ΠΑΝΕΠΙΣΤΗΜΙΟ
ΘΕΣΣΑΛΙΑΣ

ΣΧΟΛΗ ΘΕΤΙΚΩΝ ΕΠΙΣΤΗΜΩΝ

ΤΜΗΜΑ ΠΛΗΡΟΦΟΡΙΚΗΣ ΚΑΙ ΤΗΛΕΠΙΚΟΙΝΩΝΙΩΝ

Μελέτη Επίδοσης της Μη Ορθογωνικής Πολλαπλής Πρόσβασης με Αναμετάδοση σε Διαλειπτικά Κανάλια

Ευθυμιάδης Φιλάρετος

ΠΤΥΧΙΑΚΗ ΕΡΓΑΣΙΑ

ΕΠΙΒΛΕΠΩΝ

Τσιφτσής Θεόδωρος
Καθηγητής

Λαμία Νοέμβριος 2022

ΠΑΝΕΠΙΣΤΗΜΙΟ
ΘΕΣΣΑΛΙΑΣ

ΣΧΟΛΗ ΘΕΤΙΚΩΝ ΕΠΙΣΤΗΜΩΝ

ΤΜΗΜΑ ΠΛΗΡΟΦΟΡΙΚΗΣ ΚΑΙ ΤΗΛΕΠΙΚΟΙΝΩΝΙΩΝ

Μελέτη Επίδοσης της Μη Ορθογωνικής Πολλαπλής Πρόσβασης με Αναμετάδοση σε Διαλειπτικά Κανάλια

Ευθυμιάδης Φιλάρετος

ΠΤΥΧΙΑΚΗ ΕΡΓΑΣΙΑ

ΕΠΙΒΛΕΠΩΝ

Τσιφτσής Θεόδωρος
Καθηγητής

Λαμία Νοέμβριος 2022

UNIVERSITY OF THESSALY

SCHOOL OF SCIENCE

DEPARTMENT OF INFORMATICS & TELECOMMUNICATIONS

Performance Analysis of Non-Orthogonal Multiple Access for Relaying Transmissions over Fading Channels

Efthimiadis Filaretos

FINAL THESIS

ADVISOR

Tsiftsis Theodoros
Professor

Lamia November 2022

«Με ατομική μου ευθύνη και γνωρίζοντας τις κυρώσεις (1), που προβλέπονται από της διατάξεις της παρ. 6 του άρθρου 22 του Ν. 1599/1986, δηλώνω ότι:

1. Δεν παραθέτω κομμάτια βιβλίων ή άρθρων ή εργασιών άλλων αυτολεξεί ***χωρίς να τα περικλείω σε εισαγωγικά*** *και χωρίς να αναφέρω το συγγραφέα, τη χρονολογία, τη σελίδα. Η αυτολεξεί παράθεση χωρίς εισαγωγικά χωρίς αναφορά στην πηγή, είναι λογοκλοπή. Πέραν της αυτολεξεί παράθεσης, λογοκλοπή θεωρείται και η παράφραση εδαφίων από έργα άλλων, συμπεριλαμβανομένων και έργων συμφοιτητών μου, καθώς και η παράθεση στοιχείων που άλλοι συνέλεξαν ή επεξεργάσθηκαν, χωρίς αναφορά στην πηγή. Αναφέρω πάντοτε με πληρότητα την πηγή κάτω από τον πίνακα ή σχέδιο, όπως στα παραθέματα.*

2. Δέχομαι ότι η αυτολεξεί ***παράθεση χωρίς εισαγωγικά****, ακόμα κι αν συνοδεύεται από αναφορά στην πηγή σε κάποιο άλλο σημείο του κειμένου ή στο τέλος του, είναι αντιγραφή. Η αναφορά στην πηγή στο τέλος π.χ. μιας παραγράφου ή μιας σελίδας, δεν δικαιολογεί συρραφή εδαφίων έργου άλλου συγγραφέα, έστω και παραφρασμένων, και παρουσίασή τους ως δική μου εργασία.*

3. Δέχομαι ότι υπάρχει επίσης περιορισμός στο μέγεθος και στη συχνότητα των παραθεμάτων που μπορώ να εντάξω στην εργασία μου εντός εισαγωγικών. Κάθε μεγάλο παράθεμα (π.χ. σε πίνακα ή πλαίσιο, κλπ), προϋποθέτει ειδικές ρυθμίσεις, και όταν δημοσιεύεται προϋποθέτει την άδεια του συγγραφέα ή του εκδότη. Το ίδιο και οι πίνακες και τα σχέδια

4. Δέχομαι όλες τις συνέπειες σε περίπτωση λογοκλοπής ή αντιγραφής.

Ημερομηνία: ……/9/2022

Ο – Η Δηλ.

(1) «Όποιος εν γνώσει του δηλώνει ψευδή γεγονότα ή αρνείται ή αποκρύπτει τα αληθινά με έγγραφη υπεύθυνη δήλωση
του άρθρου 8 παρ. 4 Ν. 1599/1986 τιμωρείται με φυλάκιση τουλάχιστον τριών μηνών. Εάν ο υπαίτιος αυτών των πράξεων
σκόπευε να προσπορίσει στον εαυτόν του ή σε άλλον περιουσιακό όφελος βλάπτοντας τρίτον ή σκόπευε να βλάψει άλλον, τιμωρείται με κάθειρξη μέχρι 10 ετών.»

ΠΕΡΙΛΗΨΗ

Η μη-Ορθογώνια Πολλαπλή Πρόσβαση (NOMA) θεωρείται ως μια πολλά υποσχόμενη υποψήφια τεχνολογία για τα δίκτυα κινητής τηλεφωνίας πέμπτης γενιάς (5G), καθώς μπορεί να βελτιώσει την φασματική απόδοση και το ρυθμό διεκπεραίωσης του συστήματος σε σύγκριση με την ορθογώνια πολλαπλή πρόσβαση (OMA). Επιπλέον, η αναμετάδοση διευρύνει τη χωρητικότητα και τη περιοχή κάλυψης, βελτιώνοντας έτσι την επίδοση του δικτύου. Επομένως, είναι λογικό να τα συγχωνεύσουμε για να παρατηρήσουμε τα ακριβή οφέλη που παρέχει ο συνδυασμός τους. Εξάγονται εκφράσεις κλειστής μορφής για τη πιθανότητα σφάλματος και διεξάγεται ασυμπτωτική ανάλυση. Ο μέσος ρυθμός εξετάζεται επίσης ασυμπτωτικά άνω και κάτω όρια. Επίσης, όλα τα αριθμητικά αποτελέσματα έχουν προσομοιωθεί με Monte Carlo σε περιβάλλον MATLAB, προκειμένου να παρατηρήσουμε την επίδοση του συστήματος.

ABSTRACT

Non-Orthogonal Multiple Access (NOMA) is considered as a promising candidate technology for the fifth-generation (5G) mobile telecommunications networks, as it can improve the spectral efficiency and throughput of the system compared to Orthogonal Multiple Access (OMA). In addition, relaying transmission enlarges the capacity and broadens the coverage area, thus it improves the performance of the network. Therefore, it is logical to amalgamate them in order to observe the exact benefits that their combination provides. Outage probability closed form expressions are derived and asymptotic analysis is conducted. Average sum rate analysis is also examined asymptotically, providing lower and upper bounds. MATLAB numerical results using Monte Carlo simulations is also provided in order to observe the systems performance.

Table of Contents

1. INTRODUCTION

Non-Orthogonal Multiple Access

Over the last decade, wireless telecommunication systems have been evolving in an expeditious manner, as a consequence of the collective network of connective devices (IoT) and mobile internet rapid development. This is challenging us to revolutionize the entire network infrastructure. Fifth generation (5G) networks should be able to support three main technologies:

1. Enhanced Mobile Broadband (eMBB)
   a) 100 Mbps user perceived data rate.
   b) >3 times spectrum efficiency improvement over the former LTE releases to provide services including high-definition video experience, virtual reality, and augmented reality.

2. massive Machine Type Communication (mMTC)
   a) large number of IoT devices will have access to the network => connection density of 1 million devices per square kilometer.

NOMA can increase number of users by a scale factor of 5.

3. UltraReliable and Low-Latency Communication (URLLC)
   a) 0.5 ms end-to-end latency.
   b) reliability above 99.999%.

NOMA can increase number of users by a scale factor of 9. [1]

A multiple access technique that is considerably attractive for satisfying the aforementioned metrics, is the Non-Orthogonal Multiple access (NOMA). To begin with, we cannot overemphasize the superiority of NOMA over the traditional orthogonal multiple access schemes of the 1G, 2G, 3G and 4G wireless telecommunication systems such as FDMA, TDMA, CDMA, OFDMA respectively. In orthogonal multiple access (OMA), the system is transmitting the users' signals in an orthogonal way, thus, each user has its own resource block. In order to support more users than the number of available orthogonal resources, non-orthogonal multiple access is proposed. NOMA, can be divided into 2 categories, based on the method each one is implementing to perform the user's signal separation, namely, **power-domain** and **code-domain NOMA**. [1], [2] Code-domain NOMA is realized to a certain degree as the basic CDMA, sharing the available time and frequency resources; however, code-domain NOMA is different compared to CDMA due to the utilization of the user-determined sparse spreading sequences or non-orthogonal cross-correlation spreading sequences of low correlation index. It can be further classified into low-density spreading (LDS) CDMA [3], [4], low-density spreading OFDMA [5], [6] and sparse code multiple access (SCMA) [7], [8]. In SCMA, the incoming bits are directly mapped to low density multidimensional complex codewords selected from carefully designed sparse codebooks, where each codeword represents a single spread transmission layer. In contrast with the complex maximum a-posteriori probability decoding, iterative message passing algorithm (MPA) produced efficient MPA decoding with notably reduced complexity [9]. In LDS, each user is spreading his information among a set of subcarriers, in a way that every one of the latter has a small number of users compared to the total number of users. Hence, the inter-user interference is small, therefore, reliable signal detection is feasible because of the high signal to noise and interference ratio. Moreover, strong interference is circumvented since at each subcarrier different users interfere with each other, achieving interference diversity [10].

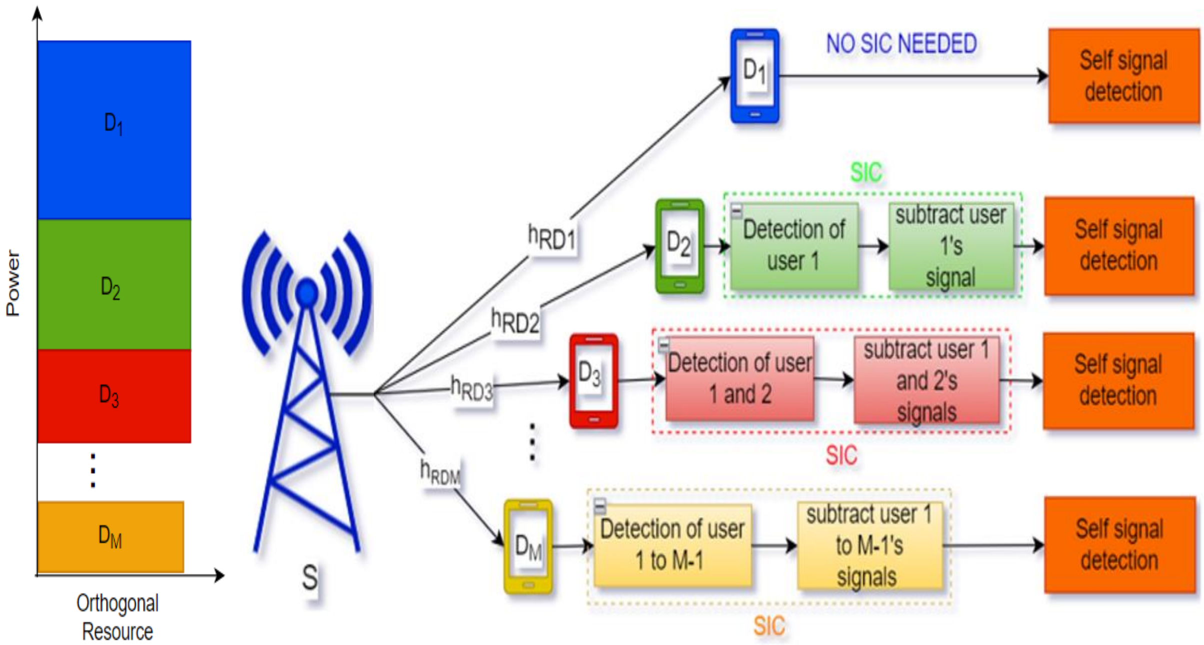

Fig.1.1 PD-NOMA network for M users + Power Domain

Power-domain NOMA users utilize the same orthogonal resources (time, frequency and code), with the multiplexing of the users realized in a non-orthogonal resource, such as the power. This type of multiplexing is performed by superposition coding (SC) and the demultiplexing by successive interference cancellation (SIC). By allocating the same resource block for many users we could solve the massive connectivity problem at the cost of a slightly more complex receiver. The receiver's additional function would be the successive interference cancelation [1]. Thus, the basic techniques used by NOMA are as follows:

- Superposition Coding - SC

At the transmitting end of the downlink system, the symbols are transmitted in different power levels according to the NOMA fairness and QoS policy; that is, assigning more power to the user with the worse channel gain to compensate for the larger channel loss (power control), if the minimum rate of the user is satisfied. The signals are then superimposed reciprocally to form a linear combination of all the users' information. A paradigm of SC is provided, showing 2 different power-constrained QPSK constellations, with the first constellation being superimposed onto the second, creating a 16-QAM.

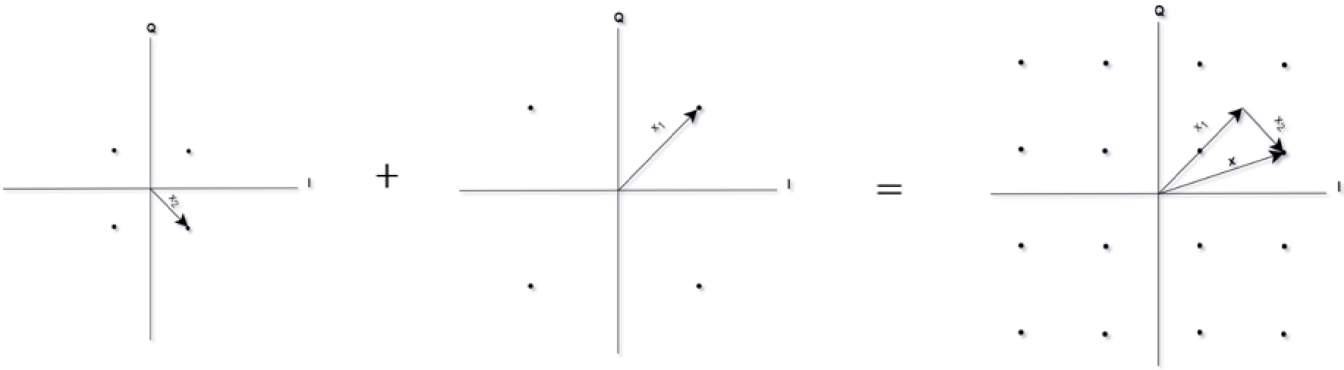

Fig 1.2 Superposition coding to transform 2 QPSK into a 16-QAM

- Successive Interference Cancelation - SIC

At the receiving ends of the downlink system, the superimposed signal is obtained by the users. The SIC mechanism will then take place and alleviate the detrimental effect of Interuser Interference. The SIC decoding order is purely contingent to the order of the perceived signal-to-noise ratio at the receivers, namely, the squared magnitude of the channel [11]. Therefore, at the ith user, the jth user's message, where $j < i$, will be detected, decoded and subtracted from the residual signal in a successive manner. The interference cancelation will be performed in descending order, starting from the users with the worst channel conditions, transmitted with the most power, treating the messages of the users with better channel conditions as noise. We should note that, even though the weak user doesn't perform SIC, the inter-user interference can still impact the efficiency of the receiver [12]. As a continuation of the SC paradigm, the weak user will decode symbol x (Fig 1.1), treating the interference of the x_2 symbol as noise, while strong user will decode symbol x_1, subtracting it from his observation and then decode x_2.

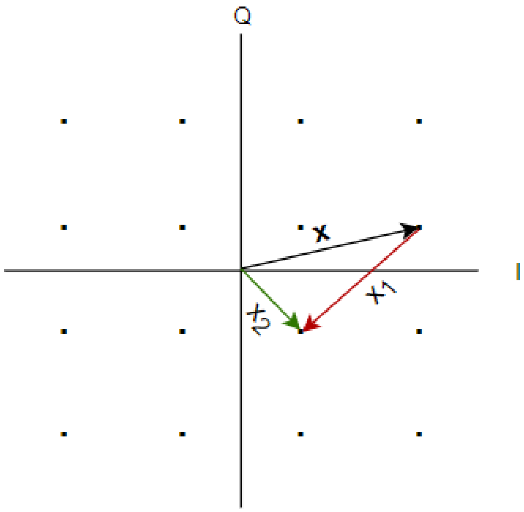

Fig 1.3 Successive Interference Cancelation by subtracting $x_1$from original observation x, in order to recover the residual information symbol, x_2.

