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Αλγόριθμοι διεπίπεδου προγραμματισμού για βέλτιστη υποβολή προσφορών παραγωγών ενέργειας σε αγορές ημερήσιου προγραμματισμού ηλεκτρικής ενέργειας

υπό

ΕΥΤΥΧΙΑΣ ΚΩΣΤΑΡΕΛΟΥ

Διπλωματούχου Μηχανικού Παραγωγής και Διοίκησης της Πολυτεχνικής Σχολής Ξάνθης, ΔΠΘ, 2006

M.Sc., Τμήμα Μηχανολόγων Μηχανικών, Πολυτεχνική Σχολή Βόλου, Πανεπιστήμιο Θεσσαλίας, 2009

Υπεβλήθη για την εκπλήρωση μέρους των

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Εγκρίθηκε από τα Μέλη της Επταμελούς Εξεταστικής Επιτροπής:

Πρώτος Εξεταστής (Επιβλέπων)	Δρ. Γεώργιος Κοζανίδης Αναπληρωτής Καθηγητής, Τμήμα Μηχανολόγων Μηχανικών, Πανεπιστήμιο Θεσσαλίας
Δεύτερος Εξεταστής	Δρ. Γεώργιος Λυμπερόπουλος Καθηγητής, Τμήμα Μηχανολόγων Μηχανικών, Πανεπιστήμιο Θεσσαλίας
Τρίτος Εξεταστής	Δρ. Αθανάσιος Ζηλιασκόπουλος Καθηγητής, Τμήμα Μηχανολόγων Μηχανικών, Πανεπιστήμιο Θεσσαλίας
Τέταρτος Εξεταστής	Δρ. Δημήτριος Παντελής Καθηγητής, Τμήμα Μηχανολόγων Μηχανικών, Πανεπιστήμιο Θεσσαλίας
Πέμπτος Εξεταστής	Δρ. Γεώργιος Κ. Σαχαρίδης Αναπληρωτής Καθηγητής, Τμήμα Μηχανολόγων Μηχανικών, Πανεπιστήμιο Θεσσαλίας
Έκτος Εξεταστής	Δρ. Αθανάσιος Μυγδαλάς Καθηγητής, Department of Business Administration, Technology and Social Sciences, Lulcá University of Technology, Sweden
Έβδομος Εξεταστής	Δρ. Παντελής Μπίσκας Αναπληρωτής Καθηγητής, Τμήμα Ηλεκτρολόγων Μηχανικών και Μηχανικών Υπολογιστών, Αριστοτέλειο Πανεπιστήμιο Θεσσαλονίκης

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ΕΥΤΥΧΙΑ ΚΩΣΤΑΡΕΛΟΥ

Πανεπιστήμιο Θεσσαλίας, Τμήμα Μηχανολόγων Μηχανικών, 2020

Επιβλέπων Καθηγητής: Δρ. Γεώργιος Κοζανίδης, Αναπληρωτής Καθηγητής

Μέθοδοι Βελτιστοποίησης Συστημάτων Παραγωγής/ Υπηρεσιών

Περίληψη

Στην παρούσα διδακτορική διατριβή, εξετάζουμε το πρόβλημα σχεδιασμού βέλτιστων τιμών-προσφορών (bids) για έναν παραγωγό ενέργειας που συμμετέχει σε μια αγορά ημερήσιου προγραμματισμού ηλεκτρικής ενέργειας, η οποία περιλαμβάνει μη κυρτότητες λόγω της διακριτής φύσης των δεσμεύσεων των μονάδων παραγωγής. Ο ορισμός του προβλήματος υποθέτει πλήρη γνώση των τεχνικών χαρακτηριστικών και των τιμών προσφορών όλων των υπόλοιπων παραγωγών. Το πρόβλημα μορφοποιείται ως διεπίπεδο μοντέλο βελτιστοποίησης με γραμμικούς περιορισμούς και στα δύο επίπεδα. Ο παραγωγός ενεργεί ως υπεύθυνος λήψης αποφάσεων στο άνω επίπεδο, στοχεύοντας στην εύρεση των βέλτιστων τιμών προσφοράς που θα μεγιστοποιήσουν το ατομικό του κέρδος μετά την εκκαθάριση της αγοράς, ενώ ένας ανεξάρτητος διαχειριστής συστήματος (ISO) ενεργεί ως υπεύθυνος λήψης αποφάσεων στο κάτω επίπεδο, στοχεύοντας στην ικανοποίηση της ζήτησης ενέργειας στο ελάχιστο συνολικό κόστος-προσφοράς του συστήματος. Το μοντέλο περιλαμβάνει διακριτές μεταβλητές για τη μοντελοποίηση της κατάστασης των μονάδων παραγωγής, οι οποίες απαγορεύουν την εφαρμογή τυπικών μεθοδολογιών για την εύρεση της βέλτιστης λύσης, όπως είναι η αντικατάσταση του προβλήματος του κάτω επιπέδου από τις συνθήκες βελτιστότητας πρώτης τάξεως KKT.

Πρώτα εξετάζουμε την εκδοχή μιας περιόδου του προβλήματος και αναπτύσσουμε έναν ακριβή αλγόριθμο για την επίλυσή του, ο οποίος χρησιμοποιεί σημαντικά αποτελέσματα από τη θεωρία του ακέραιου παραμετρικού προγραμματισμού. Παρουσιάζουμε πειραματικά αποτελέσματα που καταδεικνύουν την αποτελεσματικότητά του σε τυχαίες περιπτώσεις προβλημάτων και ολοκληρώνουμε με μια συζήτηση για διάφορα υπολογιστικά ζητήματα που σχετίζονται με τη συμπεριφορά αυτού του αλγορίθμου και μια περιγραφή του τρόπου με τον

οποίο η υποκείμενη θεωρία μπορεί να τροποποιηθεί ώστε να ταιριάζει σε αγορές με εναλλακτικό σχεδιασμό.

Στη συνέχεια, εξετάζουμε την εκδοχή πολλαπλών περιόδων του προβλήματος. Αποδεικνύουμε σημαντικές θεωρητικές ιδιότητες και τις χρησιμοποιούμε για να αναπτύξουμε τόσο μια ευρετική όσο και μια ακριβή αλγοριθμική μεθοδολογία επίλυσης για την αντιμετώπισή της. Όπως είναι αναμενόμενο, πιο αποτελεσματική μεταξύ των δύο αποδεικνύεται ότι είναι η ευρετική προσέγγιση, η οποία λειτουργεί επαναληπτικά, βελτιστοποιώντας μία μοναδική τιμή-προσφορά σε κάθε επανάληψη, υπό την προϋπόθεση ότι οι υπόλοιπες διατηρούνται σταθερές στις τρέχουσες τιμές τους. Παρουσιάζουμε πειραματικά αποτελέσματα που αποδεικνύουν ότι παρέχει λύσεις υψηλής ποιότητας, ενώ οι υπολογιστικές της απαιτήσεις είναι πολύ λογικές. Καταδεικνύουμε επίσης πώς η υποκείμενη θεωρία μπορεί να χρησιμοποιηθεί για τη δημιουργία έγκυρων ανισοτήτων σε μια κατάλληλη χαλάρωση της αρχικής μορφοποίησης, στην οποία δεν είναι εγγυημένη η λεγόμενη διεπίπεδη εφικτότητα της ληφθείσας λύσης. Αυτές οι ανισότητες μπορούν να αξιοποιηθούν, εντός ενός πλαισίου ισχύουσων ανισοτήτων, από έναν ακριβή αλγόριθμο επίλυσης για τον προσδιορισμό του ολικού βέλτιστου του προβλήματος.

Συνεχίζουμε αναπτύσσοντας μία βελτιωμένη έκδοση του ακριβούς αλγόριθμου επίλυσης για την αντιμετώπιση της εκδοχής πολλαπλών περιόδων του προβλήματος. Η σημαντική υπεροχή αυτής της αλγοριθμικής παραλλαγής έγκειται στην ενσωμάτωση ειδικών συνθηκών βελτιστότητας οι οποίες διασφαλίζουν ότι η κατανομή της ποσότητας της ενέργειας σε κάθε χρονική περίοδο του ορίζοντα προγραμματισμού είναι η βέλτιστη για το αντίστοιχο σύνολο παραγωγών που έχουν αναγνωριστεί ως ενεργοί κατά τη συγκεκριμένη χρονική περίοδο. Συνεπακόλουθα, η εύρεση της ολικά βέλτιστης λύσης του αρχικού προβλήματος ισοδυναμεί με τον προσδιορισμό του βέλτιστου συνόλου ενεργών παραγωγών σε κάθε χρονική περίοδο του ορίζοντα προγραμματισμού. Προκειμένου να αποκλείσουμε από περαιτέρω εξέταση εκείνες τις λύσεις για τις οποίες αυτά τα σύνολα είναι υπο-βέλτιστα, χρησιμοποιούμε τον ίδιο τύπο τομών (cuts) που χρησιμοποιήθηκαν στην προηγούμενη αλγοριθμική έκδοση, προσαρμοσμένες κατάλληλα να επιβάλουν τις βέλτιστες καταστάσεις των μονάδων αντί για τις αντίστοιχες ποσότητες ενέργειας. Επεξηγούμε την εφαρμογή της προτεινόμενης μεθοδολογίας σε μια μικρή μελέτη περίπτωσης και παρουσιάζουμε υπολογιστικά αποτελέσματα που δείχνουν τη συμπεριφορά και την απόδοσή της σε τυχαία προβλήματα. Αυτά τα αποτελέσματα αποκαλύπτουν ότι η προτεινόμενη μεθοδολογία είναι ικανή να χειριστεί μεσαίου μεγέθους προβλήματα χρησιμοποιώντας λογικούς υπολογιστικούς πόρους.

Για την ανάπτυξη των προτεινόμενων μοντέλων βελτιστοποίησης και των εξειδικευμένων μεθοδολογιών επίλυσης, χρησιμοποιήσαμε τη γλώσσα προγραμματισμού C/C++. Όπου ήταν απαραίτητο, η λύση των προτεινόμενων μοντέλων βελτιστοποίησης ελήφθη χρησιμοποιώντας τα εμπορικά λογισμικά βελτιστοποίησης IBM ILOG CPLEX ή/και LINGO. Η σημαντικότητα της συνεισφοράς της παρούσας έρευνας καταδεικνύεται εάν κάποιος λάβει υπόψη την έλλειψη γενικών μεθοδολογιών επίλυσης για διεπίπεδα μοντέλα βελτιστοποίησης, όπως αυτά που εξετάζονται. Συνυπολογίζοντας αυτήν την παρατήρηση, οι προτεινόμενες αλγοριθμικές μεθοδολογίες, οι οποίες συνθέτουν το αποτέλεσμα αυτής της έρευνας, είναι εξαιρετικά χρήσιμες, παρέχοντας πολύτιμα θεωρητικά θεμέλια για πρακτικές εφαρμογές καθώς και για μελλοντικές ερευνητικές αναζητήσεις.

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PhD Dissertation

Bilevel programming algorithms for optimal strategic bidding of energy producers in day-ahead electricity markets

by

EFTYCHIA KOSTARELOU

B.Sc. in Production and Management Engineering,

Democritus University of Thrace, 2006

M.Sc. in Mechanical Engineering, University of Thessaly, 2009

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The approval of this PhD Dissertation by the Department of Mechanical Engineering of the School of Engineering of the University of Thessaly does not imply acceptance of the writer's opinions (Law 5343/32 article 202 par.2).

Approved by:

First Examiner (Supervisor)	Dr. George Kozanidis Associate Professor, Department of Mechanical Engineering, University of Thessaly
Second Examiner	Dr. George Liberopoulos Professor, Department of Mechanical Engineering, University of Thessaly
Third Examiner	Dr. Athanasios Ziliaskopoulos Professor, Department of Mechanical Engineering, University of Thessaly
Fourth Examiner	Dr. Dimitris Pandelis Professor, Department of Mechanical Engineering, University of Thessaly
Fifth Examiner	Dr. Georgios K. Saharidis Associate Professor, Department of Mechanical Engineering, University of Thessaly
Sixth Examiner	Dr. Athanasios Mygdalas Professor, Department of Business Administration, Technology and Social Sciences, Lulcá University of Technology, Sweden
Seventh Examiner	Dr. Pandelis Biskas Associate Professor, School of Electrical and Computer Engineering, Aristotle University of Thessaloniki

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Eftychia Kostarelou

Bilevel programming algorithms for optimal strategic bidding of energy producers in day-ahead electricity markets

EFTYCHIA KOSTARELOU

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Dissertation Advisor: Dr. George Kozanidis, Associate Professor in Optimization Methods of Production/Service Systems

Abstract

In this dissertation, we consider the problem of devising optimal price-offers (bids) for an energy producer participating in a day-ahead electricity market which exhibits non-convexities due to the discrete nature of the generation units' commitments. The problem definition assumes perfect knowledge of the technical characteristics and bidding offers of all remaining producers. The problem is formulated as a bilevel optimization model with linear constraint sets at both levels. The producer acts as the upper-level decision maker, aiming to find the optimal bidding offers that will maximize his individual profit upon clearing of the market, while an independent system operator (ISO) acts as the lower-level decision maker, aiming to ensure satisfaction of the demand for energy at the minimum total system bid-cost. The model utilizes discrete variables to represent the commitment of the production units, which prohibits the application of typical methodologies for finding its optimal solution, such as the substitution of the lower-level problem by its first-order KKT optimality conditions.

We consider the single period variant of the problem first, and we develop an exact algorithm for its solution, which utilizes important findings from the theory of integer parametric programming. We report experimental results demonstrating its efficiency on random problem instances, and we conclude with a discussion on several computational issues pertaining to the behavior of this algorithm, and an outline of how the underlying theory can be modified to fit alternative market designs.

Next, we consider the multi-period variant of the problem. We prove several important theoretical properties, and we utilize them to develop both a heuristic as well as an exact algorithmic solution methodology for tackling it. More effective between the two naturally turns out to be the heuristic approach, which works iteratively, optimizing a single price-offer at each iteration, given that the remaining ones are kept fixed at their current values. We present experimental results demonstrating that it provides high quality solutions, while exhibiting reasonable computational requirements. We also demonstrate how the underlying

theory can be utilized for the generation of valid inequalities to a suitable relaxation of the original formulation, in which the so-called bilevel feasibility of the obtained solution is not guaranteed. These inequalities are exploited within a cutting-plane framework by the exact solution approach for identifying the global optimum of the problem.

We go on to develop an improved version of the exact solution algorithm for the treatment of the multi-period variant of the problem. The significant advancement of this algorithmic version lies in the inclusion of special optimality conditions ensuring that the energy quantity distribution in each time period of the planning horizon is optimal for the corresponding set of producers that has been identified as active in that time period. Consequently, solving the original problem to global optimality becomes equivalent to identifying the optimal set of active producers in each time period of the planning horizon. In order to exclude from further consideration those solutions for which these sets are sub-optimal, we employ the same type of cuts utilized in the previous algorithmic version, adjusted suitably to impose optimal unit commitments instead of energy quantities. We illustrate the application of the proposed methodology on a small case study and we present computational results demonstrating its behavior and performance on randomly generated problems. These results reveal that the proposed methodology is capable of handling medium sized problems using reasonable computational resources.

For the development of the proposed optimization models and the specialized solution methodologies, we utilized the C/C++ programming language. When necessary, the solution of the proposed optimization models was obtained using the commercial optimization solvers IBM ILOG CPLEX and/or LINGO. The significance of the present research contribution becomes evident when one considers the lack of generic solution methodologies for bilevel optimization models such as the one under consideration. In view of this observation, the developed methodologies, which constitute the outcome of this research, are highly beneficial, providing valuable theoretical foundation for practical applications as well as for future research pursuits.

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Chapter 1 Introduction

1.1 Motivation and background information

The deregulation of electricity markets is an important economic development that has been taking place in numerous countries worldwide in recent years. Although the particular market designs adopted by different countries occasionally vary, many of the underlying principles remain more or less the same. Most designs establish a wholesale and a retail electricity market that operate in long-term and short-term horizons. Day-ahead electricity markets, in which electricity producers submit bids for their energy generation, are present in many wholesale electricity markets.

