total}} - \sum_{i \in I_x} q_{i,t}^{\text{Imp, Total}} \leq k_{x,t}^{\text{NTC}} \qquad \forall x, t. \tag{3.52}$$

Ramp Limits:

$$q_{u,t}^{\text{G, Total}} - q_{u,t-1}^{\text{G, Total}} \leq (1 - z_{u,t}^{\text{AGC}})\bar{q}_u^{\text{G,RRU}} + z_{u,t}^{\text{AGC}} \cdot \bar{q}_u^{\text{AGC,RRU}} + z_{u,t}^{\text{SU}} \cdot \bar{k}_{u,t}^{\text{G}} \qquad \forall u, t, \tag{3.53}$$

$$q_{u,t-1}^{\text{G, Total}} - q_{u,t}^{\text{G, Total}} \leq (1 - z_{u,t}^{\text{AGC}})\bar{q}_u^{\text{G,RRD}} + z_{u,t}^{\text{AGC}} \cdot \bar{q}_u^{\text{AGC,RRU}} + z_{u,t}^{\text{SD}} \cdot \bar{k}_{u,t}^{\text{G}} \qquad \forall u, t. \tag{3.54}$$

Minimum Up/Down Times:

$$\left(y_{u,t-1}^{\text{On}} - \underline{t}_u^{\text{Up}}\right)\left(z_{u,t-1}^{\text{St}} - z_{u,t}^{\text{St}}\right) \geq 0 \qquad \forall u, t, \tag{3.55}$$

$$\left(y_{u,t-1}^{\text{Off}} - \underline{t}_u^{\text{Down}}\right)\left(z_{u,t}^{\text{St}} - z_{u,t-1}^{\text{St}}\right) \geq 0 \qquad \forall u, t. \tag{3.56}$$

Maximum Energy:

$$\sum_t q_{u,t}^{\text{G, Total}} \leq k_u^{\text{G, Daily}} \qquad \forall u. \tag{3.57}$$

Availability and AGC:

$$z_{u,t}^{\text{St}} \leq z_{u,t}^{\text{Avail}} \qquad \forall u, t, \tag{3.58}$$

$$z_{u,t}^{\text{AGC}} \leq z_{u,t}^{\text{St}} \qquad \forall u, t. \tag{3.59}$$

Dependent Variables:

$$q_{u,t}^{\text{G, Total}} = q_{u,t}^{\text{G, NonPriced}} + \sum_l q_{u,l,t}^{\text{G}} \qquad \forall u, t, \tag{3.60}$$

$$q_{o,t}^{\text{LD, Total}} = q_{o,t}^{\text{LD, NonPriced}} + \sum_l q_{o,l,t}^{\text{LD}} \qquad \forall o, t, \tag{3.61}$$

$$q_{i,t}^{\text{Imp, Total}} = q_{i,t}^{\text{Imp, NonPriced}} + \sum_l q_{i,l,t}^{\text{Imp}} \qquad \forall i, t, \tag{3.62}$$

$$q_{i,t}^{\text{Exp, Total}} = q_{i,t}^{\text{Exp, NonPriced}} + \sum_l q_{i,l,t}^{\text{Exp}} \qquad \forall i, t, \tag{3.63}$$

$$z_{u,t}^{\text{SU}} = z_{u,t}^{\text{St}}\left(1 - z_{u,t-1}^{\text{St}}\right) \qquad \forall u, t, \tag{3.64}$$

$$z_{u,t}^{\text{SD}} = z_{u,t-1}^{\text{St}}\left(1 - z_{u,t}^{\text{St}}\right) \qquad \forall u, t, \tag{3.65}$$

$$y_{u,t}^{\text{On}} = \left(y_{u,t-1}^{\text{On}} + 1\right) z_{u,t}^{\text{St}} \qquad \forall u, t, \tag{3.66}$$

$$y_{u,t}^{\text{Off}} = \left(y_{u,t-1}^{\text{Off}} + 1\right)\left(1 - z_{u,t}^{\text{St}}\right) \qquad \forall u, t. \tag{3.67}$$

Initialization:

$$z_{u,0}^{\text{St}} = z_u^{\text{St,0}} \qquad \forall u, \tag{3.68}$$

$$y_{u,0}^{\mathrm{On}} = y_u^{\mathrm{On},0} \qquad \forall u, \tag{3.69}$$

$$y_{u,0}^{\mathrm{Off}} = y_u^{\mathrm{Off},0} \qquad \forall u, \tag{3.70}$$

$$q_{u,0}^{\mathrm{G,\ Total}} = y_u^{\mathrm{G,\ Total},0} \qquad \forall u. \tag{3.71}$$

Equality (3.38) presents the energy balance constraint and inequalities (3.39)–(3.42) the requirements for ancillary services. The block limits for the generation, load declaration, imports and exports are presented in (3.43)–(3.46). Technical minimum and maximum constraints appear in (3.47)–(3.48) and the reserve availability constraints appear in (3.49)–(3.50).

Net transfer capacity constraints are listed in (3.51)–(3.52) for each interconnection. Ramp limits are listed in (3.53)–(3.54) and minimum up/down times in (3.55)–(3.56). A maximum energy constraint is presented in (3.57). Availability and AGC constraints for the status variables are listed in (3.58)–(3.59). Constraints (3.60)–(3.63) define the total quantities for energy generation, load declarations and imports/exports including a non-priced component. Constraints (3.64)–(3.67) define the start-up and shutdown variables and the counters for the time periods a generation unit has been on/off. Lastly, constraints (3.68)–(3.71) define the initial conditions.

Nonlinear constraints (3.55)–(3.56) and (3.64)–(3.67) can be replaced by linear inequalities, to turn the above MIP problem into an MILP problem.

More specifically, the definitions of the start-up and shut-down variables (3.64)–(3.65) can be replaced by the following inequalities:

$$z_{u,t}^{\mathrm{SU}} \geq z_{u,t}^{\mathrm{St}} - z_{u,t-1}^{\mathrm{St}} \qquad \forall u, t, \tag{3.72}$$

$$z_{u,t}^{\mathrm{St}} - z_{u,t-1}^{\mathrm{St}} + 1.1\left(1 - z_{u,t}^{\mathrm{SU}}\right) \geq 0.1 \qquad \forall u, t, \tag{3.73}$$

$$z_{u,t}^{\mathrm{SD}} \geq z_{u,t-1}^{\mathrm{St}} - z_{u,t}^{\mathrm{St}} \qquad \forall u, t, \tag{3.74}$$

$$z_{u,t-1}^{\mathrm{St}} - z_{u,t}^{\mathrm{St}} + 1.1\left(1 - z_{u,t}^{\mathrm{SD}}\right) \geq 0.1 \qquad \forall u, t. \tag{3.75}$$

The counters in constraints (3.66)–(3.67) can be replaced by the following inequalities, where M is a sufficiently large number.

$$y_{u,t}^{\mathrm{On}} \leq y_{u,t-1}^{\mathrm{On}} + 1 \qquad \forall u, t, \tag{3.76}$$

$$y_{u,t}^{\mathrm{On}} + (M+1)(1 - z_{u,t}^{\mathrm{St}}) \geq y_{u,t-1}^{\mathrm{On}} + 1 \qquad \forall u, t, \tag{3.77}$$

$$y_{u,t}^{\mathrm{On}} \leq M \cdot z_{u,t}^{\mathrm{St}} \qquad \forall u, t, \tag{3.78}$$

$$y_{u,t}^{\mathrm{Off}} \leq y_{u,t-1}^{\mathrm{Off}} + 1 \qquad \forall u, t, \tag{3.79}$$

$$y_{u,t}^{\mathrm{Off}} + (M+1) z_{u,t}^{\mathrm{St}} \geq y_{u,t-1}^{\mathrm{Off}} + 1 \qquad \forall u, t, \tag{3.80}$$

$$y_{u,t}^{\mathrm{Off}} \leq M(1 - z_{u,t}^{\mathrm{St}}) \qquad \forall u, t. \tag{3.81}$$

Lastly, the minimum up and down time constraints (3.55)–(3.56) can be expressed by the following inequalities, where we introduced two auxiliary integer (non-negative) variables.

$$y_{u,t-1}^{\mathrm{aux}(1)} - y_{u,t}^{\mathrm{On}} + z_{u,t}^{\mathrm{St}} - \underline{t}_u^{\mathrm{Up}} \left(z_{u,t-1}^{\mathrm{St}} - z_{u,t}^{\mathrm{St}} \right) \geq 0 \qquad \forall u, t, \tag{3.82}$$

$$\left(y_{u,t-1}^{\mathrm{Off}} - y_{u,t}^{\mathrm{Off}} + 1 - z_{u,t}^{\mathrm{St}} - y_{u,t-1}^{\mathrm{aux}(2)} - \underline{t}_u^{\mathrm{Down}} \right) \left(z_{u,t}^{\mathrm{St}} - z_{u,t-1}^{\mathrm{St}} \right) \geq 0 \qquad \forall u, t. \tag{3.83}$$

The auxiliary variables are defined by equalities (3.84) and (3.85), which are also non-linear, and can be replaced by inequalities (3.86) – (3.88) and (3.89) – (3.91), respectively.

$$y_{u,t-1}^{\mathrm{aux}(1)} = y_{u,t-1}^{\mathrm{On}} \cdot z_{u,t-1}^{\mathrm{St}} \qquad \forall u, t, \tag{3.84}$$

$$y_{u,t-1}^{\mathrm{aux}(2)} = y_{u,t-1}^{\mathrm{Off}} \cdot z_{u,t-1}^{\mathrm{St}} \qquad \forall u, t, \tag{3.85}$$

$$y_{u,t-1}^{\mathrm{aux}(1)} \leq y_{u,t-1}^{\mathrm{On}} \qquad \forall u, t, \tag{3.86}$$

$$y_{u,t-1}^{\mathrm{aux}(1)} + M(1 - z_{u,t-1}^{\mathrm{St}}) \geq y_{u,t-1}^{\mathrm{On}} \qquad \forall u, t, \tag{3.87}$$

$$y_{u,t-1}^{\mathrm{aux}(1)} \leq M \cdot z_{u,t-1}^{\mathrm{St}} \qquad \forall u, t, \tag{3.88}$$

$$y_{u,t-1}^{\mathrm{aux}(2)} \leq y_{u,t-1}^{\mathrm{Off}} \qquad \forall u, t, \tag{3.89}$$

$$y_{u,t-1}^{\mathrm{aux}(2)} + M(1 - z_{u,t-1}^{\mathrm{St}}) \geq y_{u,t-1}^{\mathrm{Off}} \qquad \forall u, t, \tag{3.90}$$

$$y_{u,t-1}^{\mathrm{aux}(2)} \leq M \cdot z_{u,t-1}^{\mathrm{St}} \qquad \forall u, t. \tag{3.91}$$

The formulation that results after those replacements is a MILP problem that can be modeled and solved with any MILP solver. Once the MILP problem is solved, an LP problem is created by fixing the integer variables at their optimal values (marked with an asterisk), and dropping the constraints that involve only integer variables. Hence, constraints (3.55)–(3.56) and (3.64)–(3.67) have been replaced by the following equalities:

$$z_{u,t}^{\mathrm{St}} = z_{u,t}^{\mathrm{St}(*)} \qquad \forall u,t, \tag{3.92}$$

$$z_{u,t}^{\mathrm{SU}} = z_{u,t}^{\mathrm{SU}(*)} \qquad \forall u,t, \tag{3.93}$$

$$z_{u,t}^{\mathrm{SD}} = z_{u,t}^{\mathrm{SD}(*)} \qquad \forall u,t. \tag{3.94}$$

The LP formulation allows for the calculation of clearing prices using marginal pricing theory [2]. The energy clearing price is then determined as the shadow price of the energy balance constraint (3.38).

3.4.3 Input Data and Test Cases

The objective of this section is to evaluate the recovery mechanism that has been implemented by the TSO with respect to an alternative bid/cost recovery mechanism. To obtain meaningful results, we evaluate this impact on an annual basis, by iteratively solving the daily market model. For each day of the year, we solve the market model twice: the first time by executing the DAS, and the second time by executing the EXPIP, as previously described. The DS and RTD processes are not considered in our analysis. In this Case Study, the sole difference between the DAS and the EXPIP lies in the system load and the RES forecast errors; hence the dispatching of the conventional generation units is also different. The remaining parameters are kept constant to facilitate the comparisons.

In what follows we list the input data, and describe the test cases that are used for the simulations.

Input Data

The input data refer to an instance representing the Greek electricity market for the year 2011.

Table 3.3: Conventional generation units (as of December 2011).

Unit Type	**Number of Units**	**Installed Capacity (MW)**	**Cost-Based Energy Offers (€/MWh)**
Lignite	18	4,456	29-45
CCGT	10	3,976	71-106
OCGT	3	147	108.8-109
Gas	2	339	108-118
Oil	4	698	112-116
Hydro	15	3,016	120
Total Capacity:		**12,632**	

System Load and Reserve Requirements For the system hourly load (DAS system load declarations, and ex post/actual system load) and the reserve requirements, we used the data of the year 2011, which are publicly available in [88]. The penalty coefficients for the violation of the constraints (3.38)–(3.42) were set at 25,000 (€/MWh) for the energy balance, 20,000 (€/MW) for the primary reserve, 15,000 (€/MW) for the secondary reserve (both up and down) and 10,000 (€/MW) for the tertiary reserve.

Conventional Generation The conventional generation units in operation are shown in Table 3.3.

The energy offers include also the emissions cost calculated with a value of 7 €/tCO2e, and as a reference reflect cost-based bidding. For the reserve bids, we assumed that the generation units bid at the average prices that were observed in 2011, i.e., 1 €/MW for the primary reserve, 4.3 €/MW for the secondary reserve up, and 6.8 €/MW for the secondary reserve down.

The maintenance schedule and the outage rate for the conventional generation units (thermal and hydro) were assumed to be the same as in the year 2011. For the needs of our analysis, we generated Bernoulli-distributed random outages for each day based on the Equivalent Demand Forced Outage Rate (EFORD) values, which provide a measure of the probability that a generation unit will not be available due to a forced outage, and assumed a 2-day outage repair time. For simplicity, we did not consider each hydro unit separately; instead we considered an aggregate unit, with a total available capacity of 2,570MW, taking into account the average EFORD of the hydro units. For the mandatory hydro production, we used the data for the hydro production in 2011.

Imports, Exports, Pumping, RES For imports, exports and the pumping profile we used the DAS data for the year 2011. For the purposes of our analysis we assumed that the DAS values remain constant in real time and therefore are the same with the ex post ones. For the RES injections, we used the forecasts in the DAS and the actual (ex post) values in the IS.

Test Cases

The test cases under consideration are listed below.

Case 1 : *All units bid at their cost.* This case serves as a reference.

Case 2 : *Bidding strategy for private units with current carbon price.* In this case, 4 lignite units and 5 Combined Cycle Gas Turbine (CCGT) units (privately-owned) employ a bidding strategy different than cost-bidding. The lignite units bid 15% higher than their variable cost during off-peak hours, whereas they bid at cost during peak hours. The CCGT units bid 15% higher than their variable cost during peak hours, whereas they bid at cost during off-peak hours. We have distinguished between peak and off-peak hours so that they both represent 50% of the total hours for the year 2011 load levels. The median was 5,909MW; values above 5,909MW are considered peak, whereas values below 5,909MW are considered off-peak. The justification for this strategy is that lignite units are more likely to be price-makers during off-peak hours, whereas the CCGT units are more likely to be price-makers during peak hours.

Case 2 refers to a normal hydro production (neither dry nor wet year), and a relatively low carbon price, equal to 7 €/tCO2e. It now becomes the "business as usual" case, on which we perform the following sensitivity analysis.

Sensitivity analysis with respect to the hydro production

Case 2D : *Case 2 for a Dry Year*

Case 2W : *Case 2 for a Wet Year*

Sensitivity analysis with respect to the carbon price

Case 2M : *Case 2 for a Medium Carbon Price*

Case 2H : *Case 2 for a High Carbon Price*

Cases 2M and 2H refer to carbon prices equal to 15 €/tCO2e and 30 €/tCO2e respectively.

Recovery Mechanisms

For each of the aforementioned six cases, we examine the performance of the following two recovery mechanisms:

1. Mechanism A: The recovery mechanism that has been in use in the Greek market, which provides:

   a) explicit compensation for the commitment costs in case these costs are incurred as a result of the market outcome (generation scheduling), and

   b) additional payments so that the generation unit ends up with a profit equal to 10% of its variable cost, in case this profit is not reached through the market revenues for energy, considering both the day-ahead market and the imbalances settlement.

2. Mechanism B: A bid/cost recovery mechanism, which compensates the generation units so that they recover their as-bid costs.

Table 3.4: Annual aggregate results.

	Row	Recov. Mech.	(Current) Case 1	(Business as Usual) Case 2	Hydro Production Sensitivity Analysis Case 2D	Case 2W	Carbon Price Sensitivity Analysis Case 2M	Case 2H
PAYMENTS (M€)								
Energy Payments	1		3279.8	3483.7	3505.2	3476.3	3650	3961.8
Recovery Payments	2a	A	155.5	125.8	124.1	125.0	125.4	147.7
	2b	B	29.8	28.1	27.9	28.5	27.8	38.8
PRICES (€/MWh)								
WASMP	3		70.67	75.11	75.58	74.92	78.72	85.53
Uplift (due to recovery)	4a	A	3.04	2.46	2.42	2.44	2.45	2.88
	4b	B	0.58	0.55	0.55	0.56	0.54	0.76
PROFITS (M€)								
Thermal Generation	5a	A	986.0	1119.5	1142.3	1106.5	912.3	557.0
	5b	B	860.3	1021.8	1046.1	1009.9	814.7	448.0
Lignite	6a	A	885.5	1013.0	1027.6	1005.9	803.5	423.3
	6b	B	832.6	958.4	974.3	949.7	750.1	370.4
4 Lignite (*)	7a	A	213.0	244.4	249.7	242.6	193.6	103.4
	7b	B	201.2	232.1	238.7	230.7	182.0	94.4
Gas	8a	A	99.7	105.6	114.0	99.7	108.0	132.9
	8b	B	27.7	63.4	71.8	60.3	64.6	77.7
5 Gas (*)	9a	A	80.1	62.4	69.5	58.0	65.4	85.9
	9b	B	21.1	46.7	52.8	44.1	49.1	62.2
ENERGY (TWh)								
Thermal Generation	10		42.55	42.51	43.34	41.69	42.53	42.56
Lignite	11		29.29	29.26	29.30	29.17	29.27	27.18
4 Lignite (*)	12		6.91	6.87	6.90	6.83	6.85	5.32
Gas	13		13.19	13.19	13.98	12.46	13.18	15.28
5 Gas (*)	14		10.80	7.82	8.54	7.27	8.01	9.93
Oil	15		0.07	0.07	0.07	0.07	0.07	0.06
Hydro Generation	16		3.66	3.69	2.86	4.51	3.69	3.69

3.4.4 Numerical Results and Discussion

We modeled the market model and the simulation procedure with GAMS 23.7.3 [86] and solved it with CPLEX 12.3 solver on an Intel Core i5 at 2.67GHz, with 6GB RAM. In Table 3.4, we present the annual aggregate results of the simulations for each Case and Mechanism. The units that employ a bidding strategy other than bidding at cost, i.e., the 4 lignite units and 5 gas (CCGT) units, are marked with an asterisk (*).

General Remarks on Recovery Mechanisms A and B

The recovery payments are significantly higher under Mechanism A (current) than under Mechanism B (proposed). They range from 124.1 to 155.5 M€ under Mechanism A, and from 27.8 to 38.8 M€ under Mechanism B (see rows 2a and 2b). Recall that the current recovery mechanism explicitly compensates for the commitment costs, which amount to about 65 M€ on average, annually (not shown in the results). In addition, the generation units are guaranteed a profit of 10% of their variable cost, which also leads to high recovery payments particularly for the extra-marginal gas units. The resulting uplifts (due to the recovery payments) that are passed onto the consumption range from 2.42 to 3.04 €/MWh for Mechanism A, whereas they range from 0.54 to 0.76 €/MWh for Mechanism B (see rows 4a and 4b). Also, note that the profits of the thermal generation units are higher under Mechanism A than under Mechanism B in all cases (see rows 5a and 5b).

Remarks on the "Business as Usual" Case (Case 2)

The "business as usual" case (Case 2) leads to higher energy prices, compared to Case 1, which is quite expected as a result of the bidding strategy of the 4 lignite and 5 gas units. The weighted average SMP (WASMP) is increased by about 4.5 €/MWh (see rows 1 and 3, Cases 1 and 2).

Case 2 also leads to reduced energy generation from the private gas units, compared to Case 1, by about 27% (see row 14, Cases 1 and 2), as a result of their higher bids. Their profits are reduced by 22% under Mechanism A, whereas they increase under Mechanism B (see rows 9a and 9b). This is an indication that the bidding strategy performs well under Mechanism B. Hence, the other gas units are likely to respond to this strategy, and this would be an interesting direction for further research.

Note that the bidding strategy that is employed by the 4 lignite units does not significantly affect their annual production; lignite still serves the base load.

Remarks on the Hydro Production Sensitivity Analysis

In the dry year case (Case 2D), the hydro production is substituted by gas (see rows 13 and 16, Cases 2 and 2D), and vice versa in the wet year case (Case 2W). The results show a small increase in the WASMP in Case 2D, and a small decrease in Case 2W (see row 3, Cases 2, 2D, 2W). The recovery payments do not exhibit significant changes (see rows 2a and 2b, Cases 2, 2D, 2W).

Remarks on the Carbon Price Sensitivity Analysis

The increased carbon price (Case 2H) reduces the profits of the lignite units, due to the high emissions cost (see rows 6a and 6b, Cases 2 and 2H). The WASMP and therefore the energy revenues increase with the carbon price (see row 3, Cases 2, 2M, 2H).

Higher energy revenues but also higher variable costs create an ambiguous outcome, as they are opposing forces in the need for recovery payments. Under the medium carbon price scenario (Case 2M), these two forces produce more or less the same outcome as under the low carbon price scenario (Case 2), whereas under the high carbon price scenario (Case 2H), the outcome is higher recovery payments (see rows 2a and 2b, Cases 2, 2M, 2H).

3.5 Conclusions

In this chapter, we illustrated the behavior of prices and profits as a function of the demand for the various pricing schemes discussed in Chapter 2. The graphs presented for the two-supplier model verified the properties mentioned in the exact analysis, and provided visual representation of the closed form expressions and the comparisons.

We also used Scarf's example, as modified by Hogan and Ring [16], as a common test-bed to elaborate on several pricing schemes that address the issue of pricing in non-convex market designs, without the need for external uplifts, i.e., the GU, MZU, AC, SLR, and PD schemes. We showed that SLR generates the highest price, which exhibits particularly high spikes at certain demand levels. The prices of GU, MZU, PD, and AC are comparable and contained, with AC being the highest. Notably, the PD price is not always greater than or equal to the MZU price, as in our two-supplier model (see Proposition 2.11). This is because in Scarf's example, the PD scheme has more flexibility in trading off price efficiency for cost efficiency, since there are more than two units and unit types to reallocate. The containment of the AC and GU prices is due to the choice of parameter values. We also showed that by modifying these values, the AC and GU prices also exhibit spikes. This is in line with our finding that the GU, AC, and SLR prices can be excessively high.

The model in Scarf's example, as modified by Hogan and Ring [16], is more general than our two-supplier model, since it involves three types of suppliers, where each type comes in a finite number of units. However, the numerical example itself, used for demonstration purposes, is only an instance of that model; hence, the results and conclusions are specific to that instance. The ability to generalize them is further limited by the assumption that the med-tech type has zero fixed cost and a marginal cost which is higher than the average cost at full capacity of the other types, and by the restricted (discretized) range of demand values for which the pricing schemes were evaluated. In fact, for the demand levels examined, all tested schemes generated prices at most equal to the highest marginal cost of med-tech. In our two-supplier model, this would be equivalent to considering only the case $f_I = 0$ and $b_I > b_i k_i + f_i$, which, from Figures 2.3 and 2.6, is degenerate, because it leads to prices at most equal to b_I under all schemes.

Finally in the context of the Greek electricity market, we compared the cost recovery mechanism that has been implemented in practice against an alternative bid/cost mechanism on a yearly basis. We also performed sensitivity analysis with respect to the hydro production and the carbon price. In this Case Study, we did not aim at evaluating the incentive compatibility of the recovery mechanisms. Instead, we adopted several assumptions on the bidding behavior of the generating units, and focused on the annual magnitude of the recovery payments under these assumptions. We also discussed the implications on the energy prices, units' profits and energy generation mix. The results showed that the recovery payments are significantly lower under the alternative bid/cost recovery mechanism. However, the latter mechanism may lead to units submitting particularly high bids, in an attempt to take advantage of the bid-based recovery payments. We will discuss these design issues in the following chapters. Lastly, we should note that in order to enhance the confidence in our results, many more scenarios should be tested with respect to other parameters, such as the fuel prices, the unit outages, the load and RES forecast errors.

Part II

Bidding in Markets with Non-Convexities under Marginal Cost Pricing

Chapter 4

Equilibrium Outcomes in a Duopoly with Non-Convex Costs

4.1 Introduction

In Chapters 2 and 3, our focus was on pricing mechanisms in markets with non-convex costs, for which we provided detailed analytical and numerical results. In this chapter, we are interested in exploring equilibrium outcomes when bidding in markets with non-convexities under marginal pricing. We study recovery mechanisms that maintain marginal pricing as the market signal, and provide additional side-payments to the market participants. In particular, we consider a) cost-based recovery mechanisms in the context of electricity auctions, and b) bid-based recovery mechanisms that involve make-whole payments on an as-bid basis. As a test-bed for our analysis we employ a duopoly model, for which we identify equilibrium outcomes. The purpose of our analysis is to shed some light and reveal interesting properties on the bidding behavior of the participants rather than to provide an exhaustive equilibria characterization for all possible cases.

Our starting point is the work of Fabra *et al.* [76] who characterized the bidding behavior and market outcomes in a basic duopoly model with asymmetric marginal costs and capacities. We extend this model by introducing a fixed cost component and we study recovery mechanisms that ensure that the suppliers will not exhibit losses while participating in an electricity auction. In effect, we introduce side-payments in the decentralized auction model, so that the market participants will no longer have to internalize their fixed costs in their offers. At first, we assume a simple fixed cost recovery (FCR) mechanism, i.e., a mechanism that fully compensates the participants for their fixed costs, whenever they occur, thus allowing the players to compete with their bids reflecting their marginal cost. We show that such a mechanism actually results in the design of [76]; for completeness, and in order to have a uniform notation, we present the equilibrium outcomes. An alternative option, which is proposed in this thesis is to compensate the players for their losses, with what we call a recovery mechanism with "loss-related profits" (LPR mechanism). Such a mechanism allows for a positive profit that is proportional to the losses. We show that this mechanism has some nice and interesting properties, and can be designed in such a way that results in lower total payments, and lower or equal equilibrium prices, compared to the FCR mechanism.

For the bid-based recovery mechanisms, on the other hand, our starting point is the IP+ pricing scheme. We consider a stylized capacity-constrained duopoly, where we add a recovery mechanism that compensates potentially incurred losses and occasionally allows for positive profits. The basic difference in the assumptions compared to the the previous duopoly is that both marginal and fixed costs are taken into account to determine the outcome, as in a traditional unit commitment problem that determines the commitment and dispatch of the generation units. This yields a rather non-trivial electricity auction. At first, we consider a standard "bid/cost recovery" mechanism, which is the standard IP+ pricing scheme that unconditionally provides make-whole payments based on the as-bid costs. The equilibrium outcomes under IP+, for the symmetric-capacity case, are presented in [77, 78]. Again, for the sake of completeness, we briefly list these outcomes for the slightly more general case where there is a price cap in the bids. We also introduce a modified version of the same mechanism in which the make-whole payments are provided under the condition that the offered bids are within a certain regulated margin from the actual marginal costs. We refer to this mechanism as "bid/cost recovery with regulated cap" or "rcIP+" for short, where "rc" denotes the "regulated cap" which is added to the standard IP+ pricing scheme. For this case, we identify and discuss equilibrium outcomes.

The remainder of this chapter is organized as follows. In Section 4.2, we present the duopoly model that is used for our analysis, for which we employ a game theoretic methodology to identify equilibria in pure strategies, whenever they exist. In Section 4.3, we examine cost-based recovery mechanisms in the context of electricity auctions. We first consider the simple FCR mechanism and we show that the equilibrium outcomes are essentially the same as in [76]. We also introduce the LPR mechanism, and we discuss the equilibrium outcomes for both of these mechanisms. In Section 4.4, we examine bid/cost recovery mechanisms, where the side-payments compensate on an as-bid basis. In Section 4.5, we conclude and provide directions for further research.

4.2 Duopoly Model Setting

We consider a single-period duopoly with two suppliers that have asymmetric constant marginal costs, c_1 and c_2, asymmetric constant fixed costs, f_1 and f_2, and asymmetric capacities, k_1 and k_2, where we assume without loss of generality that $k_1 < k_2$. For ease of exposition, in this chapter, we denote by i the supplier with the lowest marginal cost, and by I the supplier with the highest marginal cost, i.e., $c_i < c_I$. We do not consider the case of equal (symmetric) costs.

The suppliers compete to satisfy a deterministic and inelastic demand, d. The two suppliers submit bids b_1 and b_2 for their marginal costs to an auctioneer (typically a market or system operator in the context of electricity markets). These bids must be greater than or equal to their true marginal costs (c_1 and c_2) and lower than or equal to a price cap, denoted by P, i.e., $c_1 \leq b_1 \leq P$, and $c_2 \leq b_2 \leq P$.

The suppliers aim to maximize their profits by optimally selecting their bids. In the absence of a recovery mechanism, the gross profits for supplier n, $\tilde{\pi}_n$, are equal to the commodity payments, ρ_n, minus his total costs. In the presence of a recovery mechanism, the total payments, τ_n, are the sum of the commodity payments, ρ_n, and the side-payments (depending on the recovery mechanism), σ_n. The net profits π_n are equal to the total payments minus the total costs. These quantities are defined and related as follows:

$$\rho_n = \lambda q_n, \tag{4.1}$$

$$\tilde{\pi}_n = \rho_n - (c_n q_n + f_n z_n) = (\lambda - c_n) q_n - f_n z_n, \tag{4.2}$$

$$\tau_n = \rho_n + \sigma_n = \lambda q_n + \sigma_n, \tag{4.3}$$

$$\pi_n = \tau_n - (c_n q_n + f_n z_n) = (\lambda - c_n) q_n - f_n z_n + \sigma_n = \tilde{\pi}_n + \sigma_n. \tag{4.4}$$

where λ is the uniform commodity price, q_n the allocated quantity for supplier n, and z_n is a variable denoting that supplier n is committed, and hence defined as

$$z_n = \begin{cases} 1, & \text{if } q_n > 0, \\ 0, & \text{if } q_n = 0. \end{cases} \tag{4.5}$$

4.3 Cost-Based Recovery Mechanisms

In this section, we consider the cost-based recovery mechanisms. In Subsection 4.3.1, we present the model assumptions that are specific to this type of mechanisms. In Subsection 4.3.2, we present the fixed cost recovery mechanism, and in Subsection 4.3.3 the recovery mechanism with loss-related profits. In Subsection 4.3.4, we discuss design issues for the two mechanisms.

4.3.1 Model Assumptions

For a given bid profile $\mathbf{b} = (b_1, b_2)$, and demand level d, the uniform price λ will be:

$$\lambda(d; \mathbf{b}) = \begin{cases} \phi_1 b_1 + \phi_2 b_2, & \text{if } d \le k_1, \\ b_2, & \text{if } k_1 < d \le k_2, \\ \phi_2 b_1 + \phi_1 b_2, & \text{if } d > k_2, \end{cases} \tag{4.6}$$

where ϕ_n is the "ranking probability" for supplier n, $n = 1, 2$, with

$$\phi_i = \begin{cases} 1, & \text{if } b_i \le b_I, \\ 0, & \text{if } b_i > b_I, \end{cases}, \qquad \phi_I = 1 - \phi_i. \tag{4.7}$$

(Recall that i is the supplier with the least marginal cost, i.e., $c_i < c_I$; hence, in case of equal bids we assume that the tie is solved in favor of supplier i.)

For supplier $n, n = 1, 2$, the allocated quantity q_n will be:

$$q_n(d; \mathbf{b}) = \phi_n \min\{d, k_n\} + \phi_m \max\{0, d - k_m\}, m = 1, 2, n \neq m. \tag{4.8}$$

Also, for $n, m = 1, 2$ with $n \neq m$, we have

$$z_n(d; \mathbf{b}) = \begin{cases} 1, & \text{if } d \leq k_m \text{ and } \phi_n = 1 \\ & \text{(or) if } d > k_m, \\ 0, & \text{otherwise.} \end{cases} \tag{4.9}$$

The total payments (TPs) for both suppliers, including the side-payments of the recovery mechanism will be:

$$\text{TPs}(d; \mathbf{b}) = \sum_n \tau_n(d; \mathbf{b}) = \lambda(d; \mathbf{b}) \cdot d + \sum_n \sigma_n(d; \mathbf{b}). \tag{4.10}$$

In what follows, we identify Nash equilibria in pure strategies. As a general methodology, we derive the best responses $b_n^*(b_m)$ with $n, m = 1, 2$, $n \neq m$, and then we find equilibria in pure strategies. We examine three intervals for the demand:

(i) Low: $d \leq k_1$;

(ii) Intermediate: $k_1 < d \leq k_2$;

(iii) High: $d > k_2$.

For ease of exposition, we introduce the following parameters for demand levels, bids, and price intervals.

Demand levels:
$\theta^{(1)} = \frac{P-c_i}{P-c_I} k_I$, $\theta^{(2)} = k_I + \frac{c_I - c_i}{P-c_i} k_i$, $\theta^{(3)} = (1+\alpha)\frac{f_i}{c_I - c_i}$,
$\theta_n^{(4)} = k_n + (1+\alpha)\frac{f_m}{P-c_m}$, $\theta_n^{(5)} = k_n + \frac{k_m}{2\alpha}\left(\sqrt{1 + \frac{4\alpha(\alpha+1)f_m}{k_m(P-c_m)}} - 1\right)$,
with $\theta_n^{(4)} > \theta_n^{(5)}$, and $\lim_{\alpha \to 0} \theta_n^{(5)} = \theta_n^{(4)}$,
$\hat{\theta}_n^{(1)} = \frac{P-c_m}{P-c_n-\frac{f_n}{k_n}} k_n$, $\hat{\theta}_n^{(3)} = (1+\alpha)\frac{f_m}{c_n + \frac{f_n}{k_n} - c_m}$.

Bids and Price Intervals:
$b_n^{(1)}(d) = c_n + \frac{(P-c_n)(d-k_m)}{\min\{d,k_n\}}$, $b_n^{(2)}(d) = c_n + (1+\alpha)\frac{f_n}{d}$,
$b_n^{(3)}(d) = c_n + \frac{(1+\alpha)f_n}{k_n+\alpha(d-k_m)}$, $b_n^{(4)}(d) = c_n + \frac{(1+\alpha)f_n-(P-c_n)(d-k_m)}{\alpha(d-k_m)}$.
$\mathrm{B}_1 = \left[\max\{c_I, b_I^{(4)}\}, b_I^{(1)}\right]$, $\mathrm{B}_2 = \left[\max\{c_I, b_i^{(4)}\}, b_i^{(1)}\right]$,
$\mathrm{B}_3 = \left[\max\{c_I, b_i^{(1)}, b_i^{(3)}\}, \min\{b_I^{(3)}, b_I^{(4)}\}\right]$, $\mathrm{B}_4 = \left[\max\{b_I^{(1)}, b_I^{(3)}\}, \min\{b_i^{(3)}, b_i^{(4)}\}\right]$.
Note: For simplicity, the dependence on d may be implied, e.g., $b_n^{(1)}(d) \equiv b_n^{(1)}$.

4.3.2 FCR Mechanism

Under the FCR mechanism, the suppliers receive the full amount of fixed costs, whenever such costs exist. Therefore, the only impact of the fixed costs is that they are added to the payments that the auctioneer gives to the suppliers. The side-payments are given as follows:

$$\sigma_n(d;\mathbf{b}) = z_n(d;\mathbf{b})f_n \tag{4.11}$$

Therefore, from (4.4) the net profits of supplier n will be $\pi_n(d;\mathbf{b}) = (\lambda(d;\mathbf{b}) - c_n)q_n(d;\mathbf{b})$. We can rewrite the profits of supplier n, with $n,m = 1,2$, $n \neq m$, as:

$$\pi_n = \begin{cases} (b_m - c_n)q_n(d;\mathbf{b}), & \text{if } b_n \leq b_m \text{ and } d > k_n, \\ (b_n - c_n)q_n(d;\mathbf{b}), & \text{otherwise.} \end{cases} \tag{4.12}$$

As far as the equilibrium and price outcomes are concerned, the fixed costs have no influence on the bidding behavior of the suppliers. In fact, the equilibrium outcomes are described in [76]. We repeat the analysis here, in order to derive the formulas that match with the notation used in this chapter (e.g., we assume $0 < c_i < c_I$, we consider cases $k_i < k_I$ and $k_i > k_I$, and we show the results for the general case, i.e., without assuming that $c_i = 0$, as is the case in [76]).

Proposition 4.1. *The equilibrium outcomes for the FCR mechanism are presented in Table 4.1.*

Proof. The proof is found in Appendix C (Section C.1). □

Table 4.1: Equilibrium outcomes for the FCR mechanism.

No.	Equil. bids	Conditions	Price	Quantities	Total Payments
1a	$b_i^* = c_I,$	(a) $d \le k_1$ (or)	c_I	$q_i^* = d,$	$c_I d + f_i$
1b	$b_I^* = c_I$	(b) $k_I < d \le k_i, d \le \theta^{(1)}$		$q_I^* = 0$	
2a	$b_i^* \le b_I^{(1)},$	(a) $k_i < d \le k_I$ (or)	P	$q_i^* = k_i,$	$Pd + f_i + f_I$
2b	$b_I^* = P$	(b) $d > k_2$		$q_I^* = d - k_i$	
3a	$b_i^* = P,$	(a) $k_I < d \le k_i, d \ge \theta^{(1)}$ (or)	P	$q_i^* = d - k_I,$	$Pd + f_i + f_I$
3b	$b_I^* \le b_i^{(1)}$	(b) $d > k_2, d \ge \theta^{(2)}$		$q_I^* = k_I$	

Remark 4.1. *Under the FCR mechanism, in any pure strategy equilibrium the highest accepted price offer is in the set* $\{c_I, P\}$.

Proposition 4.2. *Under the FCR mechanism, there exists* $d_P = \min\left\{k_i, \theta^{(1)}\right\}$ *such that:*

(i) *if* $d < d_P$*, in the unique pure strategy equilibrium the highest accepted price offer is* c_I*, and the total payments are* $c_I d + f_i$.

(ii) *if* $d > d_P$*, a pure strategy equilibrium exists with the highest accepted price offer equal to* P*, and the total payments equal to* $Pd + f_i + f_I$.

(iii) *if* $d = d_P$*, then:*

(a) *if* $d_P = k_i$*, case (i) holds, whereas*

(b) *if* $d_P = \theta^{(1)}$ *both cases (i) and (ii) hold (the equilibrium of case (i) is no longer unique).*

Proof. From Proposition 4.1, it is seen that the unique pure strategy equilibrium in which both suppliers bid at the cost of the most expensive (in terms of marginal cost) supplier exists when the following conditions hold: (a) $d \ge k_1$ (low demand) or (b) $k_I < d \le k_i$ (intermediate demand with $k_I < k_i$), and $d \le \theta^{(1)} = \frac{P-c_i}{P-c_I} k_I$.

Note that because $c_i < c_I$, it is $\frac{P-c_i}{P-c_I}k_I < k_I$. Combining these conditions, we obtain $d \leq \min\{k_i, \frac{P-c_i}{P-c_I}k_I\}$. Therefore, the unique pure strategy equilibrium $b_i^* = b_I^* = c_I$ exists for $d \leq \min\{k_i, \frac{P-c_i}{P-c_I}k_I\} = d_P$, and the total payments are TPs $= c_I d + f_i$. If $d > d_P$, a pure strategy equilibrium exists, with one player bidding at the price cap, and the total payments are TPs $= Pd + f_i + f_I$. In the case that $\min\{k_i, \frac{P-c_i}{P-c_I}k_I\} = \frac{P-c_i}{P-c_I}k_I = d_P$, then we can have two types of equilibria: $b_i^* = b_I^* = c_I$, and $b_i^* = P$, $b_I^* = c_I$. □

The highest accepted price offer is always the price of the supplier with the highest marginal cost in the case of low demand, and the price cap in the case of high demand. The price may also reach the price cap for intermediate demand realizations.

In case the supplier with the lower marginal cost also has the smaller capacity, i.e. $k_i < k_I$, the price reaches the cap for the whole interval $k_i < d \leq k_I$. The regulator has no means to prevent this outcome.

In the opposite case, i.e. if $k_I < k_i$, the price reaches the cap when $\theta^{(1)} \leq k_i$, for $\theta^{(1)} \leq d \leq k_i$. The condition $\theta^{(1)} \leq k_i$ implies that $\frac{P-c_i}{P-c_I} \leq \frac{k_i}{k_I}$. Note that the ratios in the left and the right hand side express the cost asymmetry (with respect to the price cap) and the capacity asymmetry respectively. It follows that if the cost asymmetry is higher than the capacity asymmetry, the price will not reach the price cap (for intermediate demand realizations). We also observe that both ratios are greater than 1. Assuming that the capacity asymmetry is given, the term $\frac{P-c_i}{P-c_I}$ is decreasing in the price cap P and asymptotically reaches 1 as $P \to \infty$. We can find a value of P that ensures that if $k_I < k_i$, then $\frac{P-c_i}{P-c_I} > \frac{k_i}{k_I}$, implying that the price does not reach the cap for the middle demand. In other words, we can adjust the price cap to move $\theta^{(1)}$ out of the intermediate demand interval (see Table 4.1) or equivalently set $d_P = \min\left\{k_i, \theta^{(1)}\right\} = k_i$ (see Proposition 4.2), implying that $\theta^{(1)} > k_i$ (with $k_I < k_i$). This value is given by the following corollary.

Corollary 4.1. *Under the FCR mechanism, for intermediate demand realizations:*

(i) *If $k_I < k_i$, the price at equilibrium does not reach P if $P < \frac{c_I k_i - c_i k_I}{k_i - k_I}$.*

(ii) *If $k_i < k_I$, the price always reaches P.*

Proof. Setting $\theta^{(1)} > k_i$ we find that for $P < \frac{c_I k_i - c_i k_I}{k_i - k_I}$ (note that $\frac{c_I k_i - c_i k_I}{k_i - k_I} > c_I$) the price does not reach P for the intermediate demand. Case (ii) follows immediately from Table 4.1. □

4.3.3 LPR Mechanism

Under the LPR mechanism, the suppliers are compensated on the basis of their losses. More specifically, we assume that if a supplier exhibits losses, he will be compensated by $(1+\alpha)$ times these losses, allowing for a positive profit that equals (α) times the losses, where $\alpha > 0$; we refer to parameter α as the "loss multiplier".

We define the losses as $L_n = \min\{\tilde{\pi}_n(d;\mathbf{b}), 0\} \leq 0$. The side-payments for the recovery mechanism with loss-related profits are:

$$\sigma_n(d;\mathbf{b}) = -(1+\alpha)L_n = (1+\alpha)\max\{-\tilde{\pi}_n(d;\mathbf{b}), 0\}. \tag{4.13}$$

Therefore, the net profits will be

$$\pi_n(d;\mathbf{b}) = \tilde{\pi}_n(d;\mathbf{b}) + \sigma_n(d;\mathbf{b}) = \max\{\tilde{\pi}_n(d;\mathbf{b}), -\alpha\tilde{\pi}_n(d;\mathbf{b})\}. \tag{4.14}$$

Proposition 4.3. *The equilibrium outcomes for the LPR mechanism are presented in Table 4.2.*

Proof. The proof is found in Appendix C (Section C.2). □

Remark 4.2. *Under the LPR mechanism, the lowest accepted price is c_i. It exists surely for low demand, and may exist for intermediate demand realizations.*

This remark is revealing of a "nice" property of the proposed recovery mechanism. The supplier with the least marginal cost may now bid at his cost, in order to benefit from the loss-related profits. In fact, bidding at cost is no longer a weakly dominated strategy. This is observed in the equilibrium outcomes 1a, 1b, and 3. The key feature of the LPR mechanism — that in case of losses, the profits are proportional to the magnitude of these losses — implies that the profits actually decrease with the price,

Table 4.2: Equilibrium outcomes for the LPR mechanism.