In a N user PD-NOMA system, the ith user is detecting jth user's message, where $j < i$, with

$$SINR_{j \to i} = \frac{\gamma |h_i|^2 a_j}{\gamma |h_i|^2 \sum_{k=j+1}^{N} a_k + 1}. \tag{1.1}$$

For the ith user, SIC will iterate $i - 1$ times in order to get a signal free of weaker user interference that can be decoded with

$$SINR_i = \frac{\gamma |h_i|^2 a_i}{\gamma |h_i|^2 \sum_{k=i+1}^{N} a_k + 1} \tag{1.2}$$

treating the residual symbol interference of stronger users as noise.

The strongest user will decode and subtract all the $N-1$ weaker users' interference out of his observation. Consequently, the remaining signal can be decoded with

$$SNR_N = \gamma |h_N|^2 a_N. \tag{1.3}$$

The achievable data rate of the downlink PD-NOMA can be expressed using the $SINR$'s of the users as:

$$R_{SUM} = \sum_{i=1}^{N} \log_2(1 + SINR_i) = \sum_{i=1}^{N-1} \log_2\left(1 + \frac{\gamma |h_i|^2 a_i}{\gamma |h_i|^2 \sum_{k=i+1}^{N} a_k + 1}\right) + \log_2(1 + \gamma |h_N|^2 a_N). \tag{1.4}$$

A technique to further enhance the performance of a NOMA transmission, is cooperative NOMA. A cooperative transmission scheme for NOMA is realized by utilizing protocols such as decode and forward (DF) or amplifying and forward (AF). Relaying networks are using dedicated relays or users acting as relays or a combination, to achieve an increased capacity and broaden the coverage area while decreasing the deteriorating effects of multi-path fading. A NOMA cooperative scheme of a multiple dedicated relays and a user acting as a relay, is shown below in fig.1.4

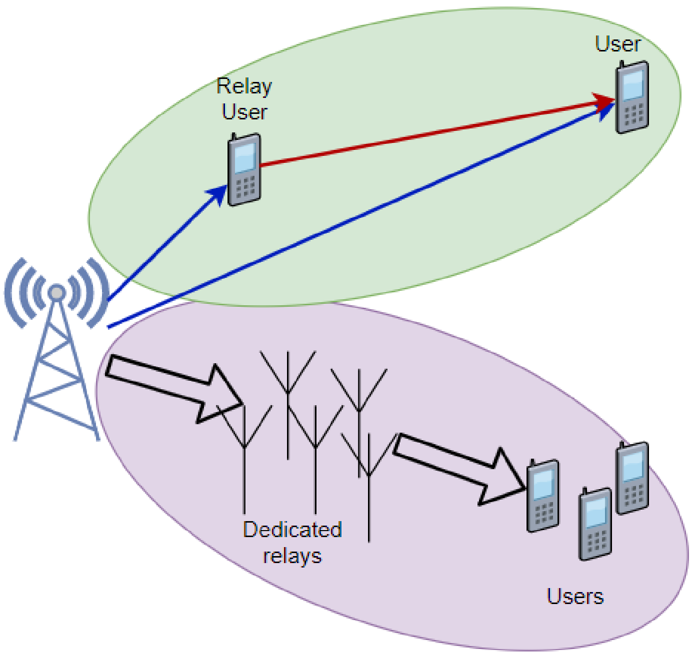

Fig.1.4 NOMA cooperative network

Conventional NOMA relying on SIC can be further enhanced by combining it with advanced MIMO techniques, improving spectral efficiency in order to get sufficient enough for the 5G requirements [13]. Users are superimposed to each other inside a space region created by base station transmitters using the aforementioned conventional NOMA technique. These regions are called beams, and this modulation is called intra-beam. More, interference exists between each beam (inter-beam interference), which is removed with spatial filtering, and intra-beam interference with SIC. The authors of [2] were suggesting that the design of beamforming vectors and spatial filtering vectors will have to be very precise, as they have decisive impact on interference cancellation (intra-beam and inter-beam).

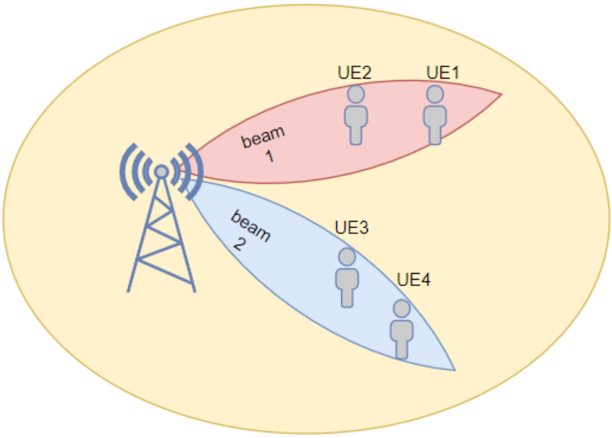

Fig.1.5 NOMA in MIMO

In [2], power domain NOMA was shown to be present in a physical-layer non-orthogonal multiplexing technology named layer-division-multiplexing (LDM), where in a 2-layered LDM scheme, bad channel enviroment (Upper Layer) mobile users were allocating higher transmission power, in addition to powerful channel coding and modulation, in order to make low-SNR detection realizable. The Lower Layer users, are providing a higher rate service, such as UHDTV and are assumed to be stationery and equipped with high antennas, compensating for the lower power allocation. These users' threshold SNR -requiring 20-30 dB- is proportional to the system performance compared to other multiplexing technologies (TDM/FDM).

NOMA can also be useful in a multi-cell scenario (NOMA in CoMP -Coordinated Multi Point-), if the negative effects of inter-cell interference is mitigated. In order to accomplish this task in the downlink, joint transmit-precoding of all users could be used. A complexity-reduced approach was proposed in [14], with authors assuming that 5G will be a cloud radio access network (C-RAN), meaning that central processing of various channel state information is bearable. A 2-cell, of 2 users-per-cell NOMA network is proposed, with the precoding method applied only to the cell-edge users, where the severe inter-cell interference is present. As a result, NOMA can be considered a candidate solution for Coordinated Systems as the cell edge user will experience an improvement in data rate.

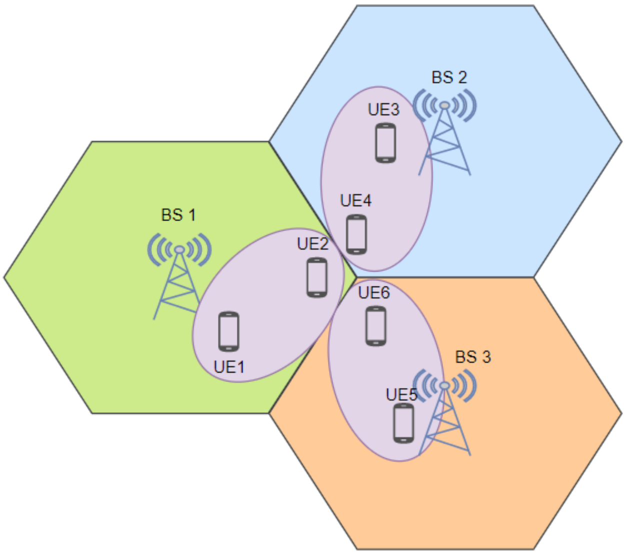

Fig.1.6 NOMA in a 3 cell CoMP scenario

Moreover, the authors of [15] are proposing a constellation domain NOMA scheme, where multiplexing is performed by rotating the constellation and transmitting only one dimension of every symbol. The constellation is rotated in such a way that every symbol has different coordinates, therefore, every symbol can be uniquely determined only from one coordinate at the receiving end. This method also utilizes Signal-Space Diversity (SSD), and thus, is a very promising NOMA technique.

2. SYSTEM MODEL

A downlink system consists of one base station S, two users D_1 and D_2, as well as an intermediate node between S and the users, working as an amplify and forward (AF) relay R. Each node has a single antenna and operates in a half-duplex mode. It is assumed that S cannot transmit directly to the user nodes in a single time slot since the distance separating them exceeds a threshold value or the direct links experience destructive interference leading to deep fading. The wireless channels are assumed to follow Rayleigh distribution in additive white Gaussian noise (AWGN). Channels between S to R and R to users D_1 and D_2 are denoted h_{SR} and h_{RD_1}, h_{RD_2} and distributed as $CN(0,\sigma^2)$ respectively. $|h_{SR}|$, $|h_{RD_1}|$, $|h_{RD_2}|$ denote the channel coefficients for the $S \to R, R \to D_1$ and $R \to D_2$ links, respectively, and the average power for the first and second slots are $\mathbb{E}\{|h_{SR}|^2\} = \Omega_1$ and $\mathbb{E}\left\{|h_{RD_1}|^2\right\} = \mathbb{E}\left\{|h_{RD_2}|^2\right\} = \Omega_2$, where $\mathbb{E}\{\cdot\}$ denotes expectation.

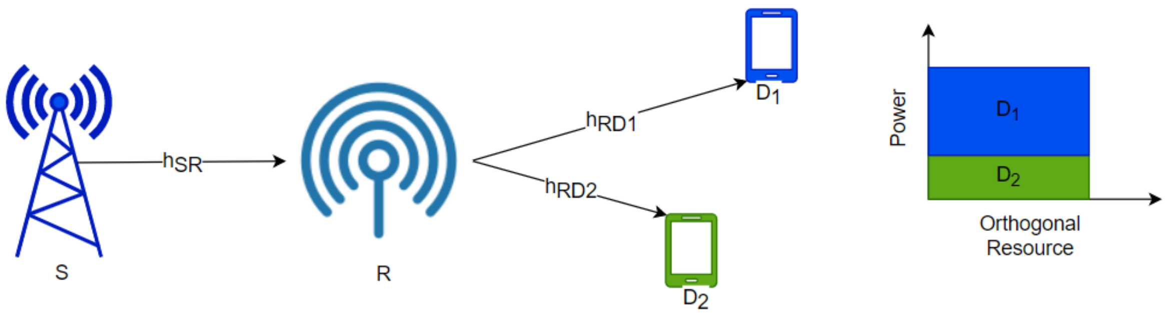

Fig.2.1 The proposed NOMA downlink system model

In PD NOMA the unit power superposition symbol $x_S = \sqrt{a_1 P_S} x_1 + \sqrt{a_2 P_S} x_2$ is transmitted from S to R, where a_1 and a_2 are the percentages of power, that information symbols x_1 and x_2 are allocating respectively and P_S is the transmission power of x_S. Without loss of generality we can assume that the second user has better channel characteristics than the first, $|h_{RD_1}|^2 \leq |h_{RD_2}|^2$. In other words, the channel gains are ordered. We also assume that $a_1 > a_2$ and $a_1 + a_2 = 1$.

In the first timeslot, the superimposed information symbol will suffer the fading and noise induced by the channel, which can be represented as $y_R = h_{SR} \cdot x_S + n_{SR}$, where n_{SR} denotes the zero mean AWGN of $S \to R$ having a σ^2 variance. In the second slot, the AF relay R will amplify the received signal by a factor of β and transmit it to the users. The received signals at user D_1 and D_2 can be expressed as

$$y_{RD_1} = \sqrt{P_R}\beta h_{RD_1} h_{SR}\left(\sqrt{a_1 P_s}x_1 + \sqrt{a_2 P_s}x_2\right) + \sqrt{P_R}\beta h_{RD_1} n_{SR} + n_{RD_1} \tag{2.1}$$

and

$$y_{RD_2} = \sqrt{P_R}\beta h_{RD_2} h_{SR}\left(\sqrt{a_1 P_s}x_1 + \sqrt{a_2 P_s}x_2\right) + \sqrt{P_R}\beta h_{RD_2} n_{SR} + n_{RD_2} \tag{2.2}$$

respectively, where $\beta = \frac{1}{\sqrt{P_S|h_{SR}|^2+\sigma^2}}$, $n_{RD} \sim CN(0,\sigma^2)$ the AWGN at $R \to D$ and P_R the transmission power at the relay node.

The two signals will be mutually interfered causing inter-user interference. In order for the users to attain the transmitted information symbols, they need to take advantage of the PD-NOMA scheme specifications on power allocation. SIC is decoding in the order specified by the users' channel magnitudes. Consequently, since D_2 is the strong user, SIC will be performed first there by decoding D_1's message and subtracting it from the received superposed signal. Then, D_2 will be able to decode its own message. On the contrary D_1 can immediately decode its own message treating D_2's interference as noise. For simplicity reasons we assume $P_R = P_S = P$ and $n_{SR} = n_{RD_1} = n_{RD_2} \sim CN(0,\sigma^2)$. In addition, the average signal-to-noise ratio will be mentioned as $\gamma \stackrel{\text{def}}{=} \frac{P}{\sigma^2}$. Signal-to-interference-and-noise ratio (SINR) is evaluated for the strong user (D_2) to decode the weak users' (D_1) message, $\gamma_{RD_{1\to 2}}$, and for both users to decode their own message γ_{RD_1} and γ_{RD_2}.

$$\gamma_{RD_{1\to 2}} = \frac{Power}{Interference + Noise} = \frac{P^2\beta^2\left|h_{RD_2}\right|^2|h_{SR}|^2 a_1}{P^2\beta^2\left|h_{RD_2}\right|^2|h_{SR}|^2 a_2 + P_R\beta^2\left|h_{RD_2}\right|^2\sigma^2 + \sigma^2}$$

$$\xRightarrow{dividing\ by\ \beta^2\sigma^4}$$

$$= \frac{\gamma^2\left|h_{RD_2}\right|^2|h_{SR}|^2 a_1}{\gamma^2\left|h_{RD_2}\right|^2|h_{SR}|^2 a_2 + \gamma\left|h_{RD_2}\right|^2 + (P_S|h_{SR}|^2+\sigma^2)/\sigma^2}$$

$$= \frac{\gamma^2\left|h_{RD_2}\right|^2|h_{SR}|^2 a_1}{\gamma^2\left|h_{RD_2}\right|^2|h_{SR}|^2 a_2 + \gamma\left|h_{RD_2}\right|^2 + \gamma|h_{SR}|^2 + 1}$$

$$\gamma_{RD_{1\to 2}} = \frac{\gamma^2 \left|h_{RD_2}\right|^2 |h_{SR}|^2 a_1}{\gamma^2 \left|h_{RD_2}\right|^2 |h_{SR}|^2 a_2 + \gamma\left(\left|h_{RD_2}\right|^2 + |h_{SR}|^2\right) + 1}.$$

(2.3*)*

The rate at which the second user is decoding the first users' message is $R_{1\to 2} = \frac{log(1+\gamma_{RD_{1\to 2}})}{2}$, noting that this rate must be greater or equal to $\tilde{R}_1 = \frac{log(1+\gamma_{th_1})}{2}$, where $\tilde{\mathrm{R}}_1$ and γ_{th_1} represents the minimum rate for user 2 to decode the first users' signal and the minimum threshold SINR that D_1's signal can be detected, respectively. When*,* D_1*'*s message is removed from D_2*'s* observation the second user can finally decode its own signal and obtain the message if $R_2 = \frac{\log(1+\gamma_{RD_2})}{2} \geq \tilde{R}_2$, where γ_{RD_2} is equal to

$$\gamma_{RD_2} = \frac{\gamma^2 \left|h_{RD_2}\right|^2 |h_{SR}|^2 a_2}{\gamma\left(\left|h_{RD_2}\right|^2 + |h_{SR}|^2\right) + 1}.$$

(2.4)

Accordingly, the SINR for the user D_1 to decode its own signal will be expressed as

$$\gamma_{RD_1} = \frac{\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2 a_1}{\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2 a_2 + \gamma\left(\left|h_{RD_1}\right|^2 + |h_{SR}|^2\right) + 1}.$$

(2.5)

The rate at which D_1 can decode its message is $R_1 = \frac{\log_2(1+\gamma_{RD_1})}{2}$ and for D_2 is $R_2 = \frac{\log_2(1+\gamma_{RD_2})}{2}$ constrained on $R_{1\to 2}, \mathrm{R}_1 \geq \tilde{\mathrm{R}}_1$ and $R_2 \geq \tilde{R}_2$, respectively. So, if the aforementioned inequalities hold, the sum rate will just be the sum of the target rates $\tilde{\mathrm{R}}_1 + \tilde{\mathrm{R}}_2 \frac{bps}{Hz}$.

3. OUTAGE PROBABILITY

In this section, the Outage Probability of the proposed NOMA AF relaying system is investigated over Rayleigh fading channels. The network's performance is analyzed in terms of outage probability, since this metric is considered ideal for measuring the systems capability of meeting the users' Quality-of-Service (QoS) requirements. Both closed-form and asymptotic expressions is derived, obtaining the systems' diversity order from the latter.