At the day-ahead level of a wholesale electricity market, energy producers bid freely their energy production. Typically, each generation unit has a fixed start-up operation cost, a fixed variable production cost, a technical minimum and a technical maximum on the allowable energy production that it can produce in each time period. The price-offers for energy are submitted in advance for each hour of the following day. An independent system operator (ISO) clears the market, allocating quantities to the participating producers so as to minimize the total system bid-cost for satisfying the demand for energy. This is carried out by solving an optimization problem whose objective minimizes the total cost of electricity production. Several alternatives can be used as objective functions in this problem, the two most common of which are the actual cost for energy production or the bid-cost ensuing from the producers' price-offers.

In this dissertation, we adopt the view of an individual (*strategic* in what follows) producer who participates in a dayahead electricity market, in which the commitment and dispatching of the generation units are determined by an ISO. Assuming that this producer has full knowledge of the technical characteristics as well as of the bidding offers of all remaining producers, we consider the problem of selecting his optimal price-bids for energy generation. Here, the word optimal pertains to the fact that after the market is cleared by the ISO, the profit that the strategic producer will realize should be the maximum possible. This problem arises naturally in open displayed tentative-market auctions, in which participants submit bids repetitively until the market is called. It is also an important subproblem in fixed-point iterative numerical procedures that aim to uncover the joint optimal bidding strategies of multiple producers in closed auctions, in which these producers submit sealed bids (Andrianesis et al., 2013a [1]).

1.2 Dissertation contribution

The main contribution of the present dissertation lies in the developement of several efficient specialized solution algorithms for the problem of devising optimal price-offers for energy producers participating in day-ahead electricity markets. A significant amount of research has also addressed this problem in the related literature. Most of the related works utilize either a suitable reformulation combined with generic optimization software, or a heuristic solution procedure in order to solve these models. In this dissertation, we follow a slightly different approach. We formulate the problem as a mixed integer bilevel optimization model, in which binary variables are utilized to model the commitment of the electricity generation units. This, in conjunction with the imposition of a strictly positive lower bound on the energy quantity of each unit should it enter the market, adds a strong combinatorial component to our model, which prohibits the application of the KKT optimality conditions in order to get an equivalent single-level formulation. Instead, we develop heuristic as well as exact algorithmic methodologies utilizing key results from the theory of integer parametric programming in order to solve the problem under consideration.

First, we consider an elementary problem variant, in which the planning horizon consists of a single time period, and, consequently, the strategic producer must submit a single price-offer to the ISO. This price-offer, as well as the energy quantities of all generation units, are treated as continuous variables in this problem variant. We develop an exact algorithm for the solution of this problem, which utilizes important findings from the theory of integer parametric programming, and we report experimental results demonstrating its efficiency on random problem instances. We conclude with a discussion on several computational issues pertaining to the behavior of this algorithm, and an outline of how the proposed methodology can be modified to fit alternative market designs.

Then, we consider the multi-period variant of the problem, in which the strategic producer must submit a price-offer for each time period of the planning horizon to the ISO. In this problem variant, both these price-offers, as well as the energy quantities of all generation units, are restricted to integer values. Utilizing the theoretical properties of this problem, we develop both a heuristic as well as an exact algorithmic solution methodology for tackling it. More effective between the two naturally turns out to be the heuristic approach, which works iteratively, optimizing a single price-offer at each iteration, given that the remaining ones are kept fixed at their current values. We present experimental results demonstrating that it provides high quality solutions, while exhibiting reasonable computational requirements. We also demonstrate how the underlying theory can be utilized for the generation of valid inequalities to a suitable relaxation of the original formulation, in which the so-called bilevel feasibility of the obtained solution is not guaranteed. These inequalities are exploited within a cutting-plane framework by the exact solution approach for identifying the global optimum of the problem.

Next, we develop an improved version of the exact solution algorithm for the multi-period variant of the problem. The significant advancement of this version lies in the incorporation of special optimality conditions for the lower-level problem, ensuring that the energy quantity distribution in each time period of the planning horizon is optimal for the corresponding set of active producers in that time period. These optimality conditions are incorporated directly in the existing formulation, guiding and expediting the search for the optimal solution. Thus, solving the original problem to global optimality becomes equivalent to identifying the optimal set of active producers in each time period of the planning horizon. In order to exclude from consideration those solutions for which these sets are not optimal, the algorithm utilizes a suitable modification of the valid inequalities employed in the previous algorithmic version. We illustrate the application of the proposed methodology on a small case study and we present computational results demonstrating its behavior and performance on randomly generated problems. These results reveal that the proposed methodology is capable of handling medium sized problems using reasonable computational resources.

The development of the proposed solution methodologies comprises an original approach to the problem under consideration, which exhibits significant research interest and can be pursued and extended in many fruitful ways. It can be utilized to overcome computational difficulties encountered in realistic problems, thus comprising a valuable tool for practitioners. Moreover, with respect to the actual design and operation of an energy market, the present research signifies important contribution which is twofold. On the one hand, it can assist electricity producers in developing bidding strategies that will maximize their individual profit; on the other hand, it allows system operators to identify potential price manipulations by individual producers, and devise rules that will prevent them.

1.3 Structure of the dissertation

The present dissertation comprises original research, part of which has been presented in international scientific conferences (Kozanidis et al., 2011[36]; Kostarelou and Kozanidis, 2013[31]; Kostarelou and Kozanidis, 2014[32]; Kostarelou and Kozanidis, 2018[33]; Kozanidis and Kostarelou, 2020[35]) and has been published in international scientific journals (Kozanidis et al., 2013[37]; Kostarelou and Kozanidis, 2020[34]). Its remainder is organized in six chapters, and one appendix, as follows:

In Chapter 2, we present a literature review on bilevel optimization models in the context of electricity market design and operation, and on specialized solution algorithms for their treatment. In Chapter 3, we present the detailed definition and the model formulation of the problem under consideration, and we provide a theoretical background on bilevel optimization. In Chapter 4, we address the single-period variant of the problem. We develop the solution methodology for its treatment, we illustrate the application of this methodology on a small case study and we evaluate its computational performance. In Chapter 5, we develop a heuristic and an exact solution algorithm for the multi-period variant of the problem, and we present extensive experimental results evaluating their relative computational performance. In Chapter 6, we present the improved algorithmic version of the exact solution methodology introduced in Chapter 5 for the multi-period variant of the problem. We illustrate the application of the proposed methodology on a small case study and we present computational results demonstrating its behavior and performance on randomly generated problems. Finally, in Chapter 7, we review the research findings of this dissertation, we summarize our conclusions, and we point to promising directions for future research.

The appendix lists the journal and conference publications that have resulted from the present dissertation to date and links them to each corresponding chapter.

Chapter 2 Literature Review

In this section, we present a literature review on previous research works relevant to the contents of this dissertation, using a suitable classification. Specifically, we review bilevel optimization models in the context of electricity markets, as well as specialized solution algorithms for their treatment. Typically, such models are utilized for devising optimal price-offers of energy producers, or for clearing an energy market fairly and compensating the generation units for any losses. The most common approach for solving them is their reformulation as a single-level optimization model through the substitution of the lower-level problem by its first order KKT optimality conditions. This leads to a suitable single-level reformulation which is commonly solved with generic optimization software. Of course, this approach is only applicable when the lower-level optimization problem is convex. Other common solution approaches include the development of heuristics/metaheuristics, as well as the discretization of the optimization model's feasible space in order to reduce the problem complexity.

While the authors of several research works illustrate the application of the proposed solution algorithms on specific case studies, often they do not present generic experimental results, making it difficult to infer the average and worst-case computational performance of these algorithms, as well as the quality of the solutions they return on random problem instances. A related difficulty arises with techniques that employ generic optimization software, since the complexity of the problem renders difficult the employment of such techniques in full scale realistic problem instances. In what follows, we review these optimization models and solution approaches in more detail.

2.1 Bilevel optimization models in energy markets

A survey on optimization models for bidding in day-ahead electricity markets was recently published by Kwon and Frances (2012)[39]. Both deterministic as well as stochastic models are reviewed in this survey, as well as models that include unit commitment decisions. The classification scheme adopted by the authors examines the degree to which competition from other producers is directly incorporated into these models.

Among others, bilevel models for optimal strategic bidding of energy producers have been proposed by Barroso et al. (2006)[8], Bakirtzis et al. (2007)[5], Hu and Ralph (2007)[26], and Ruiz and Conejo (2009)[51]. A common characteristic these models exhibit is that their lower-level problem is convex. This allows the authors to reformulate the original problem as a single-level optimization model through the substitution of the lower-level problem by its KKT optimality conditions. Barroso et al. (2006)[8] and Bakirtzis et al. (2007)[5] utilize the binary expansion approach proposed by Pereira et al. (2005)[50] in order to deal with the nonlinear non-convex formulation that arises after this substitution, resorting to commercial optimization software for solving the mixed integer linear program that results. A similar approach is adopted by Ruiz and Conejo (2009)[51], who treat the problem as a mathematical program with equilibrium constraints (MPEC). They convert the resulting nonlinear problem into a mixed integer linear program through suitable reformulations, and solve it through generic optimization software. Hu and Ralph (2007)[26], on the other hand, consider a bilevel game-theoretic model of restructured electricity markets, in which the optimization problem of each player is also reformulated as an MPEC, and establish sufficient conditions for pure-strategy Nash equilibria.

Similar bilevel optimization models have also been proposed by Hobbs et al. (2000)[25], Li and Shahidehpour (2005)[40], and Zhao et al. (2008)[67]. These authors, too, develop KKT-based solution methodologies for the treatment of these models, which nevertheless settle for local optima due to the inherent problem complexity. In Hobbs et al. (2000)[25], this is achieved with a penalty interior point algorithm that addresses the multi-firm problem as a Nash game with multiple players in a game theoretic context, in Li and Shahidehpour (2005)[40] with a primal-dual interior point method that utilizes sensitivity functions for the generator's payoff with respect to his bidding strategies, while in Zhao et al. (2008)[67] with a surrogate optimization solution methodology.

A bilevel optimization model that treats the problem with a solution approach that finds local optima is proposed by Weber and Overbye (2002)[58]. At the upper-level, this model maximizes the welfare of an individual who may control both consumer and supplier units, whereas at the lower-level, it finds the power flow that maximizes social welfare. The social welfare is expressed as the total benefit of all consumers minus the total cost of all suppliers. The proposed solution algorithm, which is an iterative search procedure that utilizes Newton-type directions of improvement, is utilized in order to determine Nash-equilibria.

Another common technique that has been proposed for the treatment of bilevel optimization models in the context of electricity markets is the discretization of the upper-level decision maker's strategy space. Of course, this does not necessarily lead to the global optimum of the problem, since it excludes a-priori certain production-level choices from consideration. Such is the case with the works of Zhang et al. (2000)[64], Li et al. (2004)[41], and Soleymani et al. (2008)[54]. The solution approach proposed by Zhang et al. (2000)[64]

is a Lagrangian relaxation based methodology that considers uncertainty for the offers of the participating producers in the form of discrete bids and corresponding probabilities. On the other hand, the solution approach proposed by Li et al. (2004)[41] is an iterative procedure that searches for Nash equilibria, while the one developed by Soleymani et al. (2008)[54] is a game-based approach. In particular, the authors of the latter work make the explicit assumption that each producer predicts the market clearing price using a technique such as neural networks, fuzzy programming or neuro-fuzzy logic, and that each producer reaches the same price forecast. For the discretization of the producers' decision space, they assume that each of them is either risk seeker or risk averse or risk indifferent, which allows them to treat the problem as a game and search for Nash-equilibria.

Vahidinasab and Jadid (2009)[56] propose a multi-objective model that incorporates the suppliers' emission of pollutants, utilize the ε-constraint reduced feasible region method in order to deal with the multiple objectives, and solve the single level problem that results after the substitution of the lower-level problem by its first order optimality conditions with generic optimization software. Gabriel and Leuthold (2010)[20] transform the problem into an MPEC first, reformulate it as a MILP using disjunctive constraints and linearization, and solve the resulting single-level model with generic optimization software, too.

Many researchers have proposed the development of heuristic/metaheuristic solution approaches for addressing the strategic bidding bilevel optimization model. Typical examples are the papers by Ma et al. (2006)[45], Bajpai and Singh (2008)[4], Zhang et al. (2009[65]; 2011[66]), and Foroud et al. (2011)[19]. The approach proposed by Bajpai and Singh (2008)[4] is a fuzzy adaptive particle swarm optimization heuristic that addresses both the single and the multi-period case of the problem. Particle swarm optimization based approaches are also the ones proposed by Ma et al. (2006)[45] and Zhang et al. (2009[65];

2011[66]), whereas Foroud et al. (2011)[19] develop a genetic algorithm and a fuzzy satisfying methodology for addressing a multiobjective formulation which maximizes the profits of the participating producers.

Other related bilevel optimization models are formulated in the works of Gross and Finlay (2000)[23], Zhao et al. (2010)[68], and Fernández-Blanco et al. (2017)[17]. The authors of the first work study a framework for the analysis and formulation of bids in competitive electricity markets and develop a solution methodology exploiting a Lagrangian relaxation based approach. The authors of the second work formulate a bilevel optimization model in order to compare different clearing schemes in a game theoretic framework. Finally, the authors of the latter work develop a nonlinear bilevel optimization model for the clearing of a day-ahead market under marginal pricing, which is reformulated as a single-level mixed integer linear program using linear programming duality and KKT optimality conditions.

In an attempt to address the inherent stochasticity that the strategic bidding optimization problem exhibits, many authors, such as Gountis and Bakirtzis (2004)[22], and Badri et al. (2008)[3], have developed stochastic models for its formulation. In the former work, the authors use a heuristic solution technique that employs Monte-Carlo simulation and genetic algorithms to obtain the optimal solution, whereas in the latter work, the authors adopt a risk management approach that takes into account bilateral contracts and transmission constraints, and solve it through a primal-dual interior point methodology

The model that we study in this dissertation exhibits similarities to the ones that have been developed by Pereira et al. (2005)[50] and Fampa et al. (2008)[16]. The lower-level of these two models, however, is convex, which allows the authors to reformulate the problem as a single-level optimization problem through the substitution of the lower-level problem by its first order optimality conditions. In the former work, the authors utilize a binary expansion scheme in order to convert the resulting nonlinear, non-convex problem into a MILP, which they then solve with generic optimization software. In the latter work, the authors treat the problem as stochastic, maximizing the expected profit of the generation company which is expressed as the sum of the profits it realizes under different scenarios multiplied by the corresponding probability of occurrence of each associated scenario. In order to reach the optimal solution, they develop a heuristic and an exact solution approach, which are both based on a mixed integer reformulation.

2.2 Specialized solution methodologies for integer bilevel programming

The specific form of an integer bilevel program depends on the presence of upper/lower level constraints or not, on the presence of upper/lower continuous/discrete variables or not, and on the association of each decision variable to the decision maker (upper or lower) who controls it. Each of these factors affects critically the properties of the problem; therefore, a solution algorithm for mixed integer bilevel programming is typically applicable only to a particular formulation with a specific configuration. Popular solution techniques that have been developed for such problems include reformulation approaches, branch and bound/cut approaches, and parametric programming approaches.

Moore and Bard (1990)[47] develop one of the earliest branch and bound algorithms for mixed integer bilevel programming, highlighting the significant differences that the underlying theory exhibits as compared to that of integer single-level programming. More specifically, the authors illustrate that out of the three standard criteria used for fathoming subproblems during a typical integer programming branch and bound algorithm, only one (the relaxed subproblem has no feasible solution) is directly applicable to the case of mixed integer bilevel programming. The second one (the optimal objective of the relaxed subproblem is no better than the value of the incumbent) needs strong modification in order to become applicable, whereas the third one (the solution of the relaxed subproblem is feasible to the original problem) must be discarded altogether.