No.	Equil. bids	Conditions	Price	Quantities	Total Payments
1a **1b**	$b_i^* = c_i$ $b_I^* \leq b_i^{(2)}$	(a) $d \leq k_1$, $d \leq \theta^{(3)}$ (or) (b) $k_I < d \leq k_i$, $d \leq \min\{\theta^{(3)}, \theta_I^{(4)}\}$	c_i	$q_i^* = d$ $q_I^* = 0$	$c_i d + (1+\alpha) f_i$
2a **2b**	$b_i^* = b_I^* = c_I$	(a) $d \leq k_1$, $d \geq \theta^{(3)}$ (or) (b) $k_I < d \leq k_i$, $\theta^{(3)} \leq d < \theta^{(1)}$	c_I	$q_i^* = d$ $q_I^* = 0$	$c_I d$
3	$b_i^* \leq c_I$ $b_I^* = c_I$	$k_i < d \leq k_I$, $d \leq \theta_i^{(4)}$	c_I	$q_i^* = k_i$ $q_I^* = d - k_i$	$c_I d + (1+\alpha)\{f_I +$ $w[f_i - (c_I - c_i) k_i]\}$
3a **(*)**	$b_i^* = \left(c_i + \frac{f_i}{k_i}\right)^+$ $b_I^* = c_i + \frac{f_i}{k_i}$	$k_i < d \leq k_I$, $\hat{\theta}_i^{(3)} \leq d \leqslant \hat{\theta}_i^{(1)}$	$c_i + \frac{f_i}{k_i}$	$q_i^* = 0$ $q_I^* = d$	$\left(c_i + \frac{f_i}{k_i}\right) d$
3b **(*)**	$b_i^* = b_I^* = c_i + \frac{f_i}{k_i}$	$k_I < d \leq k_i$, $\hat{\theta}_I^{(3)} \leq d \leq \hat{\theta}_I^{(1)}$	$c_I + \frac{f_I}{k_I}$	$q_i^* = d$ $q_I^* = 0$	$\left(c_I + \frac{f_I}{k_I}\right) d$
4	$b_i^* = b_I^* = p$	$d > k_2$, $p \in \mathrm{B}_3$	p	$q_i^* = k_i$ $q_I^* = d - k_i$	$p k_i + [c_I - \alpha(p - c_I)]$ $\times(d - k_i) + (1+\alpha) f_I$
5	$b_i^* = p$ $b_I^* = p^-$	$d > k_2$, $p \in \mathrm{B}_4$	p	$q_i^* = d - k_I$ $q_I^* = k_I$	$p k_I + [c_i - \alpha(p - c_i)]$ $\times(d - k_I) + (1+\alpha) f_i$
6	$b_i^* \leq b_I^{(1)}$ $b_I^* = P$	$k_i < d \leq k_I$, $d \geq \theta_i^{(4)}$	P	$q_i^* = k_i$ $q_I^* = d - k_i$	Pd
7	$b_i^* = P$ $b_I^* \leq b_i^{(1)}$	$k_I < d \leq k_i$, $d \geq \max\{\theta^{(1)}, \theta_I^{(4)}\}$	P	$q_i^* = d - k_I$ $q_I^* = k_I$	Pd
8	$b_i^* \in \mathrm{B}_1$ $b_I^* = P$	$d > k_2$, $d \geq \theta_i^{(5)}$	P	$q_i^* = k_i$ $q_I^* = d - k_I$	Pd
9	$b_i^* \leq c_I$ $b_I^* = P$	$d > k_2$, $d \geq \theta_i^{(4)}$	P	$q_i^* = k_i$ $q_I^* = d - k_i$	Pd
10	$b_i^* = P$ $b_I^* \in \mathrm{B}_2$	$d > k_2$, $d \geq \theta_I^{(5)}$	P	$q_i^* = d - k_I$ $q_I^* = k_I$	Pd

Note: $w = 1$, if $c_I < c_i + \frac{f_i}{k_i}$, and $w = 0$, otherwise. For (*) refer to Refinement 4.1 below.

and the supplier's interest is in bidding as low as possible (i.e. at its marginal cost). This property, a.k.a. the "revelation principle", for supplier i in case of equilibria 1a, 1b, and for supplier I in case of equilibrium 3, is a particularly attractive property of this mechanism, for the demand realizations that it exists.

Remark 4.3. *Under the LPR mechanism, in low demand realizations, the price is in the set* $\{c_i, c_I\}$.

The regulator can actually select an appropriate value for the loss multiplier α, to ensure that the price at equilibrium will be c_i for the entire low-demand region or for as large a part of the low-demand region as desired, at the expense of higher total payments.

Corollary 4.2. *Under the LPR mechanism, for the low demand realizations, the price at equilibrium is c_i for the whole low-demand region if $\alpha > \frac{c_I - c_i}{f_i} k_1 - 1$, with $\alpha > 0$.*

Proof. It should be $\theta^{(3)} = \frac{(1+\alpha) f_i}{c_I - c_i} > k_1 \Rightarrow \alpha > \frac{c_I - c_i}{f_i} k_1 - 1$. □

Corollary 4.2 implies that the price at equilibrium for low demand is always c_i if $\frac{f_i}{c_I - c_i} \geq k_1$. Therefore, the larger the fixed cost of the supplier with the least marginal cost, and the lower the cost asymmetry of the two suppliers, the more likely the price to be c_i in low demand realizations.

Remark 4.4. *Under the LPR mechanism, in intermediate demand realizations, the price is in the set $\{c_i, c_I, c_1 + \frac{f_1}{k_1}, P\}$.*

We observe that, subject to conditions, there can be more than one equilibrium outcomes for the same demand level. One of these outcomes involves a price equal to $c_1 + \frac{f_1}{k_1}$, with supplier 1 being the supplier with the smaller capacity, where supplier 2 satisfies the whole demand.

Refinement 4.1. *Under the LPR mechanism, equilibrium outcomes 3a and 3b can be ruled out as unstable and thus unlikely to occur.*

Justification. Firstly, let us note that equilibrium outcomes 3a and 3b yield a price equal to $c_1 + f_1/k_1$ (since $k_1 < d \leq k_2$, $i = 1$ in case of outcome 3a, and $i = 2$ in case of outcome 3b). Considering the potential equilibrium outcomes for the case of intermediate demand, we observe that outcomes 3a and 3b suffer from a particular unattractive property. Namely, the profits of supplier 2, who satisfies the whole demand, depend on the bid of the indifferent supplier 1. In particular, if the indifferent supplier 1 chooses a slightly lower bid, then the immediate response of supplier 2 would be to move to another equilibrium bid, according to his best response, setting the price in the set $\{c_1, c_2, P\}$. Since we have assumed a non-cooperative game, it would be "wise" and risk-averse for supplier 2, to select a bid different from $c_1 + \frac{f_1}{k_1}$, and then hope that the indifferent rival will not undercut him. It is noteworthy that a bid from supplier 2 equal to $c_1 + \frac{f_1}{k_1}$ is somewhat of a "teaser-bid" for supplier 1, since the profits of the latter are zero even if he undercuts supplier 2; in this case, if supplier 1 anticipates such a "teaser-bid", he may respond by selecting a "punishment" non-equilibrium bid and produce with zero profits instead of not producing at all (also with zero profits). □

We also note that equilibrium outcomes 3a and 3b are the only outcomes (among the ones listed in Table 4.2), in which the indifferent supplier 1 can reduce the profits of supplier 2, by undercutting and leading him to select another equilibrium bid.

Remark 4.5. *Under the LPR mechanism, for the intermediate demand realizations, the price at equilibrium reaches the price cap P if one of the following conditions holds:*

(i) $k_i < k_I$ *and* $\theta_i^{(4)} \leq k_I$,

(ii) $k_I < k_i$ *and* $\max\{\theta^{(1)}, \theta_I^{(4)}\} \leq k_i$.

Under the LPR mechanism, the design parameters are P and α. We can adjust these parameters to ensure that the price will not reach the cap for the case of intermediate demand. Such a design will also lead to efficient equilibrium outcomes.

Corollary 4.3. *Under the LPR mechanism, for the intermediate demand realizations, the price at equilibrium will not reach the price cap P if one of the following conditions holds:*

(i) $k_I < k_i$, *and* $\frac{P-c_i}{P-c_I} > \frac{k_i}{k_I}$ *or* $\alpha > \frac{k_i - k_I}{\frac{f_i}{P-c_i}} - 1$, *with* $\alpha > 0$,

(ii) $k_i < k_I$, *and* $\alpha > \frac{k_I - k_i}{\frac{f_I}{P-c_I}} - 1$, *with* $\alpha > 0$.

From this Corollary, it follows that for a given price cap we can adjust loss multiplier α in order to avoid an equilibrium price that would reach the price cap. However, as we increase α, the payments will be higher for low demand realizations.

There is also the possibility of adjusting the price cap, in order to ensure that for a given α, the price will not reach the price cap for intermediate demand realizations.

Corollary 4.4. *Under the LPR mechanism, for the intermediate demand realizations, the price at equilibrium will not reach the price cap P if one of the following conditions holds:*

(i) $k_I < k_i$, *for* $P < \max\left\{\frac{c_I k_i - c_i k_I}{k_i - k_I}, c_i + \frac{(1+\alpha) f_i}{k_i - k_I}\right\}$,

(ii) $k_i < k_I$ *for* $P < c_I + \frac{(1+\alpha) f_I}{k_I - k_i}$.

Of course, there is the possibility of adjusting simultaneously parameters α, and P to achieve the desired outcome.

Remark 4.6. *Under the LPR mechanism, in high demand realizations, the price at equilibrium may lie in price intervals with upper bounds lower than the price cap. It may also happen that these equilibria coexist with other equilibria which involve one supplier bidding at the price cap.*

Equilibrium outcomes 4 and 5 are not points but intervals. The corresponding prices, which are considered to be Nash equilibria, are also intervals, but this is not necessarily a drawback of the mechanism. In fact, these price intervals are a consequence of the main feature of the LPR mechanism that the profits are related to the losses. Such equilibria exist because one supplier is increasing his profits with increasing prices, whereas the other is increasing his profits with prices that are decreasing. These opposing forces lead to the outcomes 4 and 5, where every price in the specific price intervals B_3 and B_4 respectively can be an equilibrium. The advantage of these equilibria is that there can be equilibrium outcomes with prices less than the price cap even in high demand realizations.

Proposition 4.4. *For all demand realizations, the price at equilibrium of the LPR mechanism, considering only the refined (under refinement 4.1) equilibria, is always less than or equal to the respective price of the FCR mechanism for all* $\alpha > 0$.

The proof of this proposition is easily derived by direct comparisons of the equilibrium outcomes in Propositions 4.1 and 4.3.

Proposition 4.5. *There exists* $\alpha > 0$*, such that for all demand realizations, the total payments of the LPR mechanism, considering only the refined (under refinement 4.1) equilibria, are strictly lower than those of the FCR mechanism.*

The proof of this proposition is easily derived by direct comparisons of the equilibrium outcomes in Propositions 4.1 and 4.3.

We return to Remark 4.6 and the issue of the existence of a continuum of equilibria. We observe that in outcomes 4 and 5, the supplier who produces at full capacity, say supplier n, has increasing profits with the price, and therefore maximizes his profits for the upper bound of the interval. Supplier m, on the other hand, maximizes his profits for the lower bound of the interval. However, only supplier n has the possibility of moving from the equilibrium bid, without altering the profits. Nevertheless, supplier n is not completely indifferent because supplier m would follow this movement resulting in higher profits for m and lower profits for n. Therefore, supplier n is deterred to move if he is found in an equilibrium. Although Propositions 3 and 4 are valid for all possible equilibria in high demand, one can argue that the player who maximizes his profits with increasing prices, and whose best response is to undercut his rival, is the one who may make the equilibrium price of the upper bound more likely to occur than any lower price in the equilibrium interval. With similar arguments, one can assume that whenever such an equilibrium interval coexists with an equilibrium in which one supplier bids at the price cap, the latter equilibrium is more likely to occur.

Remark 4.7. *Under the LPR mechanism, in high demand realizations, there may exist equilibria in which either of suppliers can bid sufficiently low to induce the other bid at the price cap and serve the residual demand (see outcomes 8,9 and 10).*

These two types of equilibria are equivalent in terms of price and total payments; they differ only in terms of efficiency.

4.3.4 Design Issues

The design parameters are the price cap P for both the FCR and the LPR mechanisms, and additionally the loss multiplier α for the LPR mechanism.

Role of Price Cap P

In general, the price cap is set by the regulator so as to avoid high price spikes and mitigate market power. How to set the price cap may relate to the incentives in capacity investment, although these incentives can be provided with other means as well. Low price caps may not provide adequate incentives. Low price caps are also a sign of strong regulation, indicating that the market would not work well most probably because of the lack of competition. On the other hand, high price caps may lead to high commodity prices, overcharging the consumers. Lastly, the value of the price cap in an electricity market in which trading in the interconnections takes place, should take into account the respective price caps of the neighboring markets.

FCR mechanism: For the FCR mechanism, the price cap is the only design parameter for the regulator. As it was shown earlier, an adjustment of the price cap has no effect on the demand range for which the price reaches the cap when the supplier with the smaller capacity has also the lower marginal cost; in this case, the price at equilibrium will reach the cap for the intermediate demand realizations.

In the case that the supplier with the lower marginal cost is also the largest one, the regulator has the possibility to adjust the price cap and prevent the price from reaching the cap (see Corollary 4.1). The critical demand level above which the price reaches the cap can be controlled, so that for price caps higher than $\hat{P} = \frac{c_I k_i - c_i k_I}{k_i - k_I}$, the price reaches the cap in the intermediate demand realizations. Generally, the higher the cap, the larger the total payments for demand higher than d_P. The lower the cap, the better the outcome in terms of both total payments and price (although the latter is not affected for caps lower than $\hat{P}$).

LPR mechanism: For the LPR mechanism, it was shown in Corollary 4.4 that for a given α, the adjustment of the price cap can also ensure that for the intermediate demand realizations, the price will not reach the cap. Unlike in the FCR mechanism, this adjustment can work for all cases (i.e., either $k_i < k_I$ or $k_I < k_i$). In fact, for the case of $k_I < k_i$, the (critical) value of the price cap (e.g. $\hat{P}$) is greater than or equal to the respective value in the FCR mechanism (see Corollaries 4.1 and 4.4).

Additionally, low price caps may move the point at which the price reaches the cap beyond the lower bound of high demand (i.e. for $d > k_2$). This means that there may be a sub-interval of high demand where the multiple equilibria 4 or 5 exist, and none of equilibria 8, 9 or 10 exists. Assuming that the price is the upper bound of B_3 or B_4 (worst case in terms of price), this price is less than the cap.

We observe that if we move $\theta_n^{(5)}$, for $n = 1, 2$, in the high-demand region, by lowering the price cap (the lower the cap the higher the values of $\theta_n^{(5)}$), then we have the above outcome. We can show that since $k_1 < k_2$, this condition holds for lowering the price cap such that $P < c_2 + \frac{(1+\alpha)f_2 k_2}{(k_2-k_1)[k_2+\alpha(k_2-k_1)]}$; the lower the cap, the further the point at which the price reaches the cap.

Role of Loss Multiplier α

This is the main design parameter in the LPR mechanism.

By introducing this parameter, we can achieve equilibrium outcomes for low and occasionally for intermediate demand realizations that are equal to the least marginal cost (i.e. c_i). By relating the profits in case of losses with the magnitude of the losses in a proportional way, the supplier with the least marginal cost is actually induced to reveal his cost, in order to achieve higher profits. This outcome is achieved for any value of $\alpha > 0$; generally, the larger the value of α, the larger the demand interval for which this outcome will occur.

Corollary 4.5. *Under the LPR mechanism, the price at equilibrium is c_i (the least marginal cost) for $d \leq \min\{k_i, \theta^{(3)}, \theta_I^{(4)}\}$.*

We note that $\theta^{(3)}, \theta_I^{(4)}$ are increasing with increasing α. Therefore, we can select an appropriate value for α such that the price is c_i, for $d \leq k_i$. The trade-off from increasing α (in order to keep the price at the marginal cost) is the higher total payments.

We have also shown that when the price reaches the cap for intermediate demand, we can adjust loss multiplier α, and prevent this outcome (see Corollary 4.3). In case the cost asymmetry $\frac{P-c_i}{P-c_I}$ is less than (or equal to) the capacity asymmetry $\frac{k_i}{k_I}$, then the regulator may select $\alpha > \frac{k_2-k_1}{\frac{f_2}{P-c_2}} - 1$, with $\alpha > 0$, to ensure that the price will not reach the cap for the intermediate demand realizations. This way we move point $\theta_1^{(4)}$ out of the intermediate demand interval (i.e. $\theta_1^{(4)} > k_2$). Note that in case $k_2 - k_1 < \frac{f_2}{P-c_2}$, this is anyway achieved for any $\alpha > 0$. However, as it was mentioned, the trade-off from increasing α is the higher total payments for lower demand realizations. Nevertheless, the total payments may be reduced in intermediate demand realizations because of the lower prices, and therefore the impact on total payments may not be negative.

Lastly, it is possible that increasing α may move the point at which the price reaches the cap beyond the lower bound of high demand (i.e. for $d > k_2$). This can be achieved by moving $\min\{\theta_i^{(5)}, \theta_I^{(5)}\}$ further in the high demand interval.

4.4 Bid-Based Recovery Mechanisms

In this section we discuss the bid-based recovery mechanisms. In Subsection 4.4.1 we provide the assumptions for the duopoly model that are specific for this type of mechanisms. In Subsection 4.4.2 we present two bid/cost recovery mechanisms under consideration, one standard and the other with a regulated cap, for which we identify equilibria in Subsection 4.4.3. We discuss the results as well as design issues in Subsection 4.4.4.

4.4.1 Model Assumptions

Apart from bids b_1, and b_2 for the marginal costs, we assume that the two suppliers submit truthful bids for their fixed costs f_1 and f_2. The auctioneer accepts the offers of the suppliers after solving the following bid/cost minimization problem.

$$\underset{q_1,q_2,z_1,z_2}{\text{minimize}} \; b_1q_1 + b_2q_2 + f_1z_1 + f_2z_2, \tag{4.15}$$

subject to:

$$q_1 + q_2 = d \qquad (\lambda), \tag{4.16}$$

$$q_1 \leq k_1z_1, \quad q_2 \leq k_2z_2, \tag{4.17}$$

$$q_1, q_2 \geq 0, \quad z_1, z_2 \in \{0,1\}. \tag{4.18}$$

Problem (4.15)–(4.18) is a MILP problem, with decision variables z_n and q_n, $n = 1, 2$. Note that this problem is identical to the MILP problem presented in Chapter 2. Assuming that the integer variables z_n are set to their optimal values, the shadow price of constraint (4.16) of the resulting LP problem represents the marginal price λ the suppliers are paid for their offered quantities.

The profit maximization problem for supplier n, $n = 1, 2$, is therefore described as follows:

$$\underset{b_n}{\text{maximize}} \ \pi_n = \{(\lambda - c_n)q_n - f_n z_n + \sigma_n\}\,, \tag{4.19}$$

subject to:

$$c_n \leq b_n \leq P. \tag{4.20}$$

The problem described in (4.15)–(4.18) and (4.19)–(4.20) is a typical bilevel problem where, in the upper level, supplier n aims to maximize his profit, solving (4.19)–(4.20), and in the lower level, the auctioneer aims to minimize the system cost, solving (4.15)–(4.18). If we consider both suppliers' problems, the overall problem falls into the category of equilibrium problems with equilibrium constraints (EPECs).

In what follows, for ease of exposition, we introduce the following parameters for demand levels, and subscripts for suppliers.

We use subscripts r and R to denote the supplier with the lower and higher total cost, respectively; in case of equal total costs, r is the supplier with the lower marginal cost (supplier i), i.e.,

$r = i,\ R = I,\quad \text{if } b_i d + f_i \leq b_I d + f_I,$

$r = I,\ R = i,\quad \text{if } b_I d + f_I < b_i d + f_i.$

We use subscripts M and m to denote the supplier with the higher and lower fixed cost, respectively, as follows:

$M = \arg\max_n \ f_n, \quad m = \arg\min_n \ f_n.$

We define the following demand levels (with $\beta > 0$):

$d_L = \frac{f_M - f_m}{P - c_M},$

$\theta_n = \frac{\beta + f_n/k}{P - c_n},\ \hat{\theta}_n = \frac{f_n/k}{P - c_n - \beta},\ n = 1, 2,$

$\theta_3 = \frac{c_I - c_i}{P - c_I},\ \theta_4 = \frac{c_I - c_i}{\beta},\ \theta_5 = \frac{c_i + \beta - c_I}{\beta}.$

In what follows, we assume symmetric capacities, i.e., $k_1 = k_2 = k$, and we distinguish the following two cases for demand:

1. “Low Demand” for $d \leq k$, and

2. “High Demand” for $k < d \leq 2k$.

4.4.2 Bid/Cost Recovery Mechanisms

In this subsection we present the mechanisms that we consider for dealing with the issue of non-convexities. The recovery mechanisms that we examine are:

(i) a standard bid/cost recovery, namely IP+ pricing scheme, which unconditionally allows for make-whole payments based on the as-bid costs, as in the centralized design in [77, 78], and

(ii) a modified version of the same mechanism, referred to as rcIP+, in which the make-whole payments are provided under the condition that the offered bids are within a certain regulated margin from the actual marginal costs.

IP+ Mechanism

The IP+ mechanism provides the suppliers with side-payments, a.k.a. make-whole payments in case they do not recover their as-bid costs. Specifically, if the marginal price λ is not sufficient to provide payments that will allow supplier n to recover his as-bid costs, then (additional) side-payments will be provided so that the committed supplier is paid at least his as-bid costs. This is a standard practice met as a “revenue sufficiency guarantee” in centrally committed markets. The side-payments and profits for supplier n are as follows:

$$\sigma_n = \max\left\{0, (b_n - \lambda)q_n + f_n z_n\right\}, \tag{4.21}$$

$$\pi_n = \max\left\{(\lambda - c_n)q_n - f_n z_n, (b_n - c_n)q_n\right\}. \tag{4.22}$$

In practice, one could say that this type of mechanism results in a hybrid uniform / pay-as-bid scheme; uniform because the suppliers are paid a uniform price, and pay-as-bid because the make-whole payments provided to the suppliers that do not recover their as-bid costs reminds of (and actually results in) a pay-as-bid scheme.

It is noted that under this type of mechanism the suppliers are guaranteed their as-bid costs even if they bid at the market price cap. This characteristic is not particularly attractive from a market design point of view. To overcome this drawback, we consider the following variation.

rcIP+ Mechanism

In the rcIP+ mechanism, the (make-whole) side-payments are received only if the supplier bids below a regulated cap, which is set lower than the market price cap P. We consider this regulated cap to be set at a certain margin, say β above the supplier's cost, i.e. the regulated cap for supplier n is equal to $c_n + \beta$. The side-payments $\hat{\sigma}_n$ are now given as follows:

$$\hat{\sigma}_n = \begin{cases} \sigma_n, & \text{if } b_n \leq c_n + \beta, \\ 0, & \text{if } b_n > c_n + \beta. \end{cases} \tag{4.23}$$

This variation of the IP+ mechanism is expected to induce the suppliers to behave less speculatively. Should the supplier decide to bid higher than the regulated cap, no side-payments are granted and incurring losses becomes possible. On the other hand, should the supplier decide to behave "reasonably" within the margin provided, then recovering the as-bid costs is guaranteed.

4.4.3 Equilibrium Analysis

Equilibrium Analysis - Low Demand

In the case of low demand, since either of the suppliers can satisfy all the demand, they compete in terms of average cost (or equivalently total cost).

Proposition 4.6. *Under the IP+ and rcIP+ mechanisms, if $d \leq k$, the equilibrium outcome is always cost-efficient, i.e., $q_r^* = d, \;\; q_R^* = 0, \;$ for $d \leq k$.*

Proof. In the case of low demand ($d \leq k$), the supplier with the lowest total cost can always underbid the other and satisfy all the demand to receive non-negative profits. If supplier r does not underbid (in terms of total as-bid cost), then he will end up with zero profits. □

Proposition 4.6 applies to both recovery mechanisms. However, the price at equilibrium and the side-payments are mechanism-specific.

Proposition 4.7. *Under the IP+ mechanism, if $d \leq k$, a unique set of pure strategy Nash equilibria exists, and the price at equilibrium is $\lambda^* = \min\left\{c_R + \frac{f_R - f_r}{d},\ P\right\}$.*

Proof. In the case of low demand ($d \leq k$), when $d \leq d_L$, supplier r, where $r = m$, may well bid at the price cap and still have a lower cost than supplier R, where $R = M$, even if supplier R bids at his cost c_R. In this case, the price reaches the price cap, as supplier r takes advantage of his lower fixed cost. When $d \geq d_L$, the outcome is a Bertrand-type equilibrium, and hence the proof is trivial. Namely, supplier r bids so that his total as-bid cost equals the total cost of supplier R. The bids at equilibrium are

$$b_r^* = c_R + \frac{f_R - f_r}{d}, \quad b_R^* = c_R \text{ for } d_L \leq d \leq k. \tag{4.24}$$

□

Proposition 4.7 also appears in [78], as Proposition 7(a), but without the price cap term and the minimization.

From (4.24), we observe that at equilibrium, supplier r may actually bid lower or higher than the marginal cost of supplier R, depending on the sign of the difference of their fixed costs.

Corollary 4.6. *Under the IP+ mechanism, if $d \leq k$, the commodity payments, (make-whole) side-payments, total payments, and profits of supplier r are given by*

$$\rho_r = \lambda^* d, \tag{4.25}$$

$$\sigma_r = f_r, \tag{4.26}$$

$$\tau_r = \rho_r + \sigma_r = \min\left\{(Pd + f_r, c_R d + f_R\right\}, \tag{4.27}$$

$$\pi_r = \min\left\{(P - c_r)d, (c_R - c_r)d + (f_R - f_r)\right\}. \tag{4.28}$$

In Figure 4.1, we show the total costs (upper figures) and the prices at equilibrium (lower figures) for the IP+ mechanism. Since we assumed that $c_i < c_I$, we distinguish between different cases regarding f_i and f_I.

In the upper figures, the difference between the higher total cost (or the dashed line indicating a slope equal to the price cap, whichever is lower) and the lower total cost represents the profits of the lower cost supplier, r.

Case (a) shows a situation where $f_i >> f_I$, such that $r = I$ for the entire interval of low demand. In case (b), f_i is still greater than f_I, but the difference between the two fixed costs is small enough so that above a certain demand level, $r = i$. Case (c) represents the situation with equal fixed costs. Finally, in case (d), $f_i < f_I$, and therefore, as supplier i is cheaper in terms of both marginal and fixed costs, $r = i$.

Proposition 4.8. *Under the rcIP+ mechanism, if $d \leq k$, the price at equilibrium, denoted by $\hat{\lambda}^*$, is $\hat{\lambda}^* = \lambda^*$, if $\lambda^* \geq c_r + \beta + f_r/d$, and $c_r + \beta$, otherwise.*

Proof. In the case of low demand ($d \leq k$), when $d \leq d_L$, we have $\lambda^* = c_r + \beta$ for $d \leq f_m/(P - c_m - \beta)$, and $\lambda^* = P$ for $d \geq f_m/(P - c_m - \beta)$. When $d \geq d_L$, if the bid at equilibrium of supplier r in the standard case is smaller than or equal to the regulated cap, i.e., if $c_R + (f_R - f_r)/d \leq c_r + \beta$, then we have the same equilibrium as in the standard case. If, however, $c_R + (f_R - f_r)/d > c_r + \beta$, then supplier r must compare his profits if he bids as in standard case but with no recovery, i.e., at $b_r = c_R + (f_R - f_r)/d$ with his profits if he bids at the regulated cap, i.e., at $b_r = c_r + \beta$. In the former case, his profits are $[c_R + (f_R - f_r)/d - c_r]d - f_r$, while in the second they are βd. The conditions for $\hat{\lambda}^* = c_r + \beta$ are $c_R + (f_R - f_r)/d \leq c_r + \beta + f_r/d$, whereas for $\hat{\lambda}^* = c_R + (f_R - f_r)/d$ they are $c_R + (f_R - f_r)/d \geq c_r + \beta + f_r/d$. $\square$

Proposition 4.9. *Under the rcIP+ mechanism, if $d \leq k$, the price at equilibrium, the side-payments and the total payments are less than or equal to their respective values under the IP+ mechanism.*

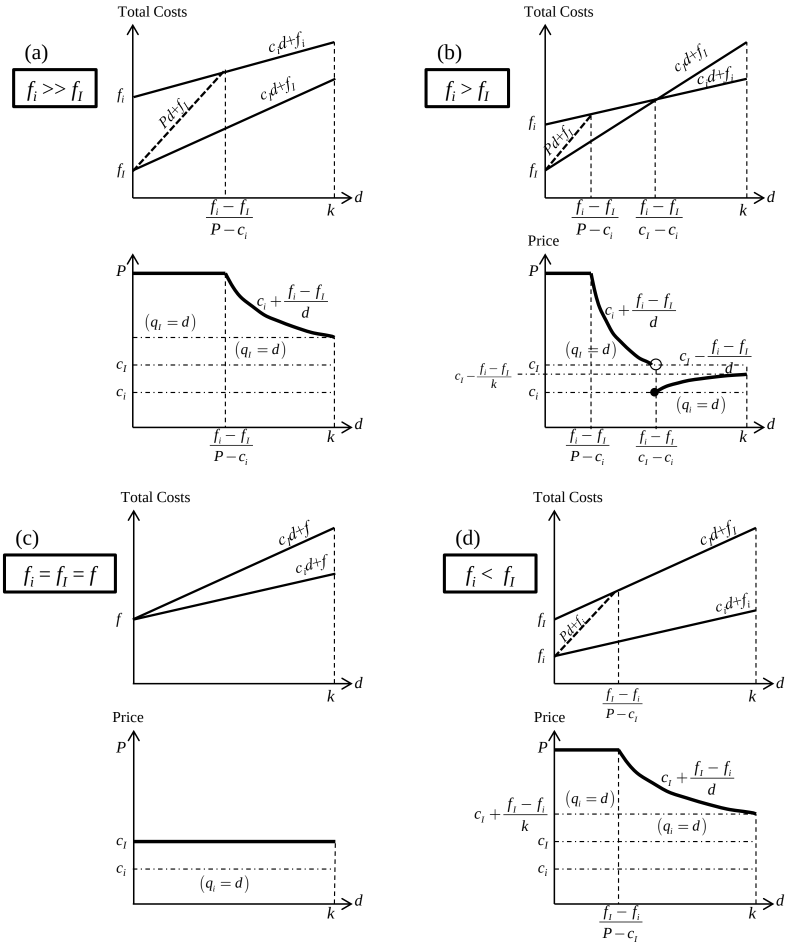

Figure 4.1: Total costs and equilibrium prices for the low demand case under the IP+ mechanism.

Proof. We only need to prove this statement for the price and the side-payments, as this would also yield the result for the total payments. For $d \leq d_L$, we have $\hat{\lambda}^* \in \{c_r + \beta, P\}$, so that $\hat{\lambda}^* \leq \lambda^* = P$. Also for $d \geq d_L$, we have $\hat{\lambda}^* \in \{c_R + (f_R - f_r)/d, c_r + \beta\}$, where $\hat{\lambda}^* = c_r + \beta$ only if $c_R = (f_R - f_r)/d > c_r + \beta$, which means that $\hat{\lambda}^* \leq \lambda^* = c_R + (f_R - f_r)/d$ always. Since the side-payments are now provided only if supplier r bids under the regulated cap, $c_r + \beta$, no side-payments will be provided whenever the price (or equivalently the bid of supplier r) is higher than the regulated cap. $\square$

Proposition 4.9 is particularly important, as it shows that for the low demand case rcIP+ results in better market performance than IP+. We shall refer to this outcome in more detail in the following section.

Corollary 4.7. *Under the rcIP+ mechanism, if $d \leq k$, the commodity payments, side (make-whole) payments, total payments, and profits of supplier r are given by*

$$\hat{\rho}_r = \hat{\lambda}^* d, \tag{4.29}$$

$$\hat{\sigma}^* = \begin{cases} \sigma_r = f_r, & \text{if } b_r \leq c_r + \beta, \\ 0, & \text{if } b_r > c_r + \beta, \end{cases} \tag{4.30}$$

$$\hat{\tau}^* = \begin{cases} \tau_r, & \text{if } b_r \leq c_r + \beta, \\ \tau_r - f_r, & \text{if } b_r > c_r + \beta, \end{cases} \tag{4.31}$$

$$\hat{\pi}^* = \begin{cases} \pi_r, & \text{if } b_r \leq c_r + \beta, \\ \pi_r - f_r, & \text{if } b_r > c_r + \beta. \end{cases} \tag{4.32}$$

Equilibrium Analysis - High Demand

In this case, both suppliers need to be committed to satisfy the demand. One supplier will be committed at full capacity, while the other will serve the residual demand. The fixed costs are not taken into account to determine the infra-marginal supplier, i.e., the one that will be committed at full capacity, since they enter objective function (4.15) as constants (note that $z_1 = z_2 = 1$). Hence, the decision of the auctioneer is based only on the marginal costs. It is expected that in this area of demand the suppliers may manipulate the price, since they both know in advance that they will be dispatched. Nevertheless, we show that the two mechanisms differ in terms of equilibrium outcomes.

Proposition 4.10. *Under the IP+ mechanism, if $d > k$, no equilibrium in pure strategies exists.*

Proof. The proof of this proposition is trivial. It can be derived using similar arguments as those used in Proposition 2 in [77]. This proposition also appears as Proposition 8 in [78]. We briefly sketch the proof for completeness, because we believe that the arguments would be useful for the reader for comparison reasons (with the rcIP+ case). First, we assume that such an equilibrium exists, and then show that such an assumption cannot hold. Since we solve ties by assuming that in case of equal bids, supplier i is considered infra-marginal, at equilibrium there will always be one supplier that will serve the residual demand (and set the price). If at equilibrium the suppliers place different bids, then bidding at the price cap will yield higher profits for the marginal supplier. An equilibrium where one supplier bids at the price cap cannot exist, because the best response of the other supplier would be to bid slightly lower than the price cap. However, this would mean that the marginal supplier would then generally have an incentive to slightly underbid and become infra-marginal. Actually, if the marginal supplier were i, it would suffice for him to bid same as supplier I, because this would render him (supplier i) the infra-marginal supplier; however, in this case, supplier I would still have an incentive to either underbid supplier i and become infra-marginal or bid at the price cap, and increase his profits. □

We note that mixed-strategy equilibria do exist in this case. We refer the interested reader to [77] and [78].

Proposition 4.11. *Under the rcIP+ mechanism, if $d > k$, equilibria in pure strategies exist under certain conditions involving either a supplier bidding at the price cap or both suppliers bidding at the regulated cap.*

Proof. The proof of Proposition 4.11 follows standard best response methodology. The equilibrium outcome involving both suppliers bidding at the regulated cap is shown in what follows:

$$b_i^* = c_i + \beta,\ b_I^* = c_I + \beta \text{ for } c_I \le c_i + \frac{f_i}{k} \text{ and } k(1+\theta_5) \le d \le k\left(1 + \min\{\theta_i, \hat{\theta}_I\}\right). \tag{4.33}$$

The equilibrium outcomes involving one supplier bidding at the price cap are shown in Table 4.3.

Table 4.3: Equilibria involving one supplier bidding at the price cap for the high-demand case under the rcIP+ mechanism.

No.	Equilibrium bids	Conditions for demand
1.	$b_i^* = P$ $b_I^* \leq c_i + \frac{(P-c_i)(d-k)}{k}$	$d \geq k\,(1 + \max\{\theta_i, \theta_3\})$
2.	$b_i^* = P$ $b_I^* \leq c_i + \beta\frac{d-k}{k}$	$d \geq k\left(1 + \max\{\hat{\theta}_i, \theta_4\}\right)$
3.	$b_I^* = P$ $b_i^* \leq c_I + \frac{(P-c_I)(d-k)}{k}$	$d \geq (1 + \theta_I)$
4.	$b_I^* = P$ $b_i^* \leq c_I + \beta\frac{d-k}{k}$	$d \geq (1 + \hat{\theta}_I)$

These outcomes hold under the assumption that $P \geq c_n + \beta + f_n/k$ or equivalently $\beta \leq P - c_n - f_n/k$, for $n = 1, 2$. This assumption ensures that the market price cap is high enough, so that the infra-marginal supplier, who is dispatched at full capacity, has higher profits if he bids at the price cap ($(P - c_n)k - f_n$) than if he bids at the regulated cap (βk). If this were not the case, the price cap would be redundant. □

4.4.4 Discussion

This section focuses on the design issues that are raised with the introduction of the regulated cap. In the IP+ mechanism, the regulator (or market designer) has limited control as the only design parameter is the price cap, P. The regulated cap parameter β provides the regulator with an additional means that can be used to mitigate market power inducing the suppliers to behave less speculatively.

Before looking into how parameter β affects the market outcome, it would be useful to define what the objectives of a market design should set. We single out the following three objectives as being important:

(i) Keep relatively low uplifts (side-payments). This creates an incentive to keep the value of β low.

(ii) Allow the suppliers to have sufficient profits. This creates an incentive to increase the value of β.

(iii) Mitigate market power. This needs to be combined with the above objectives since the value of β can be designed so as to avoid the price manipulation.

The question is, is there a value of β that satisfies all three objectives? The answer is of course not straightforward. A regulator would have to weigh the objectives subjectively, taking into account his long-term strategic goals (e.g., achieve low prices, motivate more suppliers to enter the market, etc.). The analysis of the previous section revealed some interesting properties of this modified mechanism, compared to the IP+.

Firstly, the regulated cap contributes to the reduction of the uplift, as it is clearly shown in the low-demand case (Proposition 4.9) where the comparison can be made directly for the pure-strategy equilibria.

Secondly, we observe that the introduction of the regulated cap creates a region in the area of low demand where the price does not reach the market price cap, which is the outcome in the IP+, but is set at the regulated cap. Should the regulator want to prevent the price cap as a market outcome, the value of β should be set so that

$$P \leq c_r + \beta + \frac{f_r}{d_L}, \tag{4.34}$$

which yields

$$\beta \geq P - c_m - \frac{f_m}{f_M - f_m}(P - c_M). \tag{4.35}$$

Thirdly, the introduction of the regulated cap is by its nature a regulatory action for mitigating market power, as the speculative behavior of a supplier is penalized by not receiving side-payments. We also believe that this mechanism is fair, as it guarantees positive profits that are proportional to the dispatched quantity, provided that the supplier bids within this regulated cap.

For the case of high demand, we observe from (4.33) that for an equilibrium with a price equal to the regulated cap of supplier I (the one with the highest marginal cost) to exist, condition $c_I \leq c_i + f_i/k$ must hold, i.e., the marginal cost difference (asymmetry) must be at most equal to f_i/k. In addition, condition $\theta_5 \leq \min\{\theta_i, \hat{\theta}_I\}$ must also hold. The lhs of this condition depends on β, c_i, and c_I. Note, e.g. that if $\beta \leq c_I - c_i$, then $\theta_5 \leq 0$, and therefore the condition holds; if in addition $c_I \leq c_i + f_i/k$, then the equilibrium in (4.33) exists. Also, note that the rhs increases with β and with the fixed costs. This is reasonable, since the higher the value of β, the more profitable bidding within the regulated cap should be. Also, the higher the fixed costs, the more beneficial it is for the supplier to select bidding within the regulated cap and receiving make-whole payments, than bidding outside this margin (e.g. at the price cap) but not receiving make-whole payments. Of course, for higher realizations of demand, bidding at the price cap may become more profitable for the marginal supplier.

Although an extreme value of $\beta = 0$ (or $\beta \to 0$) cannot be considered a good design, this special case may be interesting to understand the impact of β on the equilibrium outcomes. For the low demand case, as $\beta \to 0$, the equilibrium price from Proposition 4.7 is $\hat{\lambda}^* = c_r$, if $\hat{\lambda}^* \leq c_r + f_r/d$. This condition will generally hold if the demand is low enough or the fixed cost of supplier r is high enough. For the high demand case, the equilibrium outcome in (4.33), becomes $b_i^* = c_i, b_I^* = c_I$ for $c_I \leq c_i + f_i/k$ and $d \leq k\left[1 + \min\left\{f_i/(P - c_i), f_I/(P - c_I)\right\}\right]$. The (demand) region for this outcome becomes larger the higher the fixed costs and the lower the marginal costs.

Finally, we should point out that the conditions in terms of the demand, for the equilibrium bids given in (4.33) and in Table 4.3 to exist, may not cover the entire region of high demand, $k < d \leq 2k$, which means that in some parts of this region no equilibrium in pure strategies exists. If the regulator wanted to prevent the probabilistic behavior of the suppliers that is associated with mixed strategies, he could design β so as to limit or eliminate these parts.

In real markets, it is expected that the regulator would administratively select an appropriate value for β that would provide the desired economic signal to the market participants. As was discussed earlier, this value should not be very high, as this could result in high uplifts; it should also not be very low, in order to allow for some positive profits, which are substantial enough to provide an incentive to the supplier for bidding within this margin.

In Chapter 5, we study an instance of a real-sized day-ahead electricity market, with several generators, multiple periods, and numerous constraints, and examine the behavior of profit-maximizing generators under several recovery mechanisms. The simulation results demonstrate that the IP+ mechanism induces the generators to behave speculatively, and try to profit from high bids and the resulting recovery payments. On the other hand, the introduction of the regulated cap significantly reduces the uplifts and leads to a more stable bidding behavior.

4.5 Conclusions

In this chapter, we explored equilibrium outcomes for recovery mechanisms of different types. The FCR mechanism and the LPR mechanism assumed an electricity auction format where suppliers compete in terms of their bids for their marginal cost. For the latter mechanism, we should note that the selection of the loss multiplier α depends on the objective of the market designer. If the objective is to ensure low prices, we may accept high values of loss multiplier α that could over-compensate the marginal suppliers. We can take into account the probability of the demand realization in the longer-term, and select a value that will minimize the expected total payments. This value does not necessarily have to be the one that will assure that the price will not reach the price cap, but a smaller one, depending on the system parameters and the expected demand.

The insights gained from the stylized example employed in this chapter are quite encouraging. We note that we can keep the idea of the mechanism and the design tool of the loss multiplier α, and test this mechanism in real-sized systems, without necessarily having the assumptions made in the duopoly setting. In fact, we do this for a pool-market in Chapter 5, and the results are also quite encouraging; the expensive marginal and extra-marginal generation units showed bids close or equal to their cost and the uplifts associated with the recovery payments were quite low. This adds to the concept that the herein described LPR mechanism may exhibit incentive compatibility regardless of whether the fixed costs are included or not in the objective function.

One of the main outcomes of this mechanism is that it can address the problem of the compensation of the suppliers that incur losses due to their fixed costs (that are not reflected in their marginal costs offer). We showed that we can adjust the parameter of the compensation in order to avoid the price reaching the price cap in the case that the demand is not higher from the supplier with the higher capacity. We also showed that the total payments can be significantly lower under this mechanism compared with an approach that would compensate for the fixed costs whenever they exist.

As a weakness we record the existence of multiple equilibria in the case of high demand, and that we cannot know with certainty which one will actually be played; this is a general drawback of the Nash equilibrium solution concept. However, with simple refinements we can rule out some highly unlikely equilbria.

In addition, we considered the case of centrally committed electricity markets with non-convexities, that provide to suppliers make-whole payments to recover for potential as-bid losses. Apart from the IP+ mechanism that unconditionally allows for make-whole payments based on the as-bid costs, we considered a modified version, which we refer to as rcIP+, where the suppliers are entitled to the make-whole payments under the condition that the offered bids are within a certain regulated margin from the actual marginal costs (regulated cap). We examined a stylized duopoly with asymmetric costs (marginal and fixed) and symmetric capacities, and we identified equilibrium outcomes. The introduction of the regulated cap leads to equilibrium outcomes that outperform the ones of the IP+ mechanism in terms of prices and uplifts for the low demand case. It also leads to the existence of pure-strategy equilibria in the high demand case, whereas only mixed-strategy equilibria exist under IP+. We also discussed design issues for setting the value of regulated cap.

The analysis provided in this chapter is not exhaustive. In the future, it would be worthwhile to introduce capacity asymmetry in the model; this would create an intermediate demand level where the supplier with the largest capacity could satisfy all the demand. Furthermore, it would be interesting to study mixed-strategy equilibria for the rcIP+ mechanism, and consider the case of stochastic demand.

Chapter 5

Numerical Evaluation of Recovery Mechanisms in Electricity Markets

5.1 Introduction

In centralized day-ahead electricity markets with marginal pricing, unit commitment costs and capacity constraints give rise to non-convexities which may result in losses to some of the participating generating units. To compensate them for these losses, a recovery mechanism is required. In this chapter, we proceed with a numerical evaluation of recovery mechanisms with marginal cost pricing in the context of electricity markets.

We look at several alternative recovery mechanism designs that result in side-payments after the market is cleared (ex post). The mechanisms differ in the type and amount of payments with which they reimburse each generating unit that exhibits losses. The first design that we examine lets the losing units keep a fixed percentage of their variable costs. A variant of this design has been used in the Greek market. The second design lets the losing units keep a fixed percentage of their losses. The concept of this mechanism, which was referred to as recovery mechanism with loss-related profits (LPR mechanism), was discussed in Chapter 4 in the context of an electricity auction. The third design fully recovers their bids. This is the standard IP+ scheme currently deployed by System Operators in the U.S. Finally, we also look a variant of this design where the bids are recovered provided that they are within a certain set margin from their costs. This is the rcIP+ mechanism (bid/cost recovery with regulated cap), which we introduced in Chapter 4.

We also propose a methodology for evaluating the bidding strategy behavior of the participating units for each mechanism. This methodology employs an iterative numerical algorithm aimed at finding the joint optimal bidding strategies of the profit-maximizing units. We apply this methodology to evaluate the performance and incentive compatibility properties of each recovery mechanism on a test case model representing the Greek joint energy/reserve day-ahead electricity market. To make the optimization problem computationally tractable, we make certain simplifying assumptions, without loss of generality of the most important features of a realistic zonal market design. This analysis leads to results that allow us to gain insights and draw useful conclusions on the performance and incentive compatibility properties of the recovery mechanisms. Apart from their theoretical interest, these conclusions have significant practical implications, as various System Operators often modify the recovery mechanisms that they employ to attain a reasonable market outcome (e.g., see [92, 93] for proposals to modify the parameters or the rules of the recovery mechanisms used in California and Greece). Lastly, we perform sensitivity analysis with respect to key parameters and assumptions and we provide directions for further research.

The remainder of this chapter is organized as follows. In Section 5.2, we present the model of a joint energy/reserve day-ahead market problem that we use as a basis of our study. In Section 5.3, we present the main characteristics of the alternative recovery mechanisms. In Section 5.4, we develop a numerical methodology for assessing the incentive compatibility of each mechanism. In Section 5.5, we present the test case market model and we state the main assumptions used in the implementation of the methodology. We also list the performance measures used for evaluating the mechanisms and discuss relevant computational issues. In Section 5.6, we present the most important numerical results and discuss their implications regarding the performance and incentive compatibility properties of the mechanisms. We also take a closer look at the most promising mechanisms. In Section 5.7, we perform sensitivity analysis and discuss certain extensions to explore the accuracy and extendability of our results. Lastly, in Section 5.8, we summarize the most important findings, and provide directions for further research.

5.2 Joint Energy/Reserve Day-Ahead Market Problem

We consider a typical design of the joint zonal energy/ reserve day-ahead electricity market. To keep our analysis focused, we make several simplifying assumptions without loss of the most important features of a practical market design.