A. Exact Outage Behavior

Every user has a predetermined target data rate $\tilde{R}_1, \tilde{R}_2$ which is conditioned on the Quality-of-Service requirements of each. When the SINR is lower than the minimum threshold at which the signals can be decoded, the system will experience an outage event that can take one of two mathematical forms depending on the case. In the first case, user 2 cannot decode the first users' signal, so we obtain the first definition,

$$\Lambda_{1\to 2} = \{ \mathrm{R}_{1\to 2} < \tilde{\mathrm{R}}_1 \} = \{ \gamma_{\mathrm{RD}_{1\to 2}} < \gamma_{\mathrm{th}_1} \}. \tag{3.1}$$

The second case of an outage event will occur when a user cannot decode its own message, i.e.,

$$\Lambda_1 = \{ \gamma_{\mathrm{RD}_1} < \gamma_{\mathrm{th}_1} \} \tag{3.2}$$

and

$$\Lambda_2 = \{ \gamma_{\mathrm{RD}_2} < \gamma_{\mathrm{th}_2} \}. \tag{3.3}$$

We can also define the complementary events as $\Lambda_{1\to 2}^{\mathrm{c}}$, Λ_1^{c} and Λ_2^{c} of which the semantics indicates that the specified event of SIC procedure was successfully completed since the SINR values were greater than the target, γ_{th}.

The outage probability for the first and second user can be expressed as

$$P_{out}^1 = 1 - \Pr(\Lambda_1^c) = 1 - \Pr\big(\{ \gamma_{RD_1} > \gamma_{th_1} \} \big) \tag{3.4}$$

and

$$P_{out}^2 = 1 - \Pr(\ \Lambda_{1\to 2}^c \cap\ \Lambda_2^c) = 1 - \Pr(\{\ \gamma_{RD_{1\to 2}} > \gamma_{th_1}\ \} \cap \{\ \gamma_{RD_2} > \gamma_{th_2}\ \})$$

(3.5)

respectively.

We can procced by elaborating upon the outage events expressions deducting a corollary for the channel gains, $\left|h_{RD_2}\right|^2$ and $|h_{SR}|^2$ as

$$\Lambda_1^c = \left\{ \frac{\gamma^2\left|h_{RD_1}\right|^2 |h_{SR}|^2 a_1}{\gamma^2\left|h_{RD_1}\right|^2 |h_{SR}|^2 a_2 + \gamma(\left|h_{RD_1}\right|^2 + |h_{SR}|^2) + 1} > \gamma_{th_1} = 2^{2\tilde{R}_1} - 1 \right\}$$

$$= \left\{\gamma^2\left|h_{RD_1}\right|^2|h_{SR}|^2 a_1 >\ \gamma_{th_1}\gamma^2\left|h_{RD_1}\right|^2|h_{SR}|^2 a_2 + \gamma_{th_1}\gamma\left(\left|h_{RD_1}\right|^2 + |h_{SR}|^2\right) + \gamma_{th_1}\ \right\}$$

$$= \left\{\gamma^2\left|h_{RD_1}\right|^2|h_{SR}|^2(a_1 - \gamma_{th_1}a_2) - \gamma_{th_1}\gamma|h_{SR}|^2 >\ \gamma_{th_1}\gamma\left|h_{RD_1}\right|^2 + \gamma_{th_1}\ \right\}$$

$$= \left\{\left[\gamma^2\left|h_{RD_1}\right|^2(a_1 - \gamma_{th_1}a_2) - \gamma_{th_1}\gamma\right]|h_{SR}|^2 >\ \gamma_{th_1}\left(\gamma\left|h_{RD_1}\right|^2 + 1\right)\right\}$$

$$= \left\{\left[\gamma^2\left|h_{RD_1}\right|^2(a_1 - \gamma_{th_1}a_2) - \gamma_{th_1}\gamma\right] > \frac{\gamma_{th_1}\left(\gamma\left|h_{RD_1}\right|^2 + 1\right)}{|h_{SR}|^2}\right\}$$

$$= \left\{|h_{SR}|^2 > \frac{\gamma_{th_1}\left(\gamma\left|h_{RD_1}\right|^2 + 1\right)}{\gamma^2\left|h_{RD_1}\right|^2(a_1 - \gamma_{th_1}a_2) - \gamma_{th_1}\gamma}\right\}$$

(3.6)

The denominator $\gamma^2\left|h_{RD_1}\right|^2\left(a_1 - \gamma_{th_1}a_2\right) - \gamma_{th_1}\gamma$ must be greater than zero, so we can now determine that $\left|h_{RD_1}\right|^2 \geq \frac{\gamma_{th_1}}{\gamma\left(a_1 - \gamma_{th_1}a_2\right)}$

(3.7)

$$\xRightarrow{(3.6),(3.7)}\ \Lambda_1^c = \left\{ \begin{array}{c} \left|h_{RD_1}\right|^2 \geq \frac{\gamma_{th_1}}{\gamma(a_1 - \gamma_{th_1}a_2)} \triangleq \theta_1 \\ |h_{SR}|^2 > \frac{\gamma_{th_1}\left(\gamma\left|h_{RD_1}\right|^2 + 1\right)}{\gamma^2\left|h_{RD_1}\right|^2(a_1 - \gamma_{th_1}a_2) - \gamma_{th_1}\gamma} \end{array} \right\}$$

(3.8)

From **(3.7)** we can conclude that $a_1 > \gamma_{th_1}a_2$,

(3.9)

If the first user's signal was decoded successfully by the second user, we can assume that (5) is satisfied. If it isn't satisfied, no matter how high the SNR of the system is, the outage probability will always deteriorate to 1.

The outage probability of user 1 can be written as:

$$P_{out}^{1} = 1 - \Pr\left(\left\{\begin{array}{c} \left|h_{RD_1}\right|^2 \geq \dfrac{\gamma_{th_1}}{\gamma(a_1 - \gamma_{th_1} a_2)} \triangleq \theta_1 \\ |h_{SR}|^2 > \dfrac{\gamma_{th_1}\left(\gamma\left|h_{RD_1}\right|^2 + 1\right)}{\gamma^2\left|h_{RD_1}\right|^2(a_1 - \gamma_{th_1} a_2) - \gamma_{th_1}\gamma} \end{array}\right\}\right). \tag{3.10}$$

In the case of the second user, D_2 we have to elaborate on the event $\Lambda_{1\to 2}$ in order for the interference signal of user 1 to be removed, and on the event Λ_2. The complementary event of $\Lambda_{1\to 2}$ can be rewritten as:

$$\Lambda_{1\to 2}^{c} = \left\{\frac{\gamma^2\left|h_{RD_2}\right|^2|h_{SR}|^2 a_1}{\gamma^2\left|h_{RD_2}\right|^2|h_{SR}|^2 a_2 + \gamma(\left|h_{RD_2}\right|^2 + |h_{SR}|^2) + 1} > \gamma_{th_1}\right\}$$

$$= \left\{\begin{array}{c} \left|h_{RD_2}\right|^2 \geq \dfrac{\gamma_{th_1}}{\gamma(a_1 - \gamma_{th_1} a_2)} \triangleq \theta_1 \\ |h_{SR}|^2 > \dfrac{\gamma_{th_1}\left(\gamma\left|h_{RD_2}\right|^2 + 1\right)}{\gamma^2\left|h_{RD_2}\right|^2(a_1 - \gamma_{th_1} a_2) - \gamma_{th_1}\gamma} \end{array}\right\} = \left\{\begin{array}{c} \left|h_{RD_2}\right|^2 \geq \dfrac{\gamma_{th_1}}{\gamma(a_1 - \gamma_{th_1} a_2)} \triangleq \theta_1 \\ |h_{SR}|^2 > \dfrac{\theta_1\left(\gamma\left|h_{RD_2}\right|^2 + 1\right)}{\gamma\left(\left|h_{RD_2}\right|^2 - \theta_1\right)} \end{array}\right\} \tag{3.11}$$

And lastly complementary event of Λ_2 can be expressed as:

$$\Lambda_{2}^{c} = \left\{\frac{\gamma^2\left|h_{RD_2}\right|^2|h_{SR}|^2 a_2}{\gamma(\left|h_{RD_2}\right|^2 + |h_{SR}|^2) + 1} > \gamma_{th_2}\right\} = \left\{\begin{array}{c} \left|h_{RD_2}\right|^2 \geq \dfrac{\gamma_{th_2}}{\gamma a_2} \triangleq \theta_2 \\ |h_{SR}|^2 > \dfrac{\theta_2\left(\gamma\left|h_{RD_2}\right|^2 + 1\right)}{\gamma\left(\left|h_{RD_2}\right|^2 - \theta_2\right)} \end{array}\right\} \tag{3.12}$$

Moreover, the outage probability of user 2 is equivalent to

$$1 - \Pr\left(\left\{\begin{array}{c} \left|h_{RD_2}\right|^2 \geq \dfrac{\gamma_{th_1}}{\gamma(a_1 - \gamma_{th_1} a_2)} \triangleq \theta_1 \\ |h_{SR}|^2 > \dfrac{\theta_1\left(\gamma\left|h_{RD_2}\right|^2 + 1\right)}{\gamma\left(\left|h_{RD_2}\right|^2 - \theta_1\right)} \end{array}\right\} \cap \left\{\begin{array}{c} \left|h_{RD_2}\right|^2 \geq \dfrac{\gamma_{th_2}}{\gamma a_2} \triangleq \theta_2 \\ |h_{SR}|^2 > \dfrac{\theta_2\left(\gamma\left|h_{RD_2}\right|^2 + 1\right)}{\gamma\left(\left|h_{RD_2}\right|^2 - \theta_2\right)} \end{array}\right\}\right)$$

or

$$P_{out}^2 = 1 - \Pr\left(\begin{cases} \left|h_{RD_2}\right|^2 \geq \max\{\theta_1, \theta_2\} \triangleq \theta \\ |h_{SR}|^2 > \dfrac{\theta\left(\gamma\left|h_{RD_2}\right|^2 + 1\right)}{\gamma\left(\left|h_{RD_2}\right|^2 - \theta\right)} \end{cases}\right). \tag{3.13}$$

Since the channel magnitude between S and the Relay follows Rayleigh distribution, we can consider an exponential distribution for the channel gain $|h_{SR}|^2$ with probability density function (PDF):

$$f_{|h_{SR}|^2}(y) = \frac{e^{-\frac{y}{\Omega_1}}}{\Omega_1}. \tag{3.14}$$

For the unordered Random Variable $\left|\tilde{h}_{RD}\right|^2$, the PDF is given by

$$f_{\left|\tilde{h}_{RD}\right|^2}(y) = \frac{e^{-\frac{y}{\Omega_2}}}{\Omega_2}. \tag{3.15}$$

The analogous cumulative density functions (CDF) are given by

$$F_{|h_{SR}|^2}(y) = 1 - e^{-\frac{y}{\Omega_1}} \tag{3.16}$$

and

$$F_{\left|\tilde{h}_{RD}\right|^2}(y) = 1 - e^{-\frac{y}{\Omega_2}} \tag{3.17}$$

In order to derive a closed-form expression for the outage probability we can elaborate as follows

$$P_{out} = 1 - \Pr\left(\begin{cases} \left|h_{RD_m}\right|^2 \geq \theta \\ |h_{SR}|^2 > \dfrac{\theta\left(\gamma\left|h_{RD_m}\right|^2 + 1\right)}{\gamma\left(\left|h_{RD_m}\right|^2 - \theta\right)} \end{cases}\right) \tag{3.18.a}$$

$$= 1 - \int_{\theta}^{\infty} f_{\left|h_{RD_m}\right|^2}(y) e^{-\frac{\theta(\gamma y + 1)}{\gamma(y-\theta)\Omega_1}} dy \tag{3.18.b}$$

Proof: Appendix A

The probability density functions (PDF's) for the ordered channel gains $f_{|h_{RD_1}|^2}$ and $f_{|h_{RD_2}|^2}$ can be obtained by analyzing order statistics [16].

$$f_{|h_{RD_1}|^2}(y) = 2f_{|\tilde{h}_{RD_1}|^2}(y)\left[1 - F_{|\tilde{h}_{RD_1}|^2}(y)\right] = \frac{2}{\Omega_2}e^{-\frac{2y}{\Omega_2}} \tag{3.19}$$

$$f_{|h_{RD_2}|^2}(y) = 2f_{|\tilde{h}_{RD_2}|^2}(y)F_{|\tilde{h}_{RD_2}|^2}(y) = \frac{2}{\Omega_2}\left(e^{-\frac{y}{\Omega_2}} - e^{-\frac{2y}{\Omega_2}}\right) \tag{3.20}$$

The corresponding cumulative density functions (CDF's) can be easily extracted by **(3.19)** and **(3.20)** as follows

$$F_{|h_{RD_1}|^2}(y) = 2F_{|\tilde{h}_{RD_1}|^2}(y) - F^2{}_{|\tilde{h}_{RD_1}|^2}(y) = 1 - e^{-\frac{2y}{\Omega_2}} \tag{3.21}$$

$$F_{|h_{RD_2}|^2}(y) = F^2{}_{|\tilde{h}_{RD_2}|^2}(y) = 1 - 2e^{-\frac{y}{\Omega_2}} + e^{-\frac{2y}{\Omega_2}} \tag{3.22}$$

For the first user the outage probability is:

$$= 1 - \int_{\theta}^{\infty} f_{|h_{RD_1}|^2}(y)e^{-\frac{\theta(\gamma y+1)}{\gamma(y-\theta)\Omega_1}}dy \tag{3.23.a}$$

$$= 1 - \frac{2}{\Omega_2}\int_{\theta}^{\infty} e^{-\frac{2y}{\Omega_2}}e^{-\frac{\theta(\gamma y+1)}{\gamma(y-\theta)\Omega_1}}dy$$

Now by substituting $y - \theta = z$ we get

$$= 1 - \frac{2}{\Omega_2}e^{-\frac{2\theta}{\Omega_2}}\int_{0}^{\infty} e^{-\frac{2z}{\Omega_2}}e^{-\frac{\theta(\gamma z+\gamma\theta+1)}{\gamma z\Omega_1}}dz$$

And after some algebraic manipulations

$$= 1 - \frac{2}{\Omega_2}e^{-\frac{2\theta}{\Omega_2}}e^{-\frac{\theta}{\Omega_1}}\int_{0}^{\infty} e^{-\frac{2z}{\Omega_2}-\frac{\theta(\gamma\theta+1)}{\gamma z\Omega_1}}dz$$

Using [17], Eq. (3.471.9), we can obtain

$$P_{out}^{1}=1-\frac{4}{\Omega_2}e^{-\frac{2\theta}{\Omega_2}}e^{-\frac{\theta}{\Omega_1}}\sqrt{\frac{\theta(1+\gamma\theta)\Omega_2}{2\gamma\Omega_1}}K_1\left(2\sqrt{\frac{2\theta(1+\gamma\theta)}{\gamma\Omega_1\Omega_2}}\right)$$

(3.23.b)

where, $K_1(\cdot)$ is the first order modified Bessel function of second kind.

For the second user, the outage probability is:

$$P_{out}^{2}=1-\int_{\theta}^{\infty}f_{\left|h_{RD_2}\right|^2}(y)e^{-\frac{\theta(\gamma y+1)}{\gamma(y-\theta)\Omega_1}}dy$$

(3.24.a)

$$=1-\frac{2}{\Omega_2}\int_{\theta}^{\infty}\left(e^{-\frac{y}{\Omega_2}}-e^{-\frac{2y}{\Omega_2}}\right)e^{-\frac{\theta(\gamma y+1)}{\gamma(y-\theta)\Omega_1}}dy$$

$$=1+\frac{2}{\Omega_2}\int_{\theta}^{\infty}e^{-\frac{2y}{\Omega_2}}e^{-\frac{\theta(\gamma y+1)}{\gamma(y-\theta)\Omega_1}}dy-\frac{2}{\Omega_2}\underbrace{\int_{\theta}^{\infty}e^{-\frac{y}{\Omega_2}}e^{-\frac{\theta(\gamma y+1)}{\gamma(y-\theta)\Omega_1}}dy}_{\Phi}$$

$$\Phi=\int_{\theta}^{\infty}e^{-\frac{y}{\Omega_2}}e^{-\frac{\theta(\gamma y+1)}{\gamma(y-\theta)\Omega_1}}dy$$

Now by substituting $y-\theta=z$ we get

$$=e^{-\frac{\theta}{\Omega_2}}\int_{0}^{\infty}e^{-\frac{z}{\Omega_2}}e^{-\frac{\theta(\gamma z+\gamma\theta+1)}{\gamma z\Omega_1}}dz$$

And after some algebraic manipulations

$$=e^{-\frac{\theta}{\Omega_2}}e^{-\frac{\theta}{\Omega_1}}\int_{0}^{\infty}e^{-\frac{z}{\Omega_2}}e^{-\frac{\theta(1+\gamma\theta)}{\gamma z\Omega_1}}dz$$

Using [17], *Eq. (3.471.9),* we can obtain

$$=2e^{-\frac{\theta}{\Omega_2}}e^{-\frac{\theta}{\Omega_1}}\sqrt{\frac{\theta(1+\gamma\theta)\Omega_2}{\gamma\Omega_1}}K_1\left(2\sqrt{\frac{\theta(1+\gamma\theta)}{\gamma\Omega_1\Omega_2}}\right)$$

So, we can conclude that

$$P_{out}^{2}=1+\frac{4}{\Omega_2}e^{\frac{-\theta(\Omega_1+\Omega_2)}{\Omega_1\Omega_2}}\sqrt{\frac{\theta(1+\gamma\theta)\Omega_2}{\gamma\Omega_1}}\left\{\frac{1}{\sqrt{2}}e^{-\frac{\theta}{\Omega_2}}K_1\left(2\sqrt{\frac{2\theta(1+\gamma\theta)}{\gamma\Omega_1\Omega_2}}\right)-K_1\left(2\sqrt{\frac{\theta(1+\gamma\theta)}{\gamma\Omega_1\Omega_2}}\right)\right\}.$$

(3.24.b)

Note that NOMA outperforms OMA when the metric is the outage performance, presuming that users' rates are above the target rates, decided based upon the users' QoS needs, and power allocation coefficients are wisely chosen in order for the users to be able to detect and decode the interferences.