Exact and heuristic solution procedures based on the branch and bound technique for mixed integer bilevel programming with binary variables controlled by the leader and continuous variables controlled by the follower are presented by Wen and Yang (1990)[60]. The authors derive bound information on the optimal solution by solving the problem that results when the lower-level objective function is suppressed and all the decision variables are controlled by the leader. They point out that when the number of binary variables grows linearly, the computational time of the algorithm grows exponentially. For this reason, they propose a heuristic solution procedure that provides near optimal solutions in reasonable computational time. The proposed solution methodology can also handle the case in which the decision variables controlled by the leader are integer.

Bard and Moore (1992)[7] propose a solution algorithm for a class of bilevel models with binary decision variables at both levels and constraints at the lower-level only, under the assumption that all the objective and constraint coefficients are integer. The algorithm performs an enumerative branch and bound search procedure on the decision variables of the leader. This is achieved by replacing the leader's objective with a constraint that sets the value of this objective greater or equal to α, where α is a parameter, originally set equal to $-\infty$. Optimizing the follower's objective for incrementing values of α, a series of bilevel feasible solutions are obtained, which provide a monotonic improvement on the leader's objective value. The algorithm can be modified to accommodate the case in which the lower-level decision variables assume general integer values.

Wen and Huang (1996)[59] develop a tabu-search algorithm for a mixed integer bilevel programming formulation with binary upper-level and continuous lower-level decision variables. Gümüs and Floudas (2005)[24] propose solution algorithms for handling several classes of bilevel programs, including one with purely integer decision variables at both levels. In a closely related work, Domínguez and Pistikopoulos (2010)[14] develop a multi-parametric based solution algorithm for pure-integer and mixed integer bilevel programming. Multi-parametric is also the algorithm proposed by Faisca et al. (2007)[15] for the solution of bilevel programs, in which the authors express the lower-level optimization problem parametrically using the decision variables of the upper-level problem.

Tsoukalas et al. (2009)[55] develop a global optimization methodology for generalized semi-infinite, continuous minimax and bilevel optimization problems, which utilizes an auxiliary optimization problem for determining whether it is possible to attain a specific objective value. By performing a search on candidate objective values, the global optimum is identified. Global optimization is also the approach developed by Mitsos (2010)[46] for the solution of mixed integer nonlinear bilevel programming problems. It utilizes fixed value and optimal value function reformulations in order to obtain lower and upper bounds on the optimal objective.

For a class of bilevel programming problems with continuous upper-level and integer lower-level decision variables, Köppe et al. (2010)[38] develop a solution methodology that expresses the lower-level objective as a function of the upper-level decision variables. The proposed methodology is based on the theory of integer parametric programming and runs in polynomial time when the number of lower-level decision variables is fixed. If the infimum of the problem is not attained, the algorithm is able to find an ε-optimal solution whose objective value approximates the sought infimum in polynomial time, too. Wiesemann et al. (2013)[61]

examine the computational complexity of pessimistic bilevel programming problems and study the conditions under which the existence of an optimal solution is guaranteed. For a special class of bilevel models in which the feasible set of the lower-level problem does not depend on the leader's decisions, they also develop an iterative solution procedure which generates a sequence of finite dimensional semi-infinite programming approximation problems.

DeNegre and Ralphs (2009)[13] illustrate how the standard branch and cut solution methodology for integer single-level programming can be suitably extended to the case of integer bilevel programming. The proposed algorithm employs a branch and cut tree, solving a suitable relaxation at each of its nodes. If the solution obtained is bilevel feasible, then the search in the associated subtree terminates; otherwise, a suitable cut is added which excludes this solution without excluding any bilevel feasible solution. In a closely related work, Caramia and Mari (2015)[10] develop two solution algorithms for purely integer bilevel programming. The first one reformulates the model in order to relax bilevel feasibility, utilizing suitable valid cuts to eliminate the bilevel infeasible solutions encountered. The second algorithm is a branch and cut methodology, which, upon each branching decision, utilizes valid inequalities to eliminate large sets of bilevel infeasible solutions. Fischetti et al. (2017)[18] develop yet another branch and cut exact solution methodology for mixed-integer linear bilevel programs. The proposed approach applies a family of cuts to the problem relaxation in which the follower's objective is suppressed and bilevel feasibility is thus not guaranteed.

Saharidis and Ierapetritou (2010)[52] propose another algorithm for the solution of mixed integer bilevel programs, which is based on the decomposition of the initial problem into the restricted master problem (RMP) and a series of problems named slave problems

(SPs). The proposed approach is based on the Benders decomposition method where, at each iteration, the set of variables controlled by the leader is fixed, generating the SP. The RMP is a relaxation of the mixed integer bilevel program composed by all the constraints including only integer decision variables controlled by the leader. The RMP interacts at each iteration with the current SP through the addition of three types of cuts produced using Lagrangean information from the current SP. These cuts are the classical Benders cuts (optimality Benders cut and feasibility Benders cut) and a third cut referred to as exclusion cut which is used if the RMP is not restricted by the last generated Benders cut. The lower and upper bound provided (in the case of minimization) from the RMP and the (best found so far) SP are updated in each iteration, respectively. The algorithm converges when the difference between the upper and lower bound is within a small difference ε. In the case of mixed integer bilevel programming, the lower-level KKT optimality conditions cannot be used directly to transform the bilevel problem into a single-level problem. The proposed decomposition technique, however, allows the use of these conditions and transforms the mixed integer bilevel program into two single-level problems. The proposed methodology can solve mixed integer bilevel programs in which the leader controls discrete (binary or general integer) decision variables, which can appear in any constraint or in the objective function.

Xu and Wang (2014)[62] develop a branch-and-bound algorithm for mixed integer linear bilevel programming, in which each branching decision is associated with several subproblems. Kleniati and Adjiman (2015)[30] extend the global optimization solution framework they had previously developed for continuous bilevel programming (Kleniati and Adjiman, 2014a[28]; 2014b[29]) to the case of mixed integer bilevel programming. An exact solution algorithm for integer bilevel linear programming is developed by Wang and Xu

(2017)[57]. The proposed methodology is a branch and bound type search termed *watermelon algorithm*, which utilizes disjunctive cuts to eliminate bilevel infeasible solutions.

Yue et al. (2019)[63] present a reformulation and decomposition algorithm for mixed integer bilevel linear programming. The proposed algorithm implements a column and constraint generation methodology utilizing a master problem and suitable subproblems on a projection-based single-level problem reformulation. Finally, Lozano and Smith (2019)[44] present an exact solution algorithm for mixed integer bilevel programming. The proposed methodology implements a single-level value function reformulation which is used to obtain lower and upper bounds on the optimal objective.

2.3 Qualitative comparison to existing approaches

In this subsection, we perform a qualitative assessment elucidating the differences and similarities that the solution methodologies we develop in the following chapters exhibit in comparison to existing specialized solution methodologies for integer bilevel programming. In order to retain their general and wide applicability, these methodologies do not depend on peculiar assumptions regarding the structure of the underlying problem. While this renders them robust and powerful, in many occasions it comes at the price of necessitating excessive computational resources. For example, some of these methodologies require the substitution of all integer variables by pure binary ones. While this is always doable for variables with finite bounds, it may result in formulations with excessive size that cannot be handled efficiently when the problem under consideration exhibits realistic characteristics. A key observation in support of this is the fact that some of these methodologies report computational results for small sized problems only. On the other hand, our proposed algorithmic methodologies exploit a special attribute of the problem under consideration which is not always present in generic formulations, i.e., the fact that the leader's decisions affect the cost of alternative lower-level solutions but not the lower-level feasible region.

With respect to the specifics of our proposed solution methodologies, it should be noted that the exact algorithm resembles many of the generic solution methodologies in that it deals with the problem relaxation in which bilevel feasibility is suppressed. This is also the case with the works of DeNegre and Ralphs (2009)[13], Caramia and Mari (2015)[10], Fischetti et al. (2017)[18], and Wang and Xu (2017)[57]. However, whereas in our case the cuts for eliminating bilevel infeasible solutions utilize integer parametric programming theory holding true due to the independency of the lower-level feasible set from the upper-level decision variables, in the case of existing generic techniques these cuts are based on central properties holding true in more general problem formulations. More specifically, in the case of the algorithm by DeNegre and Ralphs (2009)[13], these cuts are based on the general theory of eliminating non-integer solutions from convex hulls of polyhedrons which are not integral. In the case of the algorithm by Caramia and Mari (2015)[10], these cuts are based on valid bounds on the optimal objective expressed in a nonlinear fashion. On the other hand, in the case of the algorithm by Fischetti et al. (2017)[18], these cuts are based on upper bound assertions of the lower-level optimal objective, while in the case of the algorithm by Wang and Xu (2017)[57], these cuts are based on the general optimality conditions that the optimal solution to the integer bilevel program must satisfy.

The solution methodologies that we develop in the present work remain applicable both when the price-offers of the strategic producer and the energy quantities of the generation units are continuous, as well as when they are discrete, with minor differences. This is a key advantage of our proposed approach as opposed to other existing ones, such as the ones by Köppe et al. (2010)[38], Xu and Wang (2014)[62], and Lozano and Smith

(2019)[44], whose applicability depends strongly on the integrality of the upper and/or the lower-level decision variables.

As far as the remaining generic solution methodologies are concerned, we make the following additional important observations. The algorithm of Gümüs and Floudas (2005)[24] necessitates the substitution of all integer variables by expressions involving pure binary variables, which may render the size of the problem unmanageable in large realistic cases such as the one that we address. This is also the case with the algorithm of Domínguez and Pistikopoulos (2010)[14]. The solution algorithms of Mitsos (2010)[46], Wiesemann et al. (2013)[61] and Kleniati and Adjiman (2015)[30] are tested on nominal problems only, with size considerably smaller than that of realistic problems. On the other hand, in the solution algorithm of Saharidis and Ierapetritou (2010)[52], the integer decision variables should be controlled by the upper level decision maker, although they could appear in both levels of the model formulation.

Considerable computational difficulties are also inherent in methodologies employing KKT techniques such as the ones by Gümüs and Floudas (2005)[24], Mitsos (2010)[46], and Yue et al. (2019)[63], since they necessitate the introduction of dual variables as well as big-M formulations for the treatment of the associated complementary slackness constraints. This is yet another factor that may introduce intolerable obstacles in large realistic problems. Another limitation of the approaches utilizing KKT conditions is that their applicability depends on the convexity of the lower level problem. Several of the restrictions that are present in practice, however, necessitate a formulation that utilizes discrete variables and associated integer programming modelling techniques; even in the case that the associated constraints are linear, this eliminates the convexity of the lower level problem, rendering the KKT approach inapplicable. Additionally, the utilization of the KKT optimality conditions requires the implicit assumption that suitable regularity assumptions (constraint qualifications) are valid, so that the lower level optimal solution is a KKT point; clearly, this is an assumption that cannot be always considered as valid.

The reformulation algorithm of Yue et al. (2019)[63] necessitates the introduction of a number of constraints which grows exponentially with the number of lower-level integer variables. This may render its application on large scale realistic problems intractable. For this reason, the authors propose the adoption of a decomposition approach that employs column and constraint generation in order to overcome the associated difficulties. Nevertheless, note that this algorithm, as well as that of Lozano and Smith (2019)[44], are tested on a large collection of problems of variable and considerable size with very satisfactory results.

Chapter 3 Problem Definition and Model Development

In this chapter, we present a detailed definition of the problem under consideration and we develop the optimization model framework for its formulation. Due to the fact that the resulting formulation belongs to the class of bilevel optimization models, we also provide a fundamental review on bilevel programming theory. We conclude with a short study of an alternative objective for the model formulation, which is suitable when a uniform clearing scheme is in effect.

3.1 Problem formulation

We consider a set of energy generation units participating in a multi-period day-ahead electricity market. The start-up cost and the technical characteristics (minimum and maximum output) of each production unit are fixed and known. Each corresponding producer must submit his energy price-offers (bids) for the planning horizon to an ISO, who is responsible for clearing the market and determining the unit commitments and energy dispatches that will satisfy the energy demand at the minimum total system bid-cost. With these in mind, the mathematical notation and formulation of the optimization problem the ISO is aimed to solve are as follows:

Sets:

I production units, indexed by i.

Parameters:

T number of time periods of the planning horizon,

$p_{i,t}$ price-offer of producer i for one unit of energy in time period t ($i \in I$, $t = 1,\ldots,T$),

s_i startup cost of unit i ($i \in I$),

m_i technical minimum of unit i ($i \in I$),

M_i technical maximum of unit i ($i \in I$),

d_t demand for energy in time period t ($t = 1,\ldots,T$),

$z_{i,0}$ binary parameter denoting the status of unit i at the beginning of the planning horizon ($i \in I$).

Decision Variables:

$q_{i,t}$ energy quantity of unit i in time period t ($i \in I$, $t = 1,\ldots,T$),

$z_{i,t}$ binary variable that takes the value 1 if the energy quantity of unit i in time period t is positive, and 0 otherwise ($i \in I$, $t = 1,\ldots,T$),

$y_{i,t}$ binary variable that takes the value 1 if unit i is switched on in time period t while being off in time period t-1, and 0 otherwise ($i \in I$, $t = 1,\ldots,T$).

ISO's Problem

$$\text{Min } f = \sum_{i \in I} \sum_{t=1}^{T} \left(p_{i,t} q_{i,t} + s_i y_{i,t} \right) \tag{3.1}$$

$$\text{s.t.} \quad \sum_{i \in I} q_{i,t} = d_t, \; t = 1,\ldots,T \tag{3.2}$$

$$m_i z_{i,t} \le q_{i,t} \le M_i z_{i,t}, \; i \in I, \; t = 1,\ldots,T \tag{3.3}$$

$$y_{i,t} \ge z_{i,t} - z_{i,t-1}, \; i \in I, \; t = 1,\ldots,T \tag{3.4}$$

$$y_{i,t}, \; z_{i,t} \text{ binary}, \; i \in I, \; t = 1,\ldots,T \tag{3.5}$$

$$q_{i,t} \ge 0, \; i \in I, \; t = 1,\ldots,T \tag{3.6}$$

The objective function (3.1) minimizes the total system bid-cost for providing energy. Constraint (3.2) is the market clearing constraint ensuring energy balance (production equal to demand) for each time period of the planning horizon. Constraint set (3.3) imposes the technical minima and maxima of the participating units. Constraint set (3.4) ensures correct values for the decision variables $y_{i,t}$ which are used to impose the start-up costs in the objective. More specifically, the difference ($z_{i,t}$ - $z_{i,t-1}$) can take any of the values -1, 0 and 1. Variable $y_{i,t}$ should take the value 1 in the latter case, and this is correctly imposed by this constraint. In the other two cases, both 0 and 1 are feasible for $y_{i,t}$, but the value 0 is implicitly imposed by the fact that the coefficient of $y_{i,t}$ in the objective is positive. Finally, constraints (3.5) and (3.6) impose integrality on decision variables $y_{i,t}$ and $z_{i,t}$, and nonnegativity on decision variables $q_{i,t}$, respectively. Parameters m_i, M_i and d_t are always positive integers for all i and t, with $1 < m_i < M_i$. Decision variables $q_{i,t}$ may alternatively be restricted to integer values, in which case the optimal energy dispatch will be non-fractional, keeping this way the final solution more ‘elegant’. In any case, the solution methodologies that we develop next remain applicable with minor modifications both when variables $q_{i,t}$ are continuous variables, as well as when they are integer. Finally, note that, besides constraints (3.6), the non-negativity of these variables is also implied by constraints (3.3).

After the optimal generation plan is determined, each participating producer is compensated in full for his startup cost, and is also paid a market clearing price for each MWh he contributes to the system, according to the clearing payment scheme in effect. This price may be the same for all producers under a *uniform market* clearing scheme, or the corresponding submitted price-offer under a *pay-as-bid* market clearing scheme. In the former case, the uniform clearing price is also known as *system marginal price* (*smp*), since it represents the marginal cost for energy, i.e., the additional cost that should be paid for increasing the demand by one MWh.