Specifically, we focus on thermal plants only; we do not consider hydro plants, renewable energy sources, and imports/exports. Also, we consider only one type of reserve, namely, tertiary spinning reserve; an extension to include other types of reserves (such as primary, secondary) is straightforward. The producers submit energy offers for each hour of the following day, as a step-wise function of price-quantity pairs, and reserve bids, as single price-quantity pairs. Current practices of System Operators put substantial more restrictions on the submitted unit commitment costs than on the energy bids. The reason is that market power mitigation procedures are currently used only to mitigate the energy bids, but not the unit commitment bids. With this in mind, we assume that producers submit their true start-up, shutdown, and no-load costs. Misstating the commitment costs could be examined in the context of market power mitigation methodologies. This could be an issue for further research.

We note that in practice, Market and System Operators know the true costs of the generators because the market participants are obligated to submit cost information to them. These data include the heat rate curves that are used to calculate the incremental costs of the generators as well as the unit commitment costs. System Operators have specific procedures and work with market participants to update these cost data on a periodic basis. They use these data to ensure that market participants do not exercise market power.

With these assumptions in mind, the DAS problem can be formulated as a MIP problem as follows. The nomenclature for the DAS problem appears in Appendix B.

$$\underset{q^{\mathrm{G}}_{u,l,t},q^{\mathrm{R}}_{u,l},z^{\mathrm{St}}_{u,t}}{\text{Minimize}} \left\{ \begin{array}{c} \sum_{u,l,t} b^{\mathrm{G}}_{u,l,t} \cdot q^{\mathrm{G}}_{u,l,t} + \sum_{u,t} b^{\mathrm{R}}_{u,t} \cdot q^{\mathrm{R}}_{u,t} + \\ \sum_{u,t} f^{\mathrm{SU}}_{u,t} \cdot z^{\mathrm{SU}}_{u,t} + \sum_{u,t} f^{\mathrm{SD}}_{u} \cdot z^{\mathrm{SD}}_{u,t} + \sum_{u,t} f^{\mathrm{NL}}_{u} \cdot z^{\mathrm{St}}_{u,t} \end{array} \right\}, \tag{5.1}$$

subject to: (shadow prices)

$$\sum_{u,t} q^{\mathrm{G}}_{u,l,t} = d^{\mathrm{G}}_t \qquad \forall t \qquad (\lambda^{\mathrm{G}}_t), \tag{5.2}$$

$$\sum_{u} q^{\mathrm{R}}_{u,t} \geq d^{\mathrm{R}}_t \qquad \forall t \qquad (\lambda^{\mathrm{R}}_t), \tag{5.3}$$

$$\sum_{l} q^{\mathrm{G}}_{u,l,t} \geq z^{\mathrm{St}}_{u,t} \underline{k}^{\mathrm{G}}_u \qquad \forall u,t, \tag{5.4}$$

$$\sum_{l} q^{\mathrm{G}}_{u,l,t} + q^{\mathrm{R}}_{u,t} \leq z^{\mathrm{St}}_{u,t} \bar{k}^{\mathrm{G}}_u \qquad \forall u,t, \tag{5.5}$$

$$q^{\mathrm{G}}_{u,l,t} \leq \bar{k}^{\mathrm{G}}_{u,l,t} \qquad \forall u,t, \tag{5.6}$$

$$q^{\mathrm{R}}_{u,t} \leq z^{\mathrm{St}}_{u,t} \bar{k}^{\mathrm{R}}_u \qquad \forall u,t, \tag{5.7}$$

$$\left(y^{\mathrm{On}}_{u,t-1} - t^{\mathrm{Up}}_u\right)\left(z^{\mathrm{St}}_{u,t-1} - z^{\mathrm{St}}_{u,t}\right) \geq 0 \qquad \forall u,t, \tag{5.8}$$

$$\left(y^{\mathrm{Off}}_{u,t-1} - t^{\mathrm{Down}}_u\right)\left(z^{\mathrm{St}}_{u,t} - z^{\mathrm{St}}_{u,t-1}\right) \geq 0 \qquad \forall u,t, \tag{5.9}$$

$$z^{\mathrm{SU}}_{u,t} = z^{\mathrm{St}}_{u,t}\left(1 - z^{\mathrm{St}}_{u,t-1}\right) \qquad \forall u,t, \tag{5.10}$$

$$z^{\mathrm{SD}}_{u,t} = z^{\mathrm{St}}_{u,t-1}\left(1 - z^{\mathrm{St}}_{u,t}\right) \qquad \forall u,t, \tag{5.11}$$

$$y^{\mathrm{On}}_{u,t} = \left(y^{\mathrm{On}}_{u,t-1} + 1\right) z^{\mathrm{St}}_{u,t} \qquad \forall u,t, \tag{5.12}$$

$$y^{\mathrm{Off}}_{u,t} = \left(y^{\mathrm{Off}}_{u,t-1} + 1\right)\left(1 - z^{\mathrm{St}}_{u,t}\right) \qquad \forall u,t, \tag{5.13}$$

$$z^{\mathrm{St}}_{u,0} = z^{\mathrm{St},0}_u \qquad \forall u, \tag{5.14}$$

$$y^{\mathrm{On}}_{u,0} = y^{\mathrm{On},0}_u \qquad \forall u, \tag{5.15}$$

$$y^{\mathrm{Off}}_{u,0} = y^{\mathrm{Off},0}_u \qquad \forall u, \tag{5.16}$$

with $q^{\mathrm{G}}_{u,l,t}, q^{\mathrm{R}}_{u,t} \geq 0$, $z^{\mathrm{St}}_{u,t}, z^{\mathrm{SU}}_{u,t}, z^{\mathrm{SD}}_{u,t}$ binary, and $y^{\mathrm{On}}_{u,t}, y^{\mathrm{Off}}_{u,t}$ integer, $\forall u, l, t$.

Objective function (5.1) minimizes the cost of providing energy and reserve as well as other commitment costs, namely, startup, shutdown, and no-load costs. Constraints (5.2) – (5.3) represent the market clearing constraints, i.e., the energy balance and the reserve requirements. The generation units' technical minimum/maximum, and the reserve availability constraints are given by (5.4) – (5.7); the minimum up/down-time constraints are stated by (5.8)–(5.9). To keep the formulation compact, we have not included any ramp constraints; such constraints can be easily included along with other additional constraints that may apply in any specific market design. Equalities (5.10)–(5.13) define the binary and integer variables, namely the startup/shutdown signals, and time counters of hours that a unit has been online/offline. Equalities (5.14)–(5.16) state the initial conditions of the units.

Nonlinear constraints (5.8)–(5.13) can be replaced by linear inequalities, as in Chapter 3, to turn the above MIP problem into an MILP problem. If we solve that problem and fix the integer variables at their optimal values (marked with an asterisk), we obtain an LP problem. We can then use that LP to calculate the clearing prices of the energy and reserves, as the shadow prices of the market clearing constraints (5.2) and (5.3), λ^{G}_t and λ^{R}_t, respectively.

The DAS model presented above, for simplicity, assumes a single zone. It can be expanded to include multiple zones. Further, in some markets, like the Greek electricity market, the energy pricing scheme is zonal, whereas the reserve pricing scheme currently in use is a "maximum bid accepted" scheme. Alternatively, the zonal marginal pricing for both energy and reserves can be applied, as in [94], consistent with marginal pricing theory [2].

5.3 Proposed Recovery Mechanisms

As was mentioned in the introduction, the revenues from participating in the market described in the previous section are not always sufficient to cover the costs of the participating generating units. To elaborate, let VC_n be the *variable costs* for generating energy and providing reserves, CC_u the *commitment costs*, BID_u the *bids*, and

ρ_u the *revenues* of generation unit u resulting from its participation in the day-ahead market. The above costs, revenues and bids are given by

$$\mathrm{VC}_u = \sum_{l,t} c^{\mathrm{G}}_{u,l} q^{\mathrm{G}}_{u,l,t} + \sum_t c^{\mathrm{R}}_u q^{\mathrm{R}}_{u,t}, \tag{5.17}$$

$$\mathrm{CC}_u = \sum_t f^{\mathrm{SU}}_u z^{\mathrm{SU}}_{u,t} + \sum_t f^{\mathrm{SD}}_u z^{\mathrm{SD}}_{u,t} + \sum_t f^{\mathrm{NL}}_u z^{\mathrm{St}}_{u,t}, \tag{5.18}$$

$$\mathrm{BID}_u = \sum_{l,t} b^{\mathrm{G}}_{u,l,t} q^{\mathrm{G}}_{u,l,t} + \sum_t b^{\mathrm{R}}_{u,t} q^{\mathrm{R}}_{u,t}, \tag{5.19}$$

$$\rho_u = \sum_t \left\{ \lambda^{\mathrm{G}}_t \sum_l q^{\mathrm{G}}_{u,l,t} \right\} + \sum_t \lambda^{\mathrm{R}}_t q^{\mathrm{R}}_{u,t}. \tag{5.20}$$

For the remainder of this section and for Section 5.4, we will focus our attention on an arbitrary generation unit; hence, for notational simplicity, we will omit subscript u. In what follows, we justify the need for side-payments, and we introduce several designs for the side-payments.

Let $\tilde{\pi}$ be the *gross profits* of an arbitrary generation unit, given by

$$\tilde{\pi} = \rho - (\mathrm{VC} + \mathrm{CC}). \tag{5.21}$$

From (5.21), it is obvious that the generation unit may incur losses, because its revenues from the commodities (energy and reserve) may not be sufficient to recover the commitment costs. Even if the commitment costs are explicitly compensated, however, the unit may still incur losses as follows. It may happen that in some hour(s) the unit is extra-marginal with respect to energy, i.e., its energy offer is above the marginal price, and yet the DAS solution schedules it at its technical minimum. Consequently, the unit's revenues will be lower than its bids. If, in addition, the unit's offers were truthful, i.e., equal to the true variable costs, then its revenues will be lower than its variable costs, and the unit will incur losses for that hour. If the total losses over all 24 hours are substantial, $\tilde{\pi}$ may end up being negative, which means that the unit will incur losses over the entire DAS horizon.

Based on this analysis, a recovery mechanism that provides adequate side-payments is needed to compensate for the potential losses. The side-payments should be calculated over the whole 24-hour period (as opposed to hourly side-payments) so that any volatile behavior in the commodity prices (for small changes in the demand; see [16] for a discussion) due to the non-convex nature of the optimization problem is smoothed out. In what follows, we discuss several alternative recovery payment designs.

We first consider two cases regarding the calculation of market revenue losses: cost-based and bid-based. These cases lead to two types of side-payments:

1. cost-based side-payments, and

2. bid-based side-payments.

To simplify the notation, we let $\pi(a)$ and $\pi(b)$ denote the cost-based and bid-based profits of the unit, respectively. These quantities are defined as follows:

$$\tilde{\pi}(a) = \rho - (\mathrm{VC} + \mathrm{CC}) = \tilde{\pi}, \tag{5.22}$$

$$\tilde{\pi}(b) = \rho - (\mathrm{BID} + \mathrm{CC}) = \tilde{\pi} - (\mathrm{BID} - \mathrm{VC}). \tag{5.23}$$

From (5.22) and (5.23), note that

$$\tilde{\pi}(a) = \tilde{\pi}(b) + (\mathrm{BID} - \mathrm{VC}), \tag{5.24}$$

where the quantity (BID − VC) is the difference between the as-bid based costs and the true variable costs.

A necessary condition that must be met in order for the unit to receive side-payments is that the above quantities are negative, i.e., that they correspond to market revenue losses. To further elaborate, let σ be the *side-payments* of the generation unit and π be its *net profits* after the side-payments, if any. Then

$$\pi = \begin{cases} \tilde{\pi}(i), & \text{if } \tilde{\pi}(i) \geq 0, \\ \tilde{\pi}(i) + \sigma, & \text{if } \tilde{\pi}(i) < 0, \end{cases} \quad \text{for } i = a, b. \tag{5.25}$$

Next, we derive expressions for the side-payments for each of the two cases (cost-based and bid-based), assuming that the condition $\tilde{\pi}(i) < 0$ in (5.25) holds.

5.3.1 Cost-Based Side-Payments

To be attractive, a recovery mechanism with cost-based side-payments should allow for positive net profits. To design such a mechanism, we must first define the basis of these profits in the case where $\tilde{\pi}(a) < 0$, and then derive an expression for σ that will achieve such profits. We consider two designs: one where the net profits are proportional to the unit's variable costs (design A.1) and another where the net profits are proportional to the unit's (cost-based) market revenue losses (design A.2).

Design A.1: VC-Related Profits

In this design, the final net profits, in case the unit receives side-payments, are set to a fixed percentage, say α_1, of its variable costs, namely,

$$\pi(A.1) = \alpha_1 \mathrm{VC}. \tag{5.26}$$

From (5.22), (5.25), and (5.26), the side-payments, paid ex-post, that achieve these profits are:

$$\sigma(A.1) = \alpha_1 \mathrm{VC} - \tilde{\pi}(a) = (1 + \alpha_1)\mathrm{VC} + \mathrm{CC} - \rho. \tag{5.27}$$

Apart from the fact that relating the final net profits with the variable cost seems to be a rather natural approach (in an early work of ours [95] we present a variant of design A.1, which allows explicit compensation of the commitment costs), this mechanism creates an incentive for maximizing the variable costs of a unit (in case of losses). A potential drawback of this mechanism is that the direct association of the final net profits with the variable costs, implied by equation (5.26), could favor expensive, thus inefficient units. However, as the variable cost is also a function of the scheduled quantity, there is also an incentive for maximizing production; hence it may also lead to cost-reflective bids, so that the scheduled quantity is the maximum possible. Therefore, the outcome is not obvious and needs to be investigated.

Another key feature is that the final net profits are independent of the magnitude of the losses. This creates a “discontinuity” of the net profits at the point of zero gross profits. To elaborate, think of two units with gross profits equal to 1 euro and -1 euro, respectively. The first unit will receive no σ and will end up with net profits of 1 euro, whereas the second unit will receive σ and will end up with net profits of α_1VC euro. A minimum profit condition could be applied in order to solve this discontinuity, but it could raise other discussions on the fairness of guaranteed profits, and as such it is not further examined in this work.

Design A.2: Market Revenue Loss-Related Profits

To overcome some of the drawbacks of design A.1, we propose an alternative mechanism where the final net profits that a unit is allowed to keep are set to a fixed percentage, say α_2, of its market revenue losses instead of its variable costs, namely,

$$\pi(A.2) = \alpha_2[-\tilde{\pi}(a)] = \alpha_2(\mathrm{VC} + \mathrm{CC} - \rho). \tag{5.28}$$

From (5.22), (5.25), and (5.28), the side-payments that achieve these profits are:

$$\sigma(A.2) = (1 + \alpha_2)[-\tilde{\pi}(a)] = (1 + \alpha_2)(\mathrm{VC} + \mathrm{CC} - \rho). \tag{5.29}$$

Note that design A.2 was referred to as recovery mechanism with loss-related profits (LPR mechanism) in the context of the stylized duopoly in Chapter 4. Design A.2 may prove to be a more reasonable approach, because relating net profits to losses eliminates the problem of “discontinuity” associated with design A.1, and may also result in lower side-payments. Specifically, if $\alpha_1 = \alpha_2 = \alpha$, it is easy to see, from (5.26) and (5.28), that the net profits under design A.2 are lower than those under design A.1, only if $\rho > \mathrm{CC}$.

In addition, under this design, units that are likely to be price-makers but may possibly incur losses (perhaps because they are extra-marginal in some hour(s) or because they cannot recover their commitment costs) have an incentive to submit cost-reflective bids, as they will profit from lower energy prices (the lower their revenues, the higher their losses and hence their profits). There may still be some unfairness in the margin, in the sense that a unit with negative $\tilde{\pi}$ could incur higher π than a unit with positive $\tilde{\pi}$; however, in the long run, the probability of this event should generally be low, otherwise the unit would not be profitable in the market.

Note also that if $\alpha_1 = \alpha_2 = 0$, the two mechanisms are equivalent, as the unit will receive σ to end up with zero net profits. In this case, σ represents "make-whole" payments. However, the zero-net profit condition is not attractive. In practice, and as far as the units' bidding behavior is concerned, this case would produce no different incentives than as if there were no side-payments.

To summarize, in both designs A.1 and A.2, the units may show a tendency to bid low in case they estimate market revenue losses (gross) through their market participation, to achieve higher net profits (including the side-payments) by either maximizing their scheduled quantities (therefore their VC) in design A.1, or maximizing the magnitude of their market revenue losses in design A.2. One of the potential drawbacks of both designs A.1 and A.2 is that they may not discourage high bids, because the recovery is not directly associated with the bids; therefore, these mechanisms may result in high prices and profits. An alternative design, which associates the side-payments with the bids is considered next.

5.3.2 Bid-Based Side-Payments

Under a bid-based recovery mechanism, the units are compensated with σ in order to recover their costs as they are reflected by their bids. The idea of such a design is that a unit should be able to recover its as-bid costs, without resorting to a pay-as-bid scheme. In a sense, such a recovery mechanism is a "hybrid" uniform and pay-as-bid pricing scheme. We consider two alternative designs: one where the as-bid costs are always recovered, provided that the unit incurs market revenue losses (design B.1, referred to as IP+) and another where the as-bid costs are recovered, provided that the unit incurs losses and its price offers for the commodities are within a given "reasonable" margin from the respective true costs (design B.2, referred to as rcIP+). The two mechanisms have been discussed in Chapter 4 for the case of a duopoly.

Design B.1: IP+ Pricing Mechanism

According to the IP+ pricing mechanism, the side-payments are:

$$\sigma(B.1) = -\tilde{\pi}(b) = \mathrm{BID} + \mathrm{CC} - \rho. \tag{5.30}$$

From (5.23), (5.25), and (5.30), the net profits are:

$$\pi(B.1) = \mathrm{BID} - \mathrm{VC}. \tag{5.31}$$

From the expression above and (5.24) note that the net profits are equal to the difference $\tilde{\pi}(a) - \tilde{\pi}(b)$.

This mechanism allows units that have positive bid-based profits to keep them and compensates those that exhibit market revenue losses by fully recovering their cost-reflective bids. This design is sketched in our early works [96] and [95], based on the results of [5].

A drawback of this mechanism, as is shown in [96] and [95], is that, in an oligopolistic market, the units may take advantage of the bid-recovery opportunity and place very high bids, resulting in particularly high and volatile prices. Current market designs offer bid mitigation measures to protect against such a market outcome. However, these measures require constant monitoring and adjustments, as necessary.

Design B.2: rcIP+ Pricing Mechanism

To overcome the drawback of design B.1, we propose the imposition of a regulated price cap that a unit has to respect in order to be eligible for σ given by (5.30). Specifically, if a unit has bid-based profits, then it will receive no σ. If the unit exhibits market revenue losses (again on a bid basis), then it will receive σ to reach π, given by (5.31), only if its energy (respectively, reserve) price offers lie between its true energy (respectively, reserve) cost and an upper bound, called "regulated

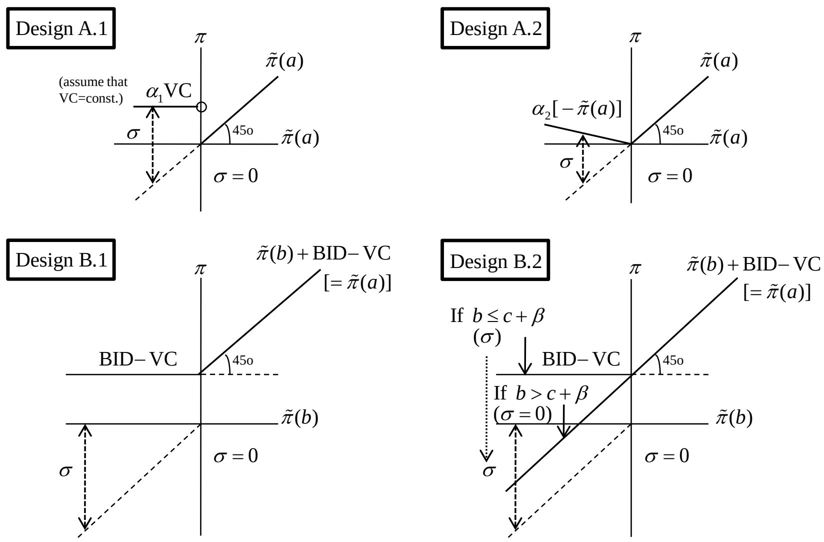

Figure 5.1: Net profits for alternative mechanism designs.

cap," which is equal to a fixed amount, say β^{G} (respectively, β^{R}), over its true energy (respectively, reserve) cost. The regulated cap should be chosen wisely to ensure proper pricing under scarcity conditions. In other words, in order for $\sigma > 0$, apart from the condition $\tilde{\pi}(b) < 0$ in (5.25), the following condition must also hold

$$b^{\mathrm{G}}_{u,l,t} \in \left[c^{\mathrm{G}}_{u,l}, c^{\mathrm{G}}_{u,l} + \beta^{\mathrm{G}}\right] \quad \text{and} \quad b^{\mathrm{R}}_{u,t} \in \left[c^{\mathrm{R}}_{u}, c^{\mathrm{R}}_{u} + \beta^{\mathrm{R}}\right]. \tag{5.32}$$

This mechanism motivates the bidder to behave less speculatively. Parameters β^{G} and β^{R} can serve as design parameters set by the regulator. A large value for either of these parameters will provide a strong incentive for the unit to bid within the recovery-eligibility margin, but may also result in large total payments; a low value, on the other hand, may not provide an adequate incentive and units may tend to bid above the upper bound.

Figure 5.1 summarizes the four designs and visualizes (5.25) to show π and σ with respect to $\tilde{\pi}(i)$, for $i = a, b$.

5.4 Numerical Evaluation Methodology

In this section, we propose a methodology for evaluating the performance of the recovery mechanisms presented in Section 5.3 to gain further insights into their incentive compatibility properties. This methodology employs an iterative numerical procedure that solves simultaneously for the optimal bidding strategies of the profit-maximizing units.

Normally, some units, such as base-load units, are price takers, bid low or self-schedule. Others, such as peak-load Open Cycle Gas Turbine (OCGT) units, may bid high to maximize their profits. In some cases, these units are only willing to sell ancillary services and produce energy only if the price is quite high. Therefore, the set of profit-maximizing units is a subset of U.

Let $U_b \subseteq U$ be the subset of units with a profit-maximizing strategy; the remaining units bid either at cost or at the price cap. Let $\mathbf{b}_u$ be the vector of energy and reserve price offers, $b^{\mathrm{G}}_{u,l,t}$ and $b^{\mathrm{R}}_{u,t}$, $\forall t$, of profit-maximizing unit u. Let $\underline{\mathbf{b}}_{-u}$ be the set of vectors $\mathbf{b}_v$, $\forall v \in U_b \setminus \{u\}$; $\mathbf{b}_u$ represents the set of offers of unit u, and $\underline{\mathbf{b}}_{-u}$ represents the set of offers of all other units except u. Finally, let $\pi_u\left(\mathbf{b}_u, \underline{\mathbf{b}}_{-u}\right)$ be the net profits after recovery of generating unit u, when its offers are $\mathbf{b}_u$ and its competitors' offers are $\underline{\mathbf{b}}_{-u}$. Each unit u will independently try to maximize its net profits, given the competitors' offers, $\underline{\mathbf{b}}_{-u}$, by setting its offers at

$$\mathbf{b}_u^{(*)}\left(\underline{\mathbf{b}}_{-u}\right) = \arg\max_{\mathbf{b}_u \in \mathbf{S}} \pi_u\left(\mathbf{b}_u, \underline{\mathbf{b}}_{-u}\right), \quad u \in U_b, \tag{5.33}$$

where $\mathbf{S}$ is the feasible space of $\mathbf{b}_u$ and is typically given by the interval $\left[c^{\mathrm{G}}_{u,l}, P^{\mathrm{G,\,CAP}}\right]$ for energy, similarly for reserve.

If all units do the same, then, theoretically, at equilibrium, the profit-maximizing units will submit offers $\mathbf{b}_u^{(*)}$, which are the solution of the following $|U_b| \times |U_b|$ system of equations:

$$\mathbf{b}_u^{(*)} = \mathbf{b}_u^{(*)}\left(\underline{\mathbf{b}}_{-u}^{(*)}\right), \quad \forall u \in U_b. \tag{5.34}$$

Equation (5.33) represents a particularly challenging bilevel optimization problem [97], which we briefly sketch below for clarity.

At the upper level, the generation unit u aims at maximizing its net profits, as follows:

$$\max_{\mathbf{b}_u} \pi_u \left(\mathbf{b}_u, \underline{\mathbf{b}}_{-u}\right), \quad \text{subject to:} \quad \mathbf{b}_u \in \mathbf{S}. \tag{5.35}$$

At the lower level, the System Operator solves the optimization problem (5.1) – (5.16) in order to minimize the total system energy cost.

The problem determined by (5.35) and (5.1) – (5.16) is a mixed integer nonlinear bilevel program. Note that to compute the objective function of the upper level problem, π (see (5.25) and (5.22) – (5.24)), which depends on the recovery mechanism in effect, one needs to compute the market revenues (ρ) first, which include (see (5.20)) products of lower level dual (λ_t^{G} and λ_t^{R}) and primal variables ($q_{u,l,t}^{\mathrm{G}}$ and $q_{u,t}^{\mathrm{R}}$, respectively). In addition, numerical experience has shown that $\pi_u \left(\mathbf{b}_u, \underline{\mathbf{b}}_{-u}\right)$ is not unimodal; therefore, maximizing it analytically is practically intractable.

If solving the optimization problem (5.33) is practically intractable, analytically unraveling the self-reference of equations (5.34) becomes impossible. In fact, the existence of a pure strategy Nash equilibrium solution is highly improbable, due to the complexity of the problem and the non-convexities.

Nonetheless, trying to numerically solve equations (5.34) by a classical scheme of successively approximating the optimal offer vectors $\mathbf{b}_u^{(*)}$ using a fixed-point iterative procedure, similar to the ones described in [60, 61, 62], is a task worth pursuing, because it can reveal patterns of bidding behavior of the individual players and the ranges and cumulative averages of values of different quantities of interest, such as the offers, side-payments, net profits, clearing prices and total payments, among others. The outline of such a procedure follows below.

Let $\mathbf{b}_u^{(n)}$ be the value of the vector of offer-values of generating unit u at the nth iteration, and let N be the maximum number of iterations we are willing to have.

Set $\mathbf{b}_u^{(0)}$ to some initial value, $\forall u \in U_b$.

For $n = 1, 2, \ldots, N$:

$$\text{Find } \mathbf{b}_u^{(n)} = \mathbf{b}_u^{(*)} \left(\underline{\mathbf{b}}_{-u}^{(n-1)}\right), \forall u \in U_b, \tag{5.36}$$

where $\mathbf{b}_u^{(*)} \left(\underline{\mathbf{b}}_{-u}^{(n-1)}\right)$ is obtained by numerically solving (5.33).

A reasonable starting point would be to assume that each unit u initially submits truthful bids, i.e., $\mathbf{b}_u^{(0)} \equiv \left\{b_{u,l,t}^{\mathrm{G}(0)}, b_{u,t}^{\mathrm{R}(0)}\right\}$ such that $b_{u,l,t}^{\mathrm{G}(0)} = c_{u,l}^{\mathrm{G}}$ and $b_{u,t}^{\mathrm{R}(0)} = c_u^{\mathrm{R}}$, $\forall l, t$.

Normally, the above procedure is terminated if the maximum number of iterations, N, is reached. It may be terminated earlier at iteration $n < N$, however, if $\mathbf{b}_u^{(n)} = \mathbf{b}_u^{(m)}$, $\forall u \in U_b$, for some $m = 0, 1, \ldots, n-1$. In fact, if $m = n-1$, then a solution of (5.34) has been found. If $m < n-1$, then the procedure has reached a "cycle" of period $n-m$, meaning that the values of the next iterations will be equal to the values of previous iterations, as follows: $\mathbf{b}_u^{(n+1)} = \mathbf{b}_u^{(m+1)}, \mathbf{b}_u^{(n+2)} = \mathbf{b}_u^{(m+2)}, \ldots, \mathbf{b}_u^{(2n-m)} = \mathbf{b}_u^{(n)}$. If the space of allowable offers of the participating units is discretized, then the number of combinations of offers of the different units is finite, and therefore a cycle will always exist, as long a period as it may have. Such cycles have been observed in numerical tests and reported in our early work [95].

The presented iterative scheme for solving what is essentially a *one-shot* (single-day) game can be viewed alternatively as a simulation procedure for solving a hypothetical, non-cooperative repetitive game with complete information, over many rounds, where in each round n, the decision variable for each player is the vector of energy and reserve offers, $\mathbf{b}_u^{(n)}$. In the first round, each player places some arbitrary initial offers. In the next round, each player determines his next offers by maximizing his net profits, assuming that the other players' offers will remain unchanged. This scheme generates a new set of offers. The game continues until either a predetermined number of rounds is reached, or the resulting set of offers has been reached in an earlier round. The implementation of this procedure reveals the bidding patterns of the players and the resulting market outcomes for each recovery mechanism.

The numerical procedure given by iteration (5.36) can be computationally extremely demanding. Even under the assumption that each unit places a single price-quantity energy offer and a single price-quantity reserve offer for each period (hour), the number of decision variables for each of the $|U_b|$ units is $2H = 2 \times 24 = 48$. In this case, solving (5.33) means optimizing a non-convex function of 48 variables.

To overcome this computational barrier, we assume that each unit places a single price-quantity energy offer (the same for all periods), and a zero-priced reserve offer that is not subject to optimization. The first assumption is not severe, as it may be the case that the units do not find it advantageous to submit multiple price-quantity offers; for example, such a behavior is sometimes observed in the Greek energy market. The second assumption helps us focus our attention on the energy bids, which determine the main volume of transactions in the day-ahead market. Note that even under zero-priced reserve offers, the reserve price can still be positive, because of marginal pricing. Both assumptions help significantly reduce the size of the problem and make it computationally tractable.

Even under the above assumptions, however, solving (5.33) is still not trivial. The way we practically solve it is by discretization and "brute force" evaluation of all feasible solutions. Namely, we assume that the decision variable $\mathbf{b}_u$ can take a finite number of discrete values, evenly distributed a certain step size apart, over the interval from the cost of energy generation to a price cap specified by the regulator. We then evaluate the net profits for each value and select as optimal the value which maximizes these profits. The selection of the step size is important as it affects the computational time and accuracy of results. Also, in some cases, the evaluation of the net profits for certain values is redundant, which helps reduce the number of computations. Finally, the optimal offer of each generation unit can be found independently of the other units, allowing the option for massive parallel computations.

The main advantage of the proposed methodology is that it can be applied in a straightforward manner by regulators and System Operators to help them predict the bidding behavior of market participants under various recovery mechanisms (ex ante evaluation). The implementation is easy, and commercial optimization platforms can be readily used. Since this is an offline procedure, the computational time is not a critical parameter.

5.5 Implementation

In this section, we present: (a) the input data of the test case (Subsection 5.5.1); (b) the assumptions of the implementation of the evaluation methodology (Subsection 5.5.2); (c) the performance measures that are deployed to evaluate the different recovery mechanisms (Subsection 5.5.3), and (d) computational issues that are related

Table 5.1: Recovery mechanisms.

Mechanism	Conditions for $\sigma \geq 0$	σ if conditions met	π with σ	Regulating Parameter
A.1	$\rho - \mathrm{VC} - \mathrm{CC} < 0$	$(1+\alpha_1)\mathrm{VC} + \mathrm{CC} - \rho$	$\alpha_1 \mathrm{VC}$	α_1
A.2	$\rho - \mathrm{VC} - \mathrm{CC} < 0$	$(1+\alpha_2)(\mathrm{VC} + \mathrm{CC} - \rho)$	$\alpha_2(\mathrm{VC} + \mathrm{CC} - \rho)$	α_2
B.1	$\rho - \mathrm{BIDCC} < 0$	$\mathrm{BID} + \mathrm{CC} - \rho$	BIDVC	N/A
B.2	$\rho - BID - \mathrm{CC} < 0$ and $b^{\mathrm{G}}_{u,t} \in \left[c^{\mathrm{G}}_u, c^{\mathrm{G}}_u + \beta\right]$	$\mathrm{BID} + \mathrm{CC} - \rho$	$\mathrm{BID} - \mathrm{VC}$	β

to the implementation of the evaluation methodology (Subsection 5.5.4).

5.5.1 Test Case Data

The test case that we used to evaluate the recovery mechanisms represents a realistic model of the Greek energy market. The mechanisms are summarized in Table 5.1. Tables 5.2 and 5.3 contain generation unit data and the hourly energy and reserve requirements that are used as input to the DAS market clearing problem. Quantities are given in MW, energy generation costs in €/MWh, and commitment costs in €. Minimum uptimes are given in hours and are considered equal to the minimum downtimes. In the Greek market model, the objective function does not include the start-up cost; it only includes the shutdown cost with a value equal to the warm start-up cost, to discourage DAS solutions from easily shutting down units [93].

The number of thermal units in Greece is about 30. The lignite units serve as base units, and actual competition is mainly limited to the gas units. With this in mind, unit U1 in Table 5.2 is an aggregate representation of the available lignite units. Units U2, U3, U4 and U5 are combined cycle units, U6 and U7 are gas units, U8 and U9 are oil units, and U10 is a "peaker," i.e., a gas unit that can provide all its capacity to the tertiary reserve market. All units are classified into three types

Table 5.2: Unit's data (DAS input).

Unit	$\bar{k}_u^{\mathrm{G}}$	$\underline{k}_u^{\mathrm{G}}$	$\bar{k}_u^{\mathrm{R}}$	c_u^{G}	t_u^{Up}	f_u^{SU}	f_u^{NL}
U1	3800	2400	250	35	24	1500000	20000
U2	377	240	137	49	3	13000	500
U3	476	144	180	52	5	10000	300
U4	550	155	180	55	5	25000	350
U5	384	240	144	57	3	15000	500
U6	151	65	45	64	16	18000	150
U7	188	105	45	65	16	27000	250
U8	287	120	10	70	8	50000	600
U9	144	60	20	72	12	24000	300
U10	141	0	141	150	0	5000	200

Table 5.3: Energy demand and reserve requirements.

t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
d_t^{G}	4200	3900	3800	3700	3700	3600	4000	4300	4800	5200	5550	5500
d_t^{R}	450	400	400	400	400	400	450	500	550	600	600	600
t	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
d_t^{G}	5450	5450	5300	5000	4950	4900	5000	5200	5100	5000	4800	4500
d_t^{R}	600	600	600	550	550	550	550	600	600	550	500	500

depending on their variable cost, as follows: (a) Type-L (low cost): U1; (b) Type-M (moderate cost): U2-U9, and (c) Type-H (high cost): U10.

Recovery mechanisms A.1, A.2 and B.2, shown in Table 5.1, depend on a mechanism-specific regulating parameter. The values of these parameters for which we evaluated these mechanisms are:

Mechanism A.1: $\alpha_1 = 0$ %, 5 %, and 10 %.

Mechanism A.2: $\alpha_2 = 5$ %, 10 %, and 100 %. We did not try out $\alpha_2 = 0$ %, because mechanism A.2 with $\alpha_2 = 0$ % is equivalent to A.1 with $\alpha_1 = 0$ %, as is mentioned in Section 5.3.

Mechanism B.2: $\beta = 3$, 6, and 9 (€/MWh).

5.5.2 Assumptions

The main assumptions concerning the implementation of the evaluation methodology are stated below. These assumptions are mild and do not influence the general applicability of the methodology. They are designed to make the computations tractable and are tailored towards the specifics of the test case market model.

Assumption 5.1. *(Initial status of the units): Initially:*

(a) *All units, except U1, are offline; unit U1 is online to ensure a feasible solution.*

(b) *All units that are initially online (offline) are assumed to be in this status long enough so that they can be shut down (started up) immediately.*

Assumption 5.1(a) is related to the aforementioned Greek market model feature that the shutdown cost is included in the objective function, whereas the startup cost is not. Given this feature, Assumption 5.1(a) allows DAS to commit a unit right from the first hour. Assumption Assumption 5.1(b) ensures that the initial values of the time counters (for the hours that the unit has been online/offline) will not affect the dispatching.

Assumption 5.2. *(Bid format): Each unit places:*

(a) *a single price-quantity energy offer, which is the same for all periods; this offer must be between the unit's cost of energy generation and a price cap equal to 150 €/MWh, which is the current cap value in the Greek market;*

(b) *a zero-priced reserve offer;*

(c) *truthful commitment costs.*

The behavior of offering the entire capacity at the same price for all periods is observed in the Greek energy market. Assumption 5.2(b) eases the computational burden of the problem, and Assumption 5.2(c) holds true if the commitment costs are auditable, which is true in most markets. Assumption 5.2, as a whole, significantly reduces the size of the problem.

Assumption 5.3. *(Tie-breaking rule): If units submit equal bids, a tie-breaking rule favors the unit with the lower variable cost.*

Assumption 5.4. *(Bidding strategies): The bidding strategy of each unit depends on its type as follows:*

(a) *Type-L units (U1) always bid at their variable cost.*

(b) *Type-M units (U2-U9) participate in the "repetitive game" described in Section 5.4 with a profit-maximizing bidding strategy. More specifically, each type-M unit:*

  (1) *in the initial round, submits truthful price offers;*

  (2) *in each subsequent round, uses "brute-force" optimization to determine its optimal price offer, as follows:*

    (i) *it evaluates its net profit for each permissible price offer value between its variable cost and the cap, using an incremental step size of 1 €/MWh, assuming the other units remain at their optimal price offers from the previous round;*

    (ii) *it selects as the optimal price offer the one that generates the highest net profit;*

    (iii) *among all possible multiple price offers that generate equal profits, it selects the lowest.*

(c) *Type-H units (U10) always bid at the price cap.*

Assumptions 5.4(a) and 5.4(c) reflect current practice in the Greek market. Unit U1 always has profits, as it has the lowest cost, so it has little interest to bid over its variable cost and risk being shut down. Unit U10 is the last unit to be dispatched for energy, due to its high cost, so it risks bidding at the price cap; its revenues come mainly from the reserve market. Assumption 5.4(b) states that type-M units try different price offers, starting from their true variable cost, as they set out to find the offer which maximizes their profits. This process is consistent with our overall aim to evaluate the incentive compatibility properties of each mechanism, i.e., the extent to which the participants fare best when they behave truthfully.

5.5.3 Performance Measures

In each round of the numerical methodology, besides the optimal bid and the DAS solution (i.e., the energy/reserve clearing prices, scheduled energy/reserve quantities), we also compute the following important performance measures:

- Net profits of each generation unit: π_u
- Total payments for energy: $\sum_t \lambda_t^{\mathrm{G}} d_t^{\mathrm{G}}$
- Total payments for reserve: $\sum_t \lambda_t^{\mathrm{R}} d_t^{\mathrm{R}}$
- Total side-payments: $\sum_u \sigma_u$
- Total uplift on the energy clearing price, due to the reserve and side-payments: $\left(\sum_t \lambda_t^{\mathrm{R}} d_t^{\mathrm{R}} + \sum_u \sigma_u\right) / \sum_t d_t^{\mathrm{G}}$
- Producers' cost: $\sum_u (\mathrm{VC}_u + \mathrm{CC}_u)$
- Producers' surplus: $\sum_u \pi_u$

The aforementioned measures are useful for comparing different mechanisms. Specifying the "best" values of these measures, however, is not obvious. The following criteria are associated with a "good" mechanism:

1. The units that submit truthful bids should not exhibit revenue losses;
2. the uplifts associated with the side-payments should not be high, and

3. the solution of the DAS problem should not be inefficient in terms of total cost (the benchmark is the DAS solution with truthful bids).

In addition, a recovery mechanism should mainly address the needs of the "marginal" and "extra-marginal" units, because these units are more likely to need to recover their costs; the units with low variable costs, which are mostly infra-marginal, will in any case recover their costs and have profits from the day-ahead market.

5.5.4 Computational Issues

For each design in Table 5.1, we ran the repetitive game described in Section 5.4 for a predetermined number of 50 rounds. In some cases, we observed "cycling" in the bidding behavior, which means that from a certain round onwards the bids of future rounds are exactly equal to the bids of previous rounds, so there is no need to run more rounds. In case a cycle was observed, the runs were terminated. This truncation procedure resulted in substantial computational savings.

The brute-force optimization procedure that the profit-maximizing type-M units perform in each round requires the solution of 724 DAS problems, as each of the type-M units searches sequentially over a set of price offers from its cost level up to the price cap; in the best case, the unit with the highest cost (U9) searches over 79 price offers, and in the worst case, the unit with the lowest cost (U2) searches over 102 price offers. During the brute-force optimization of any particular unit, if for a given price offer the DAS solution sets the unit offline for all 24 hours, then, clearly for any higher price offer it will do the same, so there is no need to examine any higher price offers. In this case the optimization can be terminated. This resulted in further computational savings.

Without accounting for the aforementioned computational savings, we had to solve 50 (724 + 1) + 1 = 36,251 DAS problem instances per design; namely, 50 rounds per design, with 724 DAS problem instances per round for determining the best price offers, plus 1 instance per round for clearing the market using the best price offers, plus the initial problem instance in which type-M units submit truthful price offers.

Before setting out to solve all these instances, however, we first solved an instance, which we refer to as the nominal case, to be used as a reference point for all other instances. In the nominal case, each unit submits bids equal to its true variable costs, and if it incurs losses, then it is compensated so as to end up with zero profits. The nominal case is in fact equivalent to the initial round (with truthful bids) of mechanisms A.1 and A.2, with $\alpha_1 = 0$ and $\alpha_2 = 0$, respectively. By exploiting the opportunities for computational savings described above, the number of DAS problem instances was reduced by 38%.

We programmed the DAS problem and the methodology for evaluating the recovery mechanism design options using the mathematical programming language AMPL [98]. We ran the program on a Pentium IV 1.8 GHz dual core processor PC with 1 GB system memory where we used the ILOG CPLEX 10.2 optimization commercial solver [99] to solve the DAS problem instances. Each DAS problem instance consists of 480 continuous variables, 1,000 general integer and 730 binary variables, and 6,158 constraints. The average time to solve a single problem instance was approximately 3.9 seconds and the average time for a single round was 30.5 minutes.

Finally, it is noteworthy that the evaluation methodology is amenable to significant parallelization, as all the market design options can be evaluated in parallel. The 724 DAS problem instances in each round can also be solved in parallel.

5.6 Numerical Results

In this section, we present the most important numerical results. In Subsection 5.6.1, we present the average aggregate results for all recovery mechanisms, while in Subsection 5.6.2, we offer a closer look at the prevailing mechanisms.

5.6.1 Average Aggregate Results for All Mechanisms

Initially, we evaluated the performance of all four recovery mechanism designs shown in Table 5.1 as well as of two other simple designs: one that explicitly compensates the commitment costs and provides no further payments, and one that provides no side-payments at all. Due to space considerations and the fact that the latter two

Table 5.4: Recovery mechanisms average aggregate results.

Mech.	Regul. Param.	Energy Payments (€/MWh) (1)	Reserve Payments (€/MWh) (2)	Side Payments (€/MWh) (3)	Total Uplift (€/MWh) (4)=(2)+(3)	Total Payments (€/MWh) (5)=(1)+(4)	Cost Increase % of Nominal Case Cost) (6)	Producers' Surplus % of Nominal Case Cost) (7)
A.1	$\alpha_1 = 0\%$	57.270	0.522	0.215	0.737	58.007	0.638%	31.707%
	$\alpha_1 = 5\%$	57.523	0.518	0.324	0.842	58.365	0.720%	32.442%
	$\alpha_1 = 10\%$	56.449	0.529	0.530	1.059	57.508	0.550%	30.656%
A.2	$\alpha_2 = 5\%$	57.396	0.532	0.164	0.696	58.092	0.631%	31.907%
	$\alpha_2 = 10\%$	57.396	0.532	0.171	0.703	58.099	0.631%	31.925%
	$\alpha_2 = 100\%$	57.937	0.572	0.382	0.954	58.891	0.759%	33.602%
B.1	N/A	77.976	1.100	0.893	1.993	79.969	1.477%	80.974%
B.2	$\beta = 3$	57.433	0.517	0.244	0.761	58.194	0.590%	32.184%
	$\beta = 6$	59.011	0.528	0.244	0.772	59.783	0.773%	35.625%
	$\beta = 9$	58.837	0.504	0.301	0.805	59.642	0.540%	35.534%
Nominal	**Case**	52.276	0.505	0.353	0.858	53.134	0%	21.226%

designs performed poorly as they resulted in negative profits for some units, we do not present results for them. It should also be noted that the first design additionally resulted in elevated uplifts and energy payments.

Table 5.4 shows the average aggregate results for the mechanisms shown in Table 5.1. To facilitate the interpretation of these results, we expressed them in relative rather than in absolute quantities. Specifically, the energy, reserve, and side-payments are normalized with the daily load; the two latter components form the total uplift, whereas the sum of all three components constitutes the total payments. The average percentage of cost increase reveals the degree of inefficiency in dispatching when compared to the nominal case. The average percentage of the producers' surplus over the cost of the nominal case provides a more comprehensive view for the aggregate profits. All averages are calculated for the total number of rounds, unless a cycle was observed, in which case, they are calculated for the cycle period.

From the results of Table 5.4, we can see that the total payments under B.1 are much higher than under the other three mechanisms. Given the comparatively poor performance of mechanism B.1, we will henceforth restrict our attention to the three more attractive mechanisms, A.1, A.2, and B.2.

5.6.2 A Closer Look at the Prevailing Mechanisms

The analysis of the results in Table 5.4 for the prevailing mechanisms, A.1, A.2, and B.2, leads to the following remarks.

Remark 5.1. *The aggregate results for the mechanisms A.1, A.2, B.2, for all the tested values of the regulating parameters shown in Table 5.4, are comparable to each other. Therefore, they are considered equivalent in terms of performance, at least based on the average aggregate results.*

Also, the tested values of the regulating parameters do not seem to have a significant or identifiable influence on the average aggregate performance.