B. Asymptotic Outage Behavior

Even though the closed-form expression for the outage probability describes exactly how the system behaves, it's difficult to extract any valuable insights, in view of the fact that the expression is very complex. So, it seems essential to investigate the systems' outage probability as an approximation in a high SNR environment in order to derive the asymptotic form for the outage probability.

The outage probability expression,

$$P_{out} = 1 - \Pr\left(|h_{SR}|^2 > \frac{\theta\left(\gamma |h_{RD}|^2 + 1\right)}{\gamma\left(|h_{RD}|^2 - \theta\right)} \right)$$

is reformulated as:

$$P_{out} = 1 - \Pr\left(\frac{|h_{SR}|^2 |h_{RD}|^2}{\left(|h_{SR}|^2 + |h_{RD}|^2\right) + \frac{1}{\gamma}} > \theta \right).$$

Under high SNR conditions, $\gamma \to \infty$, the $\frac{1}{\gamma}$ term is considered negligible.

$$P_{out} \cong 1 - \Pr\left(\frac{|h_{SR}|^2 |h_{RD}|^2}{\left(|h_{SR}|^2 + |h_{RD}|^2\right)} > \theta \right) \tag{3.25}$$

Simple lower and upper bounds can be now attained after employing the inequality $\left(\frac{1}{2}\right) \min\{a, b\} \leq \frac{a \cdot b}{a+b} \leq \min\{a, b\}$ [18].

$$\begin{cases} P_{out,LB}^{1} \\ P_{out,UB}^{1} \end{cases} = \begin{cases} 1 - \Pr\left(min\left(|h_{SR}|^2, \left|h_{RD_1}\right|^2\right) > \theta_1 \right) \\ 1 - \Pr\left(min\left(|h_{SR}|^2, \left|h_{RD_1}\right|^2\right) > 2\theta_1 \right) \end{cases} \tag{3.26}$$

$$= \begin{cases} F_{\left|h_{RD_1}\right|^2}(\theta_1) + F_{|h_{SR}|^2}(\theta_1) - F_{\left|h_{RD_1}\right|^2}(\theta_1) F_{|h_{SR}|^2}(\theta_1) \\ F_{\left|h_{RD_1}\right|^2}(2\theta_1) + F_{|h_{SR}|^2}(2\theta_1) - F_{\left|h_{RD_1}\right|^2}(2\theta_1) F_{|h_{SR}|^2}(2\theta_1) \end{cases}$$

$$= \begin{cases} 1 + e^{-\frac{\theta_1 (2\Omega_1 + \Omega_2)}{\Omega_1 \Omega_2}} \\ 1 + e^{-\frac{2\theta_1 (2\Omega_1 + \Omega_2)}{\Omega_1 \Omega_2}} \end{cases} \tag{3.27}$$

Similarly, for the second user:

$$\begin{cases} P_{out,LB}^2 \\ P_{out,UB}^2 \end{cases} = \begin{cases} 1 - \Pr\left(\ min\left(|h_{SR}|^2, |h_{RD_2}|^2\right) > \ \max\{\theta_1, \theta_2\}\right) \\ 1 - \Pr\left(\ min\left(|h_{SR}|^2, |h_{RD_2}|^2\right) > \ 2\max\{\theta_1, \theta_2\}\right) \end{cases}$$

$$= \begin{cases} F_{|h_{RD_2}|^2}(\max\{\theta_1, \theta_2\}) + F_{|h_{SR}|^2}(\max\{\theta_1, \theta_2\}) - F_{|h_{RD_2}|^2}(\max\{\theta_1, \theta_2\})F_{|h_{SR}|^2}(\max\{\theta_1, \theta_2\}) \\ F_{|h_{RD_2}|^2}(2\max\{\theta_1, \theta_2\}) + F_{|h_{SR}|^2}(2\max\{\theta_1, \theta_2\}) - F_{|h_{RD_2}|^2}(2\max\{\theta_1, \theta_2\})F_{|h_{SR}|^2}(2\max\{\theta_1, \theta_2\}) \end{cases}$$

$$= \begin{cases} 1 - 2e^{-\frac{\max\{\theta_1,\theta_2\}(\Omega_1+\Omega_2)}{\Omega_1\Omega_2}} + e^{-\frac{\max\{\theta_1,\theta_2\}(2\Omega_1+\Omega_2)}{\Omega_1\Omega_2}} \\ 1 - 2e^{-\frac{2\max\{\theta_1,\theta_2\}(\Omega_1+\Omega_2)}{\Omega_1\Omega_2}} + e^{-\frac{2\max\{\theta_1,\theta_2\}(2\Omega_1+\Omega_2)}{\Omega_1\Omega_2}} \end{cases}$$

(3.28)

The asymptotic expressions for $F_{|h_{SR}|^2}(\theta_1)$, $F_{|h_{RD_1}|^2}(\theta_1)$ and $F_{|h_{RD_2}|^2}(\theta_1)$ can be obtained by making use of the 2 first terms of the Taylor expansion and the fact that when $\gamma \to \infty$, $\theta \to 0$.

$$F_{|h_{SR}|^2}(\theta_1) \cong \frac{\theta_1}{\Omega_1}$$

(3.29)

$$F_{|h_{RD_1}|^2}(\theta_1) \cong 2\frac{\theta_1}{\Omega_2}$$

(3.30)

$$F_{|h_{RD_2}|^2}(\theta_2) \cong \left(\frac{\max\{\theta_1, \theta_2\}}{\Omega_2}\right)^2$$

(3.31)

These three approximations will let us decrease the computational complexity and perceive the system behavior.

$$\begin{cases} P_{out,LB}^1 \cong 2\frac{\theta_1}{\Omega_2} + \frac{\theta_1}{\Omega_1} \\ P_{out,UB}^1 \cong 4\frac{\theta_1}{\Omega_2} + 2\frac{\theta_1}{\Omega_1} \end{cases}$$

(3.32)

$$\begin{cases} P_{out,LB}^2 \cong \frac{\max\{\theta_1, \theta_2\}}{\Omega_1} + \left(\frac{\max\{\theta_1, \theta_2\}}{\Omega_2}\right)^2 \\ P_{out,UB}^2 \cong 2\frac{\max\{\theta_1, \theta_2\}}{\Omega_1} + 4\left(\frac{\max\{\theta_1, \theta_2\}}{\Omega_2}\right)^2 \end{cases}$$

(3.33)

As a result, the diversity order achieved by Rayleigh fading NOMA is $P_{out} \rightarrow \frac{1}{\gamma}$, meaning $G_d = 1$, which is the same diversity order as the conventional Orthogonal multiple access scheme.

4. ERGODIC SUM RATE

In this section, we suppose the targeted rates $\tilde{R}_1$ and $\tilde{R}_2$ are opportunistically allocated based upon the users' channel conditions, i.e. $\tilde{R}_1 = R_1$ and $\tilde{R}_2 = R_2$. As a consequence, we can conclude that $R_{1\to 2} \geq \tilde{R}_2 = R_2$ always holds since $|h_{RD_1}|^2 \leq |h_{RD_2}|^2$, meaning that the first users' signal can always be detected, decoded and removed from the second users' observation. For this reason, the sum rate should be $R_{sum} = R_1 + R_2$, therefore, the ergodic sum rate can be expressed by

$$R_{sum}^{ave} = E\{R_{sum}\}$$

$$= \overbrace{E\left\{\frac{1}{2}\log_2\left(1 + \frac{\gamma^2 |h_{RD_1}|^2 |h_{SR}|^2 a_1}{\gamma^2 |h_{RD_1}|^2 |h_{SR}|^2 a_2 + \gamma(|h_{RD_1}|^2 + |h_{SR}|^2) + 1}\right)\right\}}^{user\ D_1}$$

$$+ \underbrace{E\left\{\frac{1}{2}\log_2\left(1 + \frac{\gamma^2 |h_{RD_2}|^2 |h_{SR}|^2 a_2}{\gamma(|h_{RD_2}|^2 + |h_{SR}|^2) + 1}\right)\right\}}_{user\ D_2} \tag{4.1}$$

In the high-SNR regime, R_1 can be approximated by the expression $\frac{1}{2}\log_2\left(1 + \frac{a_1}{a_2}\right)$, therefore

$$R_1^{ave} = \frac{1}{2}\log_2\left(1 + \frac{a_1}{a_2}\right) \tag{4.2}$$

is a constant rate dependent on the ratio of the power allocation coefficients $\frac{a_1}{a_2}$.

Additionally, it's simple to conclude that the ergodic rate of the second user must be equal to:

$$R_2^{ave} = \frac{1}{2}\int_0^\infty \int_0^\infty \log_2\left(1 + a_2\gamma \frac{xy}{x+y}\right) f_{|h_{RD_2}|^2}(y) f_{|h_{SR}|^2}(x)\, dydx.$$

$$= \frac{1}{\Omega_1 \Omega_2}\int_0^\infty \int_0^\infty \log_2\left(1 + a_2\gamma \frac{xy}{x+y}\right)\left(e^{-\frac{y}{\Omega_2} - \frac{x}{\Omega_1}} - e^{-\frac{2y}{\Omega_2} - \frac{x}{\Omega_1}}\right) dydx \tag{4.3a}$$

$$R_1^{ave} = \frac{1}{2}\int_0^\infty \int_0^\infty \log_2\left(1 + a_2\gamma \frac{xy}{x+y}\right) f_{|h_{RD_1}|^2}(y) f_{|h_{SR}|^2}(x)\, dydx.$$

$$= \frac{1}{\Omega_1 \Omega_2}\int_0^\infty \int_0^\infty \log_2\left(1 + \frac{a_1\gamma xy}{\gamma^2 xy a_2 + \gamma(x+y) + 1}\right) e^{-\frac{2y}{\Omega_2} - \frac{x}{\Omega_1}} dydx \tag{4.3b}$$

Based on the inequality $\left(\frac{1}{2}\right)\min\{a,b\} \leq \frac{a\cdot b}{a+b} \leq \min\{a,b\}$ [18] we can bound R_2^{ave} as follows

$$R_{2,LB}^{ave} = E\left\{\frac{1}{2ln2} ln\left(1 + \frac{a_2\gamma}{2}W\right)\right\} \leq \ R_2^{ave} \leq E\left\{\frac{1}{2ln2} ln(1 + a_2\gamma W)\right\} = R_{2,UB}^{ave} \tag{4.4}$$

Were, $W \triangleq min\left(|h_{SR}|^2, \left|h_{RD_2}\right|^2\right)$ and the expression can be further analyzed as

$$\frac{1}{2ln2}\int_0^\infty ln\left(1 + \frac{a_2\gamma}{2}w\right) f_W(w)dw \leq \ R_2^{ave} \leq \frac{1}{2ln2}\int_0^\infty ln(1 + a_2\gamma w)\, f_W(w)dw \tag{4.5}$$

We can observe that $g(0) = 0$ for $g(w) = ln\left(1 + \frac{a_2\gamma}{2}w\right)$, so we can transform the inequality into:

$$\frac{a_2\gamma}{4ln2}\int_0^\infty\int_0^w \frac{1}{1 + \frac{a_2\gamma}{2}x}\, dx f_W(w)dw \leq \ R_2^{ave} \leq \frac{a_2\gamma}{2ln2}\int_0^\infty\int_0^w \frac{1}{1 + a_2\gamma x}\, dx f_W(w)dw \tag{4.6}$$

After changing the order of the integrals, we obtain the following expression

$$\frac{a_2\gamma}{4ln2}\int_0^\infty \frac{1}{1 + \frac{a_2\gamma}{2}x}\int_x^\infty f_W(w)dw\, dx \leq \ R_2^{ave} \leq \frac{a_2\gamma}{2ln2}\int_0^\infty \frac{1}{1 + a_2\gamma x}\int_x^\infty f_W(w)dw\, dx \tag{4.7}$$

And after some algebraic manipulations the result will be

$$\frac{a_2\gamma}{4ln2}\int_0^\infty \frac{1 - F_W(x)}{1 + \frac{a_2\gamma}{2}x}\, dx \leq \ R_2^{ave} \leq \frac{a_2\gamma}{2ln2}\int_0^\infty \frac{1 - F_W(x)}{1 + a_2\gamma x}\, dx \tag{4.8}$$

To procced forward, we need to evaluate the CDF $F_W(W)$, which can be calculated as

$$\begin{aligned} F_W(W) = \Pr(W \leq w) &= \Pr\left(min\left(|h_{SR}|^2, \left|h_{RD_2}\right|^2\right) \leq w\right) \\ = 1 - \Pr\left(min\left(|h_{SR}|^2, \left|h_{RD_2}\right|^2\right) \geq w\right) &= 1 - \Pr(|h_{SR}|^2 \geq w) \Pr\left(\left|h_{RD_2}\right|^2 \geq w\right) \\ &= 1 - [1 - \Pr(|h_{SR}|^2 \leq w)]\left[1 - \Pr\left(\left|h_{RD_2}\right|^2 \leq w\right)\right] \\ &= 1 - 2e^{-\frac{w(\Omega_1+\Omega_2)}{\Omega_1\Omega_2}} + e^{-\frac{w(2\Omega_1+\Omega_2)}{\Omega_1\Omega_2}} \end{aligned} \tag{4.9}$$

Substituting back to **(4.8)**, we attain:

$$R_{2,UB}^{ave} = \frac{a_2\gamma}{ln2}\int_0^\infty \frac{e^{-\frac{x(\Omega_1+\Omega_2)}{\Omega_1\Omega_2}}}{1 + a_2\gamma x}\, dx - \frac{a_2\gamma}{2ln2}\int_0^\infty \frac{e^{-\frac{x(2\Omega_1+\Omega_2)}{\Omega_1\Omega_2}}}{1 + a_2\gamma x}\, dx$$

$$= \frac{E_1(\lambda_1 + \lambda_2)e^{(\lambda_1+\lambda_2)}}{ln2} - \frac{E_1(2\lambda_1 + \lambda_2)e^{(2\lambda_1+\lambda_2)}}{2ln2}$$

$$= \frac{2E_1(\lambda_1 + \lambda_2)e^{(\lambda_1+\lambda_2)} - E_1(2\lambda_1 + \lambda_2)e^{(2\lambda_1+\lambda_2)}}{2ln2}$$

(4.10)

Where $E_1(\cdot)$ denotes the exponential integral and $\lambda_1 \stackrel{\text{def}}{=} \frac{1}{\Omega_2 a_2 \gamma}$ and $\lambda_2 \stackrel{\text{def}}{=} \frac{1}{\Omega_1 a_2 \gamma}$.

$$R_{2,LB}^{ave} = \frac{a_2\gamma}{2ln2}\int_0^\infty \frac{e^{-\frac{x(\Omega_1+\Omega_2)}{\Omega_1\Omega_2}}}{1+\frac{a_2\gamma x}{2}}\,dx - \frac{a_2\gamma}{4ln2}\int_0^\infty \frac{e^{-\frac{x(2\Omega_1+\Omega_2)}{\Omega_1\Omega_2}}}{1+\frac{a_2\gamma x}{2}}\,dx$$

$$= \frac{E_1(2(\lambda_1 + \lambda_2))e^{(2(\lambda_1+\lambda_2))}}{ln2} - \frac{E_1(4\lambda_1 + 2\lambda_2)e^{(4\lambda_1+2\lambda_2)}}{2ln2}$$

$$= \frac{2E_1(2(\lambda_1 + \lambda_2))e^{(2(\lambda_1+\lambda_2))} - E_1(4\lambda_1 + 2\lambda_2)e^{(4\lambda_1+2\lambda_2)}}{2ln2}$$

(4.11)

Resultantly, we can derive the high-SNR approximation of the ergodic sum rate bounds, combining R_1^{ave} and the inequality of R_2^{ave}.

$$R_{sum,LB}^{ave} = \frac{\log_2\left(1+\frac{a_1}{a_2}\right)}{2} + \frac{2E_1(2(\lambda_1 + \lambda_2))e^{(2(\lambda_1+\lambda_2))} - E_1(4\lambda_1 + 2\lambda_2)e^{(4\lambda_1+2\lambda_2)}}{4ln2}$$

(4.12)

$$R_{sum,UB}^{ave} = \frac{\log_2\left(1+\frac{a_1}{a_2}\right)}{2} + \frac{2E_1(\lambda_1 + \lambda_2)e^{(\lambda_1+\lambda_2)} - E_1(2\lambda_1 + \lambda_2)e^{(2\lambda_1+\lambda_2)}}{2ln2}$$

(4.13)

5. POWER ALLOCATION OPTIMIZATION

In this chapter, an optimization analysis of the power allocation coefficients selection will be introduced in order to maximize the Sum Rate of the system. By proving that the objective function is concave (Appendix B), and since the restrictions are linear, thus, can be considered either convex or concave, the problem can be classified as convex. Consequently, in order to define the optimal $a_1$and a_2 Karush-Kuhn-Tucker conditions can be used. The problem can be formulated as follows

$$\max_{\alpha} \quad R_{sum} \tag{5.1}$$

s.t.