Each producer faces the problem of selecting the optimal price-offer that he should submit to the ISO for each time period of the planning horizon, so that, after the market is cleared and the energy quantities of all the participating units are determined, his profit is maximized. Assuming perfect information of the market's technical characteristics, the corresponding profit maximization problem of an individual strategic producer is modeled as a bilevel optimization problem. For the formulation of the strategic producer's optimization problem, we introduce the following additional mathematical notation:

Parameters:

c_1 unit variable production cost of the strategic producer,

C_1 price cap for the price-offers of the strategic producer.

Decision Variables:

$p_{1,t}$ price-offer of the strategic producer for one unit of energy in time period t ($t = 1,\ldots,T$).

The strategic producer faces the bilevel optimization problem introduced next, which includes as part of its constraint set the ISO optimization problem:

Strategic Producer's Problem

$$\underset{p_{1,t}}{\text{Max}}\ F_1 = \sum_{t=1}^{T} \left(p_{1,t} - c_1 \right) q_{1,t} \tag{3.7}$$

$$\text{s.t.} \quad c_1 \leq p_{1,t} \leq C_1,\ t = 1,\ldots,T \tag{3.8}$$

$$\left(y_{i,t}, z_{i,t}, q_{i,t} \right) \in \arg \min_{y_{i,t}, z_{i,t}, q_{i,t}} f = \sum_{i \in I} \sum_{t=1}^{T} \left(p_{i,t} q_{i,t} + s_i y_{i,t} \right) \tag{3.9}$$

$$\text{s.t.} \ \sum_{i \in I} q_{i,t} = d_t,\ t = 1,\ldots,T \tag{3.10}$$

$$m_i z_{i,t} \leq q_{i,t} \leq M_i z_{i,t},\ i \in I,\ t = 1,\ldots,T \tag{3.11}$$

$$y_{i,t} \geq z_{i,t} - z_{i,t-1},\ i \in I,\ t = 1,\ldots,T \tag{3.12}$$

$$y_{i,t},\ z_{i,t} \text{ binary},\ i \in I,\ t = 1,\ldots,T \tag{3.13}$$

$$q_{i,t} \in Z^{+},\ i \in I,\ t = 1,\ldots,T \tag{3.14}$$

In the context of this bilevel programming formulation, the strategic producer's optimization problem is called the upper-level problem, while the ISO's optimization problem is called the lower-level problem. The lower-level problem is always a part of the constraint set of the upper-level problem; therefore, the upper-level problem cannot be treated in isolation. The unit index 1 is used to specify the strategic producer whose profit is maximized in the upper-level objective F_1 (3.7). This profit depends on his energy quantities, $q_{1,t}$, which are lower-level decision variables, as well as on his price-offers, $p_{1,t}$, which are upper-level decision variables. The start-up cost is not included in the upper-level objective, since producers are typically compensated in full for such costs. Constraint set (3.8) imposes a lower and an upper bound on the price-offers of the strategic producer. More specifically, typical market rules dictate that each price-offer must be at least equal to the associated unit's variable production cost, and at most equal to a *price-cap* set by the market regulator. Decision variables $p_{1,t}$ may additionally be restricted to integer values, depending on the particular problem definition. The lower-level optimization problem defined by (3.9)-(3.14) is actually the ISO's optimization problem (3.1)-(3.6) introduced before.

At first glance, it might seem unrealistic for a particular producer to have full knowledge of his competitors' bids. Note, however, that in a realistic environment, each participating producer might end-up solving a sequence of optimization models such as the above, using educated estimates of the other producers' bids, based on historic data. This would enable the comparative evaluation of alternative self-bidding strategies based on the thorough examination of different scenarios and assumptions. In addition, the above formulation and the subsequent solution methodologies developed next could also be fruitfully utilized for identifying equilibrium points within an iterative game setting, in which each producer takes turn responding to the bids of the remaining producers that have been previously announced by solving his own profit maximization problem and announcing his own bids in return. As no generic solution methodologies are available for this class of problems, the development of specialized solution approaches could turn out to be highly beneficial for such research pursuits.

3.2 Uniform clearing scheme

In the case of a uniform clearing scheme, the upper-level objective of the above formulation is expressed as follows instead:

$$\max_{p_{1,t}} F_1 = \sum_{t=1}^{T} (\lambda_t - c_1) q_{1,t} , \tag{3.15}$$

where λ_t is the *smp* of time period t ($t = 1,\ldots,T$). In order to capture the true marginal cost of the associated time period, λ_t is defined as the dual variable of the corresponding energy balance constraint (3.10). This introduces considerable difficulties because the lower-level problem is an integer program and does not possess dual variables in the traditional sense. Several approaches have been proposed for computing λ_t, many of which suffer from inequities necessitating additional uplifts and side-payments in order to reach a market equilibrium that fairly clears the market (see Andrianesis et al., 2013a[1] and 2013b[2], for example); as a consequence, the relevant research is very active. In the current work, we focus on the algorithmic aspect of the problem, and we intentionally do not deal with such market design issues. Nevertheless, we note that one of the most common methodologies for computing the *smp* is to find the optimal solution of the integer linear problem (3.9)-(3.14) first, and then to solve the continuous problem that results after the integer variables are fixed to their optimal values in this solution. Based on marginal pricing theory (Schweppe et al., 1988[53]), the energy commodities are paid at the shadow price of the market clearing constraint computed this way.

The system marginal price reflects the marginal cost of generating one additional unit of energy (typically 1 MWh for an hourly discretized problem). This is interpreted as the additional cost, in terms of the objective function, which is needed to generate this additional unit of energy. In the present formulation, the system marginal price is set by the marginal unit, which is the unit that will produce the additional MWh for energy. Note that the indivisibilities (also known as non-convexities) of the problem may result in a system marginal price which differs from the maximum bid accepted, i.e. from the highest bid that is scheduled. Extra-marginal units can be set to produce at their technical minimum, without affecting the system marginal price. This particularity of the problem may result in losses for the extra-marginal participating production units, creating the need for (make-whole) side payments with some sort of bid-cost recovery mechanism (see for e.g. Andrianesis et al., 2013a[1]; 2013b[2], for a discussion on this issue). Dealing with these difficulties is not within the scope of the present dissertation, and thus will not be addressed.

Under a uniform pricing scheme, the *smp* is not always unambiguously defined, due to the presence of the indivisibilities and the fact that alternative lower-level optimal solutions may lead to different *smp* definitions. In order to be able to eliminate these ambiguities, should they arise, the imposition of a conflict resolution set of rules is necessary and is typically in effect as a common practice in actual realistic markets. For the remainder of this dissertation, we utilize the following widely adopted set of such rules, which unambiguously determine the *smp* in any possible case that can arise:

Rule 1: If at the optimal ISO solution there is a production unit whose energy dispatch in time period t is strictly between its technical minimum and its technical maximum, then the *smp* of this time period is equal to the price-offer of this unit.

Rule 2: If at the optimal ISO solution no production unit prescribed by Rule 1 exists in time period *t* and there is at least one production unit whose energy dispatch is equal to its technical minimum, then the *smp* of this time period is equal to the minimum price-bid of any unit producing at its technical minimum.

Rule 3: If at the optimal ISO solution all participating units produce at their technical maximum in time period *t*, then the *smp* of this time period is equal to the maximum price-bid of any unit producing at its technical maximum.

To see why the above set of rules handles any possible case that might arise, note that one can easily prove that if at the optimal solution to model (3.9)-(3.14) there are more than one units producing strictly between their technical minimum and technical maximum in some time period, then the price-bids of these units for this time period must be equal, and there is an optimal solution in which this is true for at most one of them. Naturally, this unit is the optimal choice for satisfying the extra energy unit demand in this case, since the price-offer of this unit will never be greater than the price-offer of any unit producing at its technical minimum. The rationale behind Rule 2 is straightforward since the minimum price-bid unit producing at its technical minimum stems as the optimal choice for providing an extra MWh of energy when all participating units produce either at their technical minimum or at their technical maximum. Finally, the rationale behind Rule 3 becomes clear when one considers that if the energy demand is reduced by one MWh when all participating units produce at their technical maximum, then the optimal choice for accommodating the corresponding perturbation, i.e., for reducing the total energy supply by one unit, will be the maximum price-bid unit out of them. It becomes obvious from this discussion that Rules 2 and 3 exploit the right and left hand-side shadow price definitions, respectively, in order to overcome the obstacles raised by the integralities that the model formulation involves.

Note that the exact set of rules utilized for the *smp* definition is a market design issue; thus, the above rules may slightly vary from case to case. This, however, does not have any significant impact on the solution methodologies that we develop next for the treatment of the problem. In fact, these methodologies are independent of the set of rules in effect, in that they can be suitably modified to accommodate any such set. Another important market design issue is the choice of the upper-level objective function. For reasons of completeness, all the proposed methodologies that we develop next are suitably adjusted both for the case of a pay-as-bid market clearing scheme in which expression (3.7) is adopted as the upper-level objective, as well as for the case of a uniform price market clearing scheme in which expression (3.15) is adopted instead.

3.3 Bilevel programming fundamentals

The model formulation (3.7)-(3.14) fits in the general multilevel optimization modeling framework, which is a special branch of mathematical programming that deals with programs whose feasible set is implicitly determined by a sequence of nested optimization problems. The most studied case is the case of bilevel programs, a subset of the decision variables of which is required to be an optimal solution to a second mathematical program. The problem can be considered as a two-person game with the two decision makers making their decisions hierarchically. The first decision maker, referred to as the leader, controls a subset of the problem's decision variables, attempting to solve an optimization problem which includes in its constraint set a second optimization problem solved by the second decision maker (referred to as the follower), who controls the remaining decision variables. In our case, the leader corresponds to the strategic producer, while the decision variables that he controls are his energy price-offers. On the other hand, the follower corresponds to the ISO, while the decision variables that he controls are the unit commitments and the energy quantities of the participating producers. In general, a bilevel program is non-convex, and finding its global optimum is an arduous task.

Bilevel programming formulations are encountered in the context of several interdisciplinary areas, such as agricultural planning, government policy making, economic planning, financial management, warfare optimization, transportation planning, optimal pricing, ecological programming, chemical design, production planning, optimal resource allocation, etc (e.g., see Dempe, 2010[12]). This wide applicability in conjunction with the solution difficulty that bilevel programs exhibit has motivated researchers to develop specialized algorithmic methodologies for solving them. Although this has rendered the related research area highly active, none of the solution methodologies that have been developed to date is able of accommodating generic bilevel programming formulations. In fact, the large complexity of the problem makes it rather unlikely that this will be achieved, at least over the next few years.

A key characteristic of our formulation is that the upper-level decision variables do not appear at the lower-level constraint set; thus, the follower's feasible region is not influenced by the leader's decisions. However, the comparative evaluation of alternative lower-level solutions is influenced by the leader's decisions, since the upper-level decision variables appear in the follower's objective. The term *reaction set* is used to denote the set of responses of the follower for a particular leader action, i.e., the set of optimal solutions to the lower-level problem for a particular set of upper-level decision variable values. Finally, the term *inducible region* (IR) is the set of every upper-level feasible solution, and corresponding lower-level optimal solution, i.e., the set over which the leader may optimize his objective. A solution that belongs to the IR, i.e., a solution for which the lower-level decision variables constitute a lower-level optimal solution for the associated upper-level decision variable values, is called bilevel feasible. In the problem formulation (3.7)-(3.14), the leader's objective is bilinear since the generation unit quantities are determined at the lower level, while all the decision variables that he controls are integer. On the other hand, the lower-level problem is a linearly constrained integer program with a bilinear objective, since this objective becomes linear when the leader's decisions are known.

Even under the assumption that the feasible region of problem (3.7)-(3.14) is non-empty and compact, an optimal solution may not exist. This is a well-known pitfall in bilevel programming (Bard, 1998[6]) that may occur when the optimal solution of the lower problem is not unique. The basic theory of bilevel optimization (Candler and Norton, 1977[9]) prohibits the cooperation between the upper and the lower-level decision makers. Thus, it is not possible for the upper-level decision maker to force the lower-level decision maker to choose a particular lower-level optimal solution in the case of multiple optima, which, in turn, implies that the strategic producer may not always be able to attain his maximum profit.

Most approaches that have been proposed for circumventing this difficulty modify slightly the problem definition and the associated model formulation. A highly popular one called *optimistic* (*pessimistic*) approach (Loridan and Morgan, 1996[43]), suggests the selection of the most (least) favorable solution to the upper-level decision maker in case of multiple lower-level optima. This implies that there is some way for the upper-level decision maker to convince the lower-level decision maker to choose a particular lower-level optimal solution. In the particular application under study, the unit with the lowest variable production cost is typically favored in order to resolve such conflicts, mainly because such units push towards lower total system costs. This motivates the units to reduce their costs and become more competitive. In the present work, we adopt the optimistic approach for the resolution of such conflicts, because it guarantees the existence of an optimal solution under reasonable regularity assumptions (Dempe, 2002[11]). It should be clarified that this choice regarding the proposed approach for dealing with multiple lower-level optima does not affect crucially the applicability of the proposed methodology; it only affects which actual solution will be identified as optimal.

Even under this common rule, the upper-level decision maker may still be able to effectively lead the lower-level decision maker to select the most favorable (to his upper-level problem) optimal solution. More specifically, if the upper-level decision maker places an offer that is “infinitesimally” lower than the offer for which the lower-level problem exhibits multiple optima, then he may cause a “mathematical problem” to the lower-level decision maker, since the latter will not be able to find an optimal solution. One way for him to resolve this issue would be to allow the upper-level decision maker to place the offer for which the lower-level problem exhibits multiple optima, and assure him that the most favorable solution to the upper-level problem will be selected, instead of the solution that results when the least costly units are favored. We illustrate this with a small example, next.

Consider a problem with a single-period planning horizon, two production units and the data shown in Table 3-1, under a uniform clearing scheme. Assume that the unit variable production cost of the strategic producer (production unit 1) is 15 €/MWh, the price cap is 60 €/MWh, and the energy demand is equal to 450 MWh.

Table 3-1 Data of the small numerical example

Unit (*i*)	m_i	M_i	$p_{i,1}$	s_i
1	240	400	-	100
2	100	300	40	50

For $p_{1,1} \in [15, 40)$, marginal is the first unit, which implies that the system marginal price is equal to $p_{1,1}$. For $p_{1,1} = 40$, the lower problem has an infinite number of optimal solutions. More specifically, any solution with $z_{1,1} = 1$, $z_{2,1} = 1$, $q_{1,1} \in [240, 350]$ and $q_{2,1} = 450 - q_{1,1}$ is optimal with $f^* = 18{,}150$. Assuming that the second unit has lower variable production cost and that the lower-level decision maker favors the least costly unit, then for $p_{1,1} = 40$, the lower-level decision maker would select the solution $z_{1,1} = 1$, $q_{1,1} = 240$, $z_{2,1} = 1$, $q_{2,1} = 210$, which favors the least costly (second) unit, instead of the solution $z_{1,1} = 1$, $q_{1,1} = 350$, $z_{2,1} = 1$, $q_{2,1} = 100$, which is most favorable to the strategic producer. In this case, the optimal decision for the strategic producer would be to select a value for $p_{1,1}$ that is infinitesimally close to but strictly lower than 40, which would result in the original bilevel problem not having an optimal solution. To resolve this issue, the lower-level decision maker would then have to allow the strategic producer to place the offer $p_{1,1} = 40$ and select the most favorable solution to his upper-level problem, namely, $p_{1,1}^* = 40$, $z_{1,1}^* = 1$, $q_{1,1}^* = 350$, $z_{2,1}^* = 1$, $q_{2,1}^* = 100$ and $\lambda_1^* = 40$, with $f^* = 18{,}150$ and $F_1^* = 8{,}750$, instead of favoring the second unit which has the lowest variable production cost.