Remark 5.2. *The total uplifts produced by mechanisms A.1, A.2, B.2 are quite low, namely, less than 2% of the energy price for all the tested values of the regulating parameter shown in Table 5.4.*

We observe that the uplift component which is related to the provision of reserve is practically the same and close to 1% of the energy price, for all three mechanisms and all tested regulating parameter values. The uplift component due to the side-payments seems to be slightly increasing with the regulating parameter for all three mechanisms and is less than 1%. In most cases, the total uplifts in absolute numbers are smaller even than the total uplift of the nominal case.

Thus far, we have focused our attention on the average aggregate results over all units. Next, we will take a closer look at the average performance and bidding behavior for each individual generating unit.

Although not shown here for space considerations, the results indicate that the type-L lignite units (U1) have very high profits, due to their low variable cost. In fact, approximately 90% of the producers' surplus belongs to the lignite units. The results also indicate that the profits of the type-H "peaker" (U10) are also quite significant and stable; they are mostly due to the provision of reserve.

Tables 5.5 and 5.6 show the average profits and bids for each type-M generating unit for the prevailing mechanisms A.1, A.2, and B.2. Table 5.6 also shows in parentheses the difference of the average bids from the variable cost as a measure of the

Table 5.5: Type-M units' profits under different recovery mechanisms.

Mech.	Regul. Param.	U2	U3	U4	U5	U6	U7	U8	U9
A.1	$\alpha_1 = 5\%$	52,578	51,142	27,822	19,320	4,056	5,483	3,551	2,146
	$\alpha_1 = 10\%$	46,282	47,860	33,865	19,737	5,407	10,541	4,458	4,833
A.2	$\alpha_2 = 10\%$	48,484	56,279	26,357	20,834	2,736	667	437	784
	$\alpha_2 = 100\%$	56,681	51,660	30,201	23,935	3,840	6,589	7,423	6,989
B.2	$\beta = 3$	50,712	48,489	34,280	20,460	3,886	2,762	0	848
	$\beta = 6$	57,421	66,173	29,022	29,876	6,994	7,841	519	193
	$\beta = 9$	56,056	64,624	40,383	32,948	9,073	6,829	1,485	515
Nominal	**Case**	39,912	20,948	5,205	0	0	0	0	0

units' tendency to overbid. The lower this difference, the higher the level of incentive compatibility of the corresponding mechanism.

Comparisons among different units are somewhat delicate because the unit capacities are different. Nonetheless, we can distinguish between two large sub-groups of type-M units with similar characteristics and behavior: Group A (U2-U5) and group B (U6-U9). Group-A units have lower variable costs and larger capacities than group-B units. The analysis of the results contained in the Tables 5.5 and 5.6 leads to the following remarks.

Remark 5.3. *Under mechanisms A.1, A.2, and B.2, the group-A units tend to overbid and have higher profits than group-B units.*

This bidding behavior of group-A units is somewhat expected since low-cost units take advantage of their higher profit margins and try to set higher energy prices, which would result in higher profits.

Remark 5.4. *Under mechanisms A.1 and A.2, the units with higher variable costs tend to bid close to (or even at) their cost, which implies a very high level of incentive compatibility for these units.*

Table 5.6: Type-M units' bids under different recovery mechanisms.

Mech.	Regul. Param.	U2	U3	U4	U5	U6	U7	U8	U9
A.1	$\alpha_1 = 5\%$	58.04	63.78	70.86	61.28	66.90	66.10	70.04	72.00
		(9.04)	(11.78)	(15.86)	(4.28)	(2.9)	(1.10)	(0.04)	(0.00)
	$\alpha_1 = 10\%$	56.71	61.00	68.98	61.10	66.73	65.78	70.02	72.00
		(7.71)	(9.00)	(13.98)	(4.10)	(2.73)	(0.78)	(0.02)	(0.00)
A.2	$\alpha_2 = 10\%$	58.57	62.43	72.86	59.43	69.43	65.86	70.00	72.00
		(9.57)	(10.43)	(17.86)	(2.43)	(5.43)	(0.86)	(0.00)	(0.00)
	$\alpha_2 = 100\%$	57.69	64.96	70.96	62.61	67.80	66.69	70.16	72.20
		(8.69)	(12.96)	(15.96)	(5.61)	(3.80)	(1.69)	(0.16)	(0.20)
B.2	$\beta = 3$	58.24	64.14	67.08	61.14	66.69	67.43	70.18	73.98
		(9.24)	(12.14)	(12.08)	(4.14)	(2.69)	(2.43)	(0.18)	(1.98)
	$\beta = 6$	59.35	63.86	75.86	63.16	69.55	70.08	72.45	74.77
		(10.35)	(11.86)	(20.86)	(6.16)	(5.55)	(5.08)	(2.45)	(2.77)
	$\beta = 9$	60.78	62.22	72.84	63.51	71.39	71.29	73.47)	76.16
		(11.78)	(10.22)	(17.84)	(6.51)	(7.39)	(6.29)	(3.47)	(4.16)
Nominal	**Case**	49.00	52.00	55.00	57.00	64.00	65.00	70.00	72.00

Remark 5.5. *Under mechanism B.2, the units with higher variable costs bid close to their cost for low values of the cap, again implying a high level of incentive compatibility, but tend to bid higher as the cap increases.*

Recall that as the bid cap increases, the behavior of mechanism B.2 approaches that of mechanism B.1, which, as we saw earlier, is unattractive. A question that arises naturally is how often units bid over the cap. Figure 5.2 shows the frequency with which units bid over the cap. Units U8 and U9 always bid within the margin and are not shown.

Remark 5.6. *Under mechanism B.2, group-A units tend to bid over the cap more frequently than units with higher variable cost. The frequency of bidding over the cap, in general, decreases as the margin increases.*

We look next into the units' bidding behavior within the rounds.

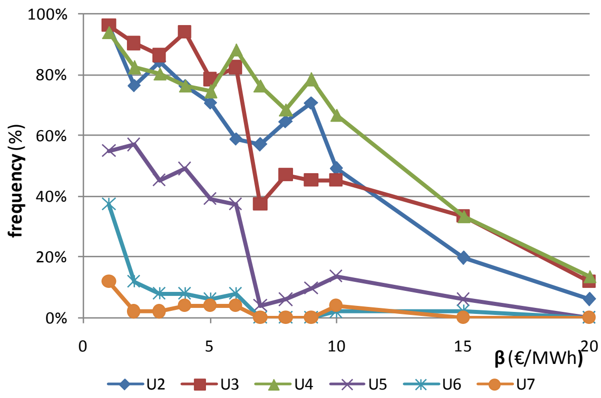

Figure 5.2: Frequency of bidding over the cap for mechanism B.2.

Figure 5.3 depicts the bidding behavior of the type-M units under mechanism B.2, which is also indicative of the other two mechanisms, and leads to the following remark.

Remark 5.7. *Under mechanisms A.1, A.2, and B.2, the group-A units exhibit a rather "volatile" bidding behavior, whereas the group-B units bid more uniformly.*

Units with lower variable costs exhibit a more speculative bidding behavior, because of their high profit margins, whereas units with higher variable costs bid more conservatively, because they have low profit margins. For mechanism B.2, in particular, the benefits of bidding more aggressively (i.e., outside the margin) diminish as the margin widens.

Figure 5.3 also shows the "cumulative" average energy payments and total payments (in €/MWh) over the first n rounds, for $n = 0, \ldots, 50$ (upper figure). It can be seen that the cumulative average "converges" in only a few rounds, which implies that the sample size of 50 rounds that we considered yields confident enough results. In the specific example, the cumulative averages over the last 10 rounds differ less than 0.1 €/MWh. The difference between the total payments and the energy payments yields the total uplift.

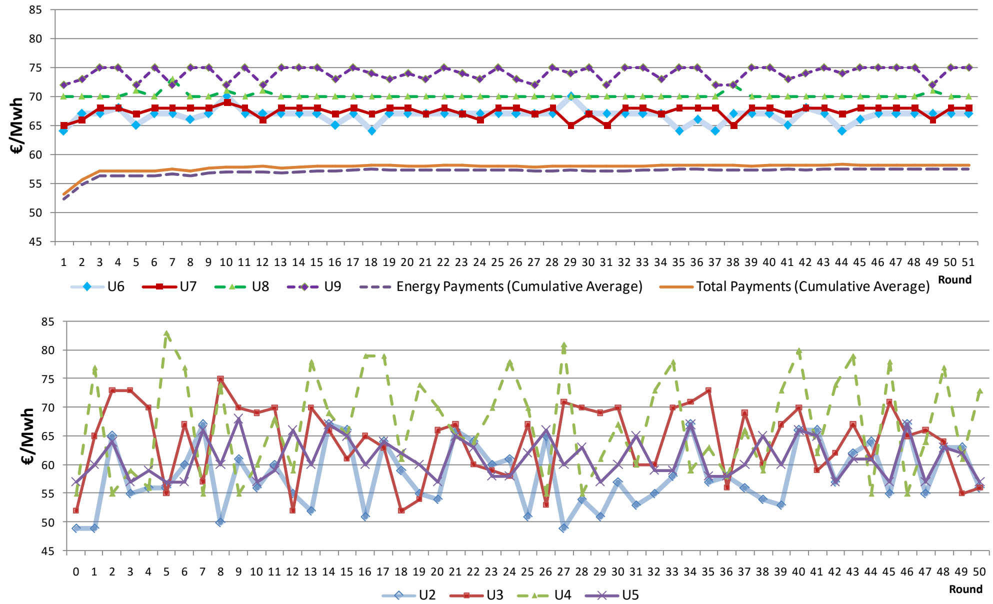

Figure 5.3: Bidding pattern for mechanism B.2, with $\beta = 3$.

5.7 Sensitivity Analysis and Extensions

To explore the accuracy and extendability of our results, we perform sensitivity analysis with respect to certain key parameters and assumptions, and discuss extensions.

In Tables 5.4 – 5.6, we presented results for some indicative values of the regulating parameters. In this section we select the values $\alpha_1 = 10\%$, $\alpha_2 = 10\%$, and $\beta = 3$ (€/MWh), for the three prevailing designs, and we perform sensitivity analysis with respect to the load (Subsection 5.7.1), and the profit maximizing bidding strategy assumptions of Type-M units (U2-U9) (Subsection 5.7.2). We also examine the impact of allowing the units to bid under their cost (Subsection 5.7.3), and we perform further sensitivity analysis for more values of the regulating parameters as well as a different bidding strategy (Subsection 5.7.4). Lastly, we discuss extension with respect to agent-based simulation methodologies (Subsection 5.7.5).

Table 5.7: Energy demand and reserve requirements (low demand scenario).

t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
d_t^{G}	3500	3200	3100	3000	3000	2900	3200	3500	3800	4200	4550	4500
d_t^{R}	300	250	250	250	250	250	300	350	400	450	450	450
t	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
d_t^{G}	4450	4450	4300	4000	3950	3900	4000	4200	4100	4000	3800	3500
d_t^{R}	450	450	450	400	400	400	400	450	450	400	350	350

5.7.1 Load Sensitivity Analysis (Low-Demand Scenario)

The demand data that we used to evaluate the recovery mechanisms correspond to a scenario where the demand is rather high, because, as is evident from the results, for the most part, 9 out of 10 generation units are committed to providing energy and/or reserve. In this subsection, we consider a low-demand scenario shown in Table 5.7, where the hourly demand and reserve requirements are approximately 20% and 30% lower, respectively, than the corresponding values in the high-demand scenario, shown in Table 5.3.

The aggregate results for the low-demand scenario are shown in Table 5.8. From these results, it can be seen that the energy payments drop by approximately 27% with respect to the high-demand scenario for all mechanisms. The uplifts, on the other hand, increase by 11-22% but still remain low. This increase is quite expected, since the revenues for the extra-marginal units are lower in the low demand case (higher competition, lower prices).

Lastly, note that the reserve payments are very low, as there is enough capacity committed to cover the need for reserves, without producing a significant marginal cost for reserve provision (in most hours).

Table 5.8: Average aggregate results (low demand scenario).

Mech.	**Regul. Param.**	**Energy Payments (€/MWh)**	**Reserve Payments (€/MWh)**	**Side-Payments (€/MWh)**
A.1	$\alpha_1 = 10\%$	41.192	0.015	1.168
A.2	$\alpha_2 = 10\%$	41.485	0.015	0.840
B.2	$\beta = 3$	42.336	0.015	0.864

5.7.2 Bidding Strategy Assumptions Sensitivity Analysis

Thus far, we have focused our attention on the performance measures of different recovery mechanisms. These measures were estimated using the evaluation methodology developed in Section 5.4, under the assumptions stated in Section 5.5. Two of these assumptions, which are related to the profit optimization procedure of each unit in each round, namely Assumptions 5.4.b(2)(i) and 5.4.b(2)(iii), may appear to be somewhat arbitrary or restrictive; for this reason, we investigate next their impact on the results.

Step Size

The purpose of Assumption 5.4.b(2)(i) is to facilitate the numerical solution of the profit maximization problem of each unit in each round, given by expression (5.36), by discretizing the theoretically continuous decision space of permissible price offers of the unit, i.e., the interval between the unit's cost of energy production and the price cap, into a number of discrete points, equally spaced by 1 €/MWh apart, then evaluating the net profit at each discrete point (brute-force). To investigate the impact of the discretization step size, we divided it by two, at the cost of having to evaluate twice as many points, and we ran the experiments again. The results show that the difference in total payments is very small, indicating that the original step size yields sufficiently accurate results. Indicatively, lines 2, 7, and 12 in Table 5.9 show the average aggregate energy payments and total uplifts for each mechanism (for an indicative regulating parameter value) for a step size of 0.5 €/MWh. These payments are only 0.9-1.7% higher (as shown in the parenthesis) than the respective payments in the original runs with a step size of 1 €/MWh, shown in lines 1, 6, and 10, respectively.

Table 5.9: Average aggregate results (sensitivity analysis on bidding strategy assumptions).

Line	**Mech.**	**Case**	**Energy Payments (€/MWh)**	**Total Payments (€/MWh)**
1	**A.1**	Original	56.449	1.059
2	$\alpha_1 = 10\%$	5.4.b(2)(i) [Step 0.5]	57.168 (+1.3%)	1.061
3		5.4.b(2)(iii) [Highest offer]	58.153 (+3.0%)	0.891
4		Random offer	56.828 (+0.7%)	1.069
5		Average offer	56.557 (+0.2%)	0.843
6	**A.2**	Original	57.396	0.703
7	$\alpha_2 = 10\%$	5.4.b(2)(i) [Step 0.5]	58.385 (+1.7%)	0.814
8		5.4.b(2)(iii) [Highest offer]	58.165 (+1.3%)	0.794
9		Random offer	57.649 (+0.4%)	0.770
10		Average offer	57.590 (+0.3%)	0.652
11	**B.2**	Original	57.433	0.762
12	$\beta = 3$	5.4.b(2)(i) [Step 0.5]	57.928 (+0.9%)	0.727
13		5.4.b(2)(iii) [Highest offer]	57.608 (+0.3%)	0.733
14		Random offer	57.793 (+0.6%)	0.803
15		Average offer	56.932 (-0.9%)	0.642

Bid Selection

The purpose of Assumption 5.4.b(2)(iii) is to resolve the situation where the brute-force optimization, which determines the optimal price offer of each unit and yields the maximum profit, results in multiple price offers. Assumption 5.4.b(2)(iii) resolves this situation by dictating that the unit chooses the lowest offer. To investigate the impact of this assumption on the results, we modified it so that it now dictates that among multiple price offers that yield the same optimum profit, a unit chooses the highest offer. With this modification, we executed the experiments again for the three mechanisms. The results show that the difference in total payments under the modified Assumption 5.4.b(2)(iii) and the original Assumption 5.4.b(2)(iii) is quite limited. Indicatively, lines 3, 8, and 13 in Table 5.8 show the average aggregate energy payments and total uplifts under the modified Assumption 5.4.b(2)(iii). These energy payments are only 0.3-3.0% higher (as shown in the parenthesis) than the respective payments in the runs under the original Assumption 5.4.b(2)(iii), shown in lines 1, 6, and 11.

Assumption 5.4.b, as a whole, is at the heart of the evaluation methodology. It states that type-M units set out to find the offers which jointly maximize their profits. In each round, each unit chooses the offer which maximizes its profits, assuming that the other units will use their offers from the previous round. The usefulness of the evaluation methodology is that it reveals patterns of bidding behavior of the individual units, the ranges and averages of different quantities of interest, such as offers, side-payments, net profits, clearing prices, total payments, etc.

Figure 5.3 is typical of the bidding behavior of the units during the execution of the evaluation methodology. Clearly, the bids oscillate from one round to the next and no pure equilibrium solution is attained. Given the apparent lack of an equilibrium solution, the natural question arises as to how a unit can use the results of the evaluation methodology to decide on its bidding strategy. We elaborate on this issue next.

A simple approach is to assume that each unit, not knowing how the other units will bid, will randomly choose one of the offers that it submitted over all rounds of the evaluation methodology with a probability that is equal to the frequency with which that offer was placed during the rounds. Essentially, under this "random offer" approach, each unit uses the *marginal frequency* of its *deterministic optimal* offers, which it extracts from the joint frequency of the offers of all units during the course of the evaluation methodology, as the *probability distribution* of its *random* offers. For clarity, we note that this probability distribution does not represent a mixed strategy Nash equilibrium. To evaluate the performance of each mechanism under the random offer approach, we solved the DAS problem for 20,000 instances, where in each instance the offers of each type-M unit were randomly generated using this approach.

Table 5.10: Average aggregate results (bidding up to 30% under cost).

Mech.	**Regul. Param.**	**Energy Payments (€/MWh)**	**Reserve Payments (€/MWh)**	**Side-Payments (€/MWh)**
A.1	$\alpha_1 = 10\%$	37.812	0.187	9.976
A.2	$\alpha_2 = 10\%$	59.366	0.907	1.079
B.2	$\beta = 3$	57.263	0.697	0.220

The results show that the difference in average aggregate total payments under the random offer approach and the original Assumption 5.4.b is very small. Indicatively, lines 4, 9, and 14 in Table 5.8 show the average aggregate energy payments and total uplifts under the random offer selection approach. The energy payments are only 0.4–0.7% higher than the respective payments in the runs under the original Assumption 4.b, shown in lines 1, 6, and 11.

An alternative approach is to assume that each unit chooses a particular deterministic offer that is representative of the optimal offers that the unit submitted over all rounds of the evaluation methodology. A natural candidate for the value of that particular offer is the average value of the optimal offers. The results show again that the difference in total payments under the average offer selection approach and the original Assumption 5.4.5.4b is quite small. Indicatively, lines 5, 10, and 15 in Table 5.8 show the average aggregate energy payments and total uplifts under the average offer selection approach. The energy payments are very close in either direction with the respective payments in the runs under the original Assumption 5.4.b, shown in lines 1, 6, and 11, and the total uplifts are lower for all mechanisms.

5.7.3 Bidding Under the Cost

An interesting inquiry with respect to Assumption 5.4.b(2)(i) is to check if any benefit can arise from allowing the generation units to bid under their variable cost. To this end, we consider the case in which the units are allowed to bid up to 30% under their variable cost. The results are shown in Table 5.10 and reveal some interesting outcomes (compared to the results in Table 5.4).

We observe that mechanism A.1 produces a particularly high uplift combined with low energy payments. As a matter of fact, this particular outcome was an equilibrium point, given the assumptions. At this point, the units bid as low as possible, under their cost, in order to maximize their quantities and benefit from the side-payments. Due to their low bids, the energy prices are low and it is not profitable to try and bid higher in order to benefit from a higher price. The outcome is characterized as a particularly bad one, as the energy price does not reflect the cost and the uplifts are high.

Mechanism A.2 exhibits some higher energy payments. A closer look at the results shows that the units tend to cycle between high and low bids, as a result of the possibility that they are given for gaming.

Mechanism B.2 seems to have the most stable performance. However, some units may exhibit negative profits, which is also not a good outcome.

In addition, we examined the above option for the low-demand scenario, described in Subsection 5.7.1, which we did not show in Table 5.10. Mechanism A.1 produced the same equilibrium point, and hence the same remarks apply. Mechanism A.2 reached the same equilibrium point with Mechanism A.1, and produced lower uplifts than A.1 (7.4 as opposed to 10.4 €/MWh) but still particularly high. Mechanism B.2 gave low uplifts (similar to the high demand case) but still resulted in negative profits for some units.

In general, we can conclude that allowing the units to bid under the cost increases their possibility for gaming and seems to have undesired properties with respect to the performance of the recovery mechanisms.

5.7.4 Further Sensitivity Analysis

Some further sensitivity analysis results are as follows.

We evaluated the three prevailing mechanisms for more values of the regulating parameters than the ones shown in Tables 5.4 –5.6. Figure 5.4 shows the uplift due to the side-payments for different regulating parameter values.

The trend-lines in each figure provide an estimate of the impact of the regulating

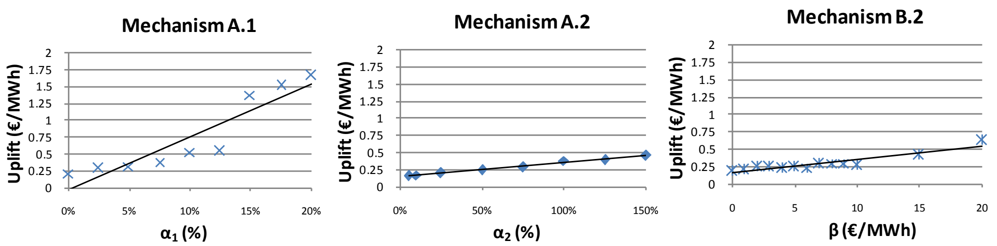

Figure 5.4: Uplifts for different regulating parameter values.

parameter under each mechanism.

We modified Assumption 5.2.a, so that instead of allowing all units to submit one bid which is valid for all hours, we allowed the profit maximizing units to submit energy offers which are equal to their cost during the time periods of low load (12 hours) and normal bids during the periods of high (peak) load (12 hours). We refer to this case as "Peak Load Pricing." The results are shown in Table 5.11.

Table 5.11: Average aggregate results (peak load pricing).

Mech.	**Regul. Param.**	**Energy Payments (€/MWh)**	**Reserve Payments (€/MWh)**	**Side-Payments (€/MWh)**
A.1	$\alpha_1 = 10\%$	57.862 (+2.5%)	0.798	0.445
A.2	$\alpha_2 = 10\%$	60.689 (+5.7%)	0.934	0.131
B.2	$\beta = 3$	58.247 (+1.4%)	0.685	0.117

Note that the uplifts due to the side-payments remain particularly low in all three mechanisms. A relatively higher energy price is observed in mechanism A.2. Generally, energy prices are higher when the units tend to offer higher bids aiming to form higher prices, as they may expect higher profits from energy payments than what they would get through the recovery mechanism. This tendency depends on the regulating parameter which determines the amount of the side-payments. Recall that the regulating parameters α_1 and α_2, even though equal, multiply different quantities and produce different outcomes. The fact that a relatively higher energy price is observed in mechanism A.2. may be due to the choice of the regulating parameter α_2. Also note that under all three mechanisms, the possibility for offering separate bids in peak and off-peak hours increases the energy payments (compare Table 5.11 with Table 5.9, lines 1, 6, 11). This is expected, because allowing units to bid above their cost during hours where the load is high gives them more flexibility to place high offers during these hours than if they had to place a single offer for all hours, where they would have to compromise their tendency to bid low when the load is low and high when the load is high.

5.7.5 Agent-Based Simulation

In the context of electricity markets, agent-based simulation models mostly focus on the market participants, and, in particular, on how their behavior is adapted over time based on their accumulated experience through their interaction with the environment (e.g., demand variations, competitors' decisions, etc). As a result, the particular market under consideration is simulated for a sufficient number of consecutive time periods, during which the market is allowed to evolve without intervention. This enables the study of the behavior patterns of individual market participants, and the recognition of the impact that these decisions have on the market dynamics. In this context, agent-based models consider some reinforcement learning algorithm to model the market reaction of the market participants, i.e., an algorithm that describes how the knowledge accumulated by each of the market participants is utilized and is put in effect as time evolves.

The primary aim of the present work is not to study the strategic behavior of market participants as time evolves, but to evaluate the performance and incentive compatibility of different recovery mechanisms. For this reason, the problem in question is solved as a single shot (one day) game, and the different recovery mechanisms are compared on the basis of the market clearing results of this single day. This implies that in our case it might not make much sense to employ a reinforcement learning algorithm for modeling the market participants' behavior, since the problem is considered over a single daily horizon. Although computationally tedious, we resort to the iterative numerical solution procedure described in the manuscript for obtaining the single-shot optimal bidding offers of the market participants (at equilibrium if one can be identified given the inherent computational limitations), since an analytical procedure that will directly achieve this is not available.

We have tried different variations of the basic methodology, which utilize features, such as stochastic choice or adaptation, from agent-based methodologies.

In one variation, which we present in Subsection 5.7.2, we assume that each unit, not knowing how the other units will bid, probabilistically chooses an offer among the offers that it submitted over all 50 rounds of the main evaluation methodology with a probability that is equal to the frequency with which that offer was placed during the rounds. The results showed that the performance of the three mechanisms remained practically unchanged. The same was true when we assumed that each unit places an offer which is equal to the average of the offers that it submitted over all 50 rounds of the main evaluation methodology.

In another variation, the offer of generating unit u at the nth iteration, $\mathbf{b}_u^{(n)}$, rather than being set equal to the optimal value given the competitors' offers in the previous iteration, $\mathbf{b}_u^{(*)}\left(\underline{\mathbf{b}}_{-u}^{(n-1)}\right)$, as is described in equation (5.36), is set equal to a weighted average of $\mathbf{b}_u^{(*)}\left(\underline{\mathbf{b}}_{-u}^{(n-1)}\right)$ and the offer of the same unit in the previous iteration, $\mathbf{b}_u^{(n-1)}$, namely, $\mathbf{b}_u^{(n)} = \alpha\left(\underline{\mathbf{b}}_{-u}^{(n-1)}\right) + (1-\alpha)\mathbf{b}_u^{(n-1)}$, where α is a smoothing coefficient between 0 and 1. This way the selection of $\mathbf{b}_u^{(n)}$ is based on the history of all the offers of the unit in the previous iterations with weight that decrease exponentially with age. The following figure shows the bidding behavior of the generators under mechanism B.2 with $\beta = 3$ using the exponential smoothing variation of the methodology just described with $\alpha = 0.2$, and is comparable with Figure 5.3.

As is expected, the results are smoother and the uplifts are still low.

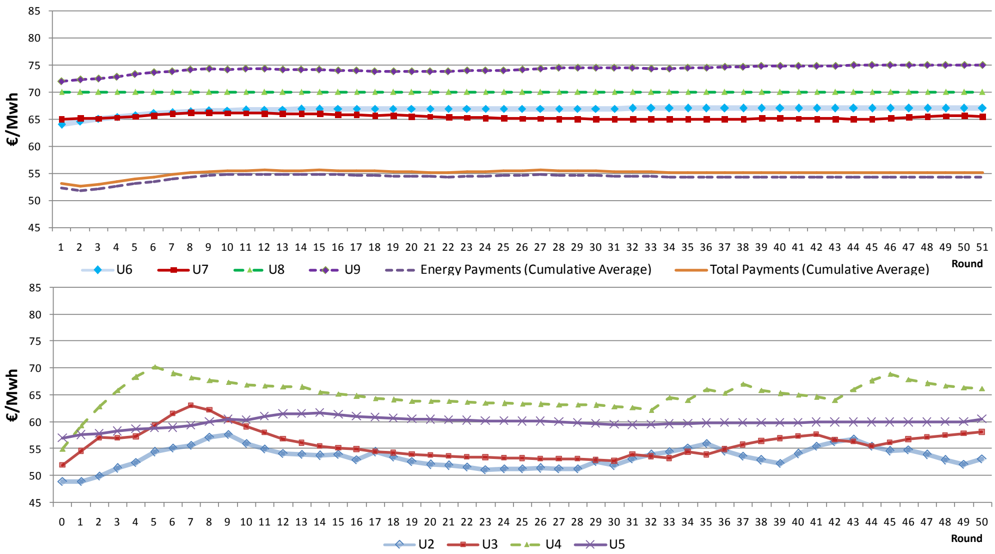

Figure 5.5: Bidding pattern for mechanism B.2, with $\beta = 3$ using exponential smoothing $\alpha = 0.2$.

5.8 Conclusions and Issues for Further Research

Many approaches in the literature propose pricing above marginal cost as a means of recovering average variable costs, in the presence of non-convexities. In this chapter, we keep classical marginal (bid-cost) pricing by solving the DAS problem, whose objective is to minimize system bid-cost, and set up an additional mechanism that recovers the commitment costs and may also provide side-payments to eliminate any market revenue losses. This approach does not directly interfere with the market design, as it provides side-payments after the market is cleared. It interferes with it only indirectly, in that the units' bidding decisions should take into account both the revenues from the market commodities and the recovery mechanism.

We consider various recovery mechanisms and discuss their advantages and disadvantages. We also propose a comprehensive methodology for evaluating these mechanisms in terms of their performance, market power and incentive compatibility properties, as the non-convexities, inter-temporal effects, and other structural elements of the market affect the players' bidding strategies in ways which are far from obvious.

The proposed methodology for evaluating the recovery mechanisms could be classified as a simulation approach that seeks to find equilibria without resorting to simplifying assumptions regarding the players' bidding options (e.g., Cournot bidding models), the competitors' response function (e.g., supply function competition models), or the dependence of the market price on the players' bids, (e.g., simulation models that are based on price-quota functions). It still refers to a static model, however, which neglects the fact that market participants base their decision on their accumulated experience through their interaction with the market environment (e.g. demand variations, competitors' decisions, etc.). A direction for further research would be to use adaptive agent-based simulation methodologies to reveal features of electricity markets that a static model ignores. Recent reviews of such methodologies can be found in [72, 73, 74].

We also presented the implementation of the proposed recovery mechanism evaluation methodology, associated practical details, and evaluated the recovery mechanisms in a realistic market model of the Greek energy market. The results can be summarized as follows.

Recovery mechanisms A.1 (variable cost related side-payments), A.2 (market revenue loss-related side-payments), and B.2 (rcIP+) prevail over mechanism B.1 (IP+) which leads to elevated uplifts and total payments. Under either of these three prevailing mechanisms, the total uplifts are quite reasonable. Group-A units (those with lower variable costs) tend to bid higher, exhibit a more "volatile" bidding behavior and have higher profits than group-B units (with higher variable costs). In fact, under mechanisms A.1 and A.2, the units with higher variable costs tend to bid close to or even at their cost. Under mechanism B.2, units with higher variable costs bid close to their cost only for low values of the cap and tend to bid higher as the cap increases. Group-A units tend to bid over the cap more frequently than units with higher variable cost. Further, the frequency of overbidding decreases with the cap.

Despite these minor variations in bidding behavior among different units, particularly those with higher variable costs, the aggregate results for the three prevailing mechanisms are comparable. A multi-criteria approach that evaluates the overall performance of each mechanism on the basis of multiple indicators described herein would be worth pursuing in future research.

Although the three prevailing mechanisms perform similarly, they differ qualitatively. The main disadvantage of mechanism A.1 is that it sets the net profits proportional to the variable costs, irrespectively of the magnitude of the market losses. This favors units with higher variable costs, i.e., inefficient units. Mechanism A.2 favors higher losses over lower losses, which may appear as counter intuitive to some participants. Mechanism B.2 seems to "naturally" align with the participants' perspective.

The results of our further numerical sensitivity analysis showed that the demand (load) level does not significantly affect the performance and still produces comparable outcomes between the three prevailing mechanisms. In addition, the assumptions of the evaluation methodology on the step size have an insignificant impact on the performance. We also saw that the average aggregate performance of the three mechanisms remains practically the same even when each unit randomly chooses its offers from a probability distribution which is equal to the marginal frequency of its deterministic optimal offers during the course of the evaluation methodology. Given that the existence of pure strategy equilibrium solutions is highly improbable, a direction for further research might be to look for mixed strategy equilibrium solutions. One should keep in mind, however, that in real markets one may prefer to settle for reasonable deterministic profits than try to maximize expected average profits.

Our results showed that the performance of the three mechanisms remains practically unchanged even when each unit chooses a deterministic offer whose value is equal to the average optimal offers during the course of the evaluation methodology. Similarly, placing offers above the regulated cap, though mathematically justifiable, carries an uncertainty that many participants may not be willing to tolerate. As such, market participants may be placing offers below the cap more often that our model predicts. Furthermore, it was seen that allowing the units to bid under their cost does not produce desirable outcomes.

We realize that the sensitivity analysis employed in this chapter can never include all aspects of such a complicated problem. It is rather used to enhance the confidence in the results and perhaps reveal some interesting findings or verify suspected outcomes. Additional sensitivity analysis might be worth performing and might be of interest to regulators and Independent System Operators (ISOs) who may wish to examine the performance of some of the proposed mechanisms. For instance, one could experiment with the technical characteristics and costs of the units which can be affected by fuel prices. Also, the proposed numerical procedure could be straightforwardly extended to accommodate more decision variables for each market participant, but the computational requirements would rise dramatically. Parallel computation could be very helpful in this respect. Another way would be to make further assumptions on the players' bidding options (e.g., Cournot bidding), or simplifying the profit maximizing units' optimization problem by assuming a competitors' response function (e.g., supply function competition), which would make the optimization problem of each profit maximizing unit easier.

Another direction for further research would be to see how the results extend to other market paradigms than the ones based on integrated co-optimized energy and ancillary services without transmission constraints.

Furthermore, we note that the solution of the bilevel problem represented in equation (5.33), which has been solved by discretization and "brute force" optimization is by itself a particularly challenging bilevel optimization problem. In a parallel to this thesis work [100], parametric integer programming has been employed to find a global optimal solution for an instance of this problem. Further elaboration and generalization of this approach is another direction for further research.

This chapter does not deal with recovery mechanisms required to limit or eliminate the expansion of side-payments, intentionally sought by market participants, above and beyond the appropriate outcome of a competitive market, by manipulating the interplay between the day-ahead and real-time markets. Neither does it deal with side-payments for lost opportunity costs, as in the case of a low-cost generator that may be scheduled to be offline even if the energy clearing price is low ([16, 17]), as this is a somewhat controversial issue which is outside the scope of this thesis. These issues could be directions for further research.

Chapter 6

Thesis Summary

Pricing in markets with non-convexities remains to this day an open challenge at the interface of economics, operations research, and engineering, featuring a mix of mechanism design, market competition, and regulation, with significant practical implications. This topic has attracted renewed interest due to the deregulation of electricity markets worldwide, and particularly in the context of the unit commitment problem in pool-based wholesale electricity markets. In these market designs, generation units submit multi-part offers for their marginal and fixed costs, and also face minimum supply requirements; hence they are characterized by non-convexities.

In recent years, various pricing schemes have been proposed to address this issue; however, to date, the connection between them has not been thoroughly studied. We distinguish between three types of approaches:

(i) Pricing schemes that provide external side-payments (uplifts). These include a scheme referred to as IP+ pricing — IP refers to Integer Programming, which involves marginal pricing plus make-whole payments, a variant of IP+ referred to as modified IP (mIP), which produces more stable prices, and an approach referred to as minimum uplift or Convex Hull (CH) pricing.

(ii) Pricing schemes that consider uplifts as internal zero-sum transfers between the suppliers. These include a scheme referred to as Generalized Uplift (GU) and a scheme proposed in this thesis, referred to as Minimum Zero-Sum Uplift (MZU).

(iii) Pricing schemes that provide revenue-adequate prices (and hence no uplifts). These include Average Cost (AC), Semi-Lagrangean Relaxation (SLR), and Primal-Dual (PD) pricing.

In the first part of this thesis, we review the aforementioned pricing schemes, by considering a basic model of two suppliers with asymmetric capacities and asymmetric marginal and fixed costs, who compete to satisfy a deterministic and inelastic demand of a commodity in a single period. The suppliers simultaneously bid their costs to an auctioneer, who determines the optimal allocation and the resulting payments. In contrast to the extant literature, we derive closed-form expressions for the price, uplifts, and profits for each scheme, and we use these expressions to compare these schemes along these three dimensions.

Our comparison shows that the mIP scheme generates the same profits as IP+ but with lower and less volatile prices and higher uplifts. CH and MZU generally generate lower uplifts and higher prices than IP+. In the case of CH, the uplifts are external; hence, the profits are higher. Under MZU, the profits remain unchanged, as the uplifts are internal zero-sum payments between the suppliers. GU also provides internal zero-sum payments, but at prices and profits which can be much higher than their MZU counterparts and are potentially unbounded. AC and SLR completely eliminate uplifts, but the resulting prices and profits can be substantial and also potentially unbounded. Finally, PD also eliminates uplifts at a possibly lower price than AC and SLR, trading off price efficiency for cost efficiency.

Our analysis identifies trade-offs between the market outcome characteristics (e.g., size of price, uplifts, and profits, efficiency, slope of price vs. demand, etc.) that are weighed differently by each scheme. We also extend some of our analytical comparisons to more than two suppliers and discuss the case of price-elastic demand.

Based on the closed-form expressions that we develop, we numerically explore and compare the quantities, prices, and profits as a function of the demand. We present several graphs for the two-supplier case for various sets of parameters, and we comment on the results and differences. We also consider an existing modification of "Scarf's example" that has been used as a benchmark for numerical evaluation of several pricing mechanisms. Given that comparisons between the IP+, mIP, CH, and

PD pricing schemes already exist, our attention is restricted to mechanisms that do not provide external uplifts, namely, GU, MZU, AC, SLR, and PD. We show that SLR generates the highest price, which exhibits particularly high spikes at certain demand levels. The prices of GU, MZU, PD, and AC are comparable and contained, with AC being the highest. Notably, the PD price is not always greater than or equal to the MZU price, as in our two-supplier model. This is because in the particular example, the PD scheme has more flexibility in trading off price efficiency for cost efficiency, since there are more than two units and unit types to reallocate. The containment of the AC and GU prices is due to the choice of parameter values. We also show that by modifying these values, the AC and GU prices also exhibit spikes. This is in line with our finding that the GU, AC, and SLR prices can be excessively high.

We close the numerical investigation by considering an actual market model that is based on the Greek wholesale electricity market. For this test case of a real market with non-convexities, we evaluate the aggregate (annual) impact of the recovery mechanism that is implemented in the Greek market against the standard bid/cost recovery (IP+) mechanism. We adopt several assumptions on the bidding behavior of the generating units, and focus on the annual magnitude of the recovery payments under these assumptions. The results show that the recovery payments are significantly lower under the standard bid/cost recovery mechanism. However, the latter mechanism may lead to situations where some units submit particularly high bids in an attempt to take advantage of the bid-based recovery payments. We address this issue in the second part of this thesis.

In the second part of the thesis, we study the bidding behavior of the market participants in markets with non-convexities and explore the implications of different pricing schemes or mechanisms on the incentives of the market participants. Following on a handful of analytical works that find equilibria in duopoly settings, we propose alternative market mechanisms and compare the equilibrium outcomes.

More specifically, we extend Fabra *et al.*'s duopoly model with asymmetric marginal costs and capacities, by introducing a fixed cost component, and we study recovery mechanisms that ensure that the suppliers will not exhibit losses while participating in an electricity auction. Firstly, we assume a mechanism that fully compensates the suppliers for their fixed costs, whenever they occur, thus allowing the players to compete with their bids based on their marginal cost. We show that this mechanism, which we refer to as fixed cost recovery (FCR), results in the same equilibrium outcomes with an already studied convex duopoly setting. Secondly, we propose an alternative mechanism, which compensates the players allowing for a positive profit that is proportional to their losses. We refer to this mechanism as loss-related profits recovery (LPR). For this mechanism, we derive equilibrium outcomes and show that it can be designed in such a way that results in lower total payments, and lower or equal equilibrium prices, compared to the FCR mechanism.

The above mechanisms can be characterized as cost-based recovery mechanisms, in the sense that the side-payments are based on the suppliers' actual costs. We also consider bid/cost recovery mechanisms that compensate the suppliers on an as-bid basis. Another basic difference in the assumptions is that both marginal and fixed costs are taken into account by the auctioneer, as in a traditional unit commitment problem; this yields a rather non trivial electricity auction.

As a starting point, we use the IP+ pricing scheme, for which equilibrium outcomes exist in the literature for a symmetric-capacity duopoly. We also propose a variant of this mechanism, which we refer to as rcIP+ (rc: regulated cap), where the suppliers are entitled to the make-whole payments under the condition that the offered bids are within a certain regulated margin from the actual marginal costs (regulated cap). Our analysis and comparison with the results for the symmetric-capacity duopoly show that the introduction of the regulated cap leads to equilibrium outcomes that outperform the ones of the standard IP+ in terms of uplifts for the low-demand case. It also leads to the existence of pure-strategy equilibria in the high-demand case, whereas only mixed-strategy equilibria exist under IP+.

The simple stylized examples, such as the aforementioned duopoly models, are useful for identifying equilibrium outcomes, and revealing the properties of the various mechanisms. Finding Nash equilibria for more complicated market designs, such as actual electricity markets, however, becomes practically infeasible. With this in mind, we try to numerically evaluate several alternative recovery mechanism designs that keep marginal pricing and provide side-payments after the market is cleared (ex post).

The mechanisms differ in the type and amount of payments with which they reimburse each generating unit that exhibits losses. The first design that we examine (design A.1) lets the losing units keep a fixed percentage of their variable costs. A variant of this design has been used in the Greek market. The second design (design A.2) lets the losing units keep a fixed percentage of their losses. The concept of this mechanism is based on the LPR mechanism. The third design (design B.1) is the IP+ scheme, which is the currently deployed bid/cost recovery scheme by System Operators in the U.S. The fourth design (design B.2) is the rcIP+ variant.

We propose a methodology for evaluating the bidding strategy behavior of the participating units for each mechanism. This methodology employs an iterative numerical algorithm aimed at finding the joint optimal bidding strategies of the profit-maximizing units. We apply this methodology to evaluate the performance and incentive compatibility properties of each recovery mechanism on a test case model representing the Greek joint energy/reserve day-ahead electricity market. To make the optimization problem computationally tractable, we make certain simplifying assumptions, without loss of generality of the most important features of a realistic zonal market design. This analysis leads to results that allow us to gain insights and draw useful conclusions on the performance and incentive compatibility properties of the recovery mechanisms.

Recovery mechanisms A.1 (variable cost related side-payments), A.2 (market revenue loss-related side-payments), and B.2 (rcIP+) prevail over mechanism B.1 (IP+) which leads to elevated uplifts and total payments. Under any of these three prevailing mechanisms, the total uplifts are quite reasonable. Units with lower variable costs tend to bid higher, exhibit a more "volatile" bidding behavior and have higher profits than units with higher variable costs. In fact, under mechanisms A.1 and A.2, the units with higher variable costs tend to bid close to or even at their cost. Under mechanism B.2, units with higher variable costs bid close to their cost only for low values of the cap and tend to bid higher as the cap increases. Units with lower variable costs tend to bid over the cap more frequently than units with higher variable cost. Further, the frequency of overbidding decreases with the cap. Lastly, we perform sensitivity analysis with respect to key parameters and assumptions and we provide directions for further research.

Appendix A

Proofs and Supplementary Material of Chapter 2

A.1 Proof of Proposition 2.1

We distinguish the following three cases:

1) If $d \leq k_1$, then any of the two suppliers can satisfy the entire demand. The optimal solution is supplier $z_{r(d)} = 1, z_{R(d)} = 0$ by definition of $r(d)$ and $R(d)$.

2) If $k_1 < d \leq k_2$, then there are two feasible solutions: Either supplier 2 satisfies the entire demand or both suppliers are dispatched. We compare the two solutions for the following two subcases: a) $i = 2$, and b) $i = 1$. For subcase a) the optimal solution is to dispatch only supplier 2. For subcase b), the solution of dispatching both suppliers is optimal if $b_2 d + f_2 > b_1 k_1 + f_1 + b_2(d - k_1) + f_2$ or, equivalently, $b_2 > b_1 + f_1/k_1$. Note that in this subcase, $k = k_1$. If, on the other hand, $b_2 \leq b_1 + f_1/k_1$, then the optimal solution is to dispatch only supplier 2, and $k = k_2$.

3) If $d > k_2$ then the only feasible solution is to dispatch both suppliers. The objective function is minimized if supplier i is dispatched at full capacity k_i, and supplier I at the residual demand $d - k_i$.

A.2 Proof of Proposition 2.3

The complicating variables of the IP problem are the relaxed commitment variables z_n, $n = 1, 2$. The Benders cut that is generated when viewing the IP problem as a Benders sub-problem, is:

$$\sum_{n=1,2} \nu_n z_n \geq \sum_{n=1,2} \nu_n z_n^{\text{MILP}}, \tag{A.1}$$

where ν_n is the dual variable of constraint $z_n = z_n^{\text{MILP}}$, $n = 1, 2$, in the IP problem.

For the case $d > k$ (high demand), the optimal solution $z_i^{\text{MILP}} = z_I^{\text{MILP}} = 1$ is the only feasible solution, and (A.1) is a supporting valid inequality, since it supports the optimal solution (by definition), and it does not exclude any other feasible solutions.

For the case $d \leq k$ (low demand), $z_{r'(d)}^{\text{MILP}} = 1$, $z_{R'(d)}^{\text{MILP}} = 0$, $\nu_{r'(d)} = f_{r'(d)}$, and, using standard duality analysis, $\nu_{R'(d)} = f_{R'(d)} - (b_{r'(d)} - b_{R'(d)})^+ k_{R'(d)}$. In this case, inequality (A.1) becomes:

$$f_{r'(d)} z_{r'(d)} + [f_{R'(d)} - (b_{r'(d)} - b_{R'(d)})^+ k_{R'(d)}] z_{R'(d)} \geq f_{r'(d)}. \tag{A.2}$$

For the sub-case $k_1 < d \leq k$ (this sub-case exists only if $k = k_2$, which from (2.1) holds only if $f_1 \geq (b_2 - b_1) k_1$), $r'(d) = 2$ by Proposition 2.1, and (A.2) becomes:

$$f_2 z_2 + [f_1 - (b_2 - b_1)^+ k_1] z_1 \geq f_2. \tag{A.3}$$

There are two feasible solutions for the integer variables: 1) $z_1^{\text{MILP}} = 0$, $z_2^{\text{MILP}} = 1$ (optimal solution) and 2) $z_1 = z_2 = 1$. Clearly, (A.3) supports the optimal solution and is also valid for $z_1 = z_2 = 1$.