$$a_1 + a_2 \leq 1$$

$$\gamma \left| h_{RD_m} \right|^2 (a_1 - \gamma_{th_1} a_2) > \frac{\gamma_{th_1} \left(\gamma \left| h_{RD_m} \right|^2 + 1 \right)}{\gamma |h_{SR}|^2} + \gamma_{th_1}, \quad m = 1{,}2$$

$$a_1, a_2 \geq 0 \tag{5.2}$$

The constraints can be represented as

$$C_1 = \frac{\gamma_{th_1} \left(\gamma \left| h_{RD_m} \right|^2 + 1 \right)}{\gamma |h_{SR}|^2} - \gamma \left| h_{RD_m} \right|^2 (a_1 - \gamma_{th_1} a_2) + \gamma_{th_1} \tag{5.11}$$

$$C_2 = a_1 + a_2 - 1 \tag{5.12}$$

Thus, the Lagrange function of the problem is

$$L(a_1, a_2, \mu_1, \mu_2) = R_{sum} - \mu_1 C_1 - \mu_2 C_2 \tag{5.13}$$

Optimality conditions (first derivatives tests)

$$\frac{\partial R_{sum}}{\partial a_1} - \mu_1 \frac{\partial C_1}{\partial a_1} - \mu_2 \frac{\partial C_2}{\partial a_1} = 0 \tag{5.14}$$

$$\frac{\partial R_{sum}}{\partial a_2} - \mu_1 \frac{\partial C_1}{\partial a_2} - \mu_2 \frac{\partial C_2}{\partial a_2} = 0$$

(5.15)

Slackness conditions are formulated as

$$\mu_1 \left\{ \frac{\gamma_{th_1}\left(\gamma \left|h_{RD_m}\right|^2 + 1\right)}{\gamma |h_{SR}|^2} - \gamma \left|h_{RD_m}\right|^2 (a_1 - \gamma_{th_1} a_2) + \gamma_{th_1} \right\} = 0$$

(5.16)

$$\mu_2 \{a_1 + a_2 - 1\} = 0$$

(5.17)

In order to prove that the Lagrange multipliers are non-negative we can elaborate on (5.14) and (5.15) as follows

$$\mu_2 = \frac{\partial R_{sum}}{\partial a_1} + \mu_1 \gamma \left|h_{RD_m}\right|^2$$

(5.18)

$$\frac{\partial R_{sum}}{\partial a_2} - \mu_1 \gamma \left|h_{RD_m}\right|^2 \gamma_{th_1} - \mu_2 = 0$$

(5.19)

By substituting (5.18) in (5.19) we can derive

$$\frac{\partial R_{sum}}{\partial a_2} - \frac{\partial R_{sum}}{\partial a_1} = \mu_1 \gamma \left|h_{RD_m}\right|^2 (\gamma_{th_1} + 1)$$

(5.20)

$\frac{\partial R_{sum}}{\partial a_2} - \frac{\partial R_{sum}}{\partial a_1}$ can be proven positive and as a result, μ_1 is positive since $\gamma \left|h_{RD_m}\right|^2 (\gamma_{th_1} + 1)$ is positive. μ_2 is also positive since $\frac{\partial R_{sum}}{\partial a_1} + \mu_1 \gamma \left|h_{RD_m}\right|^2$ is positive and thus, the Lagrange multipliers are both positive. We can calculate the closed form optimal expressions for the power allocation coefficients from the slackness conditions

$$\boxed{{a_1}^{opt} = \frac{\gamma_{th_1}}{\gamma_{th_1} + 1} \cdot \frac{\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2 + \gamma\left(\left|h_{RD_1}\right|^2 + |h_{SR}|^2\right) + 1}{\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2}}$$

(5.21)

$$\boxed{{a_2}^{opt} = \frac{\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2 - \gamma_{th_1}\left(\gamma\left(\left|h_{RD_1}\right|^2 + |h_{SR}|^2\right) + 1\right)}{\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2 \left(\gamma_{th_1} + 1\right)}}$$

(5.22)

6. NUMERICAL RESULTS

In this chapter, the previous performance analysis results are presented in terms of the system's transmit SNR and the distance between S and R, denoted as d_{SR}. The normalized distance between the base station and the mobile users is set to 1, hence the distance from the relay to each user is $1 - d_{SR}$. Thus, we can conclude that $\Omega_1 = d_{SR}{}^{-\alpha}$ and $\Omega_1 = (1 - d_{SR})^{-\alpha}$,where α symbolizes the path loss exponent and is fixed to 3. We also set $\alpha_1 = 1 - a_2$ and $a_1 > a_2$ abiding to the NOMA principle for power allocation. Outage Probability is investigated and in order to validate the derived closed-form expressions for the networks ergodic sum rate, numerical integration is performed. Monte Carlo simulations are also conducted in order to verify the closed form expressions for the outage probability and ergodic sum rates.

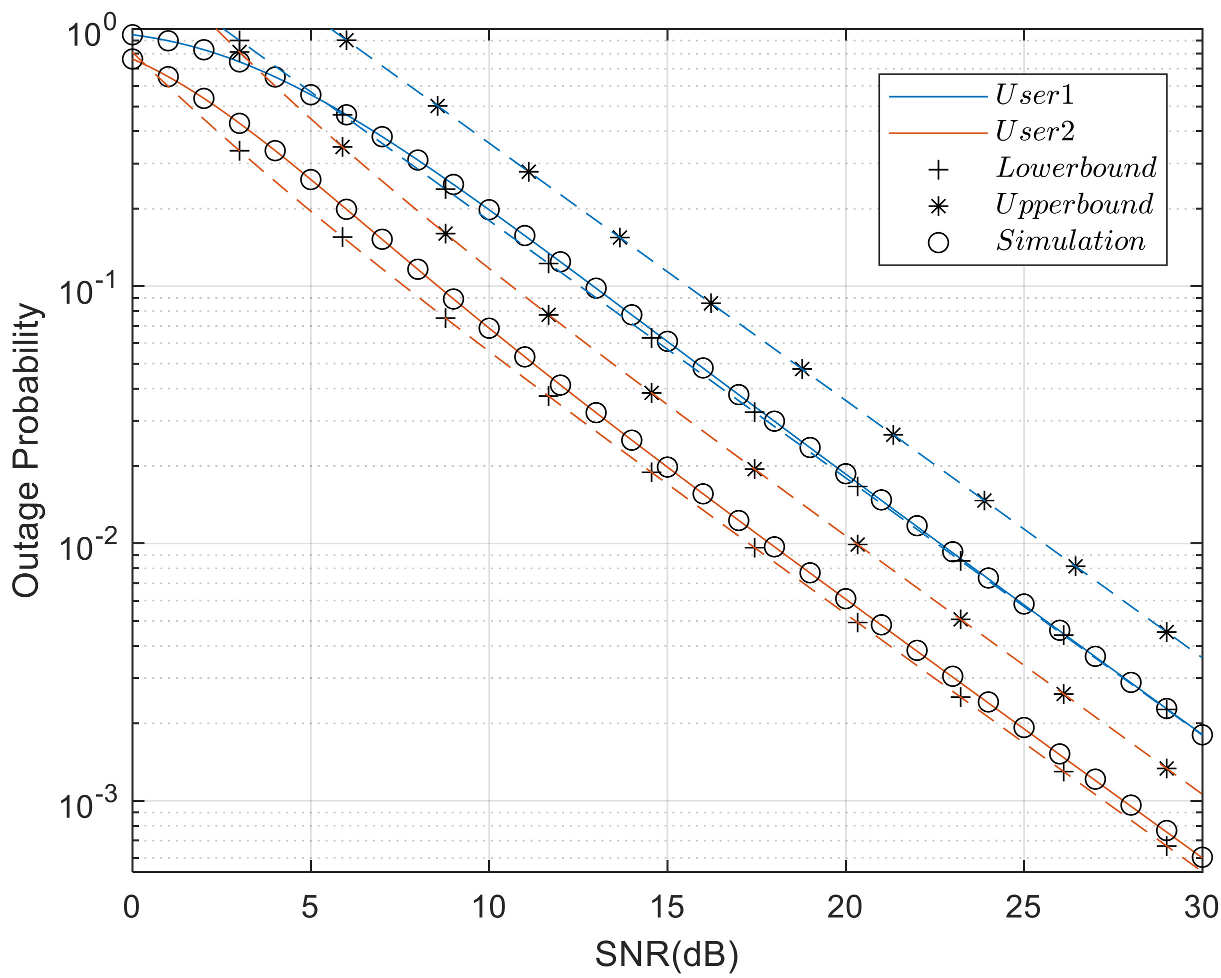

Fig 6.1. Outage probability vs. transmit SNR with $a_1 = \frac{2}{3}, a_2 = \frac{1}{3}$ $,\gamma_{th1} = 0.9(dB),\ \ \gamma_{th2} 1.5(dB)\ \ and\ d_{SR} = 0.5$

In Fig 6.1, the outage probability for each user is plotted versus the transmit SNR of the system in dB, as γ increases, the tighter the lower bound becomes to each user. In addition, we can observe that there is a significant performance gap between upper bound and exact outage probability, demonstrating how poor the performance can be. Monte Carlo simulations are matching with the exact analytical results. Moreover, the Rayleigh channels are achieving a diversity order of 1, which can be interpreted as the worst-case scenario.

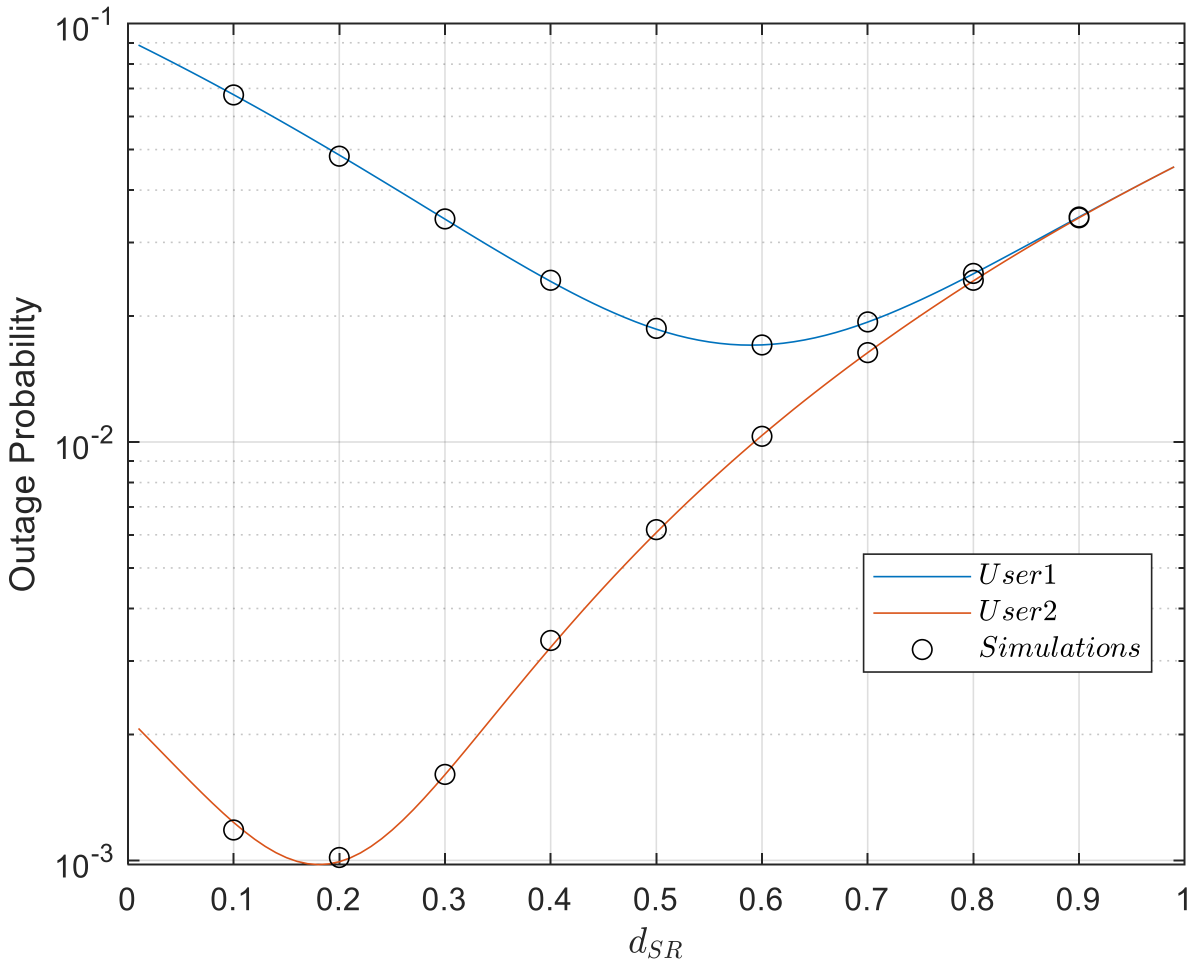

Fig 6.2. Outage probability vs. distance $S \rightarrow R$ link with $a_1 = \frac{2}{3}, a_2 = \frac{1}{3}$, $\gamma_{th1} = 0.9(dB)$, $\gamma_{th2} = 1.5(dB)$ $and\ \gamma = 20(dB)$

The Outage Probability for transmit $SNR = 20(dB)$ is illustrated in Fig 6.2 with respect to the normalized distance from the base station to the dedicated relay. The weak user (D_1), has the optimal relay location closer to the user side, whereas, the strong user (D_2), has its optimal relay location almost at the same place as the base station. The most power is allocated by the user with the worst channel conditions (D_1) and since SIC is not required after the signal reception, the location of the relay should be closer to the user side. Accordingly, since the strong user (D_2) has better channel conditions, from R to D_2, shows that D_2 should use this channel more by restricting the channel between S and R, in order to achieve better SNR at the relay. Consequently, the distance d_{SR} should be small enough to help the strong user to achieve the lowest possible outage probability. Additionally, even if OMA can outperform NOMA in terms of OP, the latter would achieve better spectral efficiency and provide user fairness.

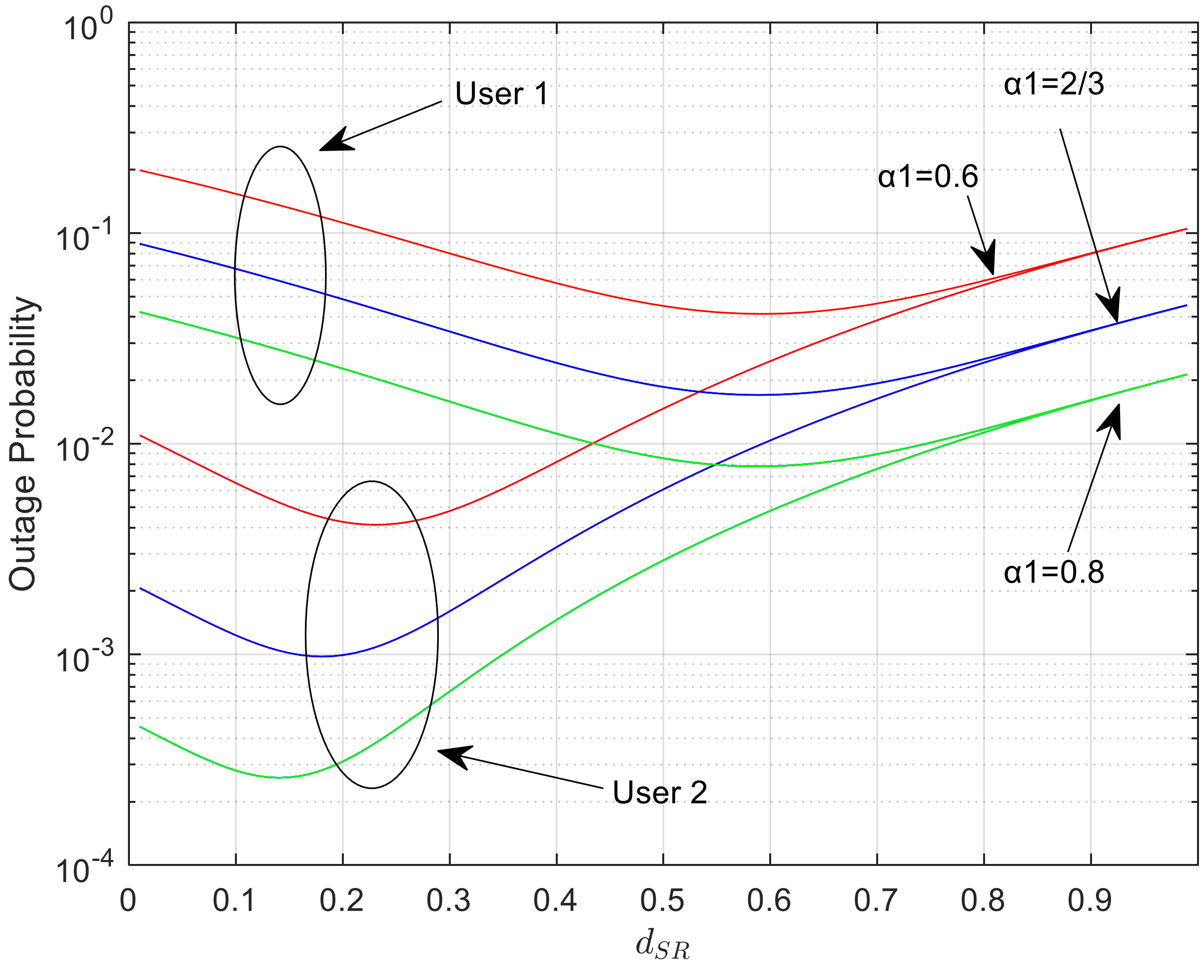

Fig 6.3. Outage probability vs. distance $S \rightarrow R$ link for various power allocation parameters with $\{a_1, a_2\} = \{0.6, 0.4\}, \left\{\frac{2}{3}, \frac{1}{3}\right\}, \{0.8, 0.2\}$ $\quad \gamma_{th1} = 0.9(dB),\ \ \gamma_{th2} = 1.5(dB)\ \ and\ \gamma = 20(dB)$.