One of the well established important results of integer bilevel programming (Moore and Bard, 1990[47]) is the fact that the optimal objective function value of an integer bilevel program's continuous relaxation does not always provide a valid bound on its optimal objective; therefore, solution procedures that are based on such relaxations may fail. To illustrate this interesting result in the context of the present work, consider the continuous relaxation of the above example, which results when the variables $z_{1,1}$ and $z_{2,1}$ are not restricted to binary values, but are allowed to take any value in the interval [0, 1]. The global optimal solution of this problem is $p_{1,1}^* \in [15, 39.916]$, $z_{1,1}^* = 1$, $q_{1,1}^* = 400$, $z_{2,1}^* = 0.167$, $q_{2,1}^* = 50$ and $\lambda_1^* = 40.167$ with $f^* = 2{,}108.333 + 400p_{1,1}$ and $F_1^* = 10{,}066.8$. Note that multiple optima depending on the exact value of variable $p_{1,1}$ exist. Thus, in this case, the optimal objective value of the continuous relaxation is larger than the optimal objective value of the mixed integer problem. Note that, in order to find the global optimal solution of the relaxed problem, we can replace the lower problem (which is linear and continuous since $p_{1,1}$ is a known parameter in it) with constraints ensuring primal and dual feasibility, as well as equality between the primal and the dual objective function values. The formulation that arises is quadratic (since it includes the product of variables λ_1 and $q_{1,1}$ in the objective) and can be solved with a typical quadratic programming solution algorithm.

Assume now that, for the same problem, the demand is equal to 400 MWh. The new global optimal solution of the problem is $p_{1,1}^* = 40.312$, $z_{1,1}^* = 1$, $q_{1,1}^* = 400$, $z_{2,1}^* = 0$, $q_{2,1}^* = 0$ and $\lambda_1^* = 40.312$, with $f^* = 16{,}224.8$ and $F_1^* = 10{,}124.8$. The global optimal solution of this problem's continuous relaxation, however, is $p_{1,1}^* = 39.916$, $z_{1,1}^* = 1$, $q_{1,1}^* = 400$, $z_{2,1}^* = 0$, $q_{2,1}^* = 0$ and $\lambda_1^* = 40.166$, with $f^* = 16{,}066.4$ and $F_1^* = 10{,}066.4$. Thus, in this case, the optimal objective value of the continuous relaxation is smaller than the optimal objective value of the mixed integer problem.

This small example illustrates that a solution procedure based on bounds obtained from continuous relaxations may fail. Note that in the case that $d_1 = 400$ MWh, this happens even though the optimal solution to the continuous relaxation is bilevel feasible (i.e., the lower-level solution is optimal for $p_{1,1} = 39.916$, resulting in a feasible solution for the original bilevel problem). The solution procedures that we develop in the next chapters do not utilize relaxations, but an important result from the theory of parametric integer programming, which has been known since the 1970s.

In our case, the feasible region of the lower-level problem is nonconvex due to the integrality of the decision variables $y_{i,t}$ and $z_{i,t}$. If this were not the case, however, then this set would be convex, since all constraints are expressed with linear functions. The existence of an optimal solution to the lower-level problem is ensured due to the fact that the feasible set is closed and compact, while all decision variables have finite bounds. Under reasonable assumptions such as that of the optimistic approach adoption, the global solution of the bilevel problem is also ensured. In fact, if decision variables $p_{1,t}$ and $q_{i,t}$ are also constrained to integer values, then the cardinality of the feasible set of both the lower as well as the upper-level problem is finite.

Chapter 4 The Single-Period Variant of the Problem

In this chapter, we consider the single-period variant of Problem (3.7)-(3.14), in which the planning horizon consists of a single time period. In this case, each producer must submit a single price-bid to the ISO, while index t is naturally suppressed as redundant. We develop an exact algorithm for the solution of this problem, which utilizes important findings from the theory of integer parametric programming, and we report experimental results demonstrating its efficiency on random problem instances. We conclude with a discussion on several computational issues pertaining to the behavior of this algorithm, and an outline of how the underlying theory can be modified to fit alternative market designs.

4.1 Solution methodology

When all the problem parameters have finite values, the feasible region of problem (3.7)-(3.14) is bounded, although non-convex. For a particular value of p_1, the feasible region of the lower level problem is non-convex, too, due to the presence of the integralities. Let $f^*(p_1)$ be the optimal objective value of the lower-level problem as a function of the price-bid of the strategic producer. The algorithm that we develop for the solution of the problem is based on the following important result:

Proposition 4.1 *The function $f^*(p_1)$ is non-decreasing, piecewise-linear and concave.*

Proof The fact that $f^*(p_1)$ is non-decreasing is trivial, since increasing the value of p_1 does not change the feasible region of the problem, but only increases the total system bid-cost of the solutions in which unit 1 participates. The fact that $f^*(p_1)$ is piecewise-linear and concave is due to Noltemeier (1970)[48]. □

The validity of Proposition 4.1 is a consequence of the fact that the feasible region of the lower-level problem can be replaced by its convex hull (the integer polyhedron) without altering the optimal solution, reducing this way the problem to the case of continuous linear programming, for which this result is well established. Proposition 4.1 can be utilized to solve the single-period variant of the problem defined by (3.7)-(3.14) parametrically, by applying a solution algorithm which employs a search procedure that has been introduced by Geoffrion and Nauss (1977)[21]. Consider the case of a uniform clearing scheme first. The aim of this algorithm is to identify all the distinct value-ranges of p_1 in the interval $[c_1, C_1]$ for which the lower-level optimal solution (in terms of the decision variables values) and the associated marginal unit remain constant. The global optimal solution of the problem is then identified as the point at which the profit of the strategic producer is maximized. For any of these distinct value-ranges of p_1, there exist only two possible cases, as described next.

The first case is when the strategic producer determines the system marginal price (unit 1 is the marginal unit), i.e., when p_1 is equal to the shadow price of the demand constraint (3.10). For that value-range of p_1, the strategic producer achieves his maximum profit when his price-bid becomes equal to the right endpoint of the corresponding value interval. To see why this is true, note that since the system marginal price is determined by the strategic producer in this range, this is the maximum possible price with which he can be compensated for each MWh of energy that he will provide to the market. Offering a larger price-bid will result in a different optimal solution for the lower-level problem, leading to a different value-range for p_1. The second case is when the system marginal price is determined by a different producer. For that value-range, the strategic producer is indifferent about the specific value of p_1, since he will be paid at that fixed system marginal price, independently of his own price-bid. In turn, his profit remains constant in that range, since his energy quantity remains constant, too.

The main difference between the two aforementioned cases is that in the first one, the profit of the strategic producer increases linearly with his price-bid within the particular value-range of p_1, whereas in the second one, it remains constant. By comparing the maximum profit that the strategic producer can achieve in any value-range of p_1 (each of which is associated with a distinct lower-level optimal solution), we can easily identify the optimal value of p_1 that results in his maximum profit. This methodology can be carried out with the procedure described next.

Suppose that we solve the lower-level problem for $p_1 = c_1$ and $p_1 = C_1$ and that the optimal values of the decision variables in the two solutions that we obtain are represented by $x^*(c_1)$ and $x^*(C_1)$, respectively. Proposition 4.1 implies that $f^*(c_1) \leq f^*(C_1)$ and that the line connecting the points $(c_1, f^*(c_1))$ and $(C_1, f^*(C_1))$ provides a lower bound on $f^*(p_1)$ for any p_1 that belongs to the interval $[c_1, C_1]$, for if there were some value k of p_1 in this interval such that the point $(k, f^*(k))$ lied below this line, the concavity of the function $f^*(p_1)$ would be contradicted (see Figure 4-1).

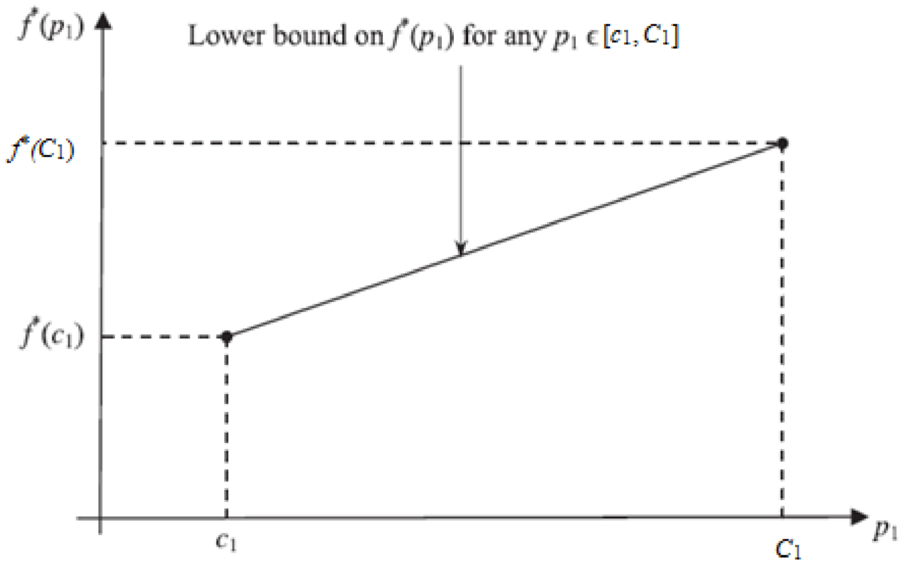

Figure 4-1: Lower bound on $f^*(p_1)$ determined by two extreme optimal solutions

Therefore, if $x^*(c_1) = x^*(C_1)$, the search terminates, concluding that the optimal solution of the lower-level problem is the same for any feasible value of p_1. In this case, the energy quantity, q_1^*, that the strategic producer provides to the system is the same for the entire interval of p_1, and his maximum profit is realized at the point of this interval for which the system marginal price is maximized. If the first unit is the marginal unit, then, according to the intuition provided above, the system marginal price is maximized for $p_1 = C_1$ and the global maximum profit of the strategic producer is equal to $(C_1 - c_1)q_1^*$. If the first unit is not marginal, then the system marginal price remains constant in the entire interval $[c_1, C_1]$, which implies that any value of p_1 in this interval is optimal, with an associated maximum profit for the strategic producer equal to $(\lambda - c_1)q_1^*$.

If $x^*(c_1) \neq x^*(C_1)$, then either these two are the only possible lower-level optimal solutions, or there exists at least one additional lower-level optimal solution, realized for some value of p_1 in the interval (c_1, C_1). Proposition 4.1 is properly utilized again to see which of the two is true, as explained next. Consider the line that represents the value of the lower-level objective function when the lower-level solution is fixed at $x^*(c_1)$ and p_1 increases above c_1, and the line that represents the value of the lower-level objective function when the lower-level solution is fixed at $x^*(C_1)$ and p_1 decreases below C_1 (see Figure 4-2).

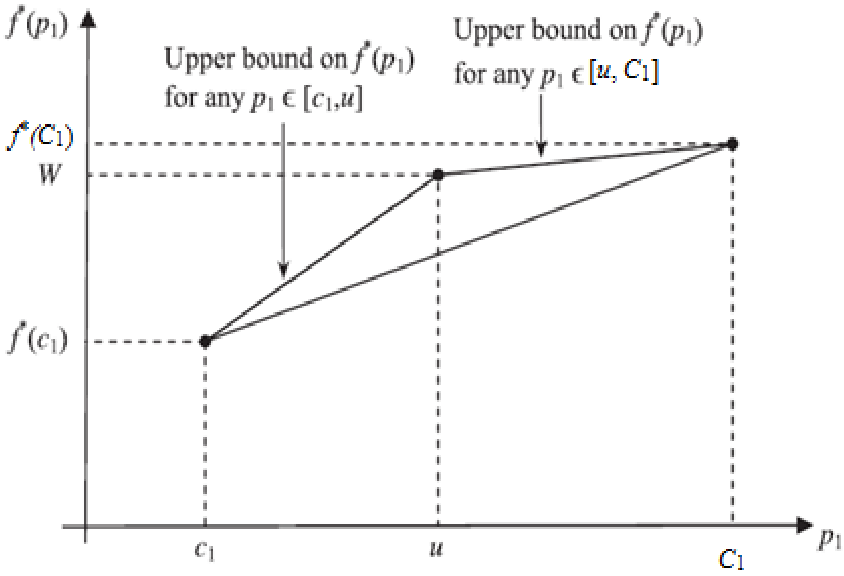

Figure 4-2: Upper bound on $f^*(p_1)$determined by two extreme optimal solutions

Let $p_1 = u$ be the point at which these two lines intersect, and suppose that the two objective values are equal to W at that point. The line connecting the point $(c_1, f^*(c_1))$ with the point (u, W) provides an upper bound on $f^*(p_1)$ for any p_1 that belongs to the interval $[c_1, u]$, for if there were some value k of p_1 in this interval such that $f^*(k)$ were above this line, we would be able to improve the optimal lower-level objective at $p_1 = k$ using the solution $x^*(c_1)$ instead, which is a contradiction. Similarly, the line connecting the point (u, W) with the point $(C_1, f^*(C_1))$ provides an upper bound on $f^*(p_1)$ for any p_1 that belongs to the interval $[u, C_1]$, for if there were some value k of p_1 in this interval such that $f^*(k)$ were above this line, we would be able to improve the lower-level objective at $p_1 = k$ using the solution $x^*(C_1)$ instead, which is a contradiction.

In order to check if, in addition to $x^*(c_1)$ and $x^*(C_1)$, another lower-level optimal solution exists for some value of p_1 between c_1 and C_1, we solve the lower-level problem for $p_1 = u$. If $f^*(u) = W$, then the procedure terminates concluding that $x^*(c_1)$ is the optimal lower-level solution for $c_1 \leq p_1 \leq u$ and $x^*(C_1)$ is the optimal lower-level solution for $u \leq p_1 \leq C_1$. Otherwise, we repeat the above procedure, considering the following two pairs of lines (see Figure 4-3). The first pair is comprised of the line that represents the lower-level objective function when the solution is fixed at $x^*(c_1)$ and p_1 increases above c_1 (this is the first line considered in Figure 4-2), and the line that represents the lower-level objective function when the solution is fixed at $x^*(u)$ and p_1 decreases below u. The second pair is comprised of the line that represents the lower-level objective function when the solution is fixed at $x^*(C_1)$ and p_1 decreases below C_1 (this is the second line considered in Figure 4-2), and the line that represents the lower-level objective function when the solution is fixed at $x^*(u)$ and p_1 increases above u. Due to the same intuition as before, these two pairs of lines provide improved upper bounds on $f^*(p_1)$ at the two corresponding intervals. At the same time, the two lines connecting the points $(c_1, f^*(c_1))$ with $(u, f^*(u))$ and $(u, f^*(u))$ with $(C_1, f^*(C_1))$ provide improved lower bounds on $f^*(p_1)$.

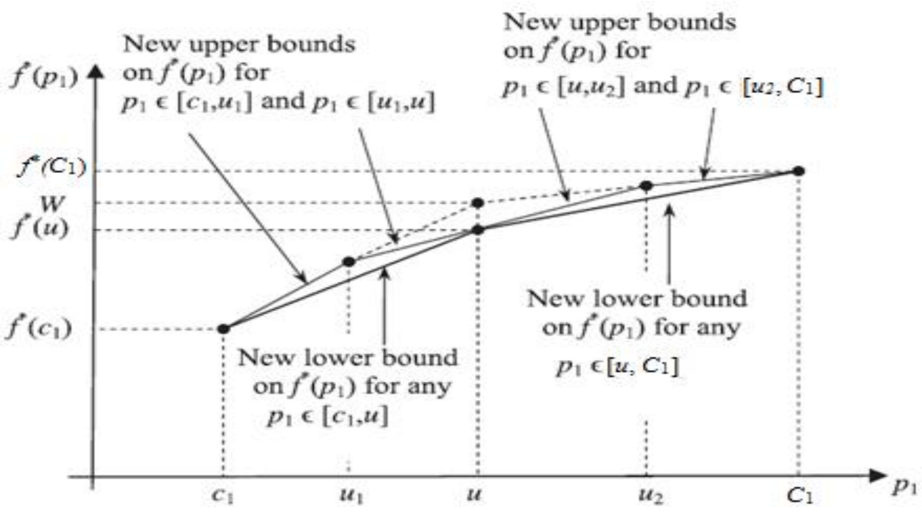

Figure 4-3: Improved lower and upper bounds on $f^*(p_1)$

The parametric search procedure continues similarly, until all the distinct value-ranges of p_1 and the associated lower-level optimal solutions are identified. At that point, the problem's global optimal solution is identified easily, by comparing the maximum profit that the strategic producer can attain in each distinct interval. Let $z_1^*(p_1)$ and $q_1^*(p_1)$ be the optimal values of decision variables z_1 and q_1, respectively, as a function of p_1. The above procedure can be significantly expedited when the following important result is exploited:

Proposition 4.2 *The function $q_1^*(p_1)$ is non-increasing.*

Proof The slope of each linear segment that comprises the function $f^*(p_1)$ is equal to the optimal energy quantity of the first unit, q_1^*, at the solution obtained when p_1 is set equal to the left endpoint of the associated interval. The validity of the proposition results from the fact that $f^*(p_1)$ is concave. □

The significance of Proposition 4.2 is that if we know the optimal value of q_1 for some $p_1 = k$, then this value can be imposed as an upper bound on the optimal value of q_1 on any instance of the problem in which p_1 is set greater than k. Moreover, whenever we identify a value k of p_1 for which $z_1^*(k) = 0$, then we do not need to apply the parametric search procedure in the interval $(k, C_1]$, since Proposition 4.2 ensures that the first unit will not participate in the market in that value-range and its corresponding profit will be equal to 0. This can lead to significant computational savings, especially for large scale problems.