For the sub-case $d \leq k_1$, $r'(d) = r(d)$, by Proposition 1, and (A.2) becomes:

$$f_{r(d)} z_{r(d)} + [f_{R(d)} - (b_{r(d)} - b_{R(d)})^+ k_{R(d)}] z_{R(d)} \geq f_{r(d)}. \tag{A.4}$$

There are three feasible solutions for the integer variables: 1) $z_{r(d)}^{\text{MILP}} = 1$, $z_{R(d)}^{\text{MILP}} = 0$ (optimal solution), 2) $z_{r(d)} = z_{R(d)} = 1$, and 3) $z_{r(d)} = 0$, $z_{R(d)} = 1$. Clearly, (A.4) supports the optimal solution. The question is whether it is also valid for $z_{r(d)} = 0$,

$z_{R(d)} = 1$ and $z_{r(d)} = z_{R(d)} = 1$. For these two solutions, (A.4) becomes:

$$f_{R(d)} - (b_{r(d)} - b_{R(d)})^+ k_{R(d)} \geq f_{r(d)}, \tag{A.5}$$

$$f_{R(d)} - (b_{r(d)} - b_{R(d)})^+ k_{R(d)} \geq 0. \tag{A.6}$$

Clearly, if (A.5) holds, then (A.6) holds as well. Let us then focus on (A.5) only. There are three cases to consider, corresponding to cases A, B, and C of Figure 2.1.

Case A: $r(d) = i$, $f_i \leq f_I$. In this case, (A.5) can be written as $f_I - f_i \geq (b_i - b_I)^+ k_I$. This inequality is valid, since its lhs is non-negative, and its rhs is zero (recall that $b_i \leq b_I$).

Case B: $f_i > f_I$, $f_i + b_i k_1 < f_I + b_I k_1$ (clearly, $f_i + b_i k_2 < f_I + b_I k_2$, as well). There are two sub-cases to consider: 1) Sub-case B1: $r(d) = I$. In this case, (A.5) can be written as $f_i + b_i k_i \geq f_I + b_I k_i$, which is valid neither for $k_i = k_1$ nor for $k_i = k_2$. 2) Sub-case B2: $r(d) = i$, $f_i > f_I$. In this case, (A.5) can be written as $f_I - f_i \geq (b_i - b_I)^+ k_I$, which is also not valid.

Case C: $r(d) = I$, $f_i > f_I$, $f_i + b_i k_1 \geq f_I + b_I k_1$. In this case, (A.5) can be written as $f_i + b_i k_i \geq f_I + b_I k_i$. This inequality is definitely valid if $k_i = k_1$, which corresponds to case (b) in Figure 2.2; however, it is not necessarily valid if $k_i = k_2$, which corresponds to case (a) in Figure 2.2.

Thus far, we showed that the only cases where (A.1) is not a valid inequality is when 1) $d \leq k_1$, $f_i > f_I$, and $f_i + b_i k_1 < f_I + b_I k_1$, which corresponds to case B of Figure 2.1 (low demand) and 2) $d \leq k_1$, $f_i > f_I$, $f_i + b_i k_1 \geq f_I + b_I k_1$, and $k_i = k_2$, which corresponds to case C of Figure 2.1 (low demand), when $i = 2$ (corresponding to case (a) of Figure 2.2). To find a supporting valid inequality for these cases, it is necessary to regard also one of the continuous variables as a complicating variable. First consider case B2 in Figure 2.1. For this case, $r(d) = i$ (hence, $R(d) = I$), and therefore (A.1), which is equivalent to (A.4) since $d \leq k_1$, can be written as $f_i z_i + f_I z_I \geq f_i$. This constraint is not valid for the feasible solution $z_i = 0$, $z_I = 1$, because in region B, $f_i > f_I$. To make it feasible, a positive term has to be added to its lhs, which can only involve continuous variable q_I, because $q_i = 0$, when $z_i = 0$, $z_I = 1$. The desired valid inequality has the form $f_i z_i + f_I z_I + x_I q_I \geq f_i$, where x_I is a coefficient such that the constraint remains valid when q_I takes its smallest possible value in case B2, which is $k_c = (f_i - f_I)/(b_I - b_i)$ (see Figure 2.1 (case B)); hence, x_I satisfies $f_I + x_I(f_i - f_I)/(b_I - b_i) \geq f_i$. The smallest value of x_I that satisfies this inequality is $b_I - b_i$, and the desired valid inequality is $f_i z_i + f_I z_I + (b_I - b_i) q_I \geq f_i$. It is straightforward to derive such an inequality also in cases B1 and C when $k_i = k_2$. The general form of the inequality for all three cases is

$$f_i z_i + f_I z_I + (b_I - b_i) q_I \geq f_i z_i^{\text{MILP}} + f_I z_I^{\text{MILP}} + (b_I - b_i) q_I^{\text{MILP}}. \tag{A.7}$$

Finally, it is straightforward to show that in these three cases (B1, B2, and C when $i = 2$), if we add constraint $q_I = q_I^{\text{MILP}}$ to the IP problem and solve the resulting mIP problem, the price, uplifts, and profits generated are $\lambda^{\text{mIP}} = \min(b_{r(k_1)}, b_2) = b_i$, $\sigma_{r(d)}^{\text{mIP}} = f_{r(d)} + (b_{r(d)} - b_i)d$, and $\pi_{r(d)}^{\text{mIP}} = 0$.

A.3 Proof of Proposition 2.5

First, consider the case $d \leq k$ (low demand). In this case, the GU solution is the optimal MILP solution, $q_{r'(d)}^{\text{MILP}} = d$, $q_{R'(d)}^{\text{MILP}} = 0$. From (2.16) we get $\lambda^{\text{GU}} = b_{r'(d)} + \Delta b_{r'(d)}$, and from (2.20) $d\Delta b_{r'(d)} + \Delta f_{r'(d)} = 0$, i.e., $\Delta b_{r'(d)} = -\Delta f_{r'(d)}/d$. Also, from (2.19), $[\lambda - (b_{r'(d)} + \Delta b_{r'(d)})]d - (f_{r'(d)} + \Delta f_{r'(d)}) \geq 0$, and because the first term in the lhs is zero, we get $\Delta f_{r'(d)} \leq -f_{r'(d)}$. The optimization problem can now be written as follows:

$$\underset{\Delta b_{r'(d)}, \Delta f_{r'(d)}}{\text{Minimize}} \; L_{\text{GU}} = (d\Delta b_{r'(d)})^2 + (\Delta f_{r'(d)})^2, \tag{A.8}$$

subject to:

$$\Delta f_{r'(d)} \leq -f_{r'(d)}, \tag{A.9}$$

$$\Delta b_{r'(d)} = -\Delta f_{r'(d)}/d. \tag{A.10}$$

The solution of this problem is $\Delta f_{r'(d)} = -f_{r'(d)}$ and $\Delta b_{r'(d)} = f_{r'(d)}/d$, so that $\lambda^{\text{GU}} = b_{r'(d)} + f_{r'(d)}/d$ and $\sigma_{r'(d)}^{\text{GU}} = -(d\Delta b_{r'(d)} + \Delta f_{r'(d)}) = -(f_{r'(d)} - f_{r'(d)}) = 0$.

Next, consider the case $d > k$ (high demand). In this case, problem (2.14)–(2.20) can be reformulated as follows. Since both suppliers must be committed, (2.17) and

(2.18) are redundant and can be omitted. By replacing n with i or I, after some simple manipulations, we obtain the following problem:

$$\underset{\Delta b_i,\Delta f_i,\Delta b_I,\Delta f_I}{\text{Minimize}} \quad L_{\text{GU}} = (\Delta b_i k_i)^2 + (\Delta f_i)^2 + [\Delta b_I(d-k_i)]^2 + (\Delta f_I)^2 \quad \text{(dual variables)}, \tag{A.11}$$

subject to:

$$\Delta b_I - \Delta b_i \geq b_i - b_I \qquad (\alpha_1 \geq 0), \tag{A.12}$$

$$d\Delta b_I + \Delta f_I \geq f_i - (b_I - b_i)k_i \qquad (\alpha_2 \geq 0), \tag{A.13}$$

$$-\Delta f_I \geq f_I \qquad (\alpha_3 \geq 0), \tag{A.14}$$

$$k_i\Delta b_i + \Delta f_i + (d-k_i)\Delta b_I + \Delta f_I = 0 \qquad (\beta \in \Re). \tag{A.15}$$

The KKT conditions, which for this type of problem (quadratic objective function and linear constraints) are necessary and sufficient, are:

$$2k_i^2\Delta b_i + \alpha_1 - \beta k_i = 0, \tag{A.16}$$

$$2\Delta f_i - \beta = 0, \tag{A.17}$$

$$2(d-k_i)^2\Delta b_I - \alpha_1 - \alpha_2 d - \beta(d-k_i) = 0, \tag{A.18}$$

$$2\Delta f_I - \alpha_2 + \alpha_3 - \beta = 0. \tag{A.19}$$

It is straightforward to prove that $\alpha_1 = 0$ by contradiction. Regarding α_2 and α_3, we distinguish between four cases. For each case, we seek a solution satisfying the KKT conditions and the constraints.

Case 1: $\alpha_2 = 0$, $\alpha_3 = 0$. Using (A.16)–(A.19), we get $k_i\Delta b_i = \Delta f_i = (d-k_i)\Delta b_I = \Delta f_I = \beta/2$. From (A.15) we get $\beta = 0$, which cannot hold since, from (A.14), $\Delta f_I \leq -f_I < 0$. Therefore, case 1 yields no solution.

Case 2: $\alpha_2 = 0$, $\alpha_3 > 0$. Since $\alpha_3 > 0$, (A.14) is binding, i.e., $\Delta f_I = -f_I$. Using the KKT conditions, from (A.15) we get $\beta = 2f_I/3$, $\Delta b_i = f_I/(3k_i)$, $\Delta f_i = f_I/3$, and $\Delta b_I = f_I/[3(d-k_i)]$. From (A.19), we get $\alpha_3 = \beta - 2\Delta f_I = (2/3)f_I + 2f_I > 0$. Also, (A.12) results in $f_I(2k_i - d)/[3k_i(d-k_i)] \geq b_i - b_I$, which is verified since the lhs is positive and the rhs negative. Lastly, (A.13) yields:

$$(b_I - b_i)k_i - f_i \geq f_I(2d - 3k_i)/[3(d-k_i)]. \tag{A.20}$$

For convenience, we let

$$\zeta = b_I k_i - (f_i + b_i k_i), \tag{A.21}$$

$$\eta = f_I(2d - 3k_i)/[3(d-k_i)], \tag{A.22}$$

so that condition (A.20) is equivalent to $\zeta \geq \eta$. To summarize, if $\zeta \geq \eta$, then $\lambda^{\mathrm{GU}} = b_I + \Delta b_I = b_I + f_I/[3(d-k_i)]$ and $\sigma_I^{\mathrm{GU}} = -(d-k_i)\Delta b_I - \Delta f_I = 2f_I/3$. Noting that $\sigma_i^{\mathrm{GU}} = -\sigma_I^{\mathrm{GU}}$ from (2.21), the resulting profits of the two suppliers, as computed from (2.7), are: $\pi_i^{\mathrm{GU}} = \zeta - \eta$; $\pi_I^{\mathrm{GU}} = 0$.

Case 3: $\alpha_2 > 0$, $\alpha_3 = 0$. Since $\alpha_2 > 0$, (A.13) is binding which from (A.21) implies

$$d\Delta b_I + \Delta f_I = -\zeta. \tag{A.23}$$

From (A.15), using (A.16) and (A.17), we obtain

$$(d-k_i)\Delta b_I + \Delta f_I + \beta = 0. \tag{A.24}$$

Solving (A.18), (A.19), (A.23) and (A.24) for α_2, β, Δb_I and Δf_I, yields:

$\alpha_2 = -[8(d-k_i)^2/(4d^2 - 4k_i d + 3k_i^2)]\zeta$, $\beta = [2(d-k_i)(2d-k_i)/(4d^2-4k_i d+3k_i^2)]\zeta$,

$\Delta f_I = -[(d-k_i)(2d-3k_i)/(4d^2 - 4k_i d + 3k_i^2)]\zeta$, and $\Delta b_I = -[(2d+k_i)/(4d^2 - 4k_i d + 3k_i^2)]\zeta$.

The above imply that $\Delta f_I = [(d-k_i)(2d-3k_i)/(2d+k_i)]\Delta b_I$. Since $\alpha_2 > 0$ and $4d^2 - 4k_i d + 3k_i^2 > 0$, it follows that $\zeta < 0$. We also need to satisfy (A.12) and (A.14); constraints (A.13) and (A.15) are already satisfied since they were used to derive (A.23) and (A.24). Substituting the solution into (A.12), we get: $-\zeta[2d + k_i + (d-k_i)(2d-k_i)/k_i]/(4d^2 - 4k_i d + 3k_i^2) \geq b_i - b_I$, which holds always, since the lhs is positive and the rhs negative. Similarly, for (A.14), we get $[(d-k_i)(2d-3k_i)/(4d^2 - 4k_i d + 3k_i^2)]\zeta \geq f_I$. Since $\zeta < 0$, it follows that $2d - 3k_i < 0$, i.e., $d < 3k_i/2$; therefore,

$\zeta \leq \theta$, where

$$\theta = f_I(4d^2 - 4k_i d + 3k_i^2)/[(2d - 3k_i)(d - k_i)]. \tag{A.25}$$

Hence, $\zeta \leq \theta$ and $d < 3k_i/2$ imply $\lambda^{\mathrm{GU}} = b_I + \Delta b_I = b_I + (f_i + b_i k_i - b_I k_i)(2d + k_i)/(4d^2 - 4k_i d + 3k_i^2)$ and $\sigma_I^{\mathrm{GU}} = -(d - k_i)\Delta b_I - \Delta f_I = 2(d - k_i)(2d - k_i)[b_I k_i - (f_i + b_i k_i)]/(4d^2 - 4k_i d + 3k_i^2)$. The resulting profits of the two suppliers, as computed from (2.7), are: $\pi_i^{\mathrm{GU}} = 0$; $\pi_I^{\mathrm{GU}} = [(\zeta/\theta) - 1]f_I$.

<u>Case 4</u>: $\alpha_2 > 0$, $\alpha_3 = 0$. Since $\alpha_2 > 0$ and $\alpha_3 > 0$, (A.13) and (A.14) are binding, and yield $\Delta f_I = -f_I$ and $\Delta b_I = (f_I - \zeta)/d$. Substituting Δb_i and Δf_i from (A.16) and (A.17) into (A.15), we get $\beta = (k_i/d)f_I + [(d - k_i)/d]\zeta$ and $k_i \Delta b_i = \Delta f_i = (1/2)\{(k_i/d)f_I + [(d-k_i)/d]\zeta\}$. (A.18)–(A.19) yield $\alpha_2 = [(2d-3k_i)f_I - 3(d-k_i)\zeta](d-k_i)/d^2$ and $\alpha_3 = f_I(4d^2 - 4k_i d + 3k_i^2)/d^2 + \zeta(d - k_i)(3k_i - 2d)/d^2$. Finally, $\alpha_2 > 0$ implies $\zeta < \eta$, and $\alpha_3 > 0$ implies $\zeta(2d - 3k_i) < [(4d^2 - 4k_i d + 3k_i^2)/(d - k_i)]f_I$, which results in the following three conditions: 1) If $d > 3k_i/2$, then $\zeta < \theta$; 2) if $d < 3k_i/2$, then $\zeta > \theta$; 3) if $d = 3k_i/2$, then the condition always holds. We must also check the validity of (A.12), since (A.13)–(A.15) have already been used in the proof. (A.12) yields $f_I/2 + [(d + k_i)/(2k_i)]f_I + (b_I - b_i)(d - k_i)/2 \geq 0$, which always holds.

Next, we explore the relationship between η and θ. We have $\eta < \theta$, for $d > 3k_i/2$, and $\eta > \theta$ for $d < 3k_i/2$. For $d = 3k_i/2$, θ is not defined and $\eta = 0$. Hence, the conditions for which the solution holds are: 1) $d \geq 3k_i/2$ and $\zeta < \eta$; 2) $d < 3k_i/2$ and $\theta < \zeta < \eta$. Under these conditions, $\lambda^{\mathrm{GU}} = b_I + \Delta b_I = b_I + (f_i + b_i k_i - b_I k_i + f_I)/d$ and $\sigma_I^{\mathrm{GU}} = -(d - k_i)\Delta b_I - \Delta f_I = \{(d - k_i)[b_I k_i - (f_i + b_i k_i)] + k_i f_I\}/d$. Note that when $\zeta = \theta$ and $d < 3k_i/2$, the solutions of cases 3 and 4 are identical. The resulting profits of the two suppliers, as computed from (2.7), are: $\pi_i^{\mathrm{GU}} = \pi_I^{\mathrm{GU}} = 0$.

To summarize, in all three valid cases (2–4), the price is given by $\lambda^{\mathrm{GU}} = b_I + \Delta b_I^{\mathrm{GU}}$ and the uplifts are given by (2.21). Table A.1 shows the expressions for the uplift parameters Δb_I^{GU}, Δf_I^{GU} and profits π_n^{GU}, $n = 1, 2$, and the conditions under which they hold, for the three cases, where ζ, η, and θ are given by (A.21), (A.22), and (A.25), respectively.

Finally, it is straightforward — although tedious — to show that

$$\Delta b_I^{\mathrm{GU}} = \max(\Delta b_I^{(1)}, \Delta b_I^{(2)}, \Delta b_I^{(3)}). \tag{A.26}$$

Table A.1: Price, uplifts and profits generated by the GU pricing scheme for the high-demand case.

<table>
<tr><th colspan="3">Conditions</th><th>Δb_I^{GU}</th><th>Δf_I^{GU}</th><th>Profits</th></tr>
<tr><td colspan="3">$\zeta \geq \eta$</td><td>$\Delta b_I^{(1)} \equiv \frac{f_I}{3(d-k_i)}$</td><td>$-f_I$</td><td>$\pi_i^{\mathrm{GU}} = \zeta - \eta; \quad \pi_I^{\mathrm{GU}} = 0$</td></tr>
<tr><td rowspan="3">$\zeta < \eta$</td><td colspan="2">$d \geq 3k_i/2$</td><td rowspan="2">$\Delta b_I^{(2)} \equiv \frac{f_i + b_i k_i - b_I k_i + f_I}{d}$</td><td rowspan="2">$-f_I$</td><td rowspan="2">$\pi_i^{\mathrm{GU}} = 0; \quad \pi_I^{\mathrm{GU}} = 0$</td></tr>
<tr><td rowspan="2">$d < 3k_i/2$</td><td>$\zeta > \theta$</td></tr>
<tr><td>$\zeta \leq \theta$</td><td>$\Delta b_I^{(3)} \equiv \frac{[f_i + b_i k_i - b_I k_i](2d+k_i)}{4d^2 - 4k_i d + 3k_i^2}$</td><td>$\frac{(d-k_i)(2d-3k_i)}{(2d+k_i)} \Delta b_I^{(3)}$</td><td>$\pi_i^{\mathrm{GU}} = 0; \quad \pi_I^{\mathrm{GU}} = \left(\frac{\zeta}{\theta} - 1\right) f_I$</td></tr>
</table>

A.4 Proof of Proposition 2.8

First, consider the case $d \leq k$ (low demand). This case can be further divided into two sub-cases.

The first sub-case is $d \leq k_1$. In this case, if $\lambda < b_{r(d)} + f_{r(d)}/d$, any supplier dispatched at d will incur losses; therefore, the optimal MILP solution ($q_{r(d)}^{\mathrm{MILP}} = d$, $q_{R(d)}^{\mathrm{MILP}} = 0$) cannot be optimal for the SLR problem. If $\lambda = b_{r(d)} + f_{r(d)}/d$, the optimal MILP solution is optimal for the SLR problem, and as λ increases beyond $b_{r(d)} + f_{r(d)}/d$, the optimal MILP solution remains the only optimal solution. Therefore, $b_{r(d)} + f_{r(d)}/d$ is the smallest price maximizing $L_{\mathrm{SLR}}^*(\lambda)$. At this price, supplier $r(d)$ merely covers his costs, i.e., $\pi_{r(d)} = 0$.

The second sub-case is $k_1 < d \leq k$. This sub-case exists only if $k = k_2$, which from (2.1) is true only if $b_2 \leq b_1 + f_1/k_1$. In this case, if $\lambda < b_2 + f_2/d$, supplier 2 will incur losses if he is dispatched at d; therefore, the optimal MILP solution ($q_1^{\mathrm{MILP}} = 0$, $q_2^{\mathrm{MILP}} = d$) cannot be optimal for the SLR problem. If $\lambda = b_2 + f_2/d$, then the optimal MILP solution yields an SLR objective function value of $b_2 d + f_2$. The solution $q_1 = k_1$, $q_2 = 0$, on the other hand, yields an SLR objective function value of $b_2 d + f_2 + b_1 k_1 + f_1 - b_2 k_1 - f_2 k_1/d$. There are two cases to consider.

If $b_1 + f_1/k_1 \geq b_2 + f_2/d$, then the solution $q_1^{\mathrm{MILP}} = 0$, $q_2^{\mathrm{MILP}} = d$ is optimal for the SLR problem. As λ increases beyond $b_2 + f_2/d$, this solution remains the only optimal solution. Therefore, $b_2 + f_2/d$ is the smallest price maximizing $L_{\mathrm{SLR}}^*(\lambda)$. At this price, $\pi_2 = 0$.

If $b_1 + f_1/k_1 < b_2 + f_2/d$, then the solution $q_1 = k_1$, $q_2 = 0$ is optimal for the SLR problem. In this case, as λ increases beyond $b_2 + f_2/d$, this solution remains the only optimal solution until λ reaches a critical value, say λ_c, at which the cost of this solution becomes equal to the cost of the optimal MILP solution, making both solutions optimal. This critical value satisfies $b_2 d + f_2 = b_1 k_1 + f_1 + \lambda(d - k_1)$. Solving for λ yields $\lambda_c = b_2 + f_2/d + [b_2 + f_2/d - (b_1 + f_1/k_1)]k_1/(d - k_1)$. As λ increases beyond λ_c, the optimal MILP solution remains the only optimal solution of the SLR problem. Hence, λ_c is the smallest price maximizing $L^*_{\text{SLR}}(\lambda)$. In this case, $\pi_2 = [b_2 k_1 + f_2 k_1/d - (b_1 k_1 + f_1)]d/(d - k_1)$.

Combining the two cases gives $\lambda = b_2 + f_2/d + [b_2 + f_2/d - (b_1 + f_1/k_1)]^+ k_1/(d - k_1)$ and $\pi_2 = [b_2 k_1 + f_2 k_1/d - (b_1 k_1 + f_1)]^+ d/(d - k_1)$.

Next, consider the case $d > k$ (high demand). If $\lambda < b_I + f_I/(d - k_i)$, supplier I will incur losses if he is dispatched at $d - k_i$; therefore, the optimal MILP solution ($q_i^{\text{MILP}} = k_i$, $q_I^{\text{MILP}} = d - k_i$) cannot be optimal for the SLR problem. If $\lambda = b_I + f_I/(d - k_i)$, then the optimal MILP solution yields an SLR objective function value of $b_i k_i + f_i + b_I(d - k_i) + f_I$. The solution $q_i = 0$, $q_I = k_I$, on the other hand, yields an SLR objective function value of $b_I k_I + f_I + [b_I + f_I/(d - k_i)](d - k_I)$. Again, there are two cases to consider.

If $b_i k_i + f_i + b_I(d - k_i) + f_I \leq b_I k_I + f_I + [b_I + f_I/(d - k_i)](d - k_I)$, which can be rewritten as $b_i + f_i/k_i \leq b_I + (f_I/k_i)(d - k_I)/(d - k_i)$, then the solution $q_i^{\text{MILP}} = k_i$, $q_I^{\text{MILP}} = d - k_i$ is optimal for the SLR problem. As λ increases beyond $b_I + f_I/(d - k_i)$, this solution remains the only optimal solution. Therefore, $b_I + f_I/(d - k_i)$ is the smallest price maximizing $L^*_{\text{SLR}}(\lambda)$. At this price, the profits of the suppliers are $\pi_i = b_I k_i + f_I k_i/(d - k_i) - (b_i k_i + f_i)$ and $\pi_I = 0$.

If $b_i + f_i/k_i > b_I + (f_I/k_i)(d - k_I)/(d - k_i)$, then the solution $q_i = 0$, $q_I = k_I$ is optimal for the SLR problem. As λ increases beyond $b_I + f_I/(d - k_i)$, this solution remains the only optimal solution until λ reaches a critical value, say λ'_c, at which the cost of this solution equals the cost of the optimal MILP solution, making both solutions optimal, i.e., $b_I k_I + f_I + \lambda(d - k_I) = b_i k_i + f_i + b_I(d - k_i) + f_I$. Solving for λ yields $\lambda'_c = b_I + (b_i + f_i/k_i - b_I)k_i/(d - k_I)$. As λ increases beyond λ'_c, the optimal MILP solution is the only optimal solution of the SLR problem. Hence λ'_c is the smallest price maximizing $L^*_{\mathrm{SLR}}(\lambda)$. At λ'_c, $\pi_i = (b_ik_i + f_i - b_Ik_i)k_i/(d-k_I) - (b_ik_i + f_i - b_Ik_i)$ and $\pi_I = (b_ik_i + f_i - b_Ik_i)(d-k_i)/(d-k_I) - f_I$.

Combining the two cases and rearranging terms gives the final expressions for the prices and profits.

A.5 Proof of Proposition 2.9

For $d \le k_1$, one supplier suffices to cover the demand. The question is which supplier and at what price? To answer this, suppose that supplier m is committed and M is not committed. From (2.29)–(2.30), clearly $q_m = d$ and $q_M = 0$. With this in mind, objective function (2.28) can be written as:

$$\underset{\lambda,\mu_n,\nu_n,n=1,2}{\text{Minimize}}\ b_md + f_m - \lambda d + \nu_m + \nu_M. \tag{A.27}$$

We can show by contradiction that $k_m\mu_m - f_m \ge 0$, $n = 1,2$, which allows us to replace ν_m and ν_M by $k_m\mu_m - f_m$ and $(k_M\mu_M - f_M)^+$, respectively, and then further replace μ_m and μ_M by $\lambda - b_m$ and $(\lambda - b_M)^+$, respectively. Objective function (A.27) can then be reformulated as follows:

$$\underset{\lambda}{\text{Minimize}}\ b_md + \lambda(k_m - d) - b_mk_m + [k_M(\lambda - b_M)^+ - f_M]^+. \tag{A.28}$$

The coefficient multiplying λ in (A.28) is clearly positive; therefore, λ should be set to the lowest feasible value in the optimal solution. Given the revenue-adequacy constraints (2.33), λ is given by:

$$\lambda = b_m + f_m/d. \tag{A.29}$$

With this in mind, the corresponding minimum value of (A.28) can be written as:

$$(b_m + f_m/d)k_m - f_m + [k_M(b_m - b_M + f_m/d)^+ - f_M]^+. \tag{A.30}$$

Expression (A.30) gives the optimal value of (2.28) if supplier m is dispatched to cover d and supplier M is not committed at all. Clearly, the supplier that yields the smallest value of (2.28) minimizes (A.30). It is easy to see that this supplier is $r(d)$ and the resulting price and objective function value are given by (A.29) and (A.30), respectively, for $m = r(d)$, $M = R(d)$.

For $d > k_2$, both suppliers are needed to cover d, and (2.28) can be reformulated as:

$$\underset{\lambda, q_n, \mu_n, \nu_n, n=1,2}{\text{Minimize}} \sum_{n=1,2} (b_n q_n + f_n) - \lambda d + \sum_{n=1,2} \nu_n. \tag{A.31}$$

Following the same arguments as in the case $d \leq k_1$, we can show that $k_n \mu_n - f_n \geq 0$, $n = 1, 2$, which allows us to replace ν_n by $k_n \mu_n - f_n$ in (A.31) and subsequently μ_n by $\lambda - b_n$, $n = 1, 2$. Objective function (A.31) can then be reformulated as follows:

$$\underset{\lambda, q_n, n=1,2}{\text{Minimize}} \sum_{n=1,2} b_n q_n + \lambda \left(\sum_{n=1,2} k_n - d \right) - \sum_{n=1,2} b_n k_n. \tag{A.32}$$

The coefficient multiplying λ in (A.32) is clearly positive; therefore, λ should be set to its lowest feasible value in the optimal solution. Given the revenue-adequacy constraints (2.33), that value is

$$\lambda = \max_{n=1,2} \{b_n + f_n / q_n\}. \tag{A.33}$$

With this in mind, (A.32) can be further reformulated as:

$$\underset{q_n, n=1,2}{\text{Minimize}} \sum_{n=1,2} b_n q_n + \left(\sum_{n=1,2} k_n - d \right) \max_{n=1,2} \{b_n + f_n / q_n\} - \sum_{n=1,2} b_n k_n. \tag{A.34}$$

Thus far, we reduced the number of decision variables to two, namely, q_n, $n = 1, 2$. Using constraint (2.29), we can further reduce the number of decision variables to only one. Without loss of generality, let us keep q_i as the decision variable and substitute q_I by $d - q_i$. In this case, the problem can be reduced as follows (after omitting the constant terms in the objective function):

$$\underset{q_i}{\text{Minimize}}\, \tilde{L}_{\text{PD}} = (b_i - b_I)q_i + \left(\sum_{n=1,2} k_n - d\right) \max[b_i + f_i/q_i,\ b_I + f_I/(d - q_i)], \quad \text{(A.35)}$$

subject to:

$$q_i \leq k_i, \quad \text{(A.36)}$$

$$q_i \geq d - k_I. \quad \text{(A.37)}$$

Objective function (A.35) consists of the linear term $(b_i - b_I)q_i$ with a negative slope (since $b_i < b_I$) and a term involving the maximum of two functions. The first function, $b_i + f_i/q_i$, is convex and decreasing in q_i, for $q_i \geq 0$, whereas the second, $b_I + f_I/(d - q_i)$, is convex and increasing in q_i, for $q_i \leq d$. These functions represent the average cost of supplier i and I, respectively.

For the moment, ignore the linear term in (A.25), and focus on the "max" term. It is easy to see that the unconstrained minimizer of that term is the value of q_i which is at the intersection of the two functions and can be found by solving the equation $b_i + f_i/q_i = b_I + f_I/(d - q_i)$. This is a second-order algebraic equation whose roots are $[\beta \pm (\beta^2 + 4\alpha f_i d)^{1/2}]/(2\alpha)$, where $\alpha = b_i - b_I$ and $\beta = \alpha d - f_i - f_I$ and $\alpha, \beta < 0$. Let δ denote the discriminant, i.e., $\delta \equiv \beta^2 + 4\alpha f_i d$. It can be shown that δ satisfies $0 < \delta < \beta^2$, which means that $0 < \delta^{1/2} < -\beta$, implying that both roots are positive. It can also be shown that the solution $(\beta - \delta^{1/2})/(2\alpha) > d$, hence it has no physical meaning. The only root left is $(\beta + \delta^{1/2})/(2\alpha) < d$. Therefore, the unconstrained minimizer of $\max[b_i + f_i/q_i, b_I + f_I/(d - q_i)]$ is $q_i' = (\beta + \delta^{1/2})/(2\alpha)$. As q_i' is at the intersection of the two convex functions $b_i + f_i/q_i$ and $b_I + f_I/(d - q_i)$, where the first is decreasing and the second increasing, it is easy to see that

$$\max[b_i + f_i/q_i,\ b_I + f_I/(d - q_i)] = \begin{cases} b_i + f_i/q_i, & \text{if } q_i \leq q_i', \\ b_I + f_I/(d - q_i), & \text{if } q_i \geq q_i'. \end{cases}$$

Hence (A.35) can be written as the following continuous, piecewise differentiable function:

$$\tilde{L}_{\text{PD}} = \begin{cases} (b_i - b_I)q_i + \left(\sum_{n=1,2} k_n - d\right)(b_i + f_i/q_i), & \text{if } q_i \le q_i', \\ (b_i - b_I)q_i + \left(\sum_{n=1,2} k_n - d\right)[b_I + f_I/(d - q_i)], & \text{if } q_i \ge q_i'. \end{cases}$$

To minimize the above function, we minimize both its parts and compare them. The first part is decreasing in q_i, so it is minimized at the rightmost endpoint of the interval in which it is valid, namely q_i'. The second part consists of a linear component which is decreasing in q_i and a non-linear convex component which is increasing in q_i. To minimize it, we set its derivative equal to zero and solve for q_i. That derivative is $(b_i - b_I) + (\sum_{n=1,2} k_n - d) f_I/(d - q_i)^2$. Setting it equal to zero yields the solution $q_i'' = d - [(\sum_{n=1,2} k_n - d) f_I/(b_I - b_i)]^{1/2}$. If $q_i'' < q_i'$, then q_i'' is smaller than the leftmost endpoint of the interval in which the second part is valid. In this case, the minimizer is the leftmost endpoint q_i'. If $q_i'' > q_i'$, the minimizer is q_i''. Hence, the minimizer of $\tilde{L}_{\text{PD}}$ is $\max(q_i', q_i'')$. Finally, if we take into account (A.36) and (A.37), the constrained optimal value of q_i, denoted by q_i^{PD}, as well as q_I^{PD}, are given by

$$q_i^{\text{PD}} = \min[\max(q_i', q_i'', d - k_I), k_i], \quad q_I^{\text{PD}} = d - q_i^{\text{PD}}. \tag{A.38}$$

The optimal price λ^{PD} is given by (A.33) after replacing quantities q_n by the optimal values given by (A.38), namely, $\lambda^{\text{PD}} = \max(b_i + f_i/q_i^{\text{PD}}, b_I + f_I/q_I^{\text{PD}})$. The profits of the suppliers, denoted by π_i^{PD} and π_I^{PD}, can be easily computed as follows: $\pi_i^{\text{PD}} = \lambda^{\text{PD}} q_i^{\text{PD}} - (f_i + b_i q_i^{\text{PD}})$ and $\pi_I^{\text{PD}} = \lambda^{\text{PD}} q_I^{\text{PD}} - (f_I + b_I q_I^{\text{PD}})$.

As can be seen from (A.38), the optimal quantity q_i^{PD} (and therefore λ^{PD}) depends on the relative ordering of $q_i', q_i'', d - k_I, k_i$. There are five cases to consider, denoted by Q1–Q5, defined in terms of the relative ordering of the above four quantities: Q1: $q_i' \ge k_i$, Q2: $q_i' \le k_i \le q_i''$, Q3: $\max(d - k_I, q_i'') \le q_i' \le k_i$, Q4: $\max(d - k_I, q_i') \le q_i'' \le k_i$, Q5: $\max(q_i', q_i'') \le d - k_I$. Each case is uniquely characterized by a set of conditions on the problem parameters. For instance, the conditions characterizing Q2 are: 1) $q_i' \le k_i$, implying $b_I - b_i \ge f_I/(d - k_i) - f_i/k_i$, and 2) $q_i'' \ge k_i$, implying $b_I - b_i \ge (\sum_{n=1,2} k_n - d) f_I/(d - k_i)^2$.

Figure A.1 shows all possible values of q_i^{PD}, λ^{PD}, and the suppliers' profits for the five different regions of the q_i' vs. q_i'' space that correspond to cases Q1–Q5. It also shows the conditions characterizing each case and defining each region. The exact expressions of the suppliers' profits in the five regions are:

$$\begin{aligned}
&\mathbf{Q1}: \quad \pi_i^{\mathrm{PD}} = 0, \quad \pi_I^{\mathrm{PD}} = (b_i + \frac{f_i}{k_i})(d-k_i) - [f_I + b_I(d-k_i)];\\
&\mathbf{Q2}: \quad \pi_i^{\mathrm{PD}} = b_I k_i + \frac{f_I k_i}{d-k_i} - (b_i k_i + f_i), \quad \pi_I^{\mathrm{PD}} = 0;\\
&\mathbf{Q3}: \quad \pi_i^{\mathrm{PD}} = 0, \quad \pi_I^{\mathrm{PD}} = 0;\\
&\mathbf{Q4}: \quad \pi_i^{\mathrm{PD}} = b_I k_I + \frac{f_I q_i''}{d-q_i''} - (b_I - b_i)(k_I - q_i'') - (b_i k_I + f_i), \quad \pi_I^{\mathrm{PD}} = 0;\\
&\mathbf{Q5}: \quad \pi_i^{\mathrm{PD}} = b_I k_I + \frac{f_I(d-k_I)}{k_I} - (b_I - b_i)\left(\sum_{n=1,2} k_n - d\right) - (b_i k_I + f_i), \quad \pi_I^{\mathrm{PD}} = 0.
\end{aligned}$$

Figure A.2(a) shows graphs of λ^{PD} vs. $b_i + f_i/k_i$ for three representative instances. Figure A.2(b) shows graphs of the suppliers' profits vs. $b_i k_i + f_i$ for the same three representative instances. The darkly shaded areas in these two graphs indicate the regions that contain λ^{PD} and the profits, respectively, and are defined in Proposition 2.11 in Chapter 2. Note that supplier I makes a profit of $(b_i + f_i/k_i)(d-k_i) - [f_I + b_I(d-k_i)]$ only when $b_i + f_i/k_i \geq b_I + f_I/(d-k_i)$, which corresponds to case Q1. Also note that the condition $b_I - b_i \geq f_I(\sum_{n=1,2} k_n - d)/(d-k_i)^2$, which defines region Q2, becomes infeasible when $d \to k_i$. This means that region Q2 does not exist when $d \to k_i$, which further implies that the PD price and profits are bounded even as $d \to k_i$ (note that in region Q2, the price is $b_I + f_I/(d-k_i)$).

Finally, consider the case $k_1 < d \leq k_2$. In this case, the demand can be covered either by committing only supplier 2 ($z_1 = 0$, $z_2 = 1$) or by committing both suppliers ($z_1 = 1$, $z_2 = 1$). Denote the first solution by "(0,1)" and the second solution by "(1,1)." The optimal allocation of the first solution is clearly $q_1^{(0,1)} = 0$, $q_2^{(0,1)} = d$, and the corresponding optimal price and objective function value, denoted by $\lambda^{(0,1)}$ and $L_{\mathrm{PD}}^{(0,1)}$, are given by (A.29) and (A.28), respectively, for $m = 2$, $M = 1$. The optimal allocation of the second solution is denoted $q_n^{(1,1)}$, $n = 1, 2$, and is given by expression (A.38). The optimal price and objective function value, denoted by

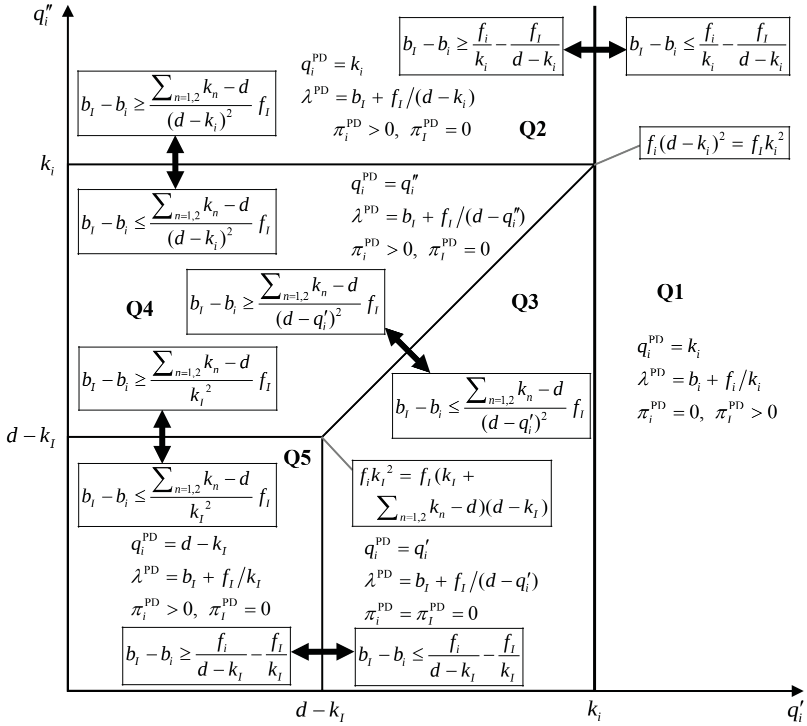

Figure A.1: Values of q_i^{PD}, λ^{PD}, and the suppliers' profits for the five possible regions of the q_i' vs. q_i'' space and the conditions that define each region.

$\lambda^{(1,1)}$ and $L_{\mathrm{PD}}^{(1,1)}$, are given by (A.33) and (A.32), respectively, after replacing q_n by $q_n^{(1,1)}$, $n = 1, 2$. It is easy to verify that the prices of the two solutions satisfy $\lambda^{(0,1)} = b_2 + f_2/d \le \max(b_2 + f_2/q_1^{(1,1)}, b_2 + f_2/q_2^{(1,1)}) = \lambda^{(1,1)}$, where $q_2^{(1,1)} = d - q_1^{(1,1)}$. To determine which of the two solutions is optimal we need to consider the difference $L_{\mathrm{PD}}^{(0,1)} - L_{\mathrm{PD}}^{(1,1)}$. This difference is given by:

$$L_{\mathrm{PD}}^{(0,1)} - L_{\mathrm{PD}}^{(1,1)} = (\lambda^{(0,1)} - \lambda^{(1,1)})(k_2 - d) + (b_2 - b_1)q_1^{(1,1)} + [k_1(\lambda^{(0,1)} - b_1)^+ - f_1]^+ - k_1(\lambda^{(1,1)} - b_1).$$

It can be easily shown that if $b_2 \le b_1 + f_1/q_1^{(1,1)}$, then $L_{\mathrm{PD}}^{(0,1)} - L_{\mathrm{PD}}^{(1,1)} \le 0$, implying

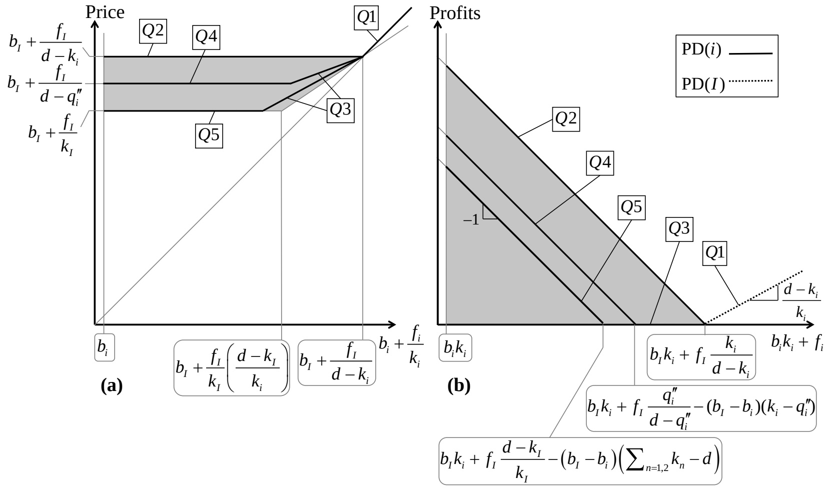

Figure A.2: (a) Price vs. $b_i + f_i/k_i$ and (b) profits vs. $b_ik_i + f_i$ for suppliers i and I, for the PD scheme for three representative instances of the problem parameters, for the case $d > k_2$.

that solution (0,1) is optimal. Note that the condition $b_2 \leq b_1 + f_1/q_1^{(1,1)}$ always holds for cases (a) and (b) of Figure 2.2, and may or may not hold in case (c). If $b_2 > b_1 + f_1/q_1^{(1,1)}$, then clearly $\lambda^{(1,1)} = b_2 + f_2/q_2^{(1,1)}$, $\lambda^{(0,1)} - \lambda^{(1,1)} = f_2/d - f_2/q_2^{(1,1)} = -(q_1^{(1,1)}/q_2^{(1,1)})(f_2/d)$, and the above difference becomes:

$$L_{\mathrm{PD}}^{(0,1)} - L_{\mathrm{PD}}^{(1,1)} = b_2 q_1^{(1,1)} - f_2(q_1^{(1,1)}/q_2^{(1,1)})[(k_1 + k_2 - d)/d] - (b_1 q_1^{(1,1)} + f_1).$$

Clearly, in this case, $L_{\mathrm{PD}}^{(0,1)} - L_{\mathrm{PD}}^{(1,1)} \geq 0$ only if the following condition holds:

$$b_2 + \frac{f_2}{q_2^{(1,1)}} \geq b_1 + \frac{f_1}{q_1^{(1,1)}} + \frac{f_2}{q_2^{(1,1)}} \frac{k_1 + k_2}{d}. \tag{A.39}$$

Inequality (A.39) represents the necessary and sufficient condition for solution (1,1) to be optimal. If (A.39) does not hold, then solution (0,1) is optimal. Solving (A.39) for d yields the critical demand level, denoted by d_c, below which solution (0,1) is optimal and above which solution (1,1) is optimal. This value is given by the following expression:

$$d_c = \left[\frac{f_2}{q_2^{(1,1)}}(k_1 + k_2)\right] \Big/ \left[b_2 + \frac{f_2}{q_2^{(1,1)}} - \left(b_1 + \frac{f_1}{q_1^{(1,1)}}\right)\right]. \tag{A.40}$$

Although expression (A.40) is seemingly simple, it is actually quite involved, given that $q_1^{(1,1)}$ and $q_2^{(1,1)}$ depend on d. Considering the constraint $k_1 < d \leq k_2$, the constrained critical value of the demand at which the optimal PD allocation switches from solution (0,1) to (1,1), denoted by k^{PD}, is given by

$$k^{\mathrm{PD}} = \max[\min(d_c, k_1), k_2]. \tag{A.41}$$

It is straightforward to find the conditions under which k^{PD} is equal to one of its three possible values indicated in (A.41). These conditions are:

$$k^{\mathrm{PD}} = \begin{cases} k_1, & \text{if } b_2 \geq b_1 + \frac{f_1}{q_1^{(1,1)}} + \frac{f_2}{q_2^{(1,1)}}\frac{k_2}{k_1}, \\ d_c, & \text{if } b_1 + \frac{f_1}{q_1^{(1,1)}} + \frac{f_2}{q_2^{(1,1)}}\frac{k_1}{k_2} < b_2 < b_1 + \frac{f_1}{q_1^{(1,1)}} + \frac{f_2}{q_2^{(1,1)}}\frac{k_2}{k_1}, \\ k_2, & \text{if } b_2 \leq b_1 + \frac{f_1}{q_1^{(1,1)}} + \frac{f_2}{q_2^{(1,1)}}\frac{k_1}{k_2}. \end{cases} \tag{A.42}$$

Finally, comparing (2.1) and (A.42), it is easy to verify that $k \leq k^{\mathrm{PD}} \leq k_2$.