In Fig 6.3. we can observe the effect of different power allocation assignments among the 2 users. When the power coefficient a_2 is increased (a_1 is decreased), both users experience an increase in outage probability, whereas, when a_2 is decreased (a_1 is increased) users experience a decrease in outage probability. This can be explained as follows: Both users have to decode the symbol of the weaker user. However, we can observe from equations **(3.8)** and **(3.11)** that it is more probable to be in outage when the $a_1 - \gamma_{th1} a_2$ term increases as it can enlarge the denominator and thus, make it more probable for the outage conditions to be satisfied. Additionally, when a_2 is decreased (a_1 is increased) the optimal relay location for the stronger user is shifted towards the base station and for the weaker user is shifted towards the user side. The distance d_{SR} is correlated with the average channel powers ($\mathbb{E}\{|h_{SR}|^2\} = d_{SR}{}^{-\alpha}$ and $\mathbb{E}\left\{\left|h_{RD_2}\right|^2\right\} = (1 - d_{SR})^{-\alpha}$), therefore for a certain value of d_{SR} outage probability is taking its minimum value, since the average channel gain is affecting the system performance.

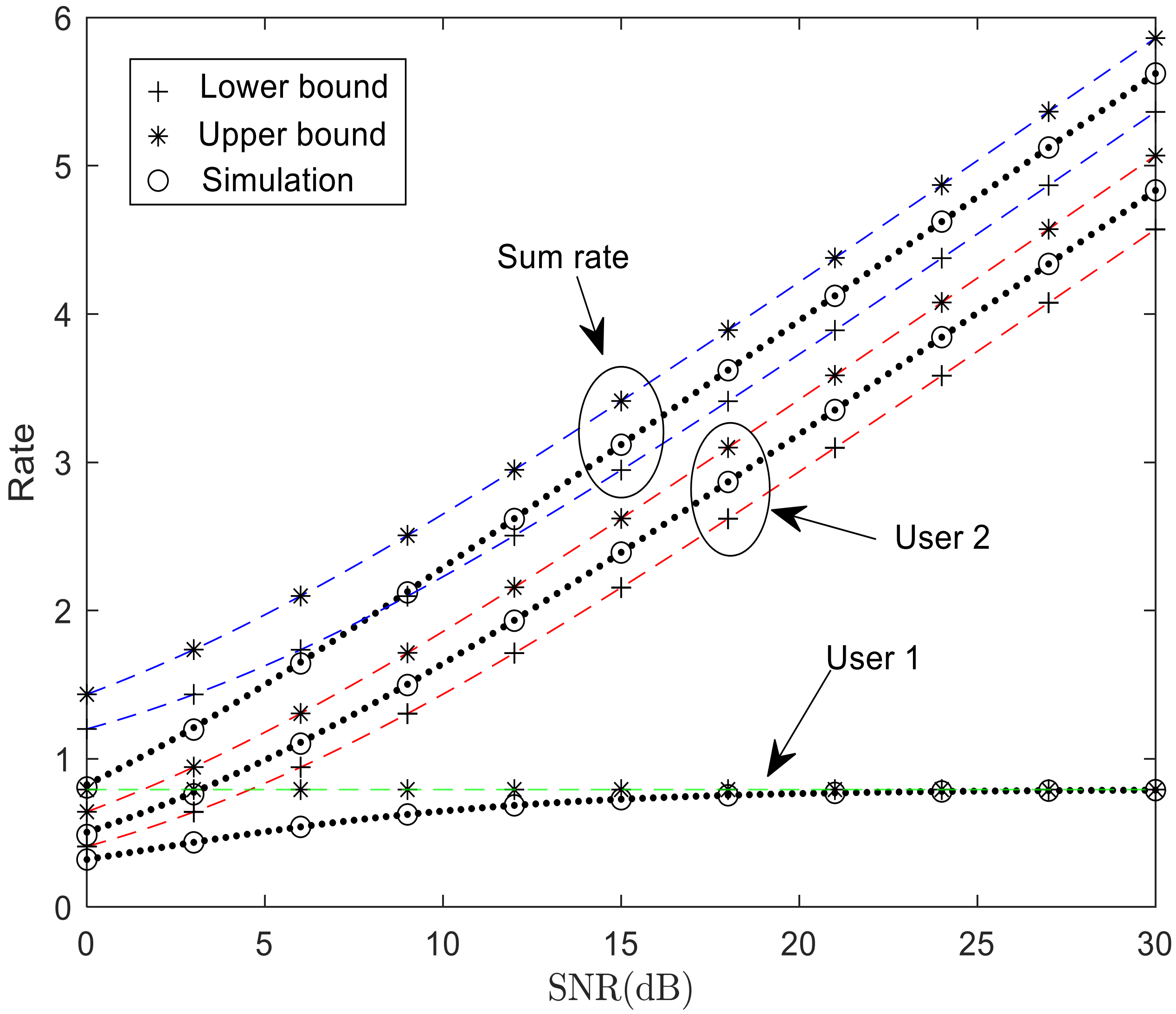

Fig 6.4. Rate vs transmit SNR with $a_1 = \frac{2}{3}, a_2 = \frac{1}{3}$, $\gamma_{th1} = 0.9(dB)$, $\gamma_{th2} = 1.5(dB)$ $and\ d_{SR} = 0.5$

In Fig. 6.4, we can observe the analytical expressions of the ergodic rates of the users in accordance to the system SNR. Firstly, we can notice that the first user has a very poor rate performance compared to the second, hence, last users' rate must be maximized, under the condition that first achieves a minimum threshold, in order to maximize the systems throughput. Additionally, we can observe that the analytic expression for the weak user is tightly bounded by the high SNR approximation in the corresponding region. The dotted plots are the exact expressions and are derived via numerical integration. Moreover, simulations are shown to be very well matched with the exact expressions.

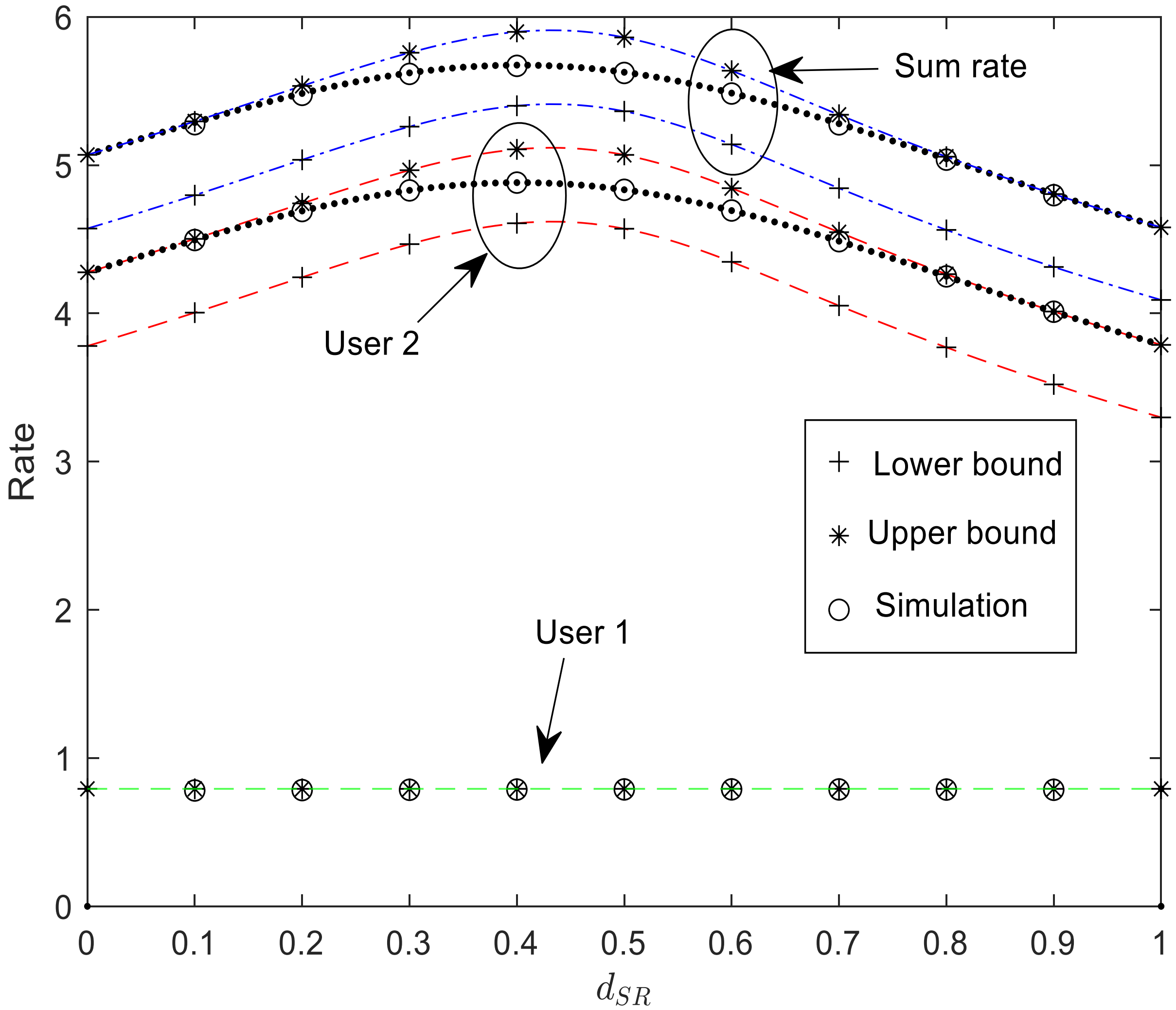

Fig 6.5. Rate vs. distance $S \rightarrow R$ link with $a_1 = \frac{2}{3}, a_2 = \frac{1}{3}$, $\gamma_{th1} = 0.9(dB)$, $\gamma_{th2} = 1.5(dB)$ $and\, \gamma = 30(dB)$

In this figure, we can see that the Rate of $User\ 1$ maintained the fixed value nature but the second $User$ seems to have a maximum value when $d_{SR} = 0.45$. In the region of maximum $Sum\ Rate$ the exact expression tends to get near to the lower bound, but when the relay is located near the user or the base station, the exact expression is shown to be tight with the upper bound, high-SNR approximation. As a consequence, for each d_{SR} different bounds would describe the system tightly.

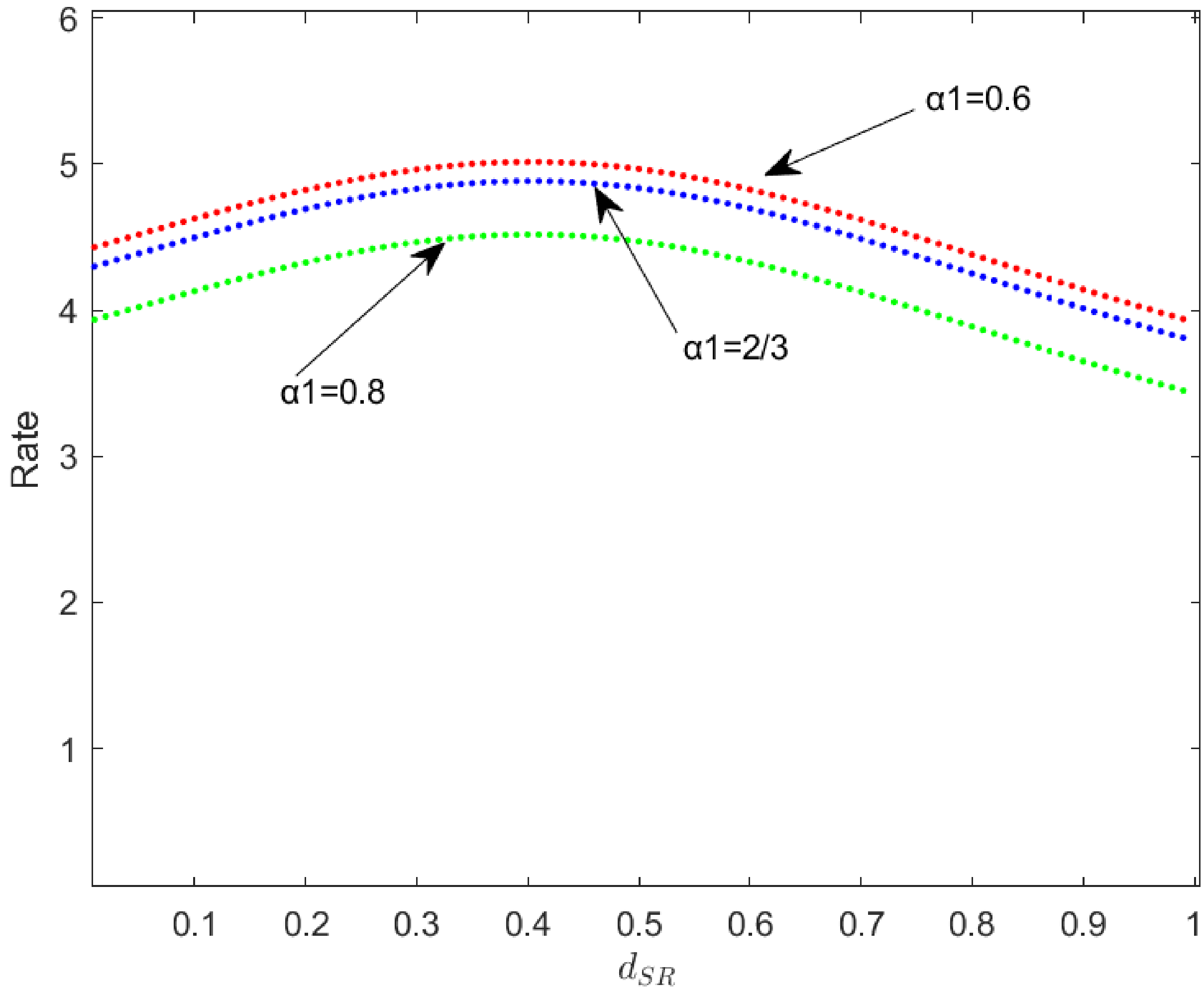

Fig 6.6. Rate vs. distance $S \rightarrow R$ link for various power allocation parameters with $\{a_1, a_2\} = \{0.6, 0.4\}, \left\{\frac{2}{3}, \frac{1}{3}\right\}, \{0.8, 0.2\}$, $\gamma_{th1} = 0.9(dB)$, $\gamma_{th2} = 1.5(dB)$ $and\, \gamma = 30(dB)$

The ergodic sum rate is degraded when user 1 is allocating an increased power coefficient, since the second users' rate is more important for the performance in terms of sum rate. As we can observe in Fig.6.3 and Fig.6.6, there is a trade-off in sum rate and both users' outage probability when the power is distributed to the users. Particularly, as the power allocated from the strong user increases the sum rate increases, whereas, outage probability of weak and strong user is increased as well.

7. CONCLUSION

The main goal was to study the power domain NOMA transmission and investigate the performance of this cooperative network under Rayleigh fading channels. Firstly, in chapter 1 we discussed about why NOMA is considered as a candidate 5G technique and how it works in much detail. In chapter 2, the model of the system is formulated, taking into account all the constraints and deriving the $SINR$'s that we'll use in chapter 3, in order to obtain the exact outage probability of $USER_1$ and $USER_2$. In the same chapter, in an effort to reduce the computational complexity, high-SNR regime bounds are derived. Moreover, in chapter 4, the ergodic sum rate of the system is also examined asymptotically. Numerical analysis was conducted as a means to verify the analytical results, giving us some insights about the behavior of the system.

References

[1] M. Aldababsa, T. Mesut, S. Gökçeli, G. K. Kurt and O. Kucur, "A Tutorial on Nonorthogonal Multiple Accessfor 5G and Beyond," *Hindawi,* 2018.

[2] L. Dai, B. Wang, Z. Ding, Z. Wang, S. Chen and L. Hanzo, "A Survey of Non-Orthogonal Multiple Access for 5G," *IEEE,* 2018.

[3] R. Hoshyar, F. P. Wathan and R. Tafazolli, "Novel low-density signature for synchronous CDMA systems over AWGN channel," *IEEE Trans. Signal Process,* vol. 56, p. 1616–1626, 2008.