4.2 Application of the algorithm

In this section, we illustrate the application of the proposed algorithm on a case study with five production units and a single-period hourly time horizon. The technical characteristics, the price-bids and the startup costs of the production units are shown in Table 4-1. The technical minima and maxima are given in MW, the startup costs in €, and the price-bids for energy in €/MWh. The unit variable production cost of the strategic producer (production unit 1) is 50 €/MWh, the price cap is 150 €/MWh, and the energy demand is equal to 1,000 MWh. The problem data are not fictitious, but correspond to factual units participating in the Greek electricity market, as described by Andrianesis et al., 2013b[2].

Table 4-1 Data of the production units

Unit (*i*)	m_i	M_i	p_i	s_i
1	240	377	-	13,000
2	144	476	52	10,000
3	240	384	57	15,000
4	105	188	65	27,000
5	60	144	72	24,000

Table 4-2 presents the results of the application of the proposed solution algorithm. For each distinct value-range of p_1, this table shows the lower-level optimal solution in the form (q_1^*, q_2^*, q_3^*, q_4^*, q_5^*), the system marginal price, the marginal unit, the optimal lower-level objective function value (f^*), and the corresponding objective value of the upper-level problem (F_1).

Table 4-2 Results of the application of the solution algorithm

Value-range of p_1	**Lower-level optimal solution**	**System marginal price**	**Marginal generation unit**	f^*	F_1
[50, 52]	(377, 383, 240, 0, 0)	52	2	71,596 + 377p_1	754
(52, 57]	(284, 476, 240, 0, 0)	p_1	1	76,432 + 284p_1	(p_1-50)284
(57, 111.58]	(240, 476, 284, 0, 0)	57	3	78,940 + 240p_1	1,680
(111.58, 150]	(0, 476, 384, 0, 140)	72	5	105,720	0

The results of Table 4-2 indicate that the maximum profit the strategic producer can attain for any value of p_1 that belongs to the interval [50, 150] is equal to 1,988 and is achieved when p_1 is equal to 57. The corresponding lower-level optimal solution is (284, 476, 240, 0, 0), resulting in a total system bid-cost of 92,620. Note that the solution (240, 476, 284, 0, 0) is also optimal to the lower-level problem for $p_1 = 57$, but is not preferred because it is less favorable to the strategic producer.

When p_1 belongs to the interval [50, 52], the best price-bid of the strategic producer is not unique, since the system marginal price is determined by unit 2. As a result, the strategic producer is indifferent for any value of p_1 in that range, since both his energy quantity (377) as well as the market clearing price (52) remain constant. The situation is similar when p_1 belongs to the interval (57, 111.58], with the system marginal price being equal to 57, and the energy quantity of the strategic producer dropping to 240. Finally, in the value-range (111.58, 150] the strategic producer does not participate in the market; therefore, he realizes zero profits. Figure 4-4, Figure 4-5 and Figure 4-6 depict the optimal lower-level objective function value (f^*), the corresponding upper-level objective function value (F_1), and the system marginal price, respectively, as a function of p_1. In accordance with Proposition 4.1, the function $f^*(p_1)$ is non-decreasing, piecewise-linear and concave, comprising of four linear segments with slopes 377, 284, 240 and 0, respectively.

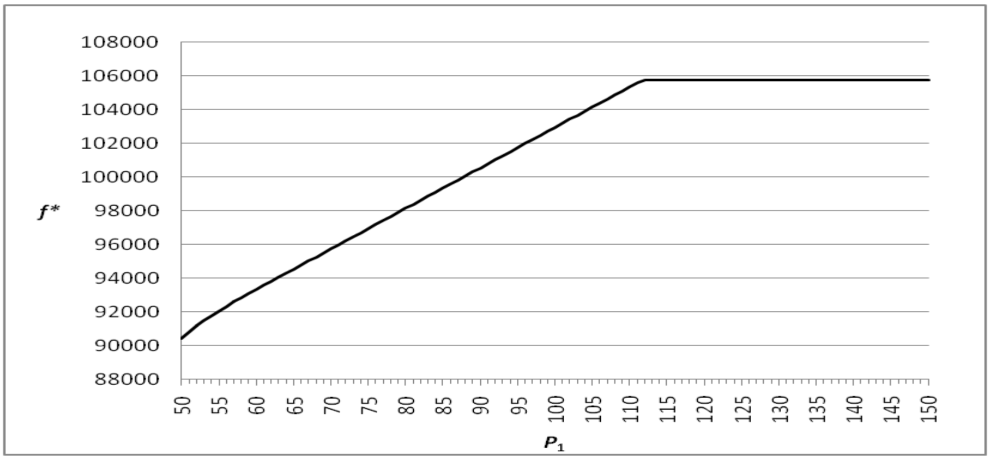

Figure 4-4: Optimal objective value of the lower level problem (f^*) as a function of p_1

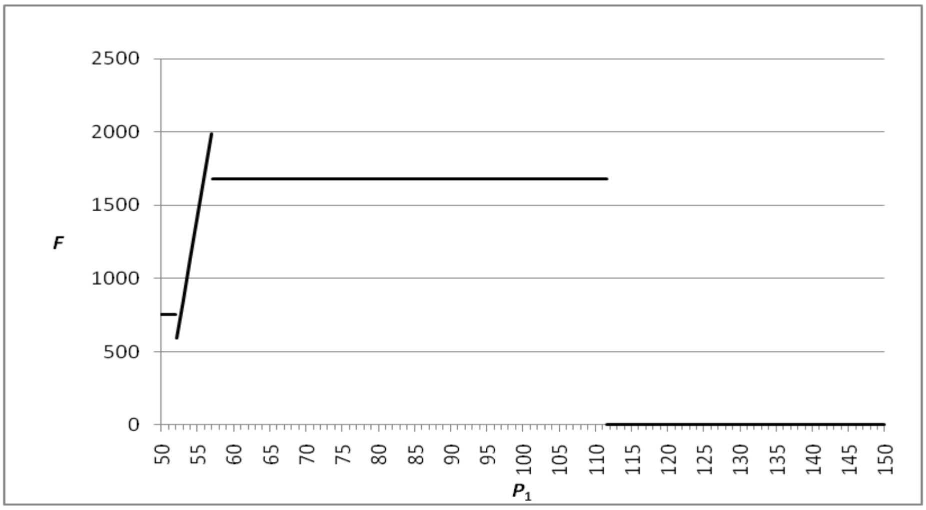

Figure 4-5: Objective value of the upper level problem (F_1) as a function of p_1

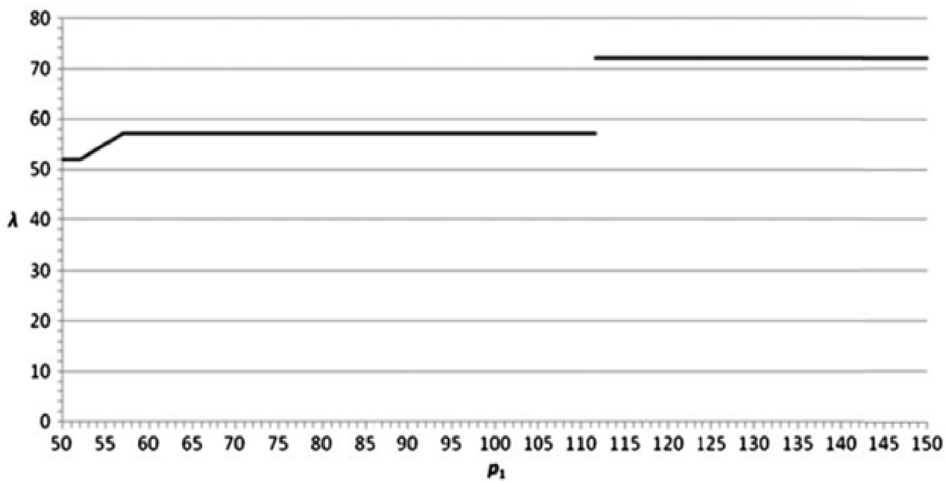

Figure 4-6: System marginal price (λ) as a function of p_1

4.3 Computational Requirements

In this subsection, we elaborate on some computational issues related to the application of the proposed solution algorithm. Problem (3.7)-(3.14) is NP-hard, since solving the lower problem for a particular value of p_1 is NP-hard, due to the presence of the integralities. The total computational effort required for the application of the proposed solution algorithm depends on the computational effort required for the solution of each lower-level problem and on the total number of such problems that need to be solved. Each of the lower-level problems contains $|I|$ continuous and $|I|$ binary decision variables, where $|I|$ is the total number of production units, and can be solved using a typical branch and bound/branch and cut solution algorithm that utilizes linear or Lagrangean relaxations. The total number of times the lower problem needs to be solved, on the other hand, is equal to $2n - 1$, where n is the number of distinct linear segments comprising the function $f^*(p_1)$, each of which corresponds to a different lower-level optimal solution (note that this relationship is valid for $n > 1$, since two distinct lower-level problems need to be solved when $n = 1$). This implies that $n - 1$ times the algorithm employed for the lower-level problem will find an optimal solution that has already been identified.

A heuristic was proposed by Jenkins (1982)[27] in order to reduce the computational effort of the parametric search procedure described above. Adapted to our setting, this heuristic adopts the rule that if the optimal commitment of the production units at two distinct values of p_1, k_1 and k_2 ($c_1 \leq k_1 < k_2 \leq C_1$) are the same, then the same commitment will be optimal for any value of p_1 in the interval $[k_1, k_2]$. Thus, at the lower-level problems that need to be solved for any value of p_1 in this interval, the corresponding binary variables are fixed in advance at these values. The small example with the data parameters displayed in Table 4-3 demonstrates that this assumption is not always valid for the problem under consideration.

Table 4-3 Data of the second numerical example

Unit (*i*)	m_i	M_i	p_i	s_i
1	70	100	-	2
2	45	55	2.1	1
3	30	70	2	10

Assume that the energy demand is equal to 130 for this problem. At the optimal solution of the lower-level problem when $p_1 = 1$, we have $z_1^* = 1$, $z_2^* = 0$ and $z_3^* = 1$; at the optimal solution of the lower-level problem when $p_1 = 2$, we have $z_1^* = 1$, $z_2^* = 1$ and $z_3^* = 0$; finally, at the optimal solution of the lower-level problem when $p_1 = 3$, we have $z_1^* = 1$, $z_2^* = 0$ and $z_3^* = 1$. This small example demonstrates that it is possible for the optimal unit commitment to be the same for two distinct values of p_1 and different for a third value that lies in-between. Thus, the above approach remains heuristic in our case.

4.4 Computational results

In this subsection, we present computational results demonstrating the performance of the proposed solution algorithm on random problem instances. We implemented the proposed solution algorithm in C/C++, utilizing the commercial optimization software LINGO 11.0 (2011) for the solution of each of the lower-level problem instances. The code that we developed feeds LINGO with the data of each lower-level problem that needs to be solved, and in turn, LINGO returns the associated optimal solution. Our computational experiments were performed on an i7-920 @ 2.7 GHz Intel processor with 3 GB system memory. We used 5 different values (i.e., 100, 200, 300, 400 and 500) for the total number of production units that participate in the market, and solved 30 random problem instances for each of them.

The random problem instances were generated as follows: For the problem parameters of the remaining (besides the first one) production units, we utilized the data of factual generation units participating in the Greek electricity market (see Andrianesis et al., 2013b[2]). More specifically, the parameters (technical minimum and maximum, startup cost and price-bid) of each generation unit i ($i = 2,\ldots,|I|$), were set equal to the corresponding parameters of one of these factual units. The specific unit with which this association was made was selected randomly.

Let r be a random number distributed uniformly in the interval (0, 1), which is renewed after being used once. After the data of every production unit i ($i = 2,\ldots,|I|$) had been generated, the data of the first production unit were generated as follows: The technical minimum of unit 1 was distributed uniformly in the interval $(\min_{i>1} m_i, \max_{i>1} M_i)$ the technical maximum of unit 1 was set equal to $m_1 + (r(\max_{i>1}(M_i - m_i) - \min_{i>1}(M_i - m_i)))$ the startup cost of unit 1 was distributed uniformly in the interval $[\min_{i>1} s_i, \max_{i>1} s_i)$ the unit variable production cost of unit 1 was set equal to $0.95(\min_{i>1} p_i)$ and the price cap was set equal to $1.05(\max_{i>1} p_i)$. This way, the feasible values of p_1 were drawn from a wide interval whose left and right endpoints were smaller and larger, respectively, than the price-bid of any other production unit. Finally, the demand for energy was distributed uniformly in the interval $[\max_i m_i, \sum_i m_i]$.

The results of our computational experiments are presented in Table 4-4. More specifically, columns 2 and 3 of this table show the proposed algorithm's average and maximum computational times over the 30 problem instances of each problem size. The next column shows the average percentage of the total computational time that was taken up by LINGO. As expected, these values are close to 100%, which confirms that most of the total computational time is spent on finding the optimal solutions of the lower-level problems, whereas the percentage of the total computational time that is devoted to the remaining steps of the parametric search procedure is negligible. The next two columns show the average and maximum number of times that the lower-level problem was solved. Finally, the last two columns show the number of distinct lower-level solutions, which is the same with the number of distinct linear segments comprising the function $f^*(p_1)$.

Table 4-4 Computational results

$\|I\|$	Times (seconds) Avg	Times (seconds) Max	% LINGO times Avg	# of lower-level problems solved Avg	# of lower-level problems solved Max	# of distinct lower-level optimal solutions Avg	# of distinct lower-level optimal solutions Max
100	2.30	10.264	99.05	2.133	5	1.132	3
200	28.08	205.888	99.57	4.433	7	2.633	4
300	125.75	392.736	99.75	4.767	7	2.833	4
400	138.72	572.426	99.78	4.433	5	2.667	3
500	175.87	672.984	99.84	4.367	5	2.694	3

The efficiency of the proposed solution algorithm becomes immediately clear, since its computational requirements are quite low, even for large scale problems. As the results of Table 4-4 demonstrate, these requirements seem to increase reasonably with problem size. The variability of the solution times appears significant, with the average time being approximately 15% of the corresponding maximum time in the worst case. Additionally, the number of times that the lower-level problem needs to be solved, which depends on the number of distinct lower-level optimal solutions, does not seem to increase as the problem size increases. This is an important observation because it demonstrates that increasing the number of production units increases the computational effort needed to solve each of the lower-level problems, as expected, but does not increase considerably the total number of distinct such problems that need to be solved. Consequently, the resulting increase in the total computational effort is mostly attributable to the larger computational requirements of the lower-level problems.