A.6 Proof of Proposition 2.10

When $d \leq k$, it is obvious from Proposition 2.1 and Proposition 2.9 that $q_n^{\mathrm{PD}} = q_n^{\mathrm{MILP}}$, $n = 1, 2$.

Next, consider the case $d > k_2$. From the analysis in Section A.5, the only regions where $q_i^{\mathrm{PD}} = q_i^{\mathrm{MILP}}$, hence the PD solution is efficient, are Q1 and Q2. The conditions defining the union of these regions are:

$$b_I + \frac{f_I}{d - k_i} \leq b_i + \frac{f_i}{k_i} \text{ or } b_I - b_i \geq \frac{f_I}{d - k_i}\frac{\sum_{n=1,2} k_n - d}{d - k_i}. \tag{A.43}$$

Finally, consider the case $k < d \leq k_2$. Note that this case exists only if $k = k_1$, which from (2.1) is true only if $b_2 > b_1 + f_1/k_1$, which in turn is true only if $i = 1$, $I = 2$ (see Figure 2.2(c)). In this case, from Proposition 2.1, the efficient allocation is $z_i^{\mathrm{MILP}} = z_I^{\mathrm{MILP}} = 1$, $q_i^{\mathrm{MILP}} = k_i$, and $q_I^{\mathrm{MILP}} = d - k_i$. In order for the optimal allocation under PD pricing to be identical to the efficient allocation, there are two requirements. The first requirement is that the optimal allocation of the solution (1,1) (i.e., $z_1 = 1$, $z_2 = 1$) must be equal to the efficient allocation. The conditions for this are the same as those that we developed above for the case $d > k_2$ and are given by (A.43), for $i = 1$ and $I = 2$. The first condition cannot be true, since $k < d \leq k_2$ implies $b_2 > b_1 + f_1/k_1$, as was mentioned above. Hence, the second condition must be true. The second requirement is that the PD objective function value corresponding to the optimal allocation of solution (1,1) must be smaller than or equal to the respective value corresponding to the optimal allocation of solution (0,1) (i.e., $z_1 = 0$, $z_2 = 1$), which is $q_1 = 0$ and $q_2 = d$. The condition for the second requirement is given by (A.39) after replacing $q_1^{(1,1)} = q_1^{\mathrm{MILP}} = k_1$, and $q_2^{(1,1)} = q_2^{\mathrm{MILP}} = d - k_1$, as dictated by the first requirement.

Putting together the conditions corresponding to the two requirements yields the combined condition: $b_2 + \frac{f_2}{d-k_1} \geq b_1 + \max\left(\frac{f_1}{k_1} + \frac{f_2}{d-k_1}\frac{k_1+k_2}{d}, \frac{f_2}{d-k_1}\frac{k_1}{d-k_1}\right)$.

A.7 Proof of Proposition 2.11

First consider the case $b_i + f_i/k_i \geq b_I + f_I/(d-k_i)$. In this case, $\lambda^{\mathrm{CH}} = \max(b_i + f_i/k_i, b_I + f_I/k_I) = b_i + f_i/k_i$ and $\lambda^{\mathrm{AC}} = \max[b_i + f_i/k_i,\ b_I + f_I/(d-k_i)] = b_i + f_i/k_i$. It is straightforward to show by contradiction that this case exists only if $d > k_2$. From Figure A.1, which holds for $d \geq k_2$ but more generally also for $d \geq k^{\mathrm{PD}}$, condition $b_i + f_i/k_i \geq b_I + f_I/(d-k_i)$ corresponds to region Q1, where $\lambda^{\mathrm{PD}} = b_i + f_i/k_i$, implying that $\lambda^{\mathrm{PD}} = \lambda^{\mathrm{CH}} = \lambda^{\mathrm{AC}}$ (see also Figure A.2(a)).

Next, consider the case $b_i + f_i/k_i < b_I + f_I/(d-k_i)$. In this case, $\lambda^{\mathrm{AC}} = \max[b_i + f_i/k_i,\ b_I + f_I/(d-k_i)] = b_I + f_I/(d-k_i)$. From Figure A.1, condition $b_i + f_i/k_i < b_I + f_I/(d-k_i)$ corresponds to regions Q2–Q5. In these regions, $\lambda^{\mathrm{PD}} = b_I + f_I/(d - q_i^{\mathrm{PD}})$, where $d - k_i \leq d - q_i^{\mathrm{PD}} \leq k_I$, implying that $b_I + f_i/k_I \leq \lambda^{\mathrm{PD}} \leq b_I + f_I/(d-k_i) = \lambda^{\mathrm{AC}}$. There are two sub-cases to consider. The fist sub-case is $k < d \leq k_2$, which, as was mentioned above, implies $k = k_1 = k_i$ (Figure 2.2(c)) and $b_I > b_i + f_i/k_i$. In this case, $\lambda^{\mathrm{CH}} = \max(b_i + f_i/k_i, b_I + f_I/k_I) = b_I + f_I/k_I$ and $\lambda^{\mathrm{MZU}} = b_I + f_I/d + (b_i + f_i/k_i - b_I)^+ k_i/d = b_I + f_I/d$. From $\lambda^{\mathrm{PD}} = b_I + f_I/(d - q_i^{\mathrm{PD}})$ and $b_I + f_I/k_I \leq \lambda^{\mathrm{PD}} \leq b_I + f_I/(d - k_i)$ it follows that $\lambda^{\mathrm{PD}} \geq \lambda^{\mathrm{CH}}$ and $\lambda^{\mathrm{PD}} \geq \lambda^{\mathrm{MZU}}$.

The second sub-case is $d > k_2$. In this sub-case, if $b_i + f_i/k_i < b_I + (f_I/k_I)(d - k_I)/k_i$, then $\lambda^{\mathrm{MZU}} < \lambda^{\mathrm{CH}} = b_I + f_I/k_I \leq \lambda^{\mathrm{PD}}$, as is graphically shown in Figure A.2(a). If $b_I + (f_I/k_I)(d - k_I)/k_i \leq b_i + f_i/k_i < b_I + f_I/(d - k_i)$, then $\lambda^{\mathrm{CH}} < \lambda^{\mathrm{MZU}}$, as is also shown in Figure A.2(a). In this case, we only need to show that λ^{PD}, which is equal to $b_I + f_I/(d - q_i^{\mathrm{PD}})$, is greater than or equal to λ^{MZU}, which is equal to $b_I + f_I/d + (b_i + f_i/k_i - b_I)k_i/d$. With simple manipulations, this can be expressed as $f_i + b_i k_i < f_I q_i^{\mathrm{PD}}/(d - q_i^{\mathrm{PD}}) + b_I k_i$. To verify that this inequality holds, we evaluate it for the four possible values of q_i^{PD}, namely $q_i^{\mathrm{PD}} = k_i, d - k_I, q_i', q_i''$.

<u>Case 1</u>: $q_i^{\mathrm{PD}} = k_i$ (Figure A.1: region Q2). In this case, the inequality in question becomes $f_i + b_i k_i < f_I k_i/(d - k_i) + b_I k_i$ which clearly holds, since we assumed that $b_i + f_i/k_i < b_I + f_I(d - k_i)$.

<u>Case 2</u>: $q_i^{\mathrm{PD}} = q_i''$, which implies that $q_i'' \geq q_i'$, where q_i' is the point of intersection of $b_i + f_i/q_i$ and $b_I + f_I/(d - q_i)$ (Figure A.1: region Q4). In this case, $b_i + f_i/q_i'' \leq b_I + f_I/(d - q_i'')$. If we multiply both sides with q_i'', add $b_i(k - q_i'')$ to the lhs, and $b_I(k_i - q_i'')$ to the rhs, where $b_i(k_i - q_i'') < b_I(k_i - q_i'')$, then $f_i + b_i k_i < f_I q_i''/(d - q_i'') + b_I k_i$. Therefore, the inequality holds.

<u>Case 3</u>: $q_i^{\mathrm{PD}} = q_i'$, which implies $q_i' \geq q_i''$ (Figure A.1: region Q3). In this case, $b_i + f_i/q_i' = b_I + f_I/(d - q_i')$. If we multiply both sides with q_i', add $b_i(k_i - q_i')$ to the lhs, and $b_I(k_i - q_i')$ to the rhs, where $b_i(k_i - q_i') < b_I(k_i - q_i')$, then $b_i k_i + f_i < b_I k_i + f_I q_i'/(d - q_i')$. Therefore, the inequality holds.

<u>Case 4</u>: $q_i^{\mathrm{PD}} = d - k_I$, which implies $d - k_I > \{q_i', q_i''\}$ (Figure A.1: region Q5). In this case, $b_i + f_i/(d - k_I) < b_I + f_I/k_I$. If we multiply both sides of this inequality with $d - k_I$, add $b_i(k_i + k_I - d)$ to the lhs, and $b_I(k_i + k_I - d)$ to the rhs, where $b_i(k_i + k_I - d) < b_I(k_i + k_I - d)$, and divide both sides by k_i, then the inequality becomes $b_i + f_i/k_i < b_I + (f_I/k_I)(d - k_I)/k_i$. However, the latter inequality violates the initial assumption $b_i + f_i/k_i \geq b_I + (f_I/k_I)(d - k_I)/k_i$; therefore, q_i^{PD} cannot equal $d - k_I$.

Finally, the relationship between π_n^{PD}, $n = 1, 2$, and the profits generated by other schemes is graphically shown in Figure A.2(b).

A.8 Proof of Proposition 2.12

Proposition 2.5 states that in the high-demand case, the GU price is given by $\lambda^{\mathrm{GU}} = b_I + \Delta b_I^{\mathrm{GU}}$, where $\Delta b_I^{\mathrm{GU}} = \max(\Delta b_I^{(1)}, \Delta b_I^{(2)}, \Delta b_I^{(3)})$ and the exact expressions of $\Delta b_I^{(1)}$, $\Delta b_I^{(2)}$, $\Delta b_I^{(3)}$, and the conditions under which each expression holds, are given by Table A.1.

It is obvious that $\lambda^{\mathrm{GU}} = b_I + \Delta b_I^{\mathrm{GU}} > b_I = \lambda^{\mathrm{IP+}} = \lambda^{\mathrm{mIP}}$, so we will proceed to compare λ^{GU} against λ^{MZU}.

First, note that by comparing the expression for λ^{MZU} given by Proposition 2.6(ii) and the expression for $\Delta b_I^{(2)}$ from Table A.1, it follows that:

$$\lambda^{\mathrm{MZU}} = \begin{cases} b_I + f_I/d = b_I + \Delta b_I^{(2)} + \zeta/d, & \text{if } \zeta \geq 0, \\ b_I + \Delta b_I^{(2)}, & \text{if } \zeta < 0. \end{cases} \tag{A.44}$$

where ζ is given by (A.21). There are three cases to consider corresponding to the cases in Table A.1.

Case 1: $\zeta \geq \eta$, where η is given by (A.22). In this case, $\lambda^{\mathrm{GU}} = b_I + \Delta b_I^{(1)} = b_I + f_I/[3(d - k_i)]$, from Table A.1, where $\Delta b_I^{(1)} \geq \{\Delta b_I^{(2)}, \Delta b_I^{(3)}\}$ from (A.26). There are two sub-cases to consider: Sub-case 1.1: $d \geq 3k_i/2$. In this case, $2d - 3k_i \geq 0$, implying $\eta \geq 0$ from (A.22) and hence $\zeta \geq \eta \geq 0$; therefore, from (A.44), $\lambda^{\mathrm{MZU}} = b_I + f_I/d$. Condition $2d - 3k_i \geq 0$ also implies $\lambda^{\mathrm{GU}} \leq \lambda^{\mathrm{MZU}}$. Sub-case 1.2: $d < 3k_i/2$. In this case, $2d - 3k_i < 0$, implying $\eta < 0$ from (A.22). There are two sub-cases. Sub-case 1.2.1: $0 > \zeta \geq \eta$. In this case, $\lambda^{\mathrm{MZU}} = b_I + \Delta b_I^{(2)}$ from (A.44). Condition $\Delta b_I^{(1)} \geq \Delta b_I^{(2)}$ implies $\lambda^{\mathrm{GU}} \geq \lambda^{\mathrm{MZU}}$. Sub-case 1.2.2: $\zeta \geq 0 > \eta$. In this case, $\lambda^{\mathrm{MZU}} = b_I + f_I/d$ from (A.44). Condition $2d - 3k_i < 0$ also implies $\lambda^{\mathrm{GU}} > \lambda^{\mathrm{MZU}}$.

Case 2: $\zeta < \eta$. There are two sub-cases to consider. Sub-case 2.1: $d \geq 3k_i/2$. In this case, $\lambda^{\mathrm{GU}} = b_I + \Delta b_I^{(2)}$ from Table A.1 and $2d - 3k_i \geq 0$, which, from (A.22), implies $\eta > 0$. There are two sub-cases to consider. Sub-case 2.1.1: $\zeta < 0$. In this case, $\lambda^{\text{MZU}} = b_I + \Delta b_I^{(2)}$ from (A.44), implying $\lambda^{\text{GU}} = \lambda^{\text{MZU}}$. Sub-case 2.1.2: $\zeta \geq 0$. In this case, $\lambda^{\text{MZU}} = b_I + \Delta b_I^{(2)} + \zeta/d$ from (A.44), implying $\lambda^{\text{GU}} \leq \lambda^{\text{MZU}}$. Sub-case 2.2: $d < 3k_i/2$. In this case, $2d - 3k_i < 0$, which from (A.22) and (A.25) implies $\eta < 0$ and $\theta < 0$, respectively. There are two sub-cases to consider. Sub-case 2.2.1: $\theta < \zeta < \eta < 0$, where θ is given by (A.25). In this case, $\lambda^{\text{GU}} = b_I + \Delta b_I^{(2)}$ from Table A.1, and $\lambda^{\text{MZU}} = b_I + \Delta b_I^{(2)}$ from (A.44); hence, $\lambda^{\text{GU}} = \lambda^{\text{MZU}}$. Sub-case 2.2.2: $\zeta \leq \theta < 0$. In this case, $\lambda^{\text{GU}} = b_I + \Delta b_I^{(3)}$ from Table A.1, where $\Delta b_I^{(3)} \geq \Delta b_I^{(1)}, \Delta b_I^{(2)}$ from (A.26). Moreover, $\lambda^{\text{MZU}} = b_I + \Delta b_I^{(2)}$ from (A.44). Condition $\Delta b_I^{(3)} \geq \Delta b_I^{(2)}$ implies $\lambda^{\text{GU}} \geq \lambda^{\text{MZU}}$.

Figure A.3 shows the commodity price vs. $b_i + f_i/k_i$ for different schemes including GU for the cases $d > 3k_i/2$ and $d < 3k_i/2$. It is similar to Figure 2.3(b), with the exclusion of PD and SLR.

First, consider the case $d > 3k_i/2$ (Figure A.3(a)). As is indicated, $b_i + f_i/k_i$ may belong to one of three regions corresponding to the above cases 1.1, 2.1.2, and 2.1.1. In the first region, $\Delta b_I^{\text{GU}} = \Delta b_I^{(1)}$, while in the other two, $\Delta b_I^{\text{GU}} = \Delta b_I^{(2)}$. The border between regions 1.1 and 2.1.2 is at the point of intersection of $b_I + \Delta b_I^{(1)}$ and $b_I + \Delta b_I^{(2)}$, satisfying $b_i + f_i/k_i = b_I - \eta/k_i = b_I - [f_I/(d-k_i)][(2d-3k_i)/(3k_i)]$. Figure A.3(a) shows that $\lambda^{\text{IP+}} = \lambda^{\text{mIP}} < \lambda^{\text{GU}} < \lambda^{\text{MZU}}$, if $b_i + f_i/k_i < b_I$ (regions 1.1 and 2.1.2), and $\lambda^{\text{GU}} = \lambda^{\text{MZU}}$, if $b_i + f_i/k_i \geq b_I$ (region 2.1.1). The lowest value of λ^{GU} is $b_I + f_I/(3k_I)$, when $d = k_i + k_I$.

Next, consider the case $k_i < d < 3k_i/2$ (Figure A.3(b)). As is indicated, $b_i + f_i/k_i$ may belong to one of four regions that correspond to cases 1.2.2, 1.2.1, 2.2.1 and 2.2.2 discussed above. In the first two regions, $\Delta b_I^{\text{GU}} = \Delta b_I^{(1)}$, in the third, $\Delta b_I^{\text{GU}} = \Delta b_I^{(2)}$, and in the fourth, $\Delta b_I^{\text{GU}} = \Delta b_I^{(3)}$. The slope of λ^{GU} in the last region (2.2.2) is denoted by s, where $s = k_i(2d+k_i)/(4d^2 - 4k_i d + 3k_i^2)$ from the last row of Table A.1. It can be shown by contradiction that $k_i/d \leq s \leq 1$. The first inequality, $s \geq k_i/d$, implies that $b_I + \Delta b_I^{(3)}$ always intersects $b_I + \Delta b_I^{(2)}$, except when $s = k_i/d$. It can be easily shown that the point of intersection of the two functions is $b_i + f_i/k_i = b_I - \theta/k_i$, as is indicated in Figure A.3(b). The second inequality, $s \leq 1$, implies that $b_I + \Delta b_I^{(3)}$ is always at or below λ^{CH}, which has a slope of 1.

Figure A.3(b) clearly illustrates that $\lambda^{\text{GU}} > \lambda^{\text{MZU}}$ in regions 1.2.2. and 1.2.1,

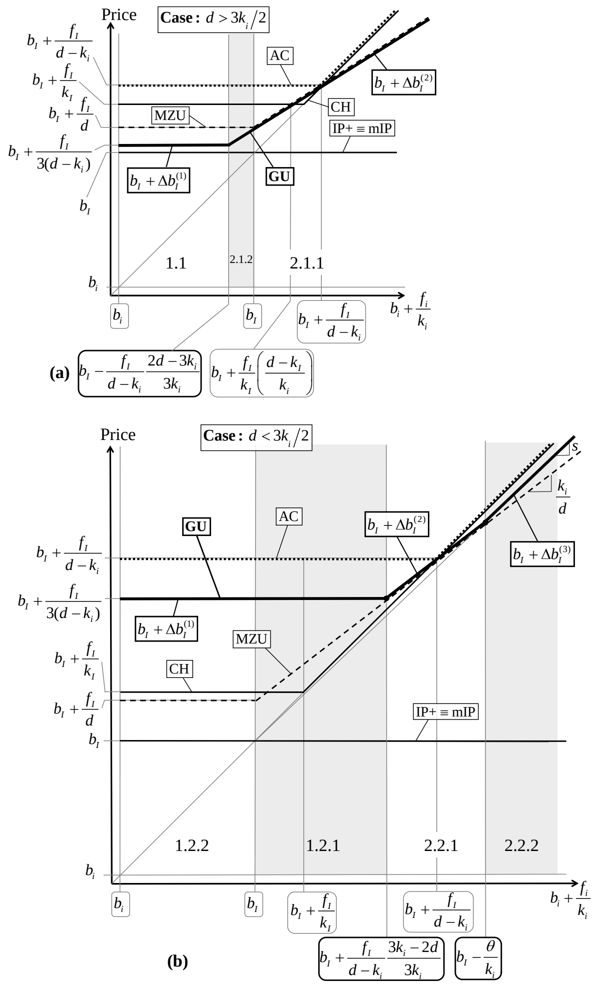

Figure A.3: Price vs. $b_i + f_i/k_i$ for the GU scheme for cases: (a) $3k_i/2 < d \leq 2k_i$; (b) $k_i \leq d < 3k_i/2$.

$\lambda^{\mathrm{GU}} = \lambda^{\mathrm{MZU}}$, in region 2.2.1, and $\lambda^{\mathrm{MZU}} < \lambda^{\mathrm{GU}} < \lambda^{\mathrm{CH}}$, in region 2.2.2. Moreover, in regions 1.2.2. and 1.2.1, λ^{GU} can be greater that λ^{CH}, if $b_I + f_I/[3(d-k_i)] > b_I + f_I/k_i$, which can be rewritten as $d < 4k_i/3$. If $4k_i/3 \leq d < 3k_i/2$, on the other hand, then $\lambda^{\mathrm{GU}} \leq \lambda^{\mathrm{CH}}$ in region 1.2.2 and part of region 1.2.1. In all cases, $\lambda^{\mathrm{GU}} < \lambda^{\mathrm{AC}}$.

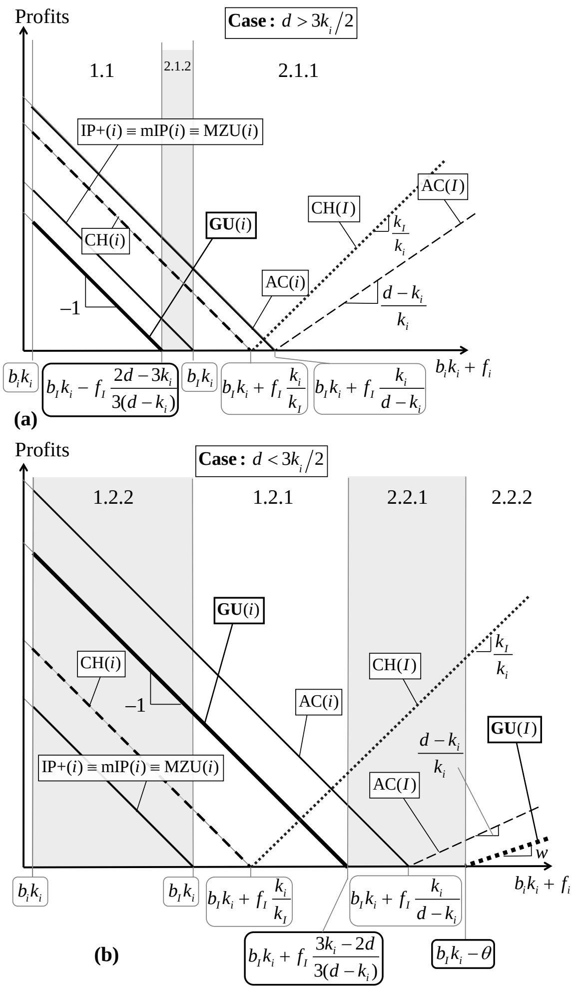

Figure A.4: Profits vs. $b_ik_i + f_i$ for the GU scheme for cases: (a) $3k_i/2 < d \leq 2k_i$; (b) $k_i \leq d < 3k_i/2$.

Figure A.4 shows the profits vs. $b_ik_i + f_i$ for different schemes including GU for $d > 3k_i/2$ and $d < 3k_i/2$. It is similar to Figure 2.6 with the exclusion of PD and SLR.

First, consider the case $d > 3k_i/2$ (Figure A.4(a)). As is indicated, $b_ik_i + f_i$ may belong to one of three regions corresponding to cases 1.1, 2.1.2, and 2.1.1. Figure A.4(a) shows that $\pi_i^{\mathrm{GU}} < \pi_i^{\mathrm{MZU}} = \pi_i^{\mathrm{mIP}} = \pi_i^{\mathrm{IP+}}$, if $b_i k_i + f_i < b_I k_i$ (regions 1.1 and 2.1.2), and $\pi_i^{\mathrm{GU}} = \pi_i^{\mathrm{MZU}} = \pi_i^{\mathrm{mIP}} = \pi_i^{\mathrm{IP+}} = 0$, if $b_i k_i + f_i \geq b_I k_i$ (region 2.1.1).

Next, consider the case $k_i < d < 3k_i/2$ (Figure A.4(b)). As is indicated, $b_i k_i + f_i$ may belong to one of four regions corresponding to cases 1.2.2, 1.2.1, 2.2.1 and 2.2.2. From Figure A.4(b), it is easy to see that $\pi_i^{\mathrm{MZU}} = \pi_i^{\mathrm{mIP}} = \pi_i^{\mathrm{IP+}} < \pi_i^{\mathrm{GU}} < \pi_i^{\mathrm{AC}}$, if $b_i k_i + f_i < b_I k_i$ (regions 1.2.2, 1.2.1), and $\pi_i^{\mathrm{GU}} = \pi_i^{\mathrm{MZU}} = \pi_i^{\mathrm{mIP}} = \pi_i^{\mathrm{IP+}} = 0$, if $b_i k_i + f_i \geq b_I k_i$ (regions 2.2.1, 2.2.2). It can also be shown that $\pi_i^{\mathrm{GU}} < \pi_i^{\mathrm{CH}}$, if $6k_i/5 < d < 3k_i/2$, and $\pi_i^{\mathrm{GU}} > \pi_i^{\mathrm{CH}}$, if $k_i < d < 6k_i/5$. Finally, if $d = 6k_i/5$, then $\pi_i^{\mathrm{GU}} = \pi_i^{\mathrm{CH}}$. The slope of π_I^{GU} in the last region (2.2.2) is denoted by w, where $w = -f_I/\theta = (3k_i - 2d)(d - k_i)/(4d^2 - 4k_i d + 3k_i^2)$ from the last row of Table A.1. We can show by contradiction that $w < (d - k_i)/k_i$, where clearly $(d - k_i)/k_i \leq k_I/k_i$ which implies that $\pi_I^{\mathrm{GU}} < \pi_I^{\mathrm{AC}} < \pi_I^{\mathrm{CH}} = \pi_I^{\mathrm{SLR}}$ in regions 2.2.1 and 2.2.2.

Finally, note that if $d = 3k_i/2$, the GU pricing scheme is identical to the MZU scheme.

Appendix B

Nomenclature for the DAS Problem

Subscripts

i	Interconnection user (importer/exporter) (set I)
l	Block
o	Load representative
t	Time period (hourly periods in the 24-hour horizon)
u	Generation unit (set U)
x	Interconnection

Superscripts

AGC	Automatic Generation Control
Aux	Auxiliary
def	Deficit
Exp	Exports
G	Energy Generation
Imp	Imports
LD	Load Declarations
NL	No-Load

PR	Primary Reserve
R	Reserve
RRD	Ramp Rate Down
RRU	Ramp Rate Up
SD	Shut-Down
SR	Secondary Reserve
SRR	Secondary Reserve Range
SRU	Secondary Reserve Up
SRD	Secondary Reserve Down
St	Status
SU	Start-Up
sur	Surplus
TR	Tertiary Reserve

Parameters

$b^{\mathrm{G}}_{u,l,t}$	Bid for marginal cost of energy generation, of generation unit u, block l, time period t.
$b^{\mathrm{Imp}}_{i,l,t}$	Bid for marginal cost for importing energy, of importer i, block l, time period t.
$b^{\mathrm{Exp}}_{i,l,t}$	Bid for marginal cost for exporting energy, of exporter i, block l, time period t.
$b^{\mathrm{LD}}_{o,l,t}$	Bid for marginal cost for consuming energy (load declaration), of load representative o, block l, time period t.
$b^{\mathrm{PR}}_{u,t}$	Bid for primary reserve, of generation unit u, time period t.
$b^{\mathrm{SRR}}_{u,t}$	Bid for secondary reserve range, of generation unit u, time period t.
$b^{\mathrm{R}}_{u,t}$	Bid for reserve, of generation unit u, time period t.
d^{G}_{t}	Demand for energy generation at time period t.
d^{PR}_{t}	Demand (requirements) for primary reserve at time period t.
d^{SRU}_{t}	Demand (requirements) for secondary reserve up at time period t.

d_t^{SRD}	Demand (requirements) for secondary reserve down at time period t.
d_t^{TR}	Demand (requirements) for tertiary reserve at time period t.
d_t^{R}	Demand (requirements) for reserve at time period t.
f_u^{SD}	Bid for the shut-down cost of generation unit u.
f_u^{SU}	Bid for the start-up cost of generation unit u.
f_u^{NL}	Bid for the no-load cost of generation unit u.
P^{G}	Penalty coefficient for energy generation.
P^{PR}	Penalty coefficient for primary reserve.
P^{SR}	Penalty coefficient for secondary reserve.
P^{TR}	Penalty coefficient for tertiary reserve.
$\bar{k}_{u,l}^{\mathrm{G}}$	Upper limit for energy generation of generation unit u, block l.
$\bar{k}_{o,l}^{\mathrm{LD}}$	Upper limit for load declaration of load representative o, block l.
$\bar{k}_{i,l}^{\mathrm{Imp}}$	Upper limit for imports of importer i, block l.
$\bar{k}_{i,l}^{\mathrm{Exp}}$	Upper limit for exports of exporter i, block l.
$\underline{k}_u^{\mathrm{G}}$	Technical minimum for energy generation of unit u.
$\underline{k}_u^{\mathrm{AGC}}$	AGC minimum for unit u.
$\bar{k}_u^{\mathrm{G}}$	Technical maximum for energy generation of unit u.
$\bar{k}_u^{\mathrm{AGC}}$	AGC maximum for unit u.
$\bar{k}_u^{\mathrm{PR}}$	Primary reserve capability for unit u.
$\bar{k}_u^{\mathrm{SRR}}$	Secondary reserve range capability for unit u.
$\bar{k}_u^{\mathrm{R}}$	Reserve capability for unit u.
$k_u^{\mathrm{G,\ Daily}}$	Maximum daily energy for unit u.
$k_{x,t}^{\mathrm{NTC}}$	Net transfer capacity for interconnection x, time period t.
M	Large number, e.g., $M = 10,000$.
$\bar{q}_u^{\mathrm{G,RRU}}$	Quantity for energy respecting ramp rate up for unit u.
$\bar{q}_u^{\mathrm{AGC,RRU}}$	Quantity for energy respecting ramp rate up for unit u on-AGC.
$\bar{q}_u^{\mathrm{G,RRD}}$	Quantity for energy respecting ramp rate down for unit u.
$\bar{q}_u^{\mathrm{AGC,RRD}}$	Quantity for energy respecting ramp rate down for unit u on-AGC.

$q_{u,t}^{\text{G, NonPriced}}$	Non-priced energy generation for unit u, time period t.
$q_{o,t}^{\text{LD, NonPriced}}$	Non-priced load declaration for load representative o, time period t.
$q_{i,t}^{\text{Imp, NonPriced}}$	Non-priced imports for importer i, time period t.
$q_{i,t}^{\text{Exp, NonPriced}}$	Non-priced exports for exporter i, time period t.
$q_{u}^{\text{G, Total,0}}$	Total energy generation scheduled for unit u at time period $t = 0$.
$\underline{t}_{u}^{\text{Up}}$	Minimum number of time periods unit u has to remain online following a start-up.
$\underline{t}_{u}^{\text{Down}}$	Minimum number of time periods unit u has to remain offline following a shut-down.
$y_{u}^{\text{On,0}}$	Number of time periods unit u has been online at time period $t = 0$.
$y_{u}^{\text{Off,0}}$	Number of time periods unit u has been offline at time period $t = 0$.
$z_{u,t}^{\text{Avail}}$	Availability for unit u, time period t; 1: available; 0: otherwise.
$z_{u}^{\text{St,0}}$	Status for unit u, at time period $t = 0$.

Variables

$q_{u,l,t}^{\text{G}}$	Quantity of energy generation scheduled for unit u, block l, time period t.
$q_{i,l,t}^{\text{Imp}}$	Quantity of imports scheduled for importer u, block l, time period t.
$q_{u,t}^{\text{PR}}$	Quantity of primary reserve scheduled for unit u, time period t.
$q_{u,t}^{\text{SRU}}$	Quantity of secondary reserve up scheduled for unit u, time period t.
$q_{u,t}^{\text{SRD}}$	Quantity of secondary reserve down scheduled for unit u, time period t.
$q_{u,t}^{\text{TR}}$	Quantity of tertiary reserve scheduled for unit u, time period t.

$q_{u,t}^{\mathrm{R}}$	Quantity of reserve scheduled for unit u, time period t.
$q_{t}^{\mathrm{G,\ def}}$	Deficit variable for energy generation at time period t.
$q_{t}^{\mathrm{G,\ sur}}$	Surplus variable for energy generation at time period t.
$q_{t}^{\mathrm{PR,\ def}}$	Deficit variable for primary reserve at time period t.
$q_{t}^{\mathrm{SRU,\ def}}$	Deficit variable for secondary reserve up at time period t.
$q_{t}^{\mathrm{SRD,\ sur}}$	Deficit variable for secondary reserve down at time period t.
$q_{t}^{\mathrm{TR,\ def}}$	Deficit variable for tertiary reserve at time period t.
$y_{u,t}^{\mathrm{On}}$	Number of time periods unit u has been online at time period t.
$y_{u,t}^{\mathrm{Off}}$	Number of time periods unit u has been offline at time period t.
$z_{u,t}^{\mathrm{St}}$	Binary variable for the unit status; 1: Unit u is online at time period t; 0:Unit u is offline at time period t .
$z_{u,t}^{\mathrm{SD}}$	Binary variable for the unit shut-down; 1: shut-down of unit u at time period t; 0: otherwise.
$z_{u,t}^{\mathrm{SU}}$	Binary variable for the unit start-up; 1: start-up of unit u at time period t; 0: otherwise.
$z_{u,t}^{\mathrm{AGC}}$	Binary variable for the unit AGC status; 1: Unit u on-AGC at time period t; 0: Unit u off-AGC at time period t.

Dependent - auxiliary variables

$q_{u,t}^{\mathrm{G,\ Total}}$	Total quantity of energy generation scheduled for unit u, time period t.
$q_{o,t}^{\mathrm{LD,\ Total}}$	Total quantity of load declarations scheduled for load representative u, time period t.
$q_{i,t}^{\mathrm{Imp,\ Total}}$	Total imports for importer i, time period t.
$q_{i,t}^{\mathrm{Exp,\ Total}}$	Total exports for exporter i, time period t.
$y_{u,t}^{\mathrm{aux}(1)}$	Integer auxiliary variable for unit u, time period t.
$y_{u,t}^{\mathrm{aux}(2)}$	Integer auxiliary variable for unit u, time period t.

Appendix C

Proofs of Chapter 4

C.1 Proof of Proposition 4.1

We consider the three cases for demand, i.e.:

1. Low demand, $d \leq k_1$,
2. Intermediate demand, $k_1 < d \leq k_2$,
3. High demand, $d > k_2$.

Case of low demand, $d \leq k_1$

Best Response of Supplier i: From (4.12), the profits of supplier i are

$$\pi_i(d; \mathbf{b}) = \begin{cases} (b_i - c_i)d, & \text{if } b_i \leq b_I, \\ 0, & \text{if } b_i > b_I. \end{cases} \tag{C.1}$$

From (C.1), it immediately follows that the best response of supplier i is

$$b_i^*(b_I) = b_I \tag{C.2}$$

Best Response of Supplier I: From (4.12), the profits of supplier I are

$$\pi_I(d;\mathbf{b}) = \begin{cases} (b_I - c_I)d, & \text{if } b_I < b_i, \\ 0, & \text{if } b_I \geq b_i. \end{cases} \tag{C.3}$$

From (C.3), it immediately follows that the best response of supplier I is

$$b_I^*(b_i) = \begin{cases} {b_i}^-, & \text{if } b_i > c_I, \\ [c_I, P], & \text{if } b_i \leq c_I. \end{cases} \tag{C.4}$$

We note that by $b_I^*(b_i) = [c_I, P]$ we mean that the best response of supplier I is to bid anywhere within the interval $[c_I, P]$, i.e., $c_I \leq b_I^*(b_i) \leq P$.

Equilibrium (1a): From the best responses (C.2) and (C.4), the equilibrium outcome is $b_i^* = b_I^* = c_I$. This is a Bertrand-type equilibrium, in which supplier I plays a weakly dominated strategy. The price at equilibrium is $\lambda^* = c_I$. The allocated quantities are $q_i^* = d, q_I^* = 0$. The total payments are $\text{TPs} = c_I d + f_i$.

Case of intermediate demand, $k_1 < d \leq k_2$

We consider two sub-cases: $k_i < k_I$, and $k_I < k_i$.

Sub-case $k_i < k_I$ (i.e., $i = 1$)

Best Response of Supplier i: From (4.12), the profits of supplier i are

$$\pi_i(d;\mathbf{b}) = \begin{cases} (b_I - c_i)k_i, & \text{if } b_i \leq b_I, \\ 0, & \text{if } b_i > b_I. \end{cases} \tag{C.5}$$

From (C.5), it immediately follows that the best response of supplier i is to bid less than or equal to the bid of supplier I, i.e.,

$$b_i^*(b_I) = [c_i, b_I]. \tag{C.6}$$

Best Response of Supplier I: From (4.12), the profits of supplier I are

$$\pi_I(d;\mathbf{b}) = \begin{cases} (b_I - c_I)d, & \text{if } b_I < b_i, \\ (b_I - c_I)(d - k_i), & \text{if } b_I \geq b_i. \end{cases} \tag{C.7}$$

From (C.7), it follows that:

— Subject to the condition $b_I < b_i$, π_I is maximized for $b_I = {b_i}^-$. (It is implied that the profits are non-negative given that $b_i > c_I$.)

— Subject to the condition $b_I \geq b_i$, π_I is maximized for $b_I = P$.

Supplier I will choose to bid at the price cap (i.e. $b_I^* = P$) and serve the residual demand if $\pi_I(d; b_I = P) \geq \pi_I(d; b_I = b_i^-) \Rightarrow (P - c_I)(d - k_i) \geq (b_i^- - c_I)d \Rightarrow b_i^- \leq c_I + \frac{(P-c_I)(d-k_i)}{d} \Rightarrow b_i - \varepsilon \leq c_I + \frac{(P-c_I)(d-k_i)}{d} \Rightarrow b_i \leq c_I + \frac{(P-c_I)(d-k_i)}{d} + \varepsilon \quad (\varepsilon \to 0^+)$.

Similarly, supplier I will choose to underbid supplier i (i.e. $b_I^* = b_i^-$) if $\pi_I(d; b_I = P) \leq \pi_I(d; b_I = b_i^-) \Rightarrow b_i \geq c_I + \frac{(P-c_I)(d-k_i)}{d} + \varepsilon \quad (\varepsilon \to 0^+)$.

Assuming that $\varepsilon \to 0^+$ and also that ε is less than the precision of the supplier's bid, the best response of supplier I is

$$b_I^*(b_i) = \begin{cases} b_i^-, & \text{if } b_i > c_I + \frac{(P-c_I)(d-k_i)}{d} = b_I^{(1)}, \\ P, & \text{if } b_i \leq b_I^{(1)}. \end{cases} \tag{C.8}$$

Equilibrium 2a: The best responses (C.6) and (C.8) yield the equilibrium outcome $b_i^* \leq b_I^{(1)}, b_I^* = P$. The price at equilibrium is $\lambda^* = P$. The allocated quantities are $q_i^* = k_i$, $q_I^* = d - k_i$. The total payments are $\text{TPs} = Pd + f_i + f_I$.

Sub-case $k_I < k_i$ (i.e., $i = 2$):

Best Response of Supplier i: From (4.12), the profits of supplier i are

$$\pi_i(d;\mathbf{b}) = \begin{cases} (b_i - c_i)d, & \text{if } b_i \leq b_I, \\ (b_i - c_i)(d - k_I), & \text{if } b_i > b_I. \end{cases} \tag{C.9}$$

From (C.9), it follows that:

— Subject to the condition $b_i \leq b_I$, π_i is maximized for $b_i = b_I$.

— Subject to the condition $b_i > b_I$, π_i is maximized for $b_i = P$.

Supplier i will choose to bid at the price cap (i.e. $b_i^* = P$) and serve the residual demand if $\pi_i(d; b_i = P) \geq \pi_i(d; b_i = b_I) \Rightarrow (P - c_i)(d - k_I) \geq (b_I - c_i)d \Rightarrow b_I \leq c_i + \frac{(P-c_i)(d-k_I)}{d}$. Since $c_I \leq b_I \leq P$, the previous inequality may hold if the RHS is greater than or equal to c_I, i.e., $c_i + \frac{(P-c_i)(d-k_I)}{d} \geq c_I \Rightarrow (P - c_i)d - (P - c_i)k_I \geq d(c_I - c_i) \Rightarrow (P - c_I)d \geq (P - c_i)k_I \Rightarrow d \geq \frac{(P-c_i)(P-c_I)}{k_I}$.

Similarly, supplier i will choose to bid $b_i^* = b_I$ if $\pi_i(d; b_i = b_I) \geq \pi_i(d; b_i = P) \Rightarrow (b_I - c_i)d \geq (P - c_i)(d - k_I) \Rightarrow b_I \geq c_i + \frac{(P-c_i)(d-k_I)}{d}$. This inequality may hold for the whole interval under consideration.

Therefore, the best response of supplier i is

$$b_i^*(b_I) = \begin{cases} b_I, & \text{if } b_I \geq c_i + \frac{(P-c_i)(d-k_I)}{d} = b_i^{(1)}, \\ P, & \text{if } b_I \leq c_i + \frac{(P-c_i)(d-k_I)}{d} = b_i^{(1)} \text{ and } d \geq \frac{P-c_i}{P-c_I}k_I = \theta^{(1)}. \end{cases} \tag{C.10}$$

Best Response of Supplier I: From (4.12) the profits of supplier i are

$$\pi_I(d; \mathbf{b}) = \begin{cases} (b_i - c_I)k_I, & \text{if } b_I < b_i, \\ 0, & \text{if } b_I \geq b_i. \end{cases} \tag{C.11}$$

From (C.11), it follows that:

— Subject to the condition $b_I < b_i$, π_I is maximized for $b_I < b_i$. This implies that it should be $b_i > c_I$, so that the profits are non-negative; otherwise supplier I is indifferent.

— Subject to the condition $b_I \geq b_i$, supplier I has always zero profits and is indifferent.

Therefore, the best response of supplier I is:

$$b_I^*(b_i) = \begin{cases} [c_I, b_i], & \text{if } b_i > c_I, \\ [c_I, P], & \text{if } b_i \leq c_I. \end{cases} \tag{C.12}$$

Equilibria 1b and 3a: The best responses (C.10) and (C.12) yield the following equilibrium outcomes.

— Equilibrium 1b: Subject to the condition $d \leq \theta^{(1)}$, $b_i^* = b_I^* = c_I$. The price at equilibrium is $\lambda^* = c_I$. The allocated quantities are $q_i = d$, $q_I = 0$. The total payments are $\text{TPs} = c_I d + f_i$.

— Equilibrium 3a: Subject to the condition $d \geq \theta^{(1)}$, $b_i^* = P$, $b_I^* \leq b_i^{(1)}$. The price at equilibrium is $\lambda^* = P$. The allocated quantities are $q_i = d - k_I$, $q_I = k_I$. The total payments are $\text{TPs} = Pd + f_i + f_I$.

Note that for $d = \theta^{(1)}$, both equilibrium outcomes exist, i.e: $b_i^* = b_I^* = c_I$, and $b_i^* = P$, $b_I^* = c_I$, since for $d = \theta^{(1)}$, we have $b_i^{(1)} = c_I$.

Case of high demand, $d > k_2$

Best Response of Supplier i: From (4.12), the profits of supplier i are

$$\pi_i(d; \mathbf{b}) = \begin{cases} (b_I - c_i)k_i, & \text{if } b_i \leq b_I, \\ (b_i - c_i)(d - k_I), & \text{if } b_i > b_I. \end{cases} \tag{C.13}$$

From (C.13), it follows that:

— Subject to the condition $b_i \leq b_I$, π_i is maximized for any $b_i \leq b_I$.

— Subject to the condition $b_i > b_I$, π_i is maximized for $b_i = P$.

Supplier i will choose to bid at the price cap (i.e. $b_i^* = P$) and serve the residual demand if $\pi_i(d; b_i = P) \geq \pi_I(d; b_i \leq b_I) \Rightarrow (P - c_i)(d - k_I) \geq (b_I - c_i)k_i \Rightarrow b_I \leq c_i + \frac{(P-c_i)(d-k_I)}{k_i}$. Since $c_I \leq b_I \leq P$, the previous inequality may hold if the RHS is greater than or equal to c_I, i.e., $c_i + \frac{(P-c_i)(d-k_I)}{k_i} \geq c_I \Rightarrow \frac{(P-c_i)(d-k_I)}{k_i} \geq c_I - c_i \Rightarrow d \geq k_I + \frac{c_I - c_i}{P - c_i} k_i$.

Similarly, supplier i will choose to bid at $b_i^* = b_I$ if $\pi_i(d; b_i \leq b_I) \geq \pi_i(d; b_i = P) \Rightarrow (b_I - c_i)k_i \geq (P - c_i)(d - k_I) \Rightarrow b_I \geq c_i + \frac{(P-c_i)(d-k_I)}{k_i}$. This inequality may hold for the whole interval under consideration.

Therefore, the best response of supplier i is:

$$b_i^*(b_I) = \begin{cases} [c_i, b_I], & \text{if } b_I \geq c_i + \frac{(P-c_i)(d-k_I)}{k_i} = b_i^{(1)} \\ P, & \text{if } b_I \leq c_i + \frac{(P-c_i)(d-k_I)}{k_i} = b_i^{(1)} \text{ and } d \geq k_I + \frac{c_I - c_i}{P - c_i} k_i = \theta^{(2)}. \end{cases} \tag{C.14}$$

Best Response of Supplier I: From (4.12), the profits of supplier I are

$$\pi_I(d;\mathbf{b}) = \begin{cases} (b_i - c_I)k_I, & \text{if } b_I < b_i, \\ (b_I - c_I)(d - k_i), & \text{if } b_I \geq b_i. \end{cases} \tag{C.15}$$

From (C.15), it follows that:

— Subject to the condition $b_I < b_i$, π_I is maximized for any $b_I < b_i$.

— Subject to the condition $b_I \geq b_i$, π_I is maximized for $b_I = P$.

Supplier I will choose to bid at the price cap (i.e. $b_I^* = P$) and serve the residual demand if $\pi_I(d; b_I = P) \geq \pi_I(d; b_I < b_i) \Rightarrow (P - c_I)(d - k_i) \geq (b_i - c_I)k_I \Rightarrow b_i \leq c_I + \frac{(P-c_I)(d-k_i)}{k_I}$.