[4] R. Razavi, R. Hoshyar, M. A. Imran and Y. Wang, "Information theoretic analysis of LDS scheme," *IEEE Commun. Lett.,* vol. 15, p. 798–800, 2011.

[5] M. Al-Imari, P. Xiao, M. A. Imran and R. Tafazolli, "Uplink nonorthogonal multiple access for 5G wireless networks," *International Symposium on Wireless Communication Systems (ISWCS2014),* p. 781–785, 2014.

[6] M. A. I. a. R. T. M. Al-Imari, "Low density spreading for next generation multicarrier cellular systems," *IEEE Int. Conf. Future Commun. Netw. (ICFCN),* pp. 52-57, 2012.

[7] H. Nikopour and H. Baligh, "Sparse code multiple access," *IEEE Int. Symp. Pers. Indoor Mobile Radio Commun. ,* p. 332–336., 2013.

[8] H. Nikopour, E. Yi, A. Bayesteh, K. Au, M. Hawryluck, H. Baligh and J. Ma, "SCMA for downlink multiple access of 5G wireless networks," *IEEE Global Communications Conference,* p. 3940–3945, 2014.

[9] S. Chaturvedi, Z. Liu, V. A. Bohara, A. Srivastava and P. Xiao, "A Tutorial on Decoding Techniques of Sparse Code Multiple Access," *IEEE Access ,* vol. 10, 2022.

[10] Z. Ding, X. Lei, G. K. Karagiannidis, R. Schober, J. Yuan and V. K. Bhargava, "A Survey on Non-Orthogonal Multiple Access for 5G Networks: Research Challenges and Future Trends," *IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS,* vol. 35, pp. 2181-2184, 2017.

[11] M. Vaezi, R. Schober, Z. Ding and H. V. Poor, "Non-Orthogonal Multiple Access: Common Myths and Critical Questions," *IEEE Wireless Communications,* vol. 26, no. 5, pp. 174 - 180, 2019.

[12] D. Wan, M. Wen, F. Ji, H. Yu and F. Chen, "Non-Orthogonal Multiple Access for Cooperative Communications: Challenges, Opportunities, and Trends," *IEEE Wireless Communications,* vol. 25, no. 2, pp. 109 - 117, 2018.

[13] K. Higuchi and Y. Kishiyama, "Non-orthogonal access with random beamforming and intra-beam SIC for cellular MIMO downlink," *Proc. IEEE Veh. Technol. Conf.,* p. 1–5, 2013.

[14] S. Han, C.-L. I, Z. Xu and Q. Sun, "Energy Efficiency and Spectrum Efficiency Co-Design: From NOMA to Network NOMA," *IEEE COMSOC MMTC E-Letter,* vol. 9, pp. 21-24, 2014.

[15] A. Chauhan and A. Jaiswal, "Non-Orthogonal Multiple Access: A Constellation Domain Multiplexing Approach," *IEEE 31st Annual International Symposium on Personal, Indoor and Mobile Radio Communications,* 2020.

[16] H.-C. Yang and M.-S. Alouini, Order Statistics in Wireless Communications: Diversity, Adaptation, and scheduling in MIMO and OFDM systems, Cambridge University Press, 2011.

[17] I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series and Products, New York,

2000.
[18] P. A. Anghel and M. Kaveh, "Exact symbol error probability of a cooperative network in a Rayleigh-fading environment," *IEEE Trans. Wireless Commun,* vol. 3, no. 5, p. 1416–1421, 2004.

Appendix A

$$P_{out} = 1 - \Pr\left(\begin{cases} |h_{RD_m}|^2 \geq \theta \\ |h_{SR}|^2 > \dfrac{\theta\left(\gamma |h_{RD_m}|^2 + 1\right)}{\gamma\left(|h_{RD_m}|^2 - \theta\right)} \end{cases}\right)$$

$$= 1 - \int_{\theta}^{\infty} \int_{\frac{\theta(\gamma x+1)}{\gamma(x-\theta)}}^{\infty} f_{|h_{RD_m}|^2, |h_{SR}|^2}(x,y) \; dxdy$$

We can rewrite $f_{|h_{RD_m}|^2, |h_{SR}|^2}(x,y)$ *as* $f_{|h_{SR}|^2 \mid |h_{RD_m}|^2}(y \mid x) \times f_{|h_{RD_m}|^2}(x)$

$$= 1 - \int_{\theta}^{\infty} \int_{\frac{\theta(\gamma x+1)}{\gamma(x-\theta)}}^{\infty} f_{|h_{SR}|^2 \mid |h_{RD_m}|^2}(y \mid x) f_{|h_{RD_m}|^2}(x) \quad dxdy$$

After changing the order of the integrals, we obtain

$$= 1 - \int_{\theta}^{\infty} f_{|h_{RD_m}|^2}(\mathrm{y}) \int_{\frac{\theta(\gamma y+1)}{\gamma(y-\theta)}}^{\infty} f_{|h_{SR}|^2}(x)\, dxdy$$

$$= 1 - \int_{\theta}^{\infty} f_{|h_{RD_m}|^2}(y) \left[1 - \int_{-\infty}^{\frac{\theta(\gamma y+1)}{\gamma(y-\theta)}} f_{|h_{SR}|^2}(x)\right] dxdy$$

$$= 1 - \int_{\theta}^{\infty} f_{|h_{RD_m}|^2}(y) \left[1 - F_{|h_{SR}|^2}\left(\frac{\theta(\gamma y + 1)}{\gamma(\mathrm{y} - \theta)}\right)\right] dy$$

$$= 1 - \int_{\theta}^{\infty} f_{|h_{RD_m}|^2}(y) dy + \int_{\theta}^{\infty} f_{|h_{RD_m}|^2}(\mathrm{y})\, F_{|h_{SR}|^2}\left(\frac{\theta(\gamma \mathrm{y} + 1)}{\gamma(\mathrm{y} - \theta)}\right) dy$$

$$= \int_{-\infty}^{\theta} f_{|h_{RD_m}|^2}(y)dy + \int_{\theta}^{\infty} f_{|h_{RD_m}|^2}(y)\, F_{|h_{SR}|^2}\left(\frac{\theta(\gamma y+1)}{\gamma(\mathrm{y}-\theta)}\right)d\mathrm{y}$$

And since $\left|h_{RD_m}\right|^2 \geq 0$

$$= \int_{0}^{\theta} f_{|h_{RD_m}|^2}(y)dy + \int_{\theta}^{\infty} f_{|h_{RD_m}|^2}(y)\, F_{|h_{SR}|^2}\left(\frac{\theta(\gamma y+1)}{\gamma(\mathrm{y}-\theta)}\right)d\mathrm{y}$$

According to **(3.16)**, we can elaborate as follows

$$= \int_{0}^{\theta} f_{|h_{RD_m}|^2}(y)d\mathrm{y} + \int_{\theta}^{\infty} f_{|h_{RD_m}|^2}(y)\left[1 - e^{-\frac{\theta(\gamma y+1)}{\gamma(\mathrm{y}-\theta)\Omega_1}}\right]d\mathrm{y}$$

$$= \int_{0}^{\theta} f_{|h_{RD_m}|^2}(y)dy + \int_{\theta}^{\infty} f_{|h_{RD_m}|^2}(y)dy - \int_{\theta}^{\infty} f_{|h_{RD_m}|^2}(y)e^{-\frac{\theta(\gamma y+1)}{\gamma(\mathrm{y}-\theta)\Omega_1}}dy$$

$$= 1 - \int_{\theta}^{\infty} f_{|h_{RD_m}|^2}(y)e^{-\frac{\theta(\gamma y+1)}{\gamma(\mathrm{y}-\theta)\Omega_1}}dy$$

Appendix B

In order to prove the concaveness of the original function R_{sum}, we prove that the 2×2 Jacobian matrix of the gradient of the original function is definite negative. Consequently, we need to prove that the hessian matrix of R_{sum} is a definite negative matrix

$$H_{R_{sum}} = J\big(\nabla R_{sum}(a_1, a_2)\big) = \begin{bmatrix} \frac{\partial^2 R_{sum}}{\partial a_1^2} & \frac{\partial^2 R_{sum}}{\partial a_1 \partial a_2} \\ \frac{\partial^2 R_{sum}}{\partial a_2 \partial a_1} & \frac{\partial^2 R_{sum}}{\partial a_2^2} \end{bmatrix} \prec 0$$

(B.1)

First order partial derivatives can be represented as follows

$$\frac{\partial R_{sum}}{\partial a_1} = \frac{\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2}{2ln2\left(\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2 (a_1 + a_2) + \gamma\left(\left|h_{RD_1}\right|^2 + |h_{SR}|^2\right) + 1\right)} \tag{B.2}$$

$$\frac{\partial R_{sum}}{\partial a_2} = \frac{\gamma^2 \left|h_{RD_2}\right|^2 |h_{SR}|^2}{2ln2\left(\gamma^2 \left|h_{RD_2}\right|^2 |h_{SR}|^2 a_2 + \gamma\left(\left|h_{RD_2}\right|^2 + |h_{SR}|^2\right) + 1\right)} - \frac{\left(\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2\right)^2 a_1}{2ln2\left(\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2 (a_1 + a_2) + \gamma\left(\left|h_{RD_1}\right|^2 + |h_{SR}|^2\right) + 1\right)} \frac{1}{\left(\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2 a_2 + \gamma\left(\left|h_{RD_1}\right|^2 + |h_{SR}|^2\right) + 1\right)} \tag{B.3}$$

Second order partial derivatives can be represented as follows

$$\frac{\partial^2 R_{sum}}{\partial a_1^2} = \frac{\partial^2 R_{sum}}{\partial a_1 \partial a_2} = \frac{\partial^2 R_{sum}}{\partial a_2 \partial a_1} = \frac{-\left(\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2\right)^2}{2ln2\left(\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2 (a_1 + a_2) + \gamma\left(\left|h_{RD_1}\right|^2 + |h_{SR}|^2\right) + 1\right)^2} \tag{B.4}$$

$$\frac{\partial^2 R_{sum}}{\partial a_2^2} = \frac{1}{2ln2}\left[\frac{2\left(\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2\right)^2 a_1\left(\left(\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2\right)^2 (a_1 + 2a_2) + 2\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2\left(\gamma\left(\left|h_{RD_1}\right|^2 + |h_{SR}|^2\right) + 1\right)\right)}{\left(\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2 (a_1 + a_2) + \gamma\left(\left|h_{RD_1}\right|^2 + |h_{SR}|^2\right) + 1\right)^2 \left(\gamma^2 \left|h_{RD_1}\right|^2 |h_{SR}|^2 a_2 + \gamma\left(\left|h_{RD_1}\right|^2 + |h_{SR}|^2\right) + 1\right)^2} - \frac{\left(\gamma^2 \left|h_{RD_2}\right|^2 |h_{SR}|^2\right)^2}{\left(\gamma^2 \left|h_{RD_2}\right|^2 |h_{SR}|^2 a_2 + \gamma\left(\left|h_{RD_2}\right|^2 + |h_{SR}|^2\right) + 1\right)^2}\right] \tag{B.5}$$

The Hessian matrix of R_{sum} after gaussian elimination takes the following form

$$H_{R_{sum}} = \begin{bmatrix} \frac{\partial^2 R_{sum}}{\partial a_1^2} & \frac{\partial^2 R_{sum}}{\partial a_1 \partial a_2} \\ 0 & \frac{\partial^2 R_{sum}}{\partial a_2^2} - \frac{\partial^2 R_{sum}}{\partial a_1^2} \end{bmatrix} \tag{B.6}$$

And can easily be proven negative definite with Sylvester's criterion.

$$\frac{\partial^2 R_{sum}}{\partial a_1^2} < 0$$

(B.7)

$$\det\left(H_{R_{sum}}\right) > 0 \Longrightarrow \frac{\partial^2 R_{sum}}{\partial a_2^2} < \frac{\partial^2 R_{sum}}{\partial a_1^2}$$

(B.8)

And thus, R_{sum} is shown to be concave.

MATLAB CODE

1.Outage probability VS SNR

```
g = 0:0.01:30;
gamma1db=0.9;
gamma1=10^(gamma1db/10);
gamma2db=1.5;
gamma2=10^(gamma2db/10);
a1=2/3;
a2=1/3;
snr=10.^(g/10);
theta1=gamma1./(snr.*(a1-gamma1*a2));
theta2=gamma2./(snr*a2);
dsr=0.5;
a=3;
omega1=(dsr).^(-a);
omega2= (1-dsr).^(-a);

% Plotting exact outage probability for each user

Prob1Out=1-(4/omega2)*exp(-2* theta1./omega2).*exp(-theta1./omega1).*
sqrt(theta1.*(1+snr.*theta1).*omega2./(2*snr.*omega1)).* besselk(1,2*sqrt(2*theta1.*(1+snr.*theta1)./(snr.*omega1.*
omega2)));

semilogy(g,Prob1Out)
hold on

Prob2Out=1+(4/omega2)*exp(-max(theta1,theta2).*(omega1+omega2)./(omega2*omega2)).*
sqrt(max(theta1,theta2).*(1+snr.* max(theta1,theta2)).*omega2./(snr.*omega1)).*((1/sqrt(2))*exp(-
max(theta1,theta2)./omega2).*besselk(1,2*sqrt(2* max(theta1,theta2).*(1+snr.* max(theta1,theta2))./(snr.*omega1.*
omega2)))-besselk(1,2*sqrt(max(theta1,theta2).*(1+snr.* max(theta1,theta2))./(snr.*omega1.* omega2))));

semilogy(g,Prob2Out)
hold on

axis([0 30 0 1])
xlabel ('SNR(dB)');
ylabel('Outage Probability')
grid on

% Plotting asymptotic bounds (--)

p1outLB=2*theta1./omega2+theta1./omega1;
p1outUB=4*theta1./omega2+2*theta1./omega1;
p1outLB_line=semilogy(g, p1outLB,'--');
hold on
p1outUB_line=semilogy(g, p1outUB,'--');
hold on
p1outLB_line.Color= "#0072BD";
p1outUB_line.Color= "#0072BD";

p2outLB=theta2./omega1+(theta2./omega2).^2;
p2outUB=2*theta2./omega1+4*(theta2/omega2).^2;
p2outLB_line=semilogy(g, p2outLB,'--');
```

```
    hold on
    p2outUB_line=semilogy(g, p2outUB,'--');
    hold on
    p2outLB_line.Color=  "#D95319";
    p2outUB_line.Color=  "#D95319";

    % Plotting asymptotic bounds + and *

    g=linspace(3,29,10);
    snr=10.^(g/10);
    theta1=gamma1./(snr.*(a1-gamma1*a2));
    theta2=gamma2./(snr*a2);
     p1outLB=2*theta1./omega2+theta1./omega1;
    p1outLB_line=semilogy(g, p1outLB,'+k');
    hold on
    p2outLB=theta2./omega1+(theta2./omega2).^2;
    p2outUB=2*theta2./omega1+4*(theta2/omega2).^2;
    p2outLB_line=semilogy(g, p2outLB,'+k');
    hold on
    p2outUB_line=semilogy(g, p2outUB,'*k');
    hold on
    g=linspace(6,29,10);
    snr=10.^(g/10);
    theta1=gamma1./(snr.*(a1-gamma1*a2));
    theta2=gamma2./(snr*a2);
    p1outUB=4*theta1./omega2+2*theta1./omega1;
    p1outUB_line=semilogy(g, p1outUB,'*k');
    hold on

    % Monte Carlo outage probability simulations for each user

    % Number of channel realizations
N=10000000;
g = 0:1:30;
snr=10.^(g/10);
theta1=gamma1./(snr.*(a1-gamma1*a2));
theta2=gamma2./(snr*a2);
outage11=zeros (1 , length ( g ) ) ;
outage21=zeros (1 , length ( g ) ) ;
no_outage1=zeros (1 , length ( g ) ) ;
no_outage2=zeros (1 , length ( g ) ) ;

    % Creating exponential distribution objects
pd1= makedist ('Gamma','a',1,'b',omega1);
pd2= makedist ('Gamma','a',1,'b',omega2);

    % Inserting randomness according to exponential distribution objects
h_SR =random ( pd1 ,1,N) ;
h2a =random ( pd2 , 1 ,N) ;
h2b =random ( pd2 , 1 ,N) ;
for i=1:length(g)
   theta(i)=max(theta1(i),theta2(i));
   for j=1:N

          % Ordering the channels
          h_RD_1=min(h2a(j),h2b(j));
          h_RD_2=max(h2a(j),h2b(j));

          % User 1
          if h_RD_1 > theta1(i)
             if h_SR(j) > theta1(i)*(1+snr(i)*h_RD_1)/(snr(i)*(h_RD_1-theta1(i)))
                    no_outage1(i)=no_outage1(i)+1;
            elseif h_SR(j) < theta1(i)*(1+snr(i)*h_RD_1)/(snr(i)*(h_RD_1-theta1(i)))
```

```
            outage11(i)=outage11(i)+1;
        end
    elseif h_RD_1 < theta1(i)
        outage11 ( i )= outage11 ( i )+1;
    end
            % User 2
    if h_RD_2 > theta(i)
        if h_SR(j) > theta(i)*(1+snr(i)*h_RD_2)/(snr(i)*(h_RD_2-theta(i)))
            no_outage2 (i)= no_outage2 (i)+1;
        elseif h_SR(j) < theta(i)*(1+snr(i)*h_RD_2)/(snr(i)*(h_RD_2-theta(i)))
            outage21 ( i )=outage21 ( i )+1;
        end
    elseif h_RD_2 < theta(i)
        outage21(i)=outage21(i)+1;
    end
  end
end
Pout1=outage11 /N;
Pout2=outage21 /N;
x=0:length(g)-1;
semilogy(x,Pout1,'ok')
hold on
semilogy(x,Pout2,'ok')
legend('$User 1$','$User 2$', '$Lower bound$','$Upper bound$', '$Simulations$','Interpreter','latex','Location','Best');
```