4.5 Alternative remuneration schemes

According to the remuneration scheme that we consider in this dissertation, strategic producers are fully compensated for their startup cost. This scheme is actually in effect in the

Greek electricity market, along with additional payments according to a cost recovery mechanism, an issue which, as already mentioned, is not within the scope of the present dissertation. A slightly different remuneration plan involves additional compensation of each unit's startup cost, based on the dual variable of the associated constraint that fixes the commitment of this unit at its optimal value. Letting π_i denote the dual variable associated with the corresponding constraint, an additional amount equal to $(\pi_i - s_i)z_i$ is paid to unit i upon the clearing of the market, to compensate it for its startup cost. The proposed algorithmic procedure remains applicable in this case too, with only a minor modification, as explained next.

Consider any of the strategic producer's energy bid intervals, for which the lower-level optimal solution remains the same in terms of energy dispatch and unit commitment. As pointed out by O'Neill et al. (2005)[49], for a mixed integer linear program (MILP) such as this one, the dual variable of the strategic producer's unit commitment constraint always takes the value that makes his *as-bid* profit equal to 0. In other words, the total amount by which the producer is compensated for his energy production and his unit commitment based on the corresponding shadow prices is equal to the sum of his bid-based production cost plus his startup cost. With this in mind, the new optimal solution can still be computed by comparing the maximum profit that the strategic producer can realize in each of these distinct intervals. Next, we illustrate this in the small example of Section 4.2.

In accordance with the above intuition, the dual variable associated with the unit commitment of the strategic producer is equal to 12246, and 12623, for p_1 = 50, and 51, respectively. For any value of p_1 in the interval [50, 52], the maximum value that this variable takes is 13000 when p_1 = 52; consequently, the producer's profit becomes maximum for p_1 = 52. Computing similarly the maximum profit that the strategic producer can attain in each of the other three intervals, we find that the maximum possible profit for the strategic producer is equal to 14779.2 for p_1 = 111.58. An interesting observation that arises is that, since the lower-level optimal solution remains the same within each distinct interval, there is an incentive for the producer to offer the maximum possible bid in each interval in order to maximize his *as-bid* cost. Of course, this value is equal to the right endpoint of the associated interval. This small example demonstrates that a market which adopts a remuneration scheme that also compensates each unit for its startup cost based on the shadow price of the corresponding unit commitment constraint is rather poorly designed, because it allows for severe price manipulations by the participating producers.

Finally, note that, in this dissertation, we adopt the assumption that the producers do not bid strategically on their startup costs. In practice, market participants submit incremental energy bids that reflect their variable production costs and their unit commitment costs (e.g. startup costs). ISOs currently put more restrictions on the submitted unit commitment costs than on the energy bids; the former cannot be frequently changed, whereas the latter are allowed to vary on an hourly basis. The rationale behind these rules is that an increase in the frequency of adjustment of startup offers could enhance the ability of price manipulation. The concern is that increasing the frequency of adjustment of unit commitment offers could enhance the ability of generator owners to withhold capacity in order to raise wholesale power prices, for example in response to a short-lived system contingency. For this reason, the assumption of bidding in terms of the variable cost only is in direct alignment with current practice.

4.6 Block bids

The assumption of fixed marginal cost is not absolutely necessary for the application of the proposed methodology. The proposed algorithm can also be utilized in the case where the offers of the energy producers are allowed to be in the form of price-quantity pairs. This practice is consistent with the market rules in current electricity markets which, taking advantage of the recent advancements of commercial MILP solvers, formulate the Unit Commitment and Economic Dispatch problem as a MILP problem, allowing the participants to bid in block-bids, i.e. submit price-quantity pairs for their energy offers. They therefore approximate the traditional quadratic cost function with a piecewise linear one, or, equivalently, the linear marginal costs, with stepwise blocks.

To elaborate more on this idea, assume that in the small numerical example presented in Subsection 4.2, the strategic producer is allowed to submit two distinct energy bids, one for the first 300 MWhs and another for the remaining 77 MWhs. We use two binary variables, $z_{1,1}$ and $z_{1,2}$, to model the commitment of the first generation unit, where $z_{1,1} = 1$ if part of this unit's first 300 MWs are injected to the system and $z_{1,2} = 1$ if part of this unit's last 77 MWs are injected to the system additionally. Additionally, the quantity variable q_1 is replaced by two corresponding variables $q_{1,1}$ and $q_{1,2}$, the production cost c_1 is replaced by two corresponding costs $c_{1,1}$ and $c_{1,2}$, and the energy bid p_1 is replaced by two corresponding energy bids $p_{1,1}$ and $p_{1,2}$. The problem formulation remains the same, except that the new upper-level objective function is expressed as $(\lambda - c_{1,1})q_{1,1} + (\lambda - c_{1,2})q_{1,2}$, the energy quantity q_1 in the energy balance constraint is replaced by expression $(q_{1,1} + q_{1,2})$, and expressions p_1q_1 and s_1z_1 in the lower objective function are replaced by expressions $(p_{1,1}q_{1,1} + p_{1,2}q_{1,2})$ and $s_1z_{1,1}$, respectively. The constraint $240z_{1,1} \leq q_{1,1} \leq 300z_{1,1}$ is added to the formulation to reflect the upper and lower bound on $q_{1,1}$, together with the constraints $q_{1,2} \leq 77z_{1,2}$ and $q_{1,1} \geq 300z_{1,2}$, imposing the upper bound on $q_{1,2}$ and ensuring that in order for $q_{1,2}$ to be positive, $q_{1,1}$ must be equal to 300. Besides the constraints ensuring that $p_{1,1}$ must lie between $c_{1,1}$ and C_1, and $p_{1,2}$ must lie between $c_{1,2}$ and C_1, additional constraints may also apply on the energy bids of the strategic producer. For example, the constraint $p_{1,2} \leq p_{1,1}+g$ is typically added to the model, to reflect the fact that, besides being non-decreasing, adjacent bids must also not differ by more than a known value, g.

Suppose that $c_{1,1}$ = 50, $c_{1,2}$ = 60 and g = 50 in the above example. When $p_{1,1}$ is fixed at its lower value, $c_{1,1}$, the structure of the problem remains unchanged; thus, the optimal lower-level objective function value remains a non-decreasing piecewise-linear and concave function of $p_{1,2}$. This implies that the proposed methodology can be utilized to compute the optimal value of $p_{1,2}$ when the value of $p_{1,1}$ remains fixed at $c_{1,1}$. For the particular example, any value of $p_{1,2}$ in the interval [60,100] is optimal with a corresponding profit for the strategic producer equal to 600. Next, we can compute the optimal value of $p_{1,1}$ when the value of $p_{1,2}$ is fixed at one of these optimal values. Continuing the same way, this procedure terminates when the values of $p_{1,1}$ and $p_{1,2}$ that are optimal for each other are identified. The quality of this solution depends on the initial solution used. Even though this procedure is heuristic, reapplying it many times with different initial solutions and choosing the best of them is expected to provide a satisfactory approximation to the problem's global optimum.

4.7 Summary

In this chapter, we addressed the problem of finding the optimal bidding strategy of an energy producer that participates in a single-period day-ahead electricity market, assuming full knowledge of the market's parameters. The use of discrete variables to represent the commitment of the production units prohibits the application of typical methodologies, such as the use of first-order KKT optimality conditions, for finding its global optimal solution. Although the feasible region of problem is non-convex, we proved interesting theoretical properties utilizing key results from the theory of parametric integer programming and the problem's special structure, and we developed an exact solution algorithm for obtaining the global optimum of this problem.

We illustrated the application of the proposed algorithm on a small case study with five production units and a single-period hourly time horizon, and we reported experimental results demonstrating its efficiency on random problem instances under a uniform clearing scheme. Although the problem is NP-hard, our computational results demonstrate the high efficiency of the proposed algorithm and its low computational requirements, even for large-scale problem instances. We also provided an outline of how the underlying theory can be modified to fit alternative market designs, such as those which include alternative remuneration schemes and/or those in which the offers of the energy producers are in the form of price-quantity pairs (block bids).

Chapter 5 The Multi-Period Variant of the Problem

5.1 Introduction

In this chapter, we consider the multi-period variant of the problem. In view of the absence of generic solution methodologies for integer bilevel programming, we utilize the theoretical properties of the optimization model under consideration to develop specialized solution methodologies for tackling it. First, we develop a heuristic solution approach, which, despite its relatively low computational requirements, appears to provide high quality solutions. This significant advantage makes this methodology suitable for the treatment of large realistic problems. It works iteratively, optimizing the bidding offer of a single time period at each iteration, while keeping all the other ones fixed at their current values. This is accomplished through the comparative evaluation of the distinct lower-level optimal solutions identified by varying parametrically the single price-offer subject to optimization. We go on to elucidate how the underlying theory can be utilized to enable the generation of valid inequalities to a suitable relaxation of the original problem in which the so-called bilevel feasibility of the obtained solution is not guaranteed. These inequalities are exploited within a cutting-plane framework by the exact solution approach for identifying the global optimum of the problem. While the considerably larger computational requirements of this methodology limit its applicability on small sized problems only, the associated framework that we develop opens up new interesting research directions towards the development of efficient exact algorithmic methodologies for the solution of the problem. In addition, we also elaborate on the conditions under which the applicability of the proposed methodologies remains valid on more complex model extensions that may involve additional problem characteristics.

Note that in this, as well as in the next chapter, we assume that the strategic producer price-offers, $p_{1,t}$ ($t = 1,\ldots,T$), and the energy quantities of the generation units, $q_{i,t}$ ($i \in I$, $t = 1,\ldots,T$) are additionally constrained to integer values, i.e., we append the following two constraints in the upper and the lower-level of the model formulation, respectively:

$$p_{1,t} \in Z^{+},\ t = 1,\ldots,T \tag{5.1}$$

$$q_{i,t} \in Z^{+},\ i \in I,\ t = 1,\ldots,T \tag{5.2}$$

These additional integrality restrictions are mainly imposed for avoiding numerical difficulties related to the modeling techniques that we employ for the solution of the associated optimization models; their inclusion does not affect the applicability of the solution methodologies that we develop next, but guards against unrealistic decision variable values with no practical meaning. In fact, it will become apparent that our proposed solution methodologies can be easily extended to the case that variables $p_{1,t}$ and/or variables q_{it} are continuous, too, with minor adjustments. In addition, note that the incorporation of these constraints leads to more meaningful and practical final solutions, thus resembling the realistic problem setting. In terms of the optimization model properties, note also that the integrality of these decision variables ensures that the cardinality of the feasible set of both the lower as well as the upper-level problem is finite.

5.2 Heuristic solution methodology

The heuristic solution methodology that we propose for the treatment of the multi-period problem variant utilizes the solution methodology developed in the previous chapter for the single period variant of the problem. Note that, given the price-offers of the participating producers, the lower-level problem remains an integer linear program. Therefore, its optimal objective is still a non-decreasing piecewise linear concave function of any single price-bid of the strategic producer. Thus, the optimal value of any single price-bid for the current values of all the remaining ones can be computed with the same exact procedure that was used in the single-period case. This involves again identifying the distinct lower-level optimal solutions that result by varying parametrically the price-offer subject to optimization, and selecting the corresponding value that results in the maximum profit.

Starting from an initial set of feasible price-bids, the proposed heuristic solution methodology works iteratively, optimizing at each iteration the price-bid of a single time period given that the remaining ones are kept fixed at their current values. Under a uniform pricing scheme, the optimal value of a price-offer, $p_{1,t}$, within a particular interval, say $[a_t, b_t]$, is not unique, unless the strategic producer is marginal. In order to break such ties, the heuristic proceeds by choosing as optimal $p_{1,t}$-value one of the two corresponding endpoints. If the previous value of $p_{1,t}$ is equal to one of the two endpoints, then the algorithm just alternates this choice by selecting as optimal $p_{1,t}$-value the other endpoint, in order to explore additional neighborhood directions for possible objective improvement. Otherwise, it selects as optimal $p_{1,t}$-value the left endpoint, which corresponds to the lowest cost, in order to increase the price-offer competitiveness of the strategic producer. The procedure terminates as soon as a full cycle in which the profit of the strategic producer remains unchanged is encountered. This design avoids the execution of meaningless cycles, which, for example, can come about when the optimal values of some price-offers are changing constantly, even though the associated ISO optimal solution and the corresponding strategic producer profit remain unaltered.

It is easy to verify that the profit of the strategic producer cannot worsen in any two consecutive iterations of the heuristic. This holds true, simply because keeping the value of any of his bids the same as before ensures that this profit will remain unchanged, too. The rationale behind the above criterion for algorithmic termination is to keep going as long as there is an improvement in this profit, and to stop as soon as a full cycle in which no such improvement has been observed is encountered. The only case in which this would not lead to termination after a finite number of iterations is if a strictly positive objective improvement could be perpetually maintained. This, however, cannot happen due to the integrality of the decision variables and the consequent finite cardinality of the problem's feasible set.

Under a pay-as-bid clearing scheme, things are more straightforward. Assuming a strictly positive energy dispatch for the strategic producer in the associated interval, the optimal $p_{1,t}$-value is unique and equal to this interval's right endpoint. If the strategic producer's dispatch is equal to 0, on the other hand, then the algorithm selects as optimal $p_{1,t}$-value the left endpoint, unless this value coincides to the previous $p_{1,t}$-value in which case the right endpoint is selected instead. The same conditions used for algorithmic termination in the case of uniform pricing are also applied in the case of a pay-as-bid clearing scheme. Repeating this iterative procedure several times with various initial sets of price-bids (seeds) provides many alternative solutions, the best of which is naturally the one the algorithm returns upon termination.

As far as the issue of addressing more complex models is concerned, note that the above procedure is straightforward and can be applied to similar strategic bidding optimization problems fitting this modeling framework under the assumption that the lower-level problem remains linear. This implies that the model formulation can be extended to incorporate additional restrictions that may be present in different applications even if the formulation of these restrictions necessitates the introduction of integer decision variables, as long as the modeling of the associated constraints remains linear.

Next, we present the proposed heuristic solution methodology for the treatment of the multi-period variant of the problem in a step-by-step basis, using pseudocode for the reader's convenience.

Heuristic Solution Algorithm

Step 0 (Initialization)

Using some educated estimate, choose an initial feasible price-offer for each time period of the planning horizon and initialize the strategic producer profit it results to.

Set $t = 1$.

Step 1 (Iteration)

While there has not been a cycle of T consecutive iterations in which the strategic producer profit remains unchanged

do {

Find the optimal value of $p_{1,t}$ while keeping all the other price-offers fixed at their current values.

Replace the old $p_{1,t}$ value with the new one and update the profit of the strategic producer.

Set $t = t + 1$. If $t > T$, set $t = 1$.

} end while

Step 2 (Report of final solution)

Return the current set of strategic producer price-offers and the corresponding ISO optimal solution as the final solution. □

5.3 Exact solution methodology

5.3.1 Motivation

The integer parametric programming theory utilized in the development of the proposed heuristic solution methodology can also be utilized within the context of a cutting-plane solution methodology for finding the exact optimum of the problem. More specifically, it can be suitably modified to enable the generation of valid inequalities for excluding solutions identified by a suitable relaxation of the original problem which do not qualify for global optimality. Note that, in bilevel programming, the theory for obtaining bound information on the optimal objective through suitable relaxations exhibits significant differences with that of typical single-level optimization problems (Bard, 1998[6]).

We consider the relaxation of the optimization model (3.7)-(3.14) after the inclusion of constraints 5.1 and 5.2, in which the restriction that the follower's response must belong to the reaction set is suppressed, i.e., we relax the requirement that the set of unit commitments and energy quantities constitutes an optimal ISO solution in conjunction with the corresponding set of strategic producer price-offers. This requirement, formally termed as bilevel feasibility, is a key prerequisite for global optimality. The following is a well-known result in the context of bilevel optimization, which is utilized in the development of the proposed exact solution algorithm for the treatment of the problem:

Proposition 5.1 *The optimal objective value to the problem that results after bilevel feasibility is relaxed from the original formulation is a valid upper bound on the optimal objective of the original problem.*

Proof The proof is trivial, since relaxing bilevel feasibility enlarges the feasible set of the upper-level problem through the inclusion of those bilevel infeasible solutions which are feasible with respect to the remaining problem constraints, without excluding any other feasible solution. □

Consider the optimal solution to the problem that results after bilevel feasibility is relaxed in the original formulation. If this solution happens to be bilevel feasible, then, naturally, it is also the exact optimal solution of the original problem. If not, then in order to pursue the search for the optimal solution, one needs to exclude this solution from further consideration. We show next how the integer parametric programming theory exploited in the development of the proposed heuristic solution approach can be suitably modified in order to accomplish this, too.