Similarly, supplier I will choose to underbid supplier 1 (i.e. $b_I^* < b_i$) if $\pi_I(d; b_I < b_i) \geq \pi_I(d; b_I = P) \Rightarrow (b_i - c_I)k_I \geq (P - c_I)(d - k_i) \Rightarrow b_i \geq c_I + \frac{(P-c_I)(d-k_i)}{k_I}$.

Therefore, the best response of supplier I is

$$b_I^*(b_i) = \begin{cases} [c_I, b_i], & \text{if } b_i \geq c_I + \frac{(P-c_I)(d-k_i)}{k_I} = b_I^{(1)}, \\ P, & \text{if } b_i \leq b_I^{(1)}. \end{cases} \tag{C.16}$$

Equilibria 2b, 3b: The best responses (C.14) and (C.16) yield the following equilibrium outcomes:

— Equilibrium 2b: For the high demand, $b_i^* \leq b_I^{(1)}$, $b_I^* = P$. The equilibrium price is $\lambda^* = P$. The allocated quantities are $q_i^* = k_i, q_I^* = d - k_i$. The total payments are $\text{TPs} = Pd + f_i + f_I$.

— Equilibrium 3b: Subject to the condition $d \geq \theta^{(2)}$, $b_i^* = P$, $b_I^* \leq b_i^{(1)}$. The equilibrium price is $\lambda^* = P$. The allocated quantities are $q_i^* = d - k_I, q_I^* = k_I$. The total payments are $\text{TPs} = Pd + f_i + f_I$.

C.2 Proof of Proposition 4.3

From (4.14), using (4.2), the profits of supplier n are

$$\pi_n = \max\{(\lambda - c_n)q_n - f_n z_n, -\alpha[(\lambda - c_n)q_n - f_n z_n]\}. \tag{C.17}$$

From (4.9) and (C.17), the profits of supplier n, $n, m = 1, 2$, with $n \neq m$ are given as follows:

If $d \leq k_m$ and $\phi_n = 1$, or if $d > k_m$, then $\pi_n = \max\{(\lambda - c_n)q_n - f_n, -\alpha[(\lambda - c_n)q_n - f_n]\}$.

If $d \leq k_m$ and $\phi_n = 0$, then $\pi_n = 0$.

Combining the above and setting for compactness

$$\psi_n = (\lambda - c_n)q_n - f_n, \tag{C.18}$$

(C.17) can be rewritten as follows:

$$\pi_n = \begin{cases} \phi_n \max\{\psi_n, -\alpha\psi_n\}, & \text{if } d \leq k_m, \\ \max\{\psi_n, -\alpha\psi_n\}, & \text{if } d > k_m. \end{cases} \tag{C.19}$$

Low demand, $d \leq k_1$

Best Response of Supplier i: From (C.19), we have

$$\pi_i(d; \mathbf{b}) = \phi_i \max\{\psi_i(d; \mathbf{b}), -\alpha\psi_i(d; \mathbf{b})\}, \tag{C.20}$$

where from (C.18), $\psi_i(d; \mathbf{b})$ is given by

$$\psi_i(d; \mathbf{b}) = [\lambda(d; \mathbf{b}) - c_i]q_i(d; \mathbf{b}) - f_i = \phi_i[b_i - c_i]d - f_i, \tag{C.21}$$

because from (4.6) and (4.8) we have $\lambda(d; \mathbf{b}) = \phi_i b_i + \phi_I b_I$, and $q_i(d; \mathbf{b}) = \phi_i d$. Note that $\phi_i \cdot \phi_i = \phi_i$, and $\phi_i \cdot \phi_I = 0$ (assumption: $c_i \neq c_I$, with $c_i < c_I$). Equation (C.20) with the aid of (C.21) yields

$$\pi_i(d; \mathbf{b}) = \phi_i \max\{(b_i - c_i)d - f_i,\ \alpha[f_i - (b_i - c_i)d]\} \tag{C.22}$$

We distinguish the 3 following cases:

— Case 1: $\pi_i(d; \mathbf{b}) = (b_i - c_i)d - f_i > 0$. It should be $\phi_i = 1 \Rightarrow b_i \leq b_I$, and $b_i > c_i + \frac{f_i}{d}$; since $\frac{\partial[(b_i - c_i)d - f_i]}{\partial b_i} > 0$, the profits are maximized for $b_i = b_I$.

— Case 2: $\pi_i(d; \mathbf{b}) = 0$. It should be $\phi_i = 0 \Rightarrow b_i > b_I$, or $b_i = c_i + \frac{f_i}{d}$.

— Case 3: $\pi_i(d; \mathbf{b}) = \alpha[f_i - (b_i - c_i)d] > 0$. It should be $\phi_i = 1 \Rightarrow b_i \leq b_I$, and $b_i < c_i + \frac{f_i}{d}$; since $\frac{\partial\{\alpha[f_i-(b_i-c_i)d]}{\partial b_i} < 0$, the profits are maximized for $b_i = c_i$.

Note: For $\alpha > 0$, Case 2, i.e., $b_i > b_I$, or $b_i = c_i + \frac{f_i}{d}$ is dominated by $b_i = c_i$ and can be eliminated.

We compare the profits of Case 1, i.e., $\pi_i(d; b_i = b_I)$, with those of Case 3, i.e., $\pi_i(d; b_i = c_i)$, and we have $\pi_i(d; b_i = b_I) \geq \pi_i(d; b_i = c_i) \Rightarrow b_I \geq c_i + (1+\alpha)\frac{f_i}{d}$. Hence, $\pi_i(d; b_i = b_I) \leq \pi_i(d; b_i = c_i) \Rightarrow b_I \leq c_i + (1+\alpha)\frac{f_i}{d}$.

Therefore, the best response of supplier i is:

$$b_i^*(b_I) = \begin{cases} c_i, & \text{if } b_I \leq c_i + (1+\alpha)\frac{f_i}{d}, \\ b_I, & \text{if } b_I \geq c_i + (1+\alpha)\frac{f_i}{d}. \end{cases} \tag{C.23}$$

Best Response of Supplier I: With similar calculations, we derive the best response of supplier I, which is as follows

$$b_I^*(b_i) = \begin{cases} [c_I, P], & \text{if } b_i \leq c_I \text{ (indifferent)}, \\ c_I, & \text{if } c_I < b_i \leq c_I + (1+\alpha)\frac{f_I}{d}, \\ b_i^-, & \text{if } b_i > c_I + (1+\alpha)\frac{f_I}{d}. \end{cases} \tag{C.24}$$

Short Proof: We have $\pi_I(d; b_I = b_i^-) \geq \pi_I(d; b_I = c_I) \Rightarrow b_i > c_I + (1+\alpha)\frac{f_I}{d}$. Similarly, $\pi_I(d; b_I = b_i^-) \leq \pi_I(d; b_I = c_I) \Rightarrow b_i \leq c_I + (1+\alpha)\frac{f_I}{d}$. Note that if $b_i = c_I + (1+\alpha)\frac{f_I}{d}$, then $b_I^* = c_I$, because $\pi_I(d; b_I = b_i^-) = (b_i - \varepsilon - c_I)d - f_I = \alpha f_I - \varepsilon d < \alpha f_I = \pi_I(d; b_I = c_I)$.

Equilibria 1a, 2a: The best responses (C.23) and (C.24) yield the following equilibrium outcomes:

— Equilibrium 1a: If $b_i^* = c_i$, for $b_I \leq c_i + (1+\alpha)\frac{f_i}{d}$, then supplier I is indifferent. It follows that in equilibrium, $b_i^* = c_i$, and $b_I^* \leq c_i + (1+\alpha)\frac{f_i}{d}$. The latter inequality can only hold if the RHS is greater than or equal to c_I, which implies that $d \leq (1+\alpha)\frac{f_i}{c_I - c_i} = \theta^{(3)}$. The price at equilibrium is $\lambda^* = c_i$. The allocated quantities are $q_i^* = d$, $q_I^* = 0$. The total payments are TPs $= c_i d + (1+\alpha) f_i$.

— Equilibrium 2a: If $b_i^* = b_I$, for $b_I \geq c_i + (1+\alpha)\frac{f_i}{d}$, then the only response of supplier I that can lead to an equilibrium is $b_I^* = [c_I, P]$, which implies that $b_i \geq c_I$. (In case that $b_I^* = b_i^-$, we cannot have an equilibrium. In case that $b_I^* = c_I$, then it should be $c_I < b_i \leq c_I + (1+\alpha)\frac{f_I}{d}$ — see best response of supplier I, and therefore the best response condition of supplier i cannot hold.) Therefore, the equilibrium is $b_i^* = b_I^* = c_I$ for $d \geq (1+\alpha)\frac{f_i}{c_I - c_i} = \theta^{(3)}$. The price at equilibrium is $\lambda^* = c_I$. The allocated quantities are $q_i^* = d$, $q_I^* = 0$. The total payments are TPs $= c_I d$.

Note that in both cases, the supplier with the least marginal cost satisfies the whole demand, i.e. $q_i^* = d$. The total cost is $c_i d + f_i$. Also, note that for $d = \theta^{(3)}$ both equilibria exist, i.e. $b_i^* = b_I^* = c_I$, and $b_i^* = c_i$, $b_I^* = c_I$.

Intermediate demand, $k_1 < d \leq k_2$

Sub-Case $k_i < k_I$ (i.e., $i = 1, I = 2$)

Best Response of Supplier i: From (C.19), we have

$$\pi_i(d; \mathbf{b}) = \phi_i \max\{\psi_i(d; \mathbf{b}), -\alpha\psi_i(d; \mathbf{b})\}, \tag{C.25}$$

where from (C.18), $\psi_i(d; \mathbf{b})$ is given by

$$\psi_i(d; \mathbf{b}) = [\lambda(d; \mathbf{b}) - c_i]q_i(d; \mathbf{b}) - f_i = \phi_i(b_I - c_i)d - f_i, \tag{C.26}$$

because from (4.6) and (4.8) we have $\lambda(d; \mathbf{b}) = b_2 = b_I$, since $I = 2$, and $q_i(d; \mathbf{b}) = \phi_i k_i$. Equation (C.25) with the aid of (C.26) yields

$$\pi_i(d; \mathbf{b}) = \phi_i \max\{(b_I - c_i)k_i - f_i,\ \alpha[f_i - (b_I - c_i)k_i]\} \tag{C.27}$$

We distinguish the following 3 cases:

— Case 1: $\pi_i(d; \mathbf{b}) = (b_I - c_i)k_i - f_i > 0$. It should be $\phi_i = 1 \Rightarrow b_i \leq b_I$, and $b_I > c_i + \frac{f_i}{k_i}$; since $\frac{\partial \pi_i}{\partial b_i} = 0$, the profits of supplier i cannot be controlled with his bid.

— Case 2: $\pi_i(d; \mathbf{b}) = 0$. It should be $\phi_i = 0 \Rightarrow b_i > b_I$, or if $b_I = c_i + \frac{f_i}{k_i}$.

— Case 3: $\pi_i(d; \mathbf{b}) = \ \alpha[f_i - (b_I - c_i)k_i] > 0$. It should be $\phi_i = 1 \Rightarrow b_i \leq b_I$, and

$b_I < c_i + \frac{f_i}{k_i}$; since $\frac{\partial \pi_i}{\partial b_i} = 0$, the profits of supplier i cannot be controlled with his bid.

Therefore, the best response of supplier i is:

$$b_i^*(b_I) = \begin{cases} [c_i, b_I], & \text{if } b_I \neq c_i + \frac{f_i}{k_i}, \\ [c_i, P], & \text{if } b_I = c_i + \frac{f_i}{k_i}. \end{cases} \tag{C.28}$$

Best Response of Supplier I: From (C.19), we have

$$\pi_I(d; \mathbf{b}) = \max\{\psi_I(d; \mathbf{b}), -\alpha\psi_I(d; \mathbf{b})\}, \tag{C.29}$$

where from (C.18), $\psi_I(d; \mathbf{b})$ is given by

$$\psi_I(d; \mathbf{b}) = (b_I - c_I)(d - \phi_i k_i) - f_I, \tag{C.30}$$

because from (4.6) and (4.8) we have $\lambda(d; \mathbf{b}) = b_2 = b_I$, $q_I(d; \mathbf{b}) = \phi_i(d - k_i) + \phi_I d$, and also $\phi_i + \phi_I = 1$. Equation (C.29) with the aid of (C.30) yields

$$\pi_I(d; \mathbf{b}) = \begin{cases} (b_I - c_I)(d - \phi_i k_i) - f_I, & \text{if } b_I > c_I + \frac{f_I}{d - \phi_i k_i}, \\ \alpha[f_I - (b_I - c_I)(d - \phi_i k_i)], & \text{if } b_I \leq c_I + \frac{f_I}{d - \phi_i k_i}. \end{cases} \tag{C.31}$$

Note that for $b_I = c_I + \frac{f_I}{d - \phi_i k_i}$, we have $\pi_I(d; \mathbf{b}) = 0$.

We distinguish the following cases:

— Case 1: $\pi_I(d; \mathbf{b}) = (b_I - c_I)(d - \phi_i k_i) - f_I$. It should be $b_I \geq c_I + \frac{f_I}{d - \phi_i k_i}$.

— Case 1a: If $\phi_i = 0$, i.e. $b_I < b_i$, then $\pi_I(d; \mathbf{b}) = (b_I - c_I)d - f_I$; since $\frac{\partial \pi_I}{\partial b_I} > 0$, $\pi_I(d; \mathbf{b})$ is maximized for $b_I = b_i^-$. In this case: $\pi_I(d; b_I = b_i^-) = (b_i^- - c_I)d - f_I$ if $b_i^- > c_I + \frac{f_I}{d}$.

— Case 1b: If $\phi_i = 1$, i.e. $b_I \geq b_i$, then $\pi_I(d; \mathbf{b}) = (b_I - c_I)(d - k_i) - f_I$; since $\frac{\partial \pi_I}{\partial b_I} > 0$, $\pi_I(d; \mathbf{b}$ is maximized for $b_I = P$. In this case: $\pi_I(d; b_I = P) = (P - c_I)(d - k_i) - f_I$ if $d > k_i + \frac{f_I}{P - c_I}$.

— Case 2: $\pi_I(d; \mathbf{b}) = \alpha[f_I - (b_I - c_I)(d - \phi_i k_i)]$. It should be $b_I \leq c_I + \frac{f_I}{d - \phi_i k_i}$. It is easily seen that $\pi_I(d; \mathbf{b})$ is maximized for $b_I = c_I$. In this case: $\pi_I(d; b_I = c_I) = \alpha f_I$.

Combining the previous results, we have for $b_I^*(b_i)$ three potential outcomes: c_I, b_i^-, and P. We compare the three outcomes in search for the conditions:

$$b_I^*(b_i) = c_I \Rightarrow \begin{cases} \pi_I(d; b_I = c_I) \geq \pi_I(d; b_I = b_i^-) \\ \pi_I(d; b_I = c_I) \geq \pi_I(d; b_I = P) \end{cases} \Rightarrow \begin{cases} b_i \leq c_I + (1+\alpha)\frac{f_I}{d} \\ d \leq k_i + (1+\alpha)\frac{f_I}{P-c_I} \end{cases}$$

$$b_I^*(b_i) = P \Rightarrow \begin{cases} \pi_I(d; b_I = P) \geq \pi_I(d; b_I = c_I) \\ \pi_I(d; b_I = P) \geq \pi_I(d; b_I = b_i^-) \end{cases} \Rightarrow \begin{cases} d \geq k_i + (1+\alpha)\frac{f_I}{P-c_I} \\ b_i \leq c_I + \frac{(P-c_I)(d-k_i)}{d} \end{cases}$$

$$b_I^*(b_i) = b_i^- \Rightarrow \begin{cases} \pi_I(d; b_I = b_i^-) \geq \pi_I(d; b_I = c_I) \\ \pi_I(d; b_I = b_i^-) \geq \pi_I(d; b_I = P) \end{cases} \Rightarrow \begin{cases} b_i > c_I + (1+\alpha)\frac{f_I}{d} \\ b_i > c_I + \frac{(P-c_I)(d-k_i)}{d} \end{cases}$$

Combining all the above, we have the best response for $b_I^*(b_i)$:

$$b_I^*(b_i) = \begin{cases} c_I, & \text{if } b_i \leq c_I + (1+\alpha)\frac{f_I}{d} \text{ and } d \leq k_i + (1+\alpha)\frac{f_I}{P-c_I}, \\ b_i^-, & \text{if } b_i > c_I + (1+\alpha)\frac{f_I}{d} \text{ and } b_i > c_I + \frac{(P-c_I)(d-k_i)}{d}, \\ P, & \text{if } b_i \leq c_I + \frac{(P-c_I)(d-k_i)}{d} \text{ and } d \geq k_i + (1+\alpha)\frac{f_I}{P-c_I}. \end{cases} \tag{C.32}$$

Equilibria 3, 6, 3a : Firstly, assume that $b_I \neq c_i + \frac{f_i}{k_i}$:

— Equilibrium 3: From the best responses (C.28) and (C.32), the equilibrium outcome for $d \leq k_i + (1+\alpha)\frac{f_I}{P-c_I} = \theta_i^{(4)}$ is $b_i^* = [c_i, c_I]$, and $b_I^* = c_I$. The price at equilibrium is $\lambda^* = c_I$. The allocated quantities are $q_i^* = k_i$, $q_I^* = d - k_i$. The total payments are $c_I d + (1+\alpha)\{f_I + [f_i - (c_I - c_i)k_i]\}$ if $c_I < c_i + \frac{f_i}{k_i}$, and $c_I d + (1+\alpha)f_I$ otherwise.

— Equilibrium 6: For $d \geq k_i + (1+\alpha)\frac{f_I}{P-c_I} = \theta_i^{(4)}$, the equilibrium outcome is $b_i \leq c_I + \frac{(P-c_I)(d-k_i)}{d} = b_I^{(1)}$, and $b_I^* = P$. The price at equilibrium is $\lambda^* = P$. The allocated quantities are $q_i^* = k_i$, $q_I^* = d - k_i$. The total payments are Pd.

Secondly, assume that $b_I = c_i + \frac{f_i}{k_i}$:

— Equilibrium 3a: Since $b_I = c_i + \frac{f_i}{k_i}$, supplier i is indifferent. Therefore, the case that $b_i = \left(c_i + \frac{f_i}{k_i}\right)^+$, so that $b_I = b_i^-$ can be an equilibrium if also the conditions of (C.32) hold, i.e., if $b_i > c_I + (1+\alpha)\frac{f_I}{d}$, and $b_i > c_I + \frac{(P-c_I)(d-k_i)}{d}$. This means that for $c_i + \frac{f_i}{k_i} \geq c_I$, it should be $c_i + \frac{f_i}{k_i} \geq c_I + (1+\alpha)\frac{f_I}{d} \Rightarrow d \geq (1+\alpha)\frac{f_I}{c_i + \frac{f_i}{k_i} - c_I}$, and also $c_i + \frac{f_i}{k_i} \geq c_I + \frac{(P-c_I)(d-k_i)}{d} \Rightarrow d \leq \frac{P-c_I}{P-c_i-\frac{f_i}{k_i}}k_i$.

In case that $c_i + \frac{f_i}{k_i} < c_I$, the above equilibrium does not exist, as the conditions in (C.32) cannot be satisfied.

Therefore, the bids at equilibrium are $b_i^* = \left(c_i + \frac{f_i}{k_i}\right)^+$, $b_I^* = c_i + \frac{f_i}{k_i}$, subject to the conditions $c_i + \frac{f_i}{k_i} \geq c_I$, and $(1+\alpha)\frac{f_I}{c_i+\frac{f_i}{k_i}-c_I} \leq d \leq \frac{P-c_I}{P-c_i-\frac{f_i}{k_i}}k_i$, i.e., $\hat{\theta}_i^{(3)} \leq d \leq \hat{\theta}_i^{(1)}$. The price at equilibrium is $c_i + \frac{f_i}{k_i}$. The allocated quantities are $q_i^* = 0$, $q_I^* = d$. The total payments are $\text{TPs} = \left(c_i + \frac{f_i}{k_i}\right) d$, with $c_I d \leq \text{TPs} < Pd$.

Note that in case that $c_i + \frac{f_i}{k_i} = c_I$, then we have the following equilibria: $b_i^* = b_I^* = c_I$, in which $\pi_i^* = 0$, $q_i^* = k_i$, $q_I^* = d - k_i$, and $b_i^* = c_I^+$, $b_I^* = c_I$, in which $\pi_i^* = 0$, $q_i^* = 0$, $q_I^* = d$.

Sub-Case $k_I < k_i$ (i.e., $i = 2, I = 1$)

Best Response of Supplier i: With similar calculations with the case $k_i < k_I$ for supplier I, we have:

$$b_i^*(b_I) = \begin{cases} c_i, & \text{if } b_I \leq c_i + (1+\alpha)\frac{f_i}{d} \text{ and } d \leq k_I + (1+\alpha)\frac{f_i}{P-c_i}, \\ b_I, & \text{if } b_I \geq c_i + (1+\alpha)\frac{f_i}{d} \text{ and } b_I \geq c_i + \frac{(P-c_i)(d-k_I)}{d}, \\ P, & \text{if } b_I \leq c_i + \frac{(P-c_i)(d-k_I)}{d} \text{ and } d \geq k_I + (1+\alpha)\frac{f_i}{P-c_i}. \end{cases} \tag{C.33}$$

We justify the equalities:

— For $b_I = c_i + (1+\alpha)\frac{f_i}{d}$, we have $\pi_i(b_i = b_I) = (b_I - c_i)d - f_i = [c_i + (1+\alpha)\frac{f_i}{d} - c_i]d - f_i = \alpha f_i = \pi_i(b_i = c_i)$.

— For $b_I = c_i + \frac{(P-c_i)(d-k_I)}{d}$, we have $\pi_i(b_i = b_I) = (b_I - c_i)d - f_i = [c_i + \frac{(P-c_i)(d-k_I)}{d} - c_i]d - f_i = (P-c_i)(d-k_I) - f_i = \pi_i(b_i = P)$.

— For $d = k_I + (1+\alpha)\frac{f_i}{P-c_i}$, we have $c_i + (1+\alpha)\frac{f_i}{d} = c_i + \frac{(P-c_i)(d-k_I)}{d}$, and $\pi_i(b_i = P) = (P-c_i)(k_I + (1+\alpha)\frac{f_i}{P-c_i} - k_I) - f_i = (P-c_i)(1+\alpha)\frac{f_i}{P-c_i} - f_i = \alpha f_i = \pi_i(b_i = c_i)$.

However, we have that $c_I \leq b_I \leq P$, and therefore we need to find the conditions for which the inequalities in (C.33) can be feasible. We have $b_I \leq c_i + \frac{(P-c_i)(d-k_I)}{d}$, so it should be $c_i + \frac{(P-c_i)(d-k_I)}{d} \geq c_I \Rightarrow d \geq \frac{P-c_i}{P-c_I}k_I$. Also, $b_I \leq c_i + (1+\alpha)\frac{f_i}{d}$, so it should be $c_i + (1+\alpha)\frac{f_i}{d} \geq c_I \Rightarrow d \leq (1+\alpha)\frac{f_i}{c_I - c_i}$.

Therefore, the best response of supplier i is:

$$b_i^*(b_I) = \begin{cases} c_i, & \text{if } b_I \leq c_i + (1+\alpha)\frac{f_i}{d} \text{ and } d \leq \min\{k_I + (1+\alpha)\frac{f_i}{P-c_i}, (1+\alpha)\frac{f_i}{c_I-c_i}\}, \\ b_I, & \text{if } b_I \geq c_i + (1+\alpha)\frac{f_i}{d} \text{ and } b_I \geq c_i + \frac{(P-c_i)(d-k_I)}{d}, \\ P, & \text{if } b_I \leq c_i + \frac{(P-c_i)(d-k_I)}{d} \text{ and } d \geq \max\{k_I + (1+\alpha)\frac{f_i}{P-c_i}, \frac{P-c_i}{P-c_I}k_I\}. \end{cases} \tag{C.34}$$

Best Response of Supplier I: With similar calculations with the case $k_i < k_I$ for supplier i, we have

$$b_I^*(b_i) = \begin{cases} \left[c_I, b_i^-\right], & \text{if } b_i > c_I \text{ with } b_i \neq c_I + \frac{f_I}{k_I}, \\ \left[c_I, P\right], & \text{if } b_i \leq c_I \text{ or } b_i = c_I + \frac{f_I}{k_I}. \end{cases} \tag{C.35}$$

Equilibria 1b, 2b, 7, 3b: From the best responses (C.34) and (C.35), the equilibrium outcomes are the following:

Firstly, assume that $b_I \neq c_i + \frac{f_i}{k_i}$.

— Equilibrium 1b: For the case $b_i^*(b_I) = c_i$ in (C.34), it should be $b_I \leq c_i + (1+\alpha)\frac{f_i}{d}$, with $d \leq \min\left\{k_I + (1+\alpha)\frac{f_i}{P-c_i}, (1+\alpha)\frac{f_i}{c_I-c_i}\right\}$. From (C.35) if $b_i = c_i$, supplier I is indifferent, therefore, we have an equilibrium with bids, $b_i^* = c_i$, ${b_I}^* \leq c_i + (1+\alpha)\frac{f_i}{d} = b_i^{(2)}$, subject to $d \leq \min\{\theta_I^{(4)}, \theta^{(3)}\}$. The price at equilibrium is $\lambda^* = c_i$. The allocated quantities are $q_i^* = d$, $q_I^* = 0$. The total payments are $\text{TPs} = c_i d + (1+\alpha)f_i$.

— Equilibrium 7: For the case $b_i^*(b_I) = P$ in (C.34), it should be $b_I \leq c_i + \frac{(P-c_i)(d-k_I)}{d}$, with $d \geq \max\left\{k_I + (1+\alpha)\frac{f_i}{P-c_i}, d \geq \frac{P-c_i}{P-c_I}k_I\right\}$. From (C.35) if $b_i = P$, supplier I should underbid supplier i, therefore, we have an equilibrium with bids, $b_i^* = P$, $b_I^* \leq c_i + \frac{(P-c_i)(d-k_I)}{d} = b_i^{(1)}$, subject to $d \geq \max\left\{k_I + (1+\alpha)\frac{f_i}{P-c_i}, \frac{P-c_i}{P-c_I}k_I\right\} = \max\left\{\theta_I^{(4)}, \theta^{(1)}\right\}$. The price at equilibrium is $\lambda^* = P$. The allocated quantities are $q_i^* = d - k_I$, $q_I^* = k_I$. The total payments are $\text{TPs} = Pd$.

— Equilibrium 2b: For the case that $b_i^*(b_I) = b_I$, it should be $b_I \geq \max\{c_i + (1+\alpha)\frac{f_i}{d}, c_i + \frac{(P-c_i)(d-k_I)}{d}\}$. If $b_I > c_I$, we have $b_i > c_I$, and the best response of supplier I is to underbid supplier i (no equilibrium). If $b_I = c_I$, then $b_i = c_I$, and supplier I is indifferent. This results in equilibrium, where $b_i^* = b_I^* = c_I$. This equilibrium is feasible if $c_I \geq c_i + (1+\alpha)\frac{f_i}{d} \Rightarrow d \geq (1+\alpha)\frac{f_i}{c_I - c_i}$ and also $c_I \geq c_i + \frac{(P-c_i)(d-k_I)}{d} \Rightarrow d \leq \frac{P-c_i}{P-c_I}k_I$, i.e., for $\theta^{(3)} \leq d \leq \theta^{(1)}$. The price at equilibrium is $\lambda^* = c_I$. The allocated quantities are $q_i^* = d$, $q_I^* = 0$. The total payments are TPs $= c_I d$.

Secondly, let us assume that $b_I = c_i + \frac{f_i}{k_i}$.

— Equilibrium 3b: If $b_i = c_I + \frac{f_I}{k_I}$, then supplier I is indifferent. In this case, we can have an equilibrium $b_i^* = b_I^* = c_I + \frac{f_I}{k_I}$ if $c_I + \frac{f_I}{k_I} \geq c_i + (1+\alpha)\frac{f_i}{d} \Rightarrow d \geq (1+\alpha)\frac{f_i}{c_I + \frac{f_I}{k_I} - c_i}$ and also $c_I + \frac{f_I}{k_I} \geq c_i + \frac{(P-c_i)(d-k_I)}{d} \Rightarrow d \leq \frac{P-c_i}{P-c_I-\frac{f_I}{k_I}}k_I$, i.e., if $\hat{\theta}_I^{(3)} \leq d \leq \hat{\theta}_I^{(1)}$. The price at equilibrium is $\lambda^* = c_I + \frac{f_I}{k_I}$. The allocated quantities are $q_i^* = d$, $q_I^* = 0$. The total payments are TPs $= \left(c_I + \frac{f_I}{k_I}\right) d$, with $c_I d <$ TPs $< Pd$.

High Demand, $d > k_2$

Best Response of Supplier i: From (C.19), we have

$$\pi_i(d;\mathbf{b}) = \max\{\psi_i(d;\mathbf{b}), -\alpha\psi_i(d;\mathbf{b})\}, \tag{C.36}$$

where from (C.18), $\psi_i(d;\mathbf{b})$ is given by

$$\psi_i(d;\mathbf{b}) = \phi_i(b_I - c_i)k_i + \phi_I(b_i - c_i)(d-k_I) - f_i, \tag{C.37}$$

since $\lambda(d;\mathbf{b}) = \phi_I b_i + \phi_i b_I$, and $q_i(d;\mathbf{b}) = \phi_i k_i + \phi_I(d-k_I)$.

Equation (C.36) because of (C.37) yields $\pi_i(d;\mathbf{b}) = \max\{\phi_i(b_I - c_i)k_i + \phi_I(b_i - c_i)(d-k_I) - f_i, a[f_i - \phi_i(b_I-c_i)k_i - \phi_I(b_i-c_i)(d-k_I)]\}$, which implies that

$$\pi_i(d;\mathbf{b}) = \begin{cases} (b_I - c_i)k_i - f_i, & \text{if } b_I \geq c_i + \frac{f_i}{k_i} \text{ and } \phi_i = 1, \\ \alpha\left[f_i - (b_I - c_i)k_i\right], & \text{if } b_I \leq c_i + \frac{f_i}{k_i} \text{ and } \phi_i = 1, \\ (b_i - c_i)(d-k_I) - f_i, & \text{if } b_i \geq c_i + \frac{f_i}{d-k_I} \text{ and } \phi_I = 1, \\ \alpha\left[f_i - (b_i - c_i)(d-k_I)\right], & \text{if } b_i \leq c_i + \frac{f_i}{d-k_I} \text{ and } \phi_I = 1. \end{cases} \tag{C.38}$$

We note that $\pi_i(d;\mathbf{b})$ for $\phi_i = 1$, i.e., $b_i \leq b_I$, does not depend on b_i.

We distinguish the following cases:

(A) If $b_I \geq c_i + \frac{f_i}{k_i}$, then:

(1) If $b_i \le b_I$, then $\pi_i^{(\mathrm{A1})} = (b_I - c_i)k_i - f_i \ge 0$.

(2) If $b_i > b_I$, then

(a) $\pi_i^{(\mathrm{A2a})} = (b_i - c_i)(d - k_I) - f_i \ge 0$ if $b_i \ge c_i + \frac{f_i}{d-k_I}$,

(b) $\pi_i^{(\mathrm{A2b})} = \alpha[f_i - (b_i - c_i)(d - k_I)] \ge 0$ if $b_i \le c_i + \frac{f_i}{d-k_I}$.

(B) If $b_I \le c_i + \frac{f_i}{k_i}$, then:

(1) If $b_i \le b_I$, then $\pi_i^{(\mathrm{B1})} = \alpha[f_i - (b_I - c_i)k_i] \ge 0$.

(2) If $b_i > b_I$, then

(a) $\pi_i^{(\mathrm{B2a})} = (b_i - c_i)(d - k_I) - f_i \ge 0$ if $b_i \ge c_i + \frac{f_i}{d-k_I}$,

(b) $\pi_i^{(\mathrm{B2b})} = \alpha[f_i - (b_i - c_i)(d - k_I)] \ge 0$ if $b_i \le c_i + \frac{f_i}{d-k_I}$.

We thoroughly check cases (A) and (B) to find the best response:

CASE (A): $b_I \ge c_i + \frac{f_i}{k_i}$

The profits are maximized as follows: $\pi_i^{(\mathrm{A1})}$ is maximized for any $b_i \le b_I$; $\pi_i^{(A2a)}$ is maximized for $b_i = P$, with $P > b_I$, and $P \ge c_i + \frac{f_i}{d-k_I} \Rightarrow d \ge k_I + \frac{f_i}{P-c_i}$; $\pi_i^{(\mathrm{A2b})} \to \max$ for $b_i = b_I^+$, with $b_I^+ \le c_i + \frac{f_i}{d-k_I}$. Therefore, we have $\pi_i^{(\mathrm{A1})} = (b_I - c_i)k_i - f_i$ for $b_i \le b_I,\ b_I \ge c_i + \frac{f_i}{k_i}$, $\pi_i^{(\mathrm{A2b})} = (P - c_i)(d - k_I) - f_i$ for $b_I < P,\ b_I \ge c_i + \frac{f_i}{k_i}, d \ge k_I + \frac{f_i}{P-c_i}$, and $\pi_i^{(\mathrm{A2b})} = \alpha[f_i - (b_I^+ - c_i)(d - k_I)]$ for $b_I^+ \le c_i + \frac{f_i}{d-k_I}$. Note: If $b_I = P$, then $b_i \le b_I$ always, and $\pi_i^{(\mathrm{A1})} = (P - c_i)k_i - f_i$. Lastly,

We compare the 3 outcomes $\pi_i^{(\mathrm{A1})}$, $\pi_i^{(\mathrm{A2a})}$, and $\pi_i^{(\mathrm{A2b})}$. We have $\pi_i^{(\mathrm{A1})} \ge \pi_i^{(\mathrm{A2a})} \Rightarrow b_I \ge c_i + \frac{(P-c_i)(d-k_I)}{k_i}$. Also, $\pi_i^{(\mathrm{A1})} \ge \pi_i^{(\mathrm{A2b})} \Rightarrow b_I \ge c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}$, where we assumed $b_I^+ = b_I + \varepsilon$, with $\varepsilon \to 0^+$, and also used the inequality $\frac{\alpha(d-k_I)}{k_i+\alpha(d-k_I)} < 1$. Lastly, $\pi_i^{(\mathrm{A2a})} \ge \pi_i^{(\mathrm{A2b})} \Rightarrow b_I \ge c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}$. For clarity, we provide the remaining conditions: $\pi_i^{(\mathrm{A1})} \le \pi_i^{(\mathrm{A2a})} \Rightarrow b_I \le c_i + \frac{(P-c_i)(d-k_I)}{k_i}$, $\pi_i^{(\mathrm{A1})} \le \pi_i^{(\mathrm{A2b})} \Rightarrow b_I < c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}$, and $\pi_i^{(\mathrm{A2a})} \le \pi_i^{(\mathrm{A2b})} \Rightarrow b_I < c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}$.

Combining the above, we have for $b_I \ge c_i + \frac{f_i}{k_i}$:

$$b_i^*(b_I) = \begin{cases} [c_i, b_I] & : b_I \ge c_i + \frac{(P-c_i)(d-k_I)}{k_i}, b_I \ge c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}, \\ P & : b_I \le c_i + \frac{(P-c_i)(d-k_I)}{k_i}, b_I \ge c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}, d \ge k_I + \frac{f_i}{P-c_i}, \\ b_I^+ & : b_I < c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}, b_I < c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}. \end{cases} \tag{C.39}$$

CASE (B): $b_I \leq c_i + \frac{f_i}{k_i}$

The profits are maximized as follows: $\pi_i^{(\mathrm{B1})}$ is maximized for any $b_i \leq b_I$; $\pi_i^{(\mathrm{B2a})}$ is maximized for $b_i = P$, with $P > b_I$, and $P \geq c_i + \frac{f_i}{d-k_I} \Rightarrow d \geq k_I + \frac{f_i}{P-c_i}$; $\pi_i^{(\mathrm{B2b})}$ is maximized for $b_i = b_I^+$, with $b_I^+ \leq c_i + \frac{f_i}{d-k_I}$. Therefore, we have $\pi_i^{(\mathrm{B1})} = \alpha[f_i - (b_I - c_i)k_i]$ for $b_i \leq b_I$, $b_I \leq c_i + \frac{f_i}{k_i}$, $\pi_i^{(\mathrm{B2a})} = (P-c_i)(d-k_I) - f_i$ for $b_I \leq c_i + \frac{f_i}{k_i}$, $d \geq k_I + \frac{f_i}{P-c_i}$, and $\pi_i^{(\mathrm{B2b})} = \alpha[f_i - (b_I^+ - c_i)(d-k_I)]$ for $b_I^+ \leq c_i + \frac{f_i}{d-k_I}$.

We compare the 3 outcomes $\pi_i^{(\mathrm{B1})}$, $\pi_i^{(\mathrm{B2a})}$, and $\pi_i^{(\mathrm{B2b})}$. We will show that $\pi_i^{(\mathrm{B1})}$ is always dominated by either $\pi_i^{(\mathrm{B2a})}$ or $\pi_i^{(\mathrm{B2b})}$.

Because $\pi_i^{(\mathrm{B1})}$ has as an upper bound αf_i, it will be $\pi_i^{(\mathrm{B2a})} > \pi_i^{(\mathrm{B1})} \Rightarrow (P-c_i)(d-k_I) - f_i > \alpha f_i \Rightarrow d > k_I + \frac{(1+\alpha)f_i}{P-c_i}$, where we assume that $k_I + \frac{(1+\alpha)f_i}{P-c_i} < k_i + k_I \Rightarrow \frac{(1+\alpha)f_i}{P-c_i} < k_i$. For values of demand not close to $k_i + k_I$, or $d \leq k_I + \frac{(1+\alpha)f_i}{P-c_i}$, we can show that it is always $\pi_i^{(\mathrm{B2b})} > \pi_i^{(\mathrm{B1})} \Rightarrow \alpha[f_i - (b_I^+ - c_i)(d-k_I)] > \alpha[f_i - (b_I - c_i)k_i] \Rightarrow -(b_I + \varepsilon - c_i)(d-k_I) > -(b_I - c_i)k_i \Rightarrow (b_I - c_i)(k_i + k_I - d) < \varepsilon(d-k_I)$. Since we have assumed that $d < k_i + k_I \Rightarrow k_i + k_I - d > 0$, we have $b_I < c_i + \varepsilon\frac{d-k_I}{k_i+k_I-d}$. However, it should be $b_I \geq c_I$, and therefore $c_I < c_i + \varepsilon\frac{d-k_I}{k_i+k_I-d}$, which cannot hold since $c_I > c_i$, and $\varepsilon \to 0^+$. We have therefore shown that $\pi_i^{(\mathrm{B1})}$ is always dominated by either $\pi_i^{(\mathrm{B2a})}$ or $\pi_i^{(\mathrm{B2b})}$.

Hence, we compare the outcomes $\pi_i^{(\mathrm{B2a})}$, and $\pi_i^{(\mathrm{B2b})}$, (the results are presented earlier), and we have for $b_I \leq c_i + \frac{f_i}{k_i}$.

$$b_i^*(b_I) = \begin{cases} P & : b_I \geq c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}, d \geq k_I + \frac{f_i}{P-c_i}, \\ b_I^+ & : b_I < c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}. \end{cases} \tag{C.40}$$

For ease of exposition, we can present both (C.39) and (C.40) in one form. We observe that the missing inequalities in (C.40) (if compared to (C.39)) for $b_i^*(b_I) = P$, and $b_i^*(b_I) = b_I^+$ hold always. Also, the inequalities of the case for $b_i^*(b_I) = [c_i, b_I]$, imply that $b_I \geq c_i + \frac{f_i}{k_i}$.

Firstly, we show that if $b_I \leq c_i + \frac{f_i}{k_i}$, and $d \geq k_I + \frac{f_i}{P-c_i}$, then $b_I \leq c_i + \frac{(P-c_i)(d-k_I)}{k_i}$. We have $d \geq k_I + \frac{f_i}{P-c_i} \Leftrightarrow (P-c_i)(d-k_I) \geq f_i \Leftrightarrow c_i + \frac{(P-c_i)(d-k_I)}{k_i} \geq c_i + \frac{f_i}{k_i}$. Therefore, since $b_I \leq c_i + \frac{f_i}{k_i}$, and $c_i + \frac{(P-c_i)(d-k_I)}{k_i} \geq c_i + \frac{f_i}{k_i}$, it is always $b_I \leq c_i + \frac{(P-c_i)(d-k_I)}{k_i}$.

Secondly, we show that if $b_I \leq c_i + \frac{f_i}{k_i}$, and $b_I < c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}$, then $b_I <$

$c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}$. Suppose that we have $c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)} > c_i + \frac{f_i}{k_i} \Leftrightarrow \frac{(1+\alpha)}{k_i+\alpha(d-k_I)} > \frac{1}{k_i} \Leftrightarrow (1+\alpha)k_i > k_i + \alpha(d-k_I) \Leftrightarrow k_i > d - k_I \Leftrightarrow d < k_i + k_I$. For $d = k_i + k_I$, we need to show that if $b_I < c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}$, with $b_I \le c_i + \frac{f_i}{k_i}$, then $b_I < c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}$, or equivalently that if $b_I < c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha k_i} \Leftrightarrow b_I < c_i + \frac{(1+\alpha)f_i-(P-c_i)k_i}{\alpha k_i}$, then it is $b_I < c_i + \frac{(1+\alpha)f_i}{k_i+\alpha k_i} \Leftrightarrow b_I < c_i + \frac{f_i}{k_i}$. Therefore, we need to show that $c_i + \frac{(1+\alpha)f_i-(P-c_i)k_i}{\alpha k_i} \le c_i + \frac{f_i}{k_i} \Leftrightarrow (1+\alpha)f_i - (P-c_i)k_i \le \alpha f_i \Leftrightarrow f_i \le (P-c_i)k_i \Leftrightarrow \frac{f_i}{P-c_i} \le k_i$. But, the last inequality is verified by the assumption that $\frac{f_i}{P-c_i} < k_i$.

Thirdly, we show that if $b_I \ge c_i + \frac{(P-c_i)(d-k_I)}{k_i}$, and $b_I \ge c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}$, then it is implied that $b_I \ge c_i + \frac{f_i}{k_i}$. We can show that $c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)} \ge c_i + \frac{f_i}{k_i} \Leftrightarrow (1+\alpha)k_i \ge k_i + \alpha(d-k_I) \Leftrightarrow k_i \ge d - k_I \Leftrightarrow d \le k_i + k_I$. Therefore if $b_I \ge c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}$, it is always $b_I \ge c_i + \frac{f_i}{k_i}$.

The best response of supplier i is given as follows:

$$b_i^*(b_I) = \begin{cases} [c_i, b_I], & \text{if } b_I \ge c_i + \frac{(P-c_i)(d-k_I)}{k_i},\ b_I \ge c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}, \\ b_I^+, & \text{if } b_I < c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)},\ b_I < c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}, \\ P, & \text{if } b_I \le c_i + \frac{(P-c_i)(d-k_I)}{k_i},\ b_I \ge c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)},\ d \ge k_I + \frac{f_i}{P-c_i}. \end{cases} \tag{C.41}$$

Note that the curves $b_I = c_i + \frac{(P-c_i)(d-k_I)}{k_i}$, $b_I = c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}$, and $b_I = c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}$ have a common point at $\theta_I^{(5)} = k_I + \frac{k_i}{2\alpha}\left(\sqrt{1 + \frac{4\alpha(\alpha+1)f_i}{k_i(P-c_i)}} - 1\right)$. Actually, the condition $d \ge k_I + \frac{f_i}{P-c_i}$, for $b_i^*(b_I) = P$ can be substituted by $d \ge \theta_I^{(5)}$.

Proof of $\theta_I^{(5)} = k_I + \frac{k_i}{2\alpha}\left(\sqrt{1 + \frac{4\alpha(\alpha+1)f_i}{k_i(P-c_i)}} - 1\right)$: With simple manipulations, the equality $c_i + \frac{(P-c_i)(d-k_I)}{k_i} = c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}$, yields $\alpha d^2 + (k_i - 2\alpha k_I)d - k_I k_i + \alpha k_I^2 - \frac{(1+\alpha)f_i k_i}{(P-c_i)} = 0$, with $d = \frac{-(k_i-2\alpha k_I) \pm \sqrt{{k_i}^2 + \frac{4\alpha(1+\alpha)f_i k_i}{(P-c_i)}}}{2\alpha} = k_I + \frac{k_i}{2\alpha}\left(\pm\sqrt{1 + \frac{4\alpha(1+\alpha)f_i}{k_i(P-c_i)}} - 1\right)$, where we keep only one solution, i.e., $d = k_I + \frac{k_i}{2\alpha}\left(\sqrt{1 + \frac{4\alpha(1+\alpha)f_i}{k_i(P-c_i)}} - 1\right)$. Similarly, we verify that $c_i + \frac{(P-c_i)(d-k_I)}{k_i} = c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}$, yields $d = k_I + \frac{k_i}{2\alpha}\left(\pm\sqrt{1 + \frac{4\alpha(1+\alpha)f_i}{k_i(P-c_i)}} - 1\right)$, and hence, $d = k_I + \frac{k_i}{2\alpha}\left(\sqrt{1 + \frac{4\alpha(1+\alpha)f_i}{k_i(P-c_i)}} - 1\right)$.

Best Response of Supplier I: With similar calculations, we have

$$\pi_I(d;\mathbf{b}) = \begin{cases} (b_i - c_I)k_I - f_I, & \text{if } b_i \geq c_I + \frac{f_I}{k_I} \text{ and } \phi_2 = 1, \\ \alpha\left[f_I - (b_i - c_I)k_I\right], & \text{if } b_i \leq c_I + \frac{f_I}{k_I} \text{ and } \phi_2 = 1, \\ (b_I - c_I)(d - k_i) - f_I, & \text{if } b_I \geq c_I + \frac{f_I}{d-k_i} \text{ and } \phi_1 = 1, \\ \alpha\left[f_I - (b_I - c_I)(d - k_i)\right], & \text{if } b_I \leq c_I + \frac{f_I}{d-k_i} \text{ and } \phi_1 = 1. \end{cases} \tag{C.42}$$

We distinguish the following cases:

(A) If $b_i \geq c_I + \frac{f_I}{k_I}$, then:

(1) If $b_I < b_i$, then $\pi_I^{(\text{A1})} = (b_i - c_I)k_I - f_I \geq 0$.