2.Outage probability VS distance(S->R)

```
    gamma1db=0.9;
    gamma1=10^(gamma1db/10);
    gamma2db=1.5;
    gamma2=10^(gamma2db/10);
    a1=2/3;
    a2=1/3;
    a=3;
    d=0:0.01:1;
    g=20;
    snr=10.^(g/10);
    theta1=gamma1./(snr.*(a1-gamma1*a2));
    theta2=gamma2./(snr*a2);
    omega1d=d.^(-a);
    omega2d= (1-d).^(-a);

    % Plotting exact outage probability for each user

    dProb1Out=1-(4./omega2d).*exp(-2* theta1./omega2d).*exp(-theta1./omega1d).*
sqrt(theta1.*(1+snr.*theta1).*omega2d./(2*snr.*omega1d)).*
    besselk(1,2*sqrt(2*theta1.*(1+snr.*theta1)./(snr.*omega1d.*omega2d)));

    semilogy(d, dProb1Out)
    hold on

    dProb2Out=1+(4./omega2d).*exp(-theta1.*(omega1d+omega2d)./(omega1d.*omega2d)).*
sqrt(theta1.*(1+snr.*theta1).*omega2d./(snr.*omega1d)).*( (1/sqrt(2))*exp(-
theta1./omega2d).*besselk(1,2*sqrt(2*theta1.*(1+snr.*theta1)./(snr.*omega1d.* omega2d)))-
besselk(1,2*sqrt(theta1.*(1+snr.*theta1)./(snr.*omega1d.* omega2d))));
    semilogy(d, dProb2Out)
    hold on
    xlabel ('$d_{SR}$','Interpreter','latex');
    ylabel('Outage Probability')
    grid on

N=1000000;
```

```
d=0:0.1:1;
omega1d=d.^(-a);
omega2d= (1-d).^(-a);
outage11=zeros (1 , length(d));
outage21=zeros (1 , length(d));
no_outage1=zeros (1 , length(d)) ;
no_outage2=zeros (1 , length(d));
for i=1:length(d)-2
    pd1(i)=makedist ('Gamma','a',1,'b',omega1d(i+1));
    pd2(i)=makedist ('Gamma','a',1,'b',omega2d(i+1));
    h_SR=random ( pd1(i) ,1, N);
    h2a=random ( pd2(i) , 1 , N) ;
    h2b=random ( pd2(i) , 1 , N) ;
    for j=1:N
      h_RD_1=min(h2a(j),h2b(j));
      h_RD_2=max(h2a(j),h2b(j));
      if h_RD_1 > theta1
        if h_SR(j) > theta1*(1+snr*h_RD_1)/(snr*(h_RD_1-theta1))
          no_outage1(i)=no_outage1(i)+1;
        elseif h_SR(j) < theta1*(1+snr*h_RD_1)/(snr*(h_RD_1-theta1))
          outage11(i)=outage11(i)+1;
        end
      elseif h_RD_1 < theta1
        outage11 ( i )= outage11 ( i )+1;
      end
      if h_RD_2 > theta1
          if h_SR(j)  > theta1*(1+snr*h_RD_2)/(snr*(h_RD_2-theta1))
              no_outage2 (i)= no_outage2 (i)+1;
          elseif h_SR(j)  < theta1*(1+snr*h_RD_2)/(snr*(h_RD_2-theta1))
              outage21 ( i )=outage21 ( i )+1;
          end
      elseif h_RD_2 < theta1
          outage21(i)=outage21(i)+1;
      end
    end
end
Pout1=outage11 /N;
Pout2=outage21 /N;
Pout1(end)=[];
Pout1(end)=[];
Pout2(end)=[];
Pout2(end)=[];
x=1:length(d)-2;
semilogy(x/10,Pout1,'ok')
hold on
semilogy(x/10,Pout2,'ok')
legend('$User 1$','$User 2$','$Simulations$','Interpreter','latex','Location','Best');
```

3. Rate VS SNR

$$\lim_{N\to\infty}\frac{1}{N}\sum_{\iota}^{N}\log_2\left(1+\frac{\gamma^2\left|h_{RD_1}\right|^2|h_{SR}|^2a_1}{\gamma^2\left|h_{RD_1}\right|^2|h_{SR}|^2a_2+\gamma\left(\left|h_{RD_1}\right|^2+|h_{SR}|^2\right)+1}\right)$$

$$+\lim_{N\to\infty}\frac{1}{N}\sum_{\iota}^{N}\log_2\left(1+\frac{\gamma^2\left|h_{RD_2}\right|^2|h_{SR}|^2a_2}{\gamma\left(\left|h_{RD_2}\right|^2+|h_{SR}|^2\right)+1}\right)$$

```
	N=1000000;
	g = 0:0.01:30;
	gamma1db=0.9;
	gamma1=10^(gamma1db/10);
	gamma2db=1.5;
	gamma2=10^(gamma2db/10);
	a1=2/3;
	a2=1/3;
	snr=10.^(g/10);
	theta1=gamma1./(snr.*(a1-gamma1*a2));
	theta2=gamma2./(snr*a2);
	dsr=0.5;
	a=3;
	omega1=(dsr).^(-a);
	omega2= (1-dsr).^(-a);
	lamda1=1./(omega1.*a2.*snr);
	lamda2=1./(omega2.*a2.*snr);

	rate2LB=expint(2*(lamda1+lamda2)).*exp(2*(lamda1+lamda2))/(log(2))-
expint(4*lamda1+2*lamda2).*exp(4*lamda1+2*lamda2)/(2*log(2));
	rate2UB=expint(lamda1+lamda2).*exp(lamda1+lamda2)/(log(2))-
expint(2*lamda1+lamda2).*exp(2*lamda1+lamda2)/(2*log(2));
	SumrateLB=(1/2)*log2(3)+expint(2*(lamda1+lamda2)).*exp(2*(lamda1+lamda2))/(log(2))-
expint(4*lamda1+2*lamda2).*exp(4*lamda1+2*lamda2)/(2*log(2));
	SumrateUB=(1/2)*log2(3)+expint(lamda1+lamda2).*exp(lamda1+lamda2)/(log(2))-
expint(2*lamda1+lamda2).*exp(2*lamda1+lamda2)/(2*log(2));

	plot(g,rate2LB,'--r')
	hold on
	plot(g,rate2UB,'--r')
	hold on
	 plot(g, SumrateLB,'--b')
	hold on
	plot(g, SumrateUB,'--b')
	hold on
	xlabel('SNR(dB)','Interpreter','latex');
	ylabel('Rate')


	user1_numerical =
@(x,y,g)(1/(omega1.*omega2))*log2(1+(a1.*((10.^(g/10)).^2).*(x.*y))./(((10.^(g/10)).^2).*x.*y.*a2+(10.^(g/10).*(x+y))+1)).
*(exp(-2.*y./omega2-x./omega1));

	user2_numerical = @(x,y,k)(1./(omega1.*omega2)).*log2(1+((10.^(k/10)).*a2).*(x.*y./(x+y))).*( exp(-y./omega2-
x./omega1)-exp(-2.*y./omega2-x./omega1));

for i=0:0.3:30
	h1 = @(x,y) user1_numerical (x,y,i);
	h2 = @(x,y) user2_numerical (x,y,i);
	num_res_1= integral2(h1,0,Inf,0,Inf);
	num_res_2= integral2(h2,0,Inf,0,Inf);
	plot(i, num_res_1,'.k');
	hold on
	plot(i, num_res_2,'.k');
	hold on
	plot(i, num_res_1+ num_res_2,'.k');
	hold on
end
rate1_ub = (1/2)*log2(1+a1/a2);
yline(rate1_ub,'--g')
hold on

sim_rate_1=zeros(1 , 11);
```

```
sim_rate_2=zeros (1 , 11);
sim_sum_rate=zeros (1 , 11);
pd1= makedist ('Gamma','a',1,'b',omega1);
pd2= makedist ('Gamma','a',1,'b',omega2);
h_SR =random ( pd1 ,1,N) ;
h2a =random ( pd2 , 1 ,N) ;
h2b =random ( pd2 , 1 ,N) ;
for i=0:3:30
   for j=1:N
      h_RD_1=min(h2a(j),h2b(j));
      h_RD_2=max(h2a(j),h2b(j));
      sim_rate_1(i/3+1)= sim_rate_1(i/3+1) + log2( 1 +  (10.^(i/10)).^2*h_RD_1* h_SR(j)*a1/((10.^(i/10)).^2*
h_RD_1*h_SR(j)*a2+10.^(i/10).*(h_RD_1+h_SR(j))+1));
      sim_rate_2(i/3+1)= sim_rate_2(i/3+1) + log2( 1 +  (10.^(i/10)).^2*h_RD_2*
h_SR(j)*a2/(10.^(i/10).*(h_RD_2+h_SR(j))+1));
    end
end
sim_rate_1=sim_rate_1/(2*N);
sim_rate_2=sim_rate_2/(2*N);
sim_sum_rate=sim_rate_1+sim_rate_2;
x=0:3:30;
plot(x,sim_rate_1,'ok')
hold on
plot(x,sim_rate_2,'ok')
hold on
plot(x,sim_sum_rate,'ok')
hold on


      g = 0:3:30;
      snr=10.^(g/10);
      theta1=gamma1./(snr.*(a1-gamma1*a2));
      theta2=gamma2./(snr*a2);
      lamda1=1./(omega1.*a2.*snr);
      lamda2=1./(omega2.*a2.*snr);
      rate2LB=expint(2*(lamda1+lamda2)).*exp(2*(lamda1+lamda2))/(log(2))-
expint(4*lamda1+2*lamda2).*exp(4*lamda1+2*lamda2)/(2*log(2));
      rate2UB=expint(lamda1+lamda2).*exp(lamda1+lamda2)/(log(2))-
expint(2*lamda1+lamda2).*exp(2*lamda1+lamda2)/(2*log(2));
      SumrateLB=(1/2)*log2(3)+expint(2*(lamda1+lamda2)).*exp(2*(lamda1+lamda2))/(log(2))-
expint(4*lamda1+2*lamda2).*exp(4*lamda1+2*lamda2)/(2*log(2));
      SumrateUB=(1/2)*log2(3)+expint(lamda1+lamda2).*exp(lamda1+lamda2)/(log(2))-
expint(2*lamda1+lamda2).*exp(2*lamda1+lamda2)/(2*log(2));
      plot(g,rate2LB,'+k')
      hold on
      plot(g,rate2UB,'*k)
      hold on
       plot(g, SumrateLB,'+k)
      hold on
      plot(g, SumrateUB,'*k')
      hold on
     plot(g,rate1_ub,'*k')

     plot(2,5.5,'+k')
     hold on
     plot(2,5.2,'*k')
     hold on
     plot(2,4.9,'ok')
```

4.Rate VS Distance

```
gamma1db=0.9;
gamma1=10^(gamma1db/10);
gamma2db=1.5;
gamma2=10^(gamma2db/10);
a1=2/3;
a2=1/3;
a=3;
d=0:0.01:1;
g=30;
snr=10.^(g/10);
theta1=gamma1./(snr.*(a1-gamma1*a2));
theta2=gamma2./(snr*a2);
omega1d=d.^(-a);
omega2d= (1-d).^(-a);
lamda1d=1./(omega2d.*a2.*snr);
lamda2d=1./(omega1d.*a2.*snr);

rate2LB=expint(2*(lamda1d+lamda2d)).*exp(2*(lamda1d+lamda2d))/(log(2))-
expint(4*lamda1d+2*lamda2d).*exp(4*lamda1d+2*lamda2d)/(2*log(2));
rate2UB=expint(lamda1d+lamda2d).*exp(lamda1d+lamda2d)/(log(2))-
expint(2*lamda1d+lamda2d).*exp(2*lamda1d+lamda2d)/(2*log(2));
plot(d,rate2LB,'--r')
hold on
plot(d,rate2UB,'--r')
hold on
drate_2 = @(x,y,k)(1./((k.^(-a))  .*((1-k).^(-a))).*log2(1+((10.^(g/10)).*a2.*x.*y)./(x+y)).*( exp(-y./ ((1-k).^(-a))-x./
(k.^(-a)) )-exp(-2.*y./ ((1-k).^(-a))-x./  (k.^(-a)) )));
drate_1 =  (1/2)*log2(1+a1/a2);
yline(drate_1,'--g')
hold on

for j=0:0.01:1
    h = @(x,y) drate_2(x,y,j);
    numeric_res=integral2(h,0,Inf,0,Inf);
    plot(j,numeric_res ,'.k');
    hold on
    plot(j, numeric_res+ drate_1,'.k');
    hold on
end


srrate2LB=(1/2)*log2(3)+expint(2*(lamda1d+lamda2d)).*exp(2*(lamda1d+lamda2d))/(log(2))-
expint(4*lamda1d+2*lamda2d).*exp(4*lamda1d+2*lamda2d)/(2*log(2));

srrate2UB=(1/2)*log2(3)+expint(lamda1d+lamda2d).*exp(lamda1d+lamda2d)/(log(2))-
expint(2*lamda1d+lamda2d).*exp(2*lamda1d+lamda2d)/(2*log(2));

plot(d, srrate2LB,'-.b')
hold on
plot(d, srrate2UB,'-.b')
hold on
ylabel('Rate')
xlabel('$d_{SR}$','Interpreter','latex');


d= linspace(0,1,11);
omega1d=d.^(-a);
```

```
omega2d= (1-d).^(-a);
lamda1d=1./(omega2d.*a2.*snr);
lamda2d=1./(omega1d.*a2.*snr);

rate2LB=expint(2*(lamda1d+lamda2d)).*exp(2*(lamda1d+lamda2d))/(log(2))-
expint(4*lamda1d+2*lamda2d).*exp(4*lamda1d+2*lamda2d)/(2*log(2));
plot(d,rate2LB,'+k')
hold on

rate2UB=expint(lamda1d+lamda2d).*exp(lamda1d+lamda2d)/(log(2))-
expint(2*lamda1d+lamda2d).*exp(2*lamda1d+lamda2d)/(2*log(2));
plot(d,rate2UB,'*k')
hold on


srrate2LB=(1/2)*log2(3)+expint(2*(lamda1d+lamda2d)).*exp(2*(lamda1d+lamda2d))/(log(2))-
expint(4*lamda1d+2*lamda2d).*exp(4*lamda1d+2*lamda2d)/(2*log(2));
plot(d, srrate2LB,'+k')
hold on

srrate2UB=(1/2)*log2(3)+expint(lamda1d+lamda2d).*exp(lamda1d+lamda2d)/(log(2))-
expint(2*lamda1d+lamda2d).*exp(2*lamda1d+lamda2d)/(2*log(2));
plot(d, srrate2UB,'*k')
hold on

plot(d, drate_1,'*k')
hold on




N=1000000;
d=0:0.1:1;
omega1d=d.^(-a);
omega2d= (1-d).^(-a);
sim_rate_1=zeros (1 , length(d)-2);
sim_rate_2=zeros (1 , length(d)-2);
sim_sum_rate=zeros (1 , length(d)-2);
for i=1:length(d)-2
    pd1(i)=makedist ('Gamma','a',1,'b',omega1d(i+1));
    pd2(i)=makedist ('Gamma','a',1,'b',omega2d(i+1));
    h_SR=random ( pd1(i) ,1, N);
    h2a=random ( pd2(i) , 1 , N) ;
    h2b=random ( pd2(i) , 1 , N) ;
    for j=1:N
       h_RD_1=min(h2a(j),h2b(j));
       h_RD_2=max(h2a(j),h2b(j));
       sim_rate_1(i)= sim_rate_1(i) + log2( 1 +  (10.^(g/10)).^2*h_RD_1* h_SR(j)*a1/((10.^(g/10)).^2*
h_RD_1*h_SR(j)*a2+10.^(g/10).*(h_RD_1+h_SR(j))+1));
      sim_rate_2(i)= sim_rate_2(i) + log2( 1 +  (10.^(g/10)).^2*h_RD_2* h_SR(j)*a2/(10.^(g/10).* (h_RD_2+h_SR(j))+1));
    end
end
sim_rate_1=sim_rate_1/(2*N);
sim_rate_2=sim_rate_2/(2*N);
sim_sum_rate=sim_rate_1+sim_rate_2;
x=0.1:0.1:(length(d)-2)/10;
plot(x,sim_rate_1,'ok')
hold on
plot(x,sim_rate_2,'ok')
hold on
plot(x,sim_sum_rate,'ok')
hold on
```

```
plot(0.7,3,'+k')
hold on
plot(0.7,2.5,'*k')
hold on
plot(0.7,2,'ok')
```