(2) If $b_I \geq b_i$, then

(a) $\pi_I^{(\text{A2a})} = (b_I - c_I)(d - k_i) - f_I \geq 0$ if $b_I \geq c_I + \frac{f_I}{d-k_i}$

(b) $\pi_I^{(\text{A2b})} = \alpha[f_I - (b_I - c_I)(d - k_i)] \geq 0$ if $b_I \leq c_I + \frac{f_I}{d-k_i}$

(B) If $b_i \leq c_I + \frac{f_I}{k_I}$, then:

(1) If $b_I < b_i$, then $\pi_I^{(\text{B1})} = \alpha[f_I - (b_i - c_I)k_I]$

(2) If $b_I \geq b_i$, then

(a) $\pi_I^{(\text{B2a})} = (b_I - c_I)(d - k_i) - f_I$ if $b_I \geq c_I + \frac{f_I}{d-k_i}$

(b) $\pi_I^{(\text{B2b})} = \alpha[f_I - (b_I - c_I)(d - k_i)]$ if $b_I \leq c_I + \frac{f_I}{d-k_i}$

We thoroughly check cases (A) and (B) to find the best response:

CASE (A): $b_i \geq c_I + \frac{f_I}{k_I}$

The profits are maximized as follows: $\pi_I^{(\text{A1})}$ is maximized for any $b_I < b_i$; $\pi_I^{(\text{A2a})}$ is maximized for $b_I = P$, with $P \geq b_i$, and $P \geq c_I + \frac{f_I}{d-k_i} \Rightarrow d \geq k_i + \frac{f_I}{P-c_I}$; $\pi_I^{(\text{A2b})}$ is maximized for $b_i = b_I$, with $b_i \leq c_I + \frac{f_I}{d-k_i}$. Therefore, we have $\pi_I^{(\text{A1})} = (b_i - c_I)k_I - f_I$ for $b_I < b_i$, $b_i \geq c_I + \frac{f_I}{k_I}$, $\pi_I^{(\text{A2a})} = (P - c_I)(d - k_i) - f_I$ for $b_i \geq c_I + \frac{f_I}{k_I}$, $d \geq k_i + \frac{f_I}{P-c_I}$, and $\pi_I^{(\text{A2b})} = \alpha[f_I - (b_i - c_I)(d - k_i)]$ for $b_i \leq c_I + \frac{f_I}{d-k_i}$.

We compare the 3 outcomes $\pi_I^{(\text{A1})}$, $\pi_I^{(\text{A2a})}$, and $\pi_I^{(\text{A2b})}$. We have $\pi_I^{(\text{A1})} \geq \pi_I^{(\text{A2a})} \Rightarrow b_i \geq c_I + \frac{(P-c_I)(d-k_i)}{k_I}$. Also, $\pi_I^{(\text{A1})} \geq \pi_I^{(\text{A2b})} \Rightarrow b_i \geq c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)}$. Lastly, $\pi_I^{(\text{A2a})} \geq \pi_I^{(\text{A2b})} \Rightarrow b_i \geq c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}$.

Combining the above, we have for $b_i \geq c_I + \frac{f_I}{k_I}$

$$b_I^*(b_i) = \begin{cases} [c_I, b_i) & : b_i \leq c_I + \frac{(P-c_I)(d-k_i)}{k_I}, b_i \geq c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}, d \geq k_i + \frac{f_I}{P-c_I}, \\ P & : b_i \leq c_I + \frac{(P-c_I)(d-k_i)}{k_I}, b_i \geq c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}, d \geq k_i + \frac{f_I}{P-c_I}, \\ b_i & : b_i \leq c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)}, b_i \leq c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}. \end{cases} \tag{C.43}$$

CASE (B): $b_i \leq c_I + \frac{f_I}{k_I}$

In this case, we observe that the strategy $b_I < b_i$ is possible only if $b_i > c_I$. Therefore, assuming that $b_i > c_I$, we have $\pi_I^{(\text{B1})}$ maximized for any $b_I < b_i$. Also, $\pi_I^{(\text{B2a})}$ is maximized for $b_I = P$, with $P \geq b_i$, and $P \geq c_I + \frac{f_I}{d-k_i} \Rightarrow d \geq k_i + \frac{f_I}{P-c_I}$. We also observe that $\pi_I^{(\text{B2b})}$ is maximized for the minimum value of b_i, subject to $b_i \geq b_I$. In the case that $b_i \leq c_I$ we have $b_I = c_I$, whereas if $b_i > c_I$, then $b_i = b_I$. Therefore, we have $\pi_I^{(\text{B2b})}$ maximized for $b_I = \max\{b_i, c_I\}$, with $\max\{b_i, c_I\} \leq c_I + \frac{f_I}{d-k_i}$. Therefore, we have $\pi_I^{(\text{B1})}(b_I < b_i) = \alpha[f_I - (b_i - c_I)k_I]$ for $b_I < b_i, c_I < b_i \leq c_I + \frac{f_I}{k_I}$, $\pi_I^{(\text{B2a})}(b_I = P) = (P - c_I)(d - k_i) - f_I$ for $b_i \leq c_I + \frac{f_I}{k_I}, d \geq k_i + \frac{f_I}{P-c_I}$, and $\pi_I^{(\text{B2b})}(b_I = b_i) = \alpha[f_I - (b_i - c_I)(d - k_i)]$ for $c_I \leq b_i \leq c_I + \frac{f_I}{d-k_i}$. (Note: $b_i \leq c_I + \frac{f_I}{d-k_i}$ is always true since $b_i \leq c_I + \frac{f_I}{k_I}$ as $c_I + \frac{f_I}{k_I} \leq c_I + \frac{f_I}{d-k_i}$). Lastly, $\pi_I^{(\text{B2b})}(b_I = c_I) = \alpha f_I$ for $b_i \leq c_I$.

We show that $\pi_I^{(\text{B1})}$ is always dominated by either $\pi_I^{(\text{B2a})}$ or $\pi_I^{(\text{B2b})}$. For $c_I \leq b_i \leq c_I + \frac{f_I}{d-k_i}$ it is $\pi_I^{(\text{B1})}(b_I < b_i) = \alpha[f_I - (b_i - c_I)k_I] < \pi_I^{(\text{B2b})}(b_I = b_i) = \alpha[f_I - (b_i - c_I)(d - k_i)]$ if $d < k_i + k_I$. If $d = k_i + k_I$, we have $\pi_I^{(\text{B1})}(b_I < b_i) = \pi_I^{(\text{B2b})}(b_I = b_i)$; we show that in this case it is $\pi_I^{(\text{B2a})} > \pi_I^{(\text{B1})}$. If $d = k_i + k_I$, then $\pi_I^{(\text{B1})}$ has an upper bound the value αf_I. Therefore, it will be $\pi_I^{(\text{B2a})} > \pi_I^{(\text{B1})} \Rightarrow (P - c_I)(d - k_i) - f_I > \alpha f_I \Rightarrow d > k_i + \frac{(1+\alpha)f_I}{P-c_I}$, where we assume that $k_i + \frac{(1+\alpha)f_I}{P-c_I} < k_i + k_I \Rightarrow \frac{(1+\alpha)f_I}{P-c_I} < k_I$. Therefore, we have shown that $\pi_I^{(\text{B1})}$ is always dominated by either $\pi_I^{(\text{B2a})}$ or $\pi_I^{(\text{B2b})}$.

Hence, we compare the outcomes $\pi_I^{(\text{B2a})}$, and $\pi_I^{(\text{B2b})}$, and actually the outcome for $b_i \leq c_I$, and we obtain $pi_I^{(\text{B2a})}(b_I = P) \geq \pi_I^{(\text{B2b})}(b_I = c_I) \Rightarrow d \geq k_i + \frac{(1+\alpha)f_I}{P-c_I}$. (Note: $d \geq k_i + \frac{f_I}{P-c_I}$ is implied since we found the condition: $d \geq k_i + \frac{(1+\alpha)f_I}{P-c_I}$.)

Combining the above, the best response for $b_i \leq c_I + \frac{f_I}{k_I}$ is:

$$b_I^*(b_i) = \begin{cases} P & : b_i \geq c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}, b_i \geq c_I, d \geq k_i + \frac{f_I}{P-c_I} \text{ (or) }, \\ & \quad b_i \leq c_I, d \geq k_i + \frac{(1+\alpha)f_I}{P-c_I}, \\ b_i & b_i \leq c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}, b_i \geq c_I, \\ c_I & : b_i \leq c_I, d \leq k_i + \frac{(1+\alpha)f_I}{P-c_I}. \end{cases} \tag{C.44}$$

For ease of exposition, we can present both (C.43) and (C.44) in one form. We observe that the missing inequalities in (C.44) (if compared to (C.43)) for $b_I^*(b_i) = P$, and $b_I^*(b_i) = b_i$ hold always. Also, the inequalities of the case for $b_I^*(b_i) = [c_I, b_i)$, imply that $b_i \geq c_I + \frac{f_I}{k_I}$.

Firstly, we show that if $b_i \leq c_I + \frac{f_I}{k_I}$, and $d \geq k_i + \frac{f_I}{P-c_I}$, then $b_i \leq c_I + \frac{(P-c_I)(d-k_i)}{k_I}$. We have $d \geq k_i + \frac{f_I}{P-c_I} \Leftrightarrow (P-c_I)(d-k_i) \geq f_I \Leftrightarrow c_I + \frac{(P-c_I)(d-k_i)}{k_I} \geq c_I + \frac{f_I}{k_I}$. Therefore, since $b_i \leq c_I + \frac{f_I}{k_I}$, and $c_I + \frac{(P-c_I)(d-k_i)}{k_I} \geq c_I + \frac{f_I}{k_I}$, it is always $b_i \leq c_I + \frac{(P-c_I)(d-k_i)}{k_I}$.

Secondly, we show that if $b_i \leq c_I + \frac{f_I}{k_I}$, and $b_i \leq c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}$, then $b_i \leq c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)}$. Suppose that we have $c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)} \geq c_I + \frac{f_I}{k_I} \Leftrightarrow \frac{(1+\alpha)}{k_I+\alpha(d-k_i)} \geq \frac{1}{k_I} \Leftrightarrow (1+\alpha)k_I \geq k_I + \alpha(d-k_i) \Leftrightarrow k_I \geq d - k_i \Leftrightarrow d \leq k_i + k_I$.

Thirdly, we show that if $b_i \geq c_I + \frac{(P-c_I)(d-k_i)}{k_I}$, and $b_i \geq c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)}$, then it is implied that $b_i \geq c_I + \frac{f_I}{k_I}$. We can show that $c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)} \geq c_I + \frac{f_I}{k_I} \Leftrightarrow (1+\alpha)k_I \geq k_I + \alpha(d-k_i) \Leftrightarrow k_I \geq d-k_i \Leftrightarrow d \leq k_i + k_I$. Therefore if $b_i \geq c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)}$, it is always $b_i \geq c_I + \frac{f_I}{k_I}$.

The best response of supplier i is given as follows:

$$b_I^*(b_i) = \begin{cases} c_I, & \text{if } b_i \leq c_I, d \leq k_i + \frac{(1+\alpha)f_I}{P-c_I}, \\ [c_I, b_i), & \text{if } b_i \geq c_I + \frac{(P-c_I)(d-k_i)}{k_I}, b_i \geq c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)}, \\ b_i, & \text{if } b_i \leq c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)}, b_i \leq c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}, b_i \geq c_I, \\ P, & \text{if } b_i \leq c_I + \frac{(P-c_I)(d-k_i)}{k_I}, b_i \geq c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}, b_i \geq c_I, \\ & d \geq k_i + \frac{f_I}{P-c_I}, \quad \text{(or) } b_i \leq c_I, d \geq k_i + \frac{(1+\alpha)f_I}{P-c_I}. \end{cases} \tag{C.45}$$

Note: The curves $b_i = c_I + \frac{(P-c_I)(d-k_i)}{k_I}$, $b_i = c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)}$, and $b_i = c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}$ have a common point at: $\theta_i^{(5)} = k_i + \frac{k_I}{2\alpha}\left(\sqrt{1 + \frac{4\alpha(\alpha+1)f_I}{k_I(P-c_I)}} - 1\right)$. Actually, the condition $d \geq k_i + \frac{f_I}{P-c_I}$, for can be substituted by $d \geq \theta_i^{(5)}$. [The proof is similar to the previous case.]

Equilibria 8, 9, 10, 4, 5: Reviewing the best responses of the twos suppliers, we can show that for the case that $b_I^* = P$, it should be $d \geq k_i + \frac{k_I}{2\alpha}\left(\sqrt{1+\frac{4\alpha(\alpha+1)f_I}{k_I(P-c_I)}}-1\right)$. Subject to the condition: $k_I > \frac{f_I}{P-c_I}$, we have that $k_i + \frac{k_I}{2\alpha}\left(\sqrt{1+\frac{4\alpha(\alpha+1)f_I}{k_I(P-c_I)}}-1\right) > k_i + \frac{f_I}{P-c_I}$. Therefore the condition $d \geq k_i + \frac{f_I}{P-c_I}$, when $b_I^* = P$ can be omitted. With similar calculations, we can omit the condition $d \geq k_I + \frac{f_i}{P-c_i}$, when $b_i^* = P$.

— Equilibria 8, 9: Assume that $b_I^* = P$. This is the best response of supplier I subject to the conditions: $b_i \leq c_I + \frac{(P-c_I)(d-k_i)}{k_I}$, $b_i \geq c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}$, $b_i \geq c_I$ or $b_i \leq c_I$, $d \geq k_i + \frac{(1+\alpha)f_I}{P-c_I}$.

From the best response of supplier i, we see that when $b_I = P$, supplier i is practically indifferent and can bid from its cost to the price cap. Therefore, we have the following equilibria: $b_I^* = P$, and $b_i^* \leq c_I + \frac{(P-c_I)(d-k_i)}{k_I}$, $b_i^* \geq c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}$, $b_i^* \geq c_I$, or $b_i^* \leq c_I$, $d \geq k_i + \frac{(1+\alpha)f_I}{P-c_I}$. Note: We can include the condition: $d \geq \theta_i^{(5)}$ for the first case, i.e. when $b_i^* \geq c_I$.

Hence, we have Equilibrium 8, where the bids are $b_I^* = P$, and $b_i^* \leq b_I^{(1)}$, $b_i^* \geq b_I^{(4)}$, $b_i^* \geq c_I$, i.e., $\max\{c_I, b_I^{(4)}\} \leq b_i^* \leq b_I^{(1)}$, i.e., $b_i^* \in \mathrm{B}_1$, subject to the condition $d \geq \theta_i^{(5)}$. The price at equilibrium is $\lambda^* = P$. The allocated quantities are $q_i^* = k_i$, $q_I^* = d - k_i$. The total payments are $\mathrm{TPs} = Pd$.

Similarly, we have Equilibrium 9, where the bids are $b_I^* = P$, and $b_i^* \leq c_I$, subject to the condition $d \geq \theta_i^{(4)}$. The price at equilibrium is $\lambda^* = P$. The allocated quantities are $q_i^* = k_i$, $q_I^* = d - k_i$. The total payments are $\mathrm{TPs} = Pd$.

— Equilibrium 10: With similar arguments, we have an equilibrium where $b_i^* = P$, $b_I^* \leq c_i + \frac{(P-c_i)(d-k_I)}{k_i}$, $b_I^* \geq c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}$, $b_I^* \geq c_I$, i.e., $b_I^* \leq b_i^{(1)}$, $b_I^* \geq b_i^{(4)}$, $b_I^* \geq c_I$, i.e., $\max\{c_I,\ b_i^{(4)}\} \leq b_I^* \leq b_i^{(1)}$, i.e., $b_I^* \in \mathrm{B}_2$, subject to the condition $d \geq \theta_I^{(5)}$. The price at equilibrium is $\lambda^* = P$. The allocated quantities are $q_i^* = d - k_I$, $q_I^* = k_I$. The total payments are $\mathrm{TPs} = Pd$.

— Equilibrium 4: We observe that the combination of $b_i^*(b_I) = [c_i, b_I]$ with $b_I^*(b_i) = b_i$ can result in an equilibrium subject to the following conditions: $b_I \geq c_i + \frac{(P-c_i)(d-k_I)}{k_i}$, $b_I \geq c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}$, and $b_i \leq c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)}$, $b_i \leq c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}$, $b_i \geq c_I$.

The equilibrium is $b_i^* = b_I^* = p$, if $p \geq c_i + \frac{(P-c_i)(d-k_I)}{k_i}$, $p \geq c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}$, $p \leq c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)}$, $p \leq c_I - \frac{P-c_I}{\alpha} + \frac{(1+\alpha)f_I}{\alpha(d-k_i)}$, $p \geq c_I$. The conditions can be rewritten as $\max\{c_I, b_i^{(1)}, b_i^{(3)}\} \leq p \leq \min\{b_I^{(3)}, b_I^{(4)}\}$, or shortly as $p \in \mathrm{B}_3$. The price at equilibrium is $\lambda^* = p$. The allocated quantities are $q_i^* = k_i$, $q_I^* = d - k_i$. The total payments are $\mathrm{TPs} = pk_i + [c_I - \alpha(p-c_I)](d-k_i) + (1+\alpha)f_I$. (The total payments are obtained by adding the profits of supplier i $\pi_i = (p-c_i)k_i - f_i$, and those of supplier I $\pi_I = \alpha[f_I - (p-c_I)(d-k_i)]$ with the total cost of both suppliers.)

— Equilibrium 5: We also observe that the combination $b_i^*(b_I) = b_I^+$ with $b_I^*(b_i) = [c_I, b_i)$ can result in an equilibrium, subject to the conditions $b_I < c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}$, $b_I < c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}$, and $b_i \geq c_I + \frac{(P-c_I)(d-k_i)}{k_I}$, $b_i \geq c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)}$. The equilibrium is $b_i^* = p$, $b_I^* = p^-$, if $p \leq c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}$, $p \leq c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}$, $p \geq c_I + \frac{(P-c_I)(d-k_i)}{k_I}$, $p \geq c_I + \frac{(1+\alpha)f_I}{k_I+\alpha(d-k_i)}$. (They are derived since $b_I^* = p^- \Leftrightarrow b_I^* = p - \varepsilon$, with $\varepsilon \to 0^+$, and the conditions $b_I \leq c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)} - \varepsilon\frac{\alpha(d-k_I)}{k_i+\alpha(d-k_I)} \Rightarrow p \leq c_i + \frac{(1+\alpha)f_i}{k_i+\alpha(d-k_I)}$, and $b_I \leq c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)} - \varepsilon\frac{\alpha(d-k_I)}{k_i+\alpha(d-k_I)} \Rightarrow p \leq c_i - \frac{P-c_i}{\alpha} + \frac{(1+\alpha)f_i}{\alpha(d-k_I)}$, where we have $\frac{k_i}{k_i+\alpha(d-k_I)} < 1$). The conditions can be rewritten as $\max\{b_I^{(1)}, b_I^{(3)}\} \leq p \leq \min\{b_i^{(3)}, b_i^{(4)}\}$, and shortly as $p \in \mathrm{B}_4$. The price at equilibrium is $\lambda^* = p$. The allocated quantities are $q_i^* = d - k_I$, $q_I^* = k_I$. The total payments are $\mathrm{TPs} = pk_I + [c_i - \alpha(p-c_i)](d-k_I) + (1+\alpha)f_i$. (The total payments are obtained by adding the profits of supplier i $\pi_i = \alpha[f_i - (p-c_i)(d-k_I)]$, and those of supplier I $\pi_I = (p-c_I)k_I - f_I$ with the total cost of both suppliers.)

Note: There exist demand levels in which more than one equilibrium exists.

Appendix D

List of Thesis Publications

Parts of the work presented in this thesis have been published and presented as follows:

Book Chapter

[B.1] P. Andrianesis, G. Liberopoulos, and G. Kozanidis, "Modeling the day-ahead scheduling problem in Greece's wholesale electricity market," Scientific Analects: Anniversary Vol. for the 20 Years of the Univ. of Thessaly. M. Zouboulakis (ed.). Univ. Publications of Thessaly, Volos, Greece, 2010, 271 – 284.

Journal Papers

[J.1] G. Liberopoulos and P. Andrianesis, "Critical review of pricing schemes in markets with non-convex costs," Operations Research, vol. 64, no. 1, pp. 17–31, 2016.

[J.2] P. Andrianesis, G. Liberopoulos, G. Kozanidis, and A. Papalexopoulos, "Recovery mechanisms in day-ahead electricity markets with non-convexities — Part I: Design and evaluation methodology," IEEE Transactions on Power Systems, Vol. 28, no. 2, pp. 960–968, 2013.

[J.3] P. Andrianesis, G. Liberopoulos, G. Kozanidis, and A. Papalexopoulos, "Recovery mechanisms in day-ahead electricity markets with non-convexities — Part

II: Implementation and numerical evaluation," IEEE Transactions on Power Systems, Vol. 28, no. 2, pp. 969–977, 2013.

[J.4] P. Andrianesis, P. Biskas, and G. Liberopoulos, "An overview of Greece's wholesale electricity market with emphasis on ancillary services," Electric Power Systems Research, vol. 81, pp. 1631–1642, 2011.

Papers in International Conferences

[C.1] P. Andrianesis, G. Liberopoulos, "Comparison of pricing mechanisms in markets with non-convexities," in Proceedings of the 3rd International Symposium and 25th National Conference on Operational Research (HELORS 2014), Volos, Greece, 26 – 28 June 2014, 254–259.

[C.2] P. Andrianesis and G. Liberopoulos, "Revenue-adequate pricing mechanisms in non-convex electricity markets: A comparative study," in Proceedings of the 11th International Conference on the European Energy Market (EEM'14), Krakow, Poland, 28 – 30 May 2014.

[C.3] P. Andrianesis and G. Liberopoulos, "On the design of electricity auctions with non-convexities and make-whole payments." in Proceedings of the 10th International Conference on the European Energy Market (EEM'13), Stockholm, Sweden, 27 – 31 May 2013.

[C.4] P. Andrianesis, G. Liberopoulos, A. Papalexopoulos, "Greek wholesale electricity market: Forthcoming market changes and bid/cost recovery," in Proceedings of MedPower 2012, Cagliari, Italy, 1 – 3 October 2012.

[C.5] P. Andrianesis, G. Liberopoulos, G. Kozanidis, A. Papalexopoulos, "A recovery mechanism with loss-related profits in a day-ahead electricity market with non-convexities," in Proceedings of IEEE PowerTech 2011, Trondheim, Norway, 19 – 23 June 2011.

[C.6] P. Andrianesis, G. Liberopoulos, G. Kozanidis, A. Papalexopoulos, "Recovery mechanisms in a joint energy/reserve day-ahead electricity market with

non-convexities," in Proceedings of the 7th International Conference on the European Energy Market (EEM'10), 2010, Madrid, 27 – 29 June 2010.

[C.7] P. Andrianesis, G. Liberopoulos, G. Kozanidis, "Energy-reserve markets with non-convexities: an empirical analysis," in Proceedings of IEEE PowerTech 2009, Bucharest, Romania, 28 June – 2 July 2009.

Abstracts and Presentations in International Conferences

[P.1] P. Andrianesis and G. Liberopoulos, "Equilibrium analysis in markets with non-convex costs," presented at the 2nd International Conference on Energy, Sustainability, and Climate Change, Chania, Greece, 21 – 27 June 2015.

[P.2] P. Andrianesis and G. Liberopoulos, and A. Papalexopoulos, "Non-convexities in Electricity Markets: Theoretical and Practical Implications," presented at the INFORMS Annual Meeting 2014, San Francisco, CA, 9 – 12 November, 2014.

[P.3] P. Andrianesis and G. Liberopoulos, "Bidding in markets with non-convex costs: A comparison of market outcomes under different pricing mechanisms," presented at the 20th Conference of the International Federation of the Operational Research Societies, (IFORS 2014), Barcelona, Spain, 13 – 18 July, 2014.

[P.4] G. Liberopoulos and P. Andrianesis, "Comparative Analysis of Pricing Schemes in Markets with Non-Convex Costs," presented at the 20th Conference of the International Federation of the Operational Research Societies, (IFORS 2014), Barcelona, Spain, 13 – 18 July, 2014.

[P.5] P. Andrianesis, and G. Liberopoulos, "Commodity pricing in markets with non-convexities: Lessons from a duopoly," presented at the 26th European Conference on Operational Research, Rome 1 – 4 July, 2013.

[P.6] P. Andrianesis and G. Liberopoulos, "Equilibria characterization in a two-player auction with maximum and minimum capacity constraints," presented

at the 3rd Meeting of the EURO Working Group on Stochastic Modeling, Nafplio, Greece, 7 – 9 June 2010.

In Table D.1, we relate each of the above works to the Chapters of this thesis. For each chapter, the publications are listed in chronological order, with the most recent one at the top.

Table D.1: Relation of thesis publications to main chapters.

Chapter 2	Chapter 3	Chapter 4	Chapter 5
[J.1]: entire chapter	[P.3]: Section 3.3	[P.1]: Section 4.4	[J.2]: Sections 5.2 – 5.4
[P.4]: early work	[C.1]: Section 3.3	[P.2]: Section 4.4	[J.3]: Sections 5.5 – 5.7
[P.5]: early work	[C.2]: Section 3.3	[P.5]: Section 4.4	[C.5]: early work
	[C.4]: Section 3.4	[C.3]: Section 4.4	[C.6]: early work
	[J.4]: part, Section 3.4	[P.6]: early work, Section 4.3	[C.7]: early work
	[B.1]: early work, Section 3.4		

Bibliography

[1] S. Stoft, *Power System Economics: Designing Markets for Electricity*, IEEE/Wiley, 2002.

[2] F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R. E. Bohn, *Spot Pricing of Electricity*, Kluwer, Boston, MA, 1988.

[3] R. B. Johnson, S. S. Oren, and A. J. Svoboda, "Equity and efficiency of unit commitment in competitive electricity markets," *Utilities Policy*, vol. 6, no. 1, pp. 9–19, 1997.

[4] W. Elmaghraby, R. O'Neill, M. Rothkopf, and W. Stewart, "Pricing and efficiency in "lumpy" energy markets," *The Electricity Journal*, vol. 17, no. 5, pp. 54–64, 2004.

[5] R. P. O'Neill, P. M. Sotkiewicz, B. F. Hobbs, M. H. Rothkopf, and W. R. Stewart Jr, "Efficient market-clearing prices in markets with nonconvexities," *European Journal of Operational Research*, vol. 164, no. 1, pp. 269–285, 2005.

[6] R. Baldick, U. Helman, B. F. Hobbs, and R. P. O'Neill, "Design of efficient generation markets," *Proceedings of IEEE*, vol. 93, no. 11, pp. 1998–2012, 2005.

[7] R. M. Starr, "Quasi-equilibria in markets with non-convex preferences," *Econometrica*, vol. 37, no. 1, pp. 25–38, 1969.

[8] R. E. Gomory and W. J. Baumol, "Integer programming and pricing," *Econometrica*, vol. 28, no. 3, pp. 521–550, 1960.

[9] L. A. Wolsey, "Integer programming duality: Price functions and sensitivity analysis," *Mathematical Programming*, vol. 20, no. 1, pp. 173–195, 1981.

[10] H. E. Scarf, "Mathematical programming and economic theory," *Operations Research*, vol. 38, no. 3, pp. 377–385, 1990.

[11] H. E. Scarf, "The allocation of resources in the presence of indivisibilities," *Journal of Economic Perspectives*, vol. 8, no. 4, pp. 111–128, 1994.

[12] B. Wingfield and D. Kopecki, "JP Morgan to pay $410 million in U.S. FERC settlement," Bloomberg. [Online] 30th July. Available from: http://www.bloomberg.com/. [Accessed: 30th June 2015].

[13] Federal Energy Regulatory Commission (FERC), "Order approving stipulation and consent agreement in make-whole payments and related bidding strategies," Docket Nos. IN11-8-000 IN 13-5-000, 2013.

[14] M. Bjørndal and K. Jörnsten, "Equilibrium prices supported by dual price functions in markets with non-convexities," *European Journal of Operational Research*, vol. 190, no. 3, pp. 768–789, 2008.

[15] M. Bjørndal and K. Jörnsten, *A partitioning method that generates interpretable prices for integer programming problems*, vol. Handbook of Power Systems II of *Energy Systems*, pp. 337–350, Springer, New York, NY, 2010.

[16] W. W. Hogan and B. J. Ring, "On minimum-uplift pricing for electricity markets," Working Paper, John F. Kennedy School of Government, Harvard University, 2003.

[17] P. R. Gribik, W. W. Hogan, and S. L. Pope, "Market-clearing electricity prices and energy uplift," Working Paper, John F. Kennedy School of Government, Harvard University, 2007.

[18] A. L. Motto and F. D. Galiana, "Equilibrium of auction markets with unit commitment: the need for augmented pricing," *IEEE Transactions on Power Systems*, vol. 17, no. 3, pp. 798–805, 2002.

[19] F. D. Galiana, A. L. Motto, and F. Bouffard, "Reconciling social welfare, agent profits, and consumer payments in electricity pools," *IEEE Transactions on Power Systems*, vol. 18, no. 2, pp. 452–459, 2003.

[20] M. Van Vyve, "Linear price for non-convex electricity markets: Models and algorithms," CORE — UCL Discussion Paper 2011/50, Belgium, 2011.

[21] V. Araoz V and K. Jörnsten, "Semi-Lagrangean approach for price discovery in markets with non-convexities," *European Journal of Operational Research*, vol. 214, no. 2, pp. 411–417, 2011.

[22] C. Ruiz, A. J. Conejo, and S. A. Gabriel, "Pricing non-convexities in an electricity pool," *IEEE Transactions on Power Systems*, vol. 27, no. 3, pp. 1334–1342, 2012.

[23] S. A. Gabriel, A. J. Conejo, C. Ruiz, and S. Siddiqui, "Solving discretely-constrained, mixed linear complementarity problems with applications in energy," *Computers and Operations Research*, vol. 40, no. 5, pp. 1339–1350, 2013.

[24] P. Cramton and S. Stoft, "Why we need to stick with uniform-price auctions in electricity markets," *The Electricity Journal*, vol. 20, no. 1, pp. 26–37, 2007.

[25] B. F. Hobbs, M. H. Rothkopf, L. C. Hyde, and R. P. O' Neill, "Evaluation of a truthful revelation auction in the context of energy markets with nonconcave benefits," *Journal of Regulatory Economics*, vol. 18, no. 1, pp. 5–32, 2000.

[26] A. L. Motto and F. D. Galiana, "Coordination in markets with nonconvexities as a mathematical program with equilibrium constraints — Part I: A solution procedure," *IEEE Transactions on Power Systems*, vol. 19, no. 1, pp. 308–316, 2004.

[27] A. L. Motto and F. D. Galiana, "Coordination in markets with nonconvexities as a mathematical program with equilibrium constraints — Part II: Case studies," *IEEE Transactions on Power Systems*, vol. 19, no. 1, pp. 317–324, 2004.

[28] G. Muratore, "Equilibria in markets with non-convexities and a solution to the missing money phenomenon in energy markets," CORE — UCL Discussion Paper 2008/6, Belgium, 2008.

[29] E. Toczylowski and I. Zoltowska, "A new pricing scheme for a multi-period pool-based electricity auction," *European Journal of Operational Research*, vol. 197, no. 3, pp. 1051–1062, 2009.

[30] A. Martin, J. C. Müller, and S. Pokutta, "Strict linear prices in non-convex European day-ahead electricity markets," *Optimization Methods and Software*, vol. 29, no. 1, pp. 189–221, 2014.

[31] M. Madani and M. Van Vyve, "A MIP framework for non-convex uniform price day-ahead electricity auctions," pp. 1–22, 2015.

[32] R. P. O'Neill, P. M. Sotkiewicz, and M. H. Rothkopf, "Equilibrium prices in power exchanges with non-convex bids," Working Paper, 2007.

[33] R. P. O'Neill, "Nonconvex electricity market design," in *Proceedings of the 2009 IEEE Power and Energy Society General Meeting*, Calgary, AB, July 2009.

[34] B. Zhang, P. B. Luh, E. Litvinov, T. Zheng, and F. Zhao, "On reducing uplift payment in electricity markets," in *Proceedings of the 2009 IEEE/PES Power Systems Conference and Exposition*, Seattle, Washington, 2009.

[35] G. Wang, U. V. Shanbhag, T. Zheng, E. Litvinov, and S. Meyn, "An extreme-point subdifferential method for convex hull pricing in energy and reserve markets — Part I: Algorithm structure," vol. 28, no. 3, pp. 2111–2120, 2013.

[36] G. Wang, U. V. Shanbhag, T. Zheng, E. Litvinov, and S. Meyn, "An extreme-point subdifferential method for convex hull pricing in energy and reserve markets — Part II: Convergence analysis and numerical performance," vol. 28, no. 3, pp. 2121–2127, 2013.

[37] F. Bouffard and F. D. Galiana, *Generalized uplifts in pool-based electricity markets: Coordinating time-dynamic markets*, vol. Analysis, Control and Optimization of Complex Dynamic Systems, pp. 193–214, Springer, 2010.

[38] F. Abbaspourtorbati, A. J. Conejo, J. Wang, and R. Cherkaoui, "Pricing electricity through a stochastic non-convex market-clearing model," *IEEE Transactions on Power Systems*, 2016.

[39] C. Grigg, P. Wong, P. Albrecht, R. Allan, M. Bhavaraju, R. Billinton, Q. Chen, C. Fong, S. Haddad, and S. Kuruganty, "The IEEE reliability test system—1996. a report prepared by the reliability test system task force of the application of probability methods subcommittee," *IEEE Transactions on Power Systems*, vol. 14, no. 3, pp. 1010–1020, 1999.

[40] J. M. Arroyo and A. J. Conejo, "Optimal response of a thermal unit to an electricity spot market," *IEEE Transactions on Power Systems*, vol. 15, no. 3, pp. 1098–1104, 2000.

[41] R. P. O'Neill, U. Helman, P. Sotkiewicz, M. H. Rothkopf, and W. R. Stewart Jr., *Regulatory evolution, market design and unit commitment*, vol. The Next Generation of Electric Power Unit Commitment Models, pp. 15–37, Kluwer, 2001.

[42] N. P. Padhy, "Unit commitment — A bibliographical survey," *IEEE Transactions on Power Systems*, vol. 19, no. 2, pp. 1196–1205, 2004.

[43] H. Y. Yamin, "Review on methods of generation scheduling in electric power systems," *Electric Power Systems Research*, vol. 69, no. 2–3, pp. 227–248, 2004.

[44] J. Ostrowski, M. F. Anjos, and A. Vannelli, "Tight Mixed Integer Linear Programming Formulations for the Unit Commitment Problem," *IEEE Transactions on Power Systems*, vol. 27, no. 1, pp. 39–46, 2012.

[45] L. L. Garver, "Power generation scheduling by integer programming — Development and theory," *Transactions of the American Institute of Electrical*

Engineers. Part III: Power Apparatus and Systems, vol. 81, no. 3, pp. 730–734, 1962.

[46] M. Carrión and J. M. Arroyo, "A computationally efficient mixed integer linear formulation for the thermal unit commitment problem," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1371–1378, 2006.

[47] G. Morales-Espana, J. M. Latorre, and A. Ramos, "Tight and compact MILP formulation of start-up and shut-down ramping in unit commitment," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1288–1296, 2013.

[48] R. Sioshansi, R. O' Neill, and S. S. Oren, "Economic consequences of alternative solution methods for centralized unit commitment in day-ahead electricity markets," vol. 23, no. 2, pp. 344–352, 2008.

[49] P. Cramton, "Competitive bidding behavior in uniform-price auction markets," in *Proceedings of the 37th Annual Hawaii International Conference on System Sciences*, Hawaii, Jan. 2004.

[50] M. Ventosa, A. Baillo, A. Ramos, and M. Rivier, "Electricity market modeling trends," *Energy Policy*, vol. 33, no. 7, pp. 897–913, 2005.

[51] A. J. Conejo, F. J. Nogales, and J. M. Arroyo, "Price-taker bidding strategy under price uncertainty," *IEEE Transactions on Power Systems*, vol. 17, no. 4, pp. 1081–1088, 2002.

[52] D. De Ladurantaye, M. Gendreau, and J.-Y. Potvin, "Strategic bidding for price-taker hydroelectricity producers," vol. 22, no. 4, pp. 2187–2203, 2007.

[53] C. K. Simoglou, P. N. Biskas, and A. G. Bakirtzis, "Optimal self-scheduling of a thermal producer in short-term electricity markets by MILP," vol. 25, no. 4, pp. 1965–1977, 2010.

[54] F. J. Díaz, J. Contreras, J. I. Muñoz, and D. Pozo, "Optimal scheduling of a price-taker cascaded reservoir system in a pool-based electricity market," *IEEE Transactions on Power Systems*, vol. 26, no. 2, pp. 604–615, 2011.

[55] L. Baringo and A. J. Conejo, "Offering Strategy Via Robust Optimization," *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 599–604, 2011.

[56] E. J. Anderson and A. B. Philpott, "Optimal offer construction in electricity markets," *Mathematics of Operations Research*, vol. 27, no. 1, pp. 82–100, 2002.

[57] S. de la Torre, J. M. Arroyo, A. J. Conejo, and J. Contreras, "Price maker self-scheduling in a pool-based electricity market: A Mixed-Integer LP approach," vol. 17, no. 4, pp. 1037–1042, 2002.

[58] A. G. Bakirtzis, N. P. Ziogos, A. C. Tellidou, and G. A. Bakirtzis, "Electricity producer offering strategies in day-ahead energy market with step-wise offers," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 1804–1818, 2007.

[59] H. M. I. Pousinho, J. Contreras, A. G. Bakirtzis, and J. P. S. Catalão, "Risk-constrained scheduling and offering strategies of a price-maker hydro producer under uncertainty," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1879–1887, 2013.

[60] B. F. Hobbs, C. B. Metzler, and J.-S. Pang, "Strategic gaming analysis for electric power systems: An MPEC approach," *IEEE Transactions on Power Systems*, vol. 15, no. 2, pp. 638–645, 2000.

[61] S. de la Torre, J. Contreras, and A. J. Conejo, "Finding multiperiod Nash equilibria in pool-based electricity markets," vol. 19, no. 1, pp. 643–651, 2004.

[62] H. Haghighat, H. Seifi, and A. R. Kian, "Gaming analysis in joint energy and spinning reserve markets," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 2074–2085, 2007.

[63] L. A. Barroso, R. D. Carneiro, S. Granville, M. V. Pereira, and M. H. C. Fampa, "Nash equilibrium in strategic bidding: A binary expansion approach," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 629–638, 2006.

[64] E. Hasan, F. D. Galiana, and A. J. Conejo, "Electricity markets cleared by merit order — Part I: Finding the market outcomes supported by pure strategy

Nash equilibria," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 361–371, 2008.

[65] E. Hasan and F. D. Galiana, "Electricity markets cleared by merit order — Part II: Strategic offers and market power," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 372–379, 2008.

[66] E. Hasan and F. D. Galiana, "Fast computation of pure strategy Nash equilibria in electricity markets cleared by merit order," *IEEE Transactions on Power Systems*, vol. 25, no. 2, pp. 722–728, 2010.

[67] P. D. Klemperer and M. A. Meyer, "Supply function equilibria in oligopoly under uncertainty," *Econometrica*, vol. 57, no. 6, pp. 1243–1277, 1989.

[68] R. J. Green and D. M. Newbery, "Competition in the British electricity spot market," *Journal of Political Economy*, vol. 100, no. 5, pp. 929–953, 1992.

[69] E. J. Anderson and A. B. Philpott, "Using supply functions for offering generation into an electricity market," *Operations Research*, vol. 50, no. 3, pp. 477–489, 2002.

[70] E. J. Anderson and H. Xu, "Optimal supply functions in electricity markets with option contracts and non-smooth costs," *Mathematical Methods of Operations Research*, vol. 63, no. 3, pp. 387–411, 2006.

[71] I. Otero-Novas, C. Meseguer, C. Batlle, and J. J. Alba, "A simulation model for a competitive generation market," *IEEE Transactions on Power Systems*, vol. 15, no. 1, pp. 250–256, 2000.

[72] T. Krause, E. V. Beck, R. Cherkaoui, A. Germond, G. Andersson, and D. Ernst, "A comparison of Nash equilibria analysis and agent-based modelling for power markets," *IEEE Transactions on Power Systems*, vol. 28, no. 9, pp. 599–607, 2002.

[73] A. Weidlich and D. Veit, "A critical survey of agent-based wholesale electricity market models," *Energy Economics*, vol. 30, no. 4, pp. 1728–1759, 2008.

[74] E. Guerci, M. A. Rasregar, and S. Cincotti, *Agent-based Modeling and Simulation of Competitive Wholesale Electricity Markets*, vol. Handbook of Power Systems II of *Energy Systems*, pp. 241–286, Springer, 2010.

[75] N. H. M. von der Fehr and D. Harbord, "Spot market competition in the UK electricity industry," *The Economic Journal*, vol. 103, no. 418, pp. 531–546, 1993.

[76] N. Fabra, N.-H. von der Fehr, and D. Harbord, "Designing electricity auctions," *RAND Journal of Economics*, vol. 37, no. 1, pp. 23–46, 2006.

[77] R. Sioshansi and E. Nicholson, "Towards equilibrium offers in unit commitment auctions with nonconvex costs," *Journal of Regulatory Economics*, vol. 40, no. 1, pp. 41–61, 2011.

[78] G. Wang, U. V. Shanbhag, and S. P. Meyn, "On Nash equilibria in duopolistic power markets subject to make-whole uplift," in *Proceedings of the 51st IEEE Conference on Decision Control (CDC 2012)*, Maui, Hawaii, 2012, pp. 472–477.

[79] G. Wang, "Design and operation of electricity markets: Dynamics, uncertainty, pricing and completion," PhD Dissertation, Dept Electr Comp Engr, U Illinois at Urbana-Champaign, 2013.

[80] P. B. Luh, W. E. Blankson, Y. Chen, J. H. Yan, G. A. Stern, S.-C. Chang, and F. Zhao, "Payment cost minimization auction for deregulated electricity markets using surrogate optimization," vol. 21, no. 2, pp. 568–578, 2006.

[81] F. Zhao, P. B. Luh, J. H. Yan, G. A. Stern, and S.-C. Chang, "Payment cost minimization auction for deregulated electricity markets with transmission capacity constraints," vol. 23, no. 2, pp. 532–544, 2008.

[82] F. Zhao, P. B. Luh, J. H. Yan, G. A. Stern, and S.-C. Chang, "Bid cost minimization versus payment cost minimization: A game theoretic study of electricity auctions," vol. 25, no. 1, pp. 181–194, 2010.

[83] E. Maskin, "A comparison of Nash equilibria analysis and agent-based modelling for power markets," *Games and Economic Behavior*, vol. 71, no. 1, pp. 9–11, 2011.

[84] C. Beltran, C. Tadonki, and J.-P.H. Vial, "Solving the p-median problem with a semi-Lagrangian relaxation," *Computational Optimization and Applications*, vol. 35, no. 2, pp. 239–260, 2006.

[85] D. Klabjan, "A new subadditive approach to integer programming," 2002.

[86] A. Brooke, D. Kendrick, and A. Meeraus, *GAMS User's Guide*, The Scientific Press, Redwood City, CA, 1990.

[87] Regulatory Authority for Energy, "RAE," www.rae.gr.

[88] Hellenic Transmission System Operator, "DESMIE," www.desmie.gr.

[89] Independent Power Transmission Operator, "ADMIE," www.admie.gr.

[90] Hellenic Electricity Market Operator, "LAGIE," www.lagie.gr.

[91] P. Andrianesis, G. Liberopoulos, P. Biskas, and A. Bakirtzis, "Medium-Term Unit Commitment in a pool market," in *Proceedings of the 8th International Conference on the European Energy Market (EEM'11)*, Zagreb, Croatia, May 2011.

[92] California Independent System Operator (CaISO), "Tariff revision and request for waiver of sixty day notice requirements," [Online]. Available: http://www.caiso.com/2ba5/2ba5d37337b50.pdf, 2011.

[93] P. Andrianesis, P. Biskas, and G. Liberopoulos, "An overview of Greece's wholesale electricity market with emphasis on ancillary services," *Electric Power Systems Research*, vol. 81, no. 8, pp. 1631–1642, 2011.

[94] P. Andrianesis, G. Liberopoulos, and A. Papalexopoulos, "Application of zonal pricing in Greeces electricity market," in *Proceedings of the 6th International*

Conference on the European Energy Market (EEM'09), Leuven, Belgium, May 2009.

[95] P. Andrianesis, G. Liberopoulos, G. Kozanidis, and A. Papalexopoulos, "Recovery mechanisms in a joint energy/reserve day-ahead electricity market with non-convexities," in *Proceedings of the 7th International Conference on the European Energy Market (EEM'10)*, Madrid, Spain, 2010.

[96] P. Andrianesis, G. Liberopoulos, and G. Kozanidis, "Energy-reserve markets with non-convexities: An empirical analysis," in *Proceedings of the IEEE/PES PowerTech 2009 Conference*, Bucharest, Romania, 2009.

[97] J. F. Bard, *Practical Bilevel Optimization: Algorithms and Applications*, Kluwer Academic Publishers, USA, 1998.

[98] R. Fourer, D. M. Gay, and B. W. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*, Boyd & Fraser, Danvers, MA, 1993.

[99] "ILOG AMPL CPLEX System Version 10.0 User's Guide," [Online]. Available: www.ampl.com/BOOKLETS/amplcplex100userguide.pdf, 2006.

[100] G. Kozanidis, E. Kostarelou, P. Andrianesis, and G. Liberopoulos, "Mixed integer parametric bilevel programming for optimal strategic bidding of energy producers in day-ahead electricity markets with indivisibilities," *Optimization*, vol. 62, no. 8, pp. 1045–1068, 2013.