The price-offers of the strategic producer appear as objective coefficients of his energy quantity variables in the ISO optimization problem. For a particular set of values of these price-offers, it is trivial to solve the lower-level problem and identify its optimal solution. Based on fundamental integer parametric programming theory, this solution remains optimal for a sufficiently small simultaneous perturbation of some of the decision variables' objective coefficients. More specifically, Geoffrion and Nauss (1977)[21] have showed that when the objective coefficients of a minimization integer program are linearly perturbed through a single scalar parameter, then its optimal objective is piecewise-linear, continuous, and concave on its finite domain as a function of this parameter. In our case, by solely perturbing the strategic producer's price-offers, this property allows us to identify interval ranges, such that, when each of these bids lies in its corresponding interval, the lower-level optimal solution remains unchanged. After identifying these interval ranges, we use typical integer programming modeling techniques to generate a valid inequality imposing the truly optimal lower-level solution. Besides excluding the previously identified bilevel infeasible solution from further consideration, this procedure also enforces the truly optimal lower-level solution for large value-combinations of the price-offers. The specifics of this procedure are explained next.

For $t = 1,\ldots,T$, assume that $p_{1,t}^{b}$ is the strategic producer's price-offer for time period t in the identified bilevel infeasible solution. It is trivial to find the truly optimal ISO solution for ($p_{1,1}{}^b$, $p_{1,2}{}^b$,…, $p_{1,T}{}^b$). We seek maximum θ_1 and θ_2 values, such that this solution remains optimal when each price-offer $p_{1,t}$ belongs to the interval [$p_{1,t}{}^b$ - θ_1, $p_{1,t}{}^b$ + θ_2]. The left endpoint of this interval is identified by finding the maximum θ_1 value for which the slope of the function f^* remains unchanged when, starting from their initial values, these price-offers are simultaneously decreased by θ_1. Similarly, the right endpoint of this interval is identified by finding the maximum θ_2 value for which the slope of the function f^* remains unchanged when, starting from their initial values, these price-offers are simultaneously increased by θ_2. Of course, only integer values are of interest in each of these intervals. Once the maximum θ_1 and θ_2 values have been identified, the following crucial result justifies the validity of the proposed valid inequality:

Proposition 5.2 *The optimal ISO objective for (*$p_{1,1}{}^b$*,* $p_{1,2}{}^b$*,…,* $p_{1,T}{}^b$*) remains unchanged when each price-offer* $p_{1,t}$ *belongs to the interval [*$p_{1,t}{}^b$ *-* θ_1*,* $p_{1,t}{}^b$ *+* θ_2*].*

Proof Consider the ISO optimal objective for ($p_{1,1}{}^b$, $p_{1,2}{}^b$,…, $p_{1,T}{}^b$), say $f_1{}^*$, and assume that the ISO optimal objective is also equal to $f_1{}^*$ when each price-offer $p_{1,t}$ is equal to [$p_{1,t}{}^b$ - θ_1], as well as when each price-offer $p_{1,t}$ is equal to [$p_{1,t}{}^b$ + θ_2]. If there exists some combination of $p_{1,t}$-values with each $p_{1,t}$ belonging to [$p_{1,t}{}^b$ - θ_1, $p_{1,t}{}^b$ + θ_2], such that the optimal ISO solution, say $f_2{}^*$, is different than $f_1{}^*$, then this is a contradiction, since it directly negates the concavity and monotonicity of objective f^* as a function of the scalar parameter that linear perturbs it. Therefore, the fact that the optimal ISO objective is the same when each price-offer $p_{1,t}$ is equal to [$p_{1,t}{}^b$ - θ_1], as well as when each price-offer $p_{1,t}$ is equal to [$p_{1,t}{}^b$ + θ_2], implies that it will also be the same for any value-combination of the price-offers within these intervals. □

After incorporating the ensuing valid inequality into the model formulation, the relaxed problem is solved again and its next optimal solution is identified. The procedure continues similarly, eventually terminating as soon as the first bilevel feasible solution is encountered, which naturally constitutes the problem's exact optimum. An elucidation of the valid inequalities generation procedure is presented next in sufficient detail, in order to ensure material completeness and comprehension.

5.3.2 Valid inequalities' generation

The valid inequality that we want to introduce in order to exclude a bilevel infeasible solution must impose the restriction that if each of the strategic producer's bids, $p_{1,t}$, belongs to the interval it has been associated with, then a particular unit commitment and energy quantity distribution must comprise the corresponding ISO optimal solution. Assume that the particular interval in question for price offer $p_{1,t}$ is denoted by $[a_t, b_t]$. If both $a_t \neq c_1$ and $b_t \neq C_1$, then the generation of the cut necessitates the introduction of two binary variables, say W_l^t and W_r^t, denoting whether $p_{1,t}$ is greater or equal to a_t and less or equal to b_t, respectively. Mathematically, this is expressed through the following four constraints:

$$p_{1,t} \leq (C_1 - a_t + 1)W_l^t + (a_t - 1) \quad (5.3)$$

$$p_{1,t} \geq (a_t - c_1)W_l^t + c_1 \quad (5.4)$$

$$p_{1,t} \leq (C_1 - b_t)(1 - W_r^t) + b_t \quad (5.5)$$

$$p_{1,t} \geq c_1 + (b_t + 1 - c_1)(1 - W_r^t) \quad (5.6)$$

Constraints (5.3) and (5.4) impose the restriction pertaining to the left endpoint of the interval, i.e., $W_l^t = 1$ if and only if $p_{1,t} \geq a_t$. More specifically, if $W_l^t = 0$ then $p_{1,t} \leq a_t - 1$ from constraint (5.3), while constraint (5.4) becomes redundant. On the other hand, if $W_l^t = 1$ then $p_{1,t} \geq a_t$ from constraint (5.4), while constraint (5.3) becomes redundant. Similarly, constraints (5.5) and (5.6) impose the restriction pertaining to the right endpoint of the interval, i.e., $W_r^t = 1$ if and only if $p_{1,t} \leq b_t$. More specifically, if $W_r^t = 1$ then $p_{1,t} \leq b_t$ from constraint (5.5), while constraint (5.6) becomes redundant. On the other hand, if $W_r^t = 0$ then $p_{1,t} \geq b_t + 1$ from constraint (5.6), while constraint (5.5) becomes redundant. Of course, no corresponding binary variable needs to be introduced if the corresponding endpoint coincides with c_1 or C_1, respectively.

After the required binary variables have been properly defined for all price-offers, the imposition of a particular energy quantity, say Q, for production unit i in period t is accomplished by introducing the following two constraints:

$$q_{i,t} \leq Q + (M_i - Q)\sum_t (2 - W_l^t - W_r^t) \tag{5.7}$$

$$q_{i,t} \geq Q - Q\sum_t (2 - W_l^t - W_r^t) \tag{5.8}$$

If $W_l^t = W_r^t = 1$ for $t = 1,\ldots,T$, then the two summations in constraints (5.7) and (5.8) are eliminated and $q_{i,t}$ is set equal to Q. If at least one of these auxiliary variables is equal to 0, which implies that the corresponding price-offer does not belong to its associated interval, then both these constraints become redundant. Using such a pair of constraints for the energy quantity of each energy producer, we can impose a specific ISO optimal solution, thus eliminating a bilevel infeasible solution. Note that for units which are constrained to 0-quantity in the associated solution, the above two constraints can be replaced by the following equivalent constraint that directly fixes the status of unit i in period t:

$$z_{i,t} \leq \sum_t (2 - W_l^t - W_r^t) \tag{5.9}$$

5.3.3 Relaxing bilevel feasibility

The most typical approach for relaxing bilevel feasibility in general bilevel optimization problems is the suppression of the lower-level objective; this transforms the problem into a single-level optimization model. In the case of a pay-as-bid clearing scheme, this can be accomplished straightforwardly by removing the ISO objective from the original model formulation. Note that the objective of this relaxed single-level optimization model is quadratic, since it involves the product of two decision variables both treated at the same level. In the case of uniform pricing, on the other hand, an explicit representation of the system marginal price is not present in the problem formulation and needs to be incorporated. In order to accomplish this, we introduce extra constraints enforcing the correct *smp* definition for each time period of the planning horizon, according to the actual set of rules in effect. The exact procedure for doing this is illustrated next.

The procedure we adopt for relaxing bilevel feasibility in the case of uniform pricing necessitates the introduction of the following two binary decision variables for each generation unit i and time period t:

$w_{i,t}$ binary decision variable that takes the value 1 if and only if the output of unit i in time period t is strictly greater than m_i, and 0 otherwise, $i \in I$, $t = 1,\dots,T$,

$v_{i,t}$ binary decision variable that takes the value 1 if and only if the output of unit i in time period t is strictly less than M_i, and 0 otherwise, $i \in I$, $t = 1,\dots,T$.

Variable $w_{i,t}$ takes the value 1 if and only if $q_{i,t} > m_i$, and 0 otherwise, while variable $v_{i,t}$ takes the value 1 if and only if $q_{i,t} < M_i$, and 0 otherwise. We can modify accordingly constraints (5.3)-(5.6) to ensure that $w_{i,t}$ and $v_{i,t}$ correctly depict these two conditions as follows:

$$q_{i,t} \leq (M_i - m_i)w_{i,t} + m_i \tag{5.10}$$

$$q_{i,t} \geq (m_i + 1)w_{i,t} \tag{5.11}$$

$$q_{i,t} \leq M_i - v_{i,t} \tag{5.12}$$

$$q_{i,t} \geq M_i(1 - v_{i,t}) \tag{5.13}$$

When $q_{i,t} > m_i$, constraint (5.10) makes $w_{i,t}$ equal to 1, while constraint (5.11) becomes redundant. On the other hand, when $q_{i,t} \leq m_i$, constraint (5.11) makes $w_{i,t}$ equal to 0, while constraint (5.10) becomes redundant. Similarly, when $q_{i,t} = M_i$, constraint (5.12) makes $v_{i,t}$ equal to 0, while constraint (5.13) is redundant. When $q_{i,t} < M_i$, on the other hand, constraint (5.13) makes $v_{i,t}$ equal to 1, while constraint (5.12) becomes redundant. After the introduction of these constraints, Rule 1 is enforced by adding the following two constraints for each unit *i*, in which *K* is a sufficiently large number:

$$\lambda_t \leq p_{i,t} + (2 - w_{i,t} - v_{i,t})K \tag{5.14}$$

$$\lambda_t \geq p_{i,t} - (2 - w_{i,t} - v_{i,t})K \tag{5.15}$$

If $m_i < q_{i,t} < M_i$, then $w_{i,t} = v_{i,t} = 1$, so these two constraints set the *smp* equal to the price-offer of unit *i*; in any other case, they are both redundant. In order to express Rule 2, we additionally introduce a binary variable $u_{j,t}$ for each unit $j > 1$ and time period *t*, which takes the value 1 if and only if $p_{1,t} < p_{j,t}$, and 0 otherwise. Correct values for variables $u_{j,t}$ are ensured through the introduction of the following two constraints which are analogous to constraints (5.5) and (5.6):

$$p_{1,t} + 1 \leq (C_1 - p_{j,t} + 1)(1 - u_{j,t}) + p_{j,t} \tag{5.16}$$

$$p_{1,t} \geq c_1 + (p_{j,t} - c_1)(1 - u_{j,t}) \tag{5.17}$$

For each unit $j > 1$ and time period *t*, we also introduce a binary variable $b_{j,t}$ which takes the value 1 if and only if both $q_{j,t} = m_j$ (i.e., $z_{j,t} = 1$ and $w_{j,t} = 0$) and $p_{j,t} \leq p_{1,t}$ (i.e., $u_{j,t} = 0$) hold, and 0 otherwise, as well as a binary variable $g_{j,t}$, which takes the value 1 if and only if both $q_{1,t} = m_1$ (i.e., $z_{1,t} = 1$ and $w_{1,t} = 0$) and $p_{j,t} > p_{1,t}$ (i.e., $u_{j,t} = 1$) hold, and 0 otherwise. Correct values for variables $b_{j,t}$ and $g_{j,t}$ are ensured through the following eight constraints:

$$b_{j,t} \geq z_{j,t} - w_{j,t} - u_{j,t} \tag{5.18}$$

$$b_{j,t} \leq z_{j,t} \tag{5.19}$$

$$b_{j,t} \leq 1 - w_{j,t} \tag{5.20}$$

$$b_{j,t} \leq 1 - u_{j,t} \tag{5.21}$$

$$g_{j,t} \geq z_{1,t} + u_{j,t} - w_{1,t} - 1 \quad (5.22)$$

$$g_{j,t} \leq z_{1,t} \quad (5.23)$$

$$g_{j,t} \leq u_{j,t} \quad (5.24)$$

$$g_{j,t} \leq 1 - w_{1,t} \quad (5.25)$$

Rule 2 is then expressed through the introduction of the set of constraints (5.26)-(5.29) that follows. Constraints (5.26)-(5.27) are introduced only once as they pertain to unit 1, while constraints (5.28)-(5.29) are introduced once for each unit i > 1.

$$\lambda_t \leq p_{1,t} + K(1 - z_{1,t} + w_{1,t}) + \sum_{j>1} Kb_{j,t} + \sum_{j>1} K(w_{j,t} + v_{j,t} - 1) \quad (5.26)$$

$$\lambda_t \geq p_{1,t} - K(1 - z_{1,t} + w_{1,t}) - \sum_{j>1} Kb_{j,t} - \sum_{j>1} K(w_{j,t} + v_{j,t} - 1) \quad (5.27)$$

$$\lambda_t \leq p_{i,t} + K(1 - z_{i,t} + w_{i,t}) + \sum_{j>1,\, j\neq i:\, p_j < p_i} K(z_{j,t} - w_{j,t}) + Kg_{i,t} + \sum_{j \neq i} K(w_{j,t} + v_{j,t} - 1) \quad (5.28)$$

$$\lambda_t \geq p_{i,t} - K(1 - z_{i,t} + w_{i,t}) - \sum_{j>1,\, j\neq i:\, p_j < p_i} K(z_{j,t} - w_{j,t}) - Kg_{i,t} - \sum_{j \neq i} K(w_{j,t} + v_{j,t} - 1) \quad (5.29)$$

Constraint (5.28) sets λ_t less than or equal to $p_{i,t}$ plus a summation of non-negative terms, each of which involves a multiplication with the sufficiently large number K. Similarly, constraint (5.29) sets λ_t greater than or equal to $p_{i,t}$ minus the summation of the same exact terms. Thus, if all these terms are equal to 0, then λ_t is set equal to $p_{i,t}$ by these two constraints; otherwise, they are both redundant. The term $(1 - z_{i,t} + w_{i,t})$ is equal to 0 if unit i produces at its technical minimum in time period t; otherwise, it is equal to 1. The term $(z_{j,t} - w_{j,t})$ is equal to 1 if unit j produces at its technical minimum in time period t; otherwise, it is equal to 0. The term $g_{i,t}$ is equal to 1 if unit 1 has smaller price-offer than unit i and produces at its technical minimum in time period t; otherwise, it is equal to 0. Finally, the term $(w_{j,t} + v_{j,t} - 1)$ is equal to 1 if the output of unit j is strictly between its technical minimum and its technical maximum in time period t; otherwise, it is equal to 0. Thus, these two constraints set λ_t equal to the price-offer of unit i in time period t if unit i produces at its technical minimum, and in addition there is no other unit with smaller price-offer producing at its technical minimum, and no other unit producing strictly between its technical minimum and its technical maximum in the same time period; in any other case, these constraints are both redundant. Constraints (5.26)-(5.27) are similar to constraints (5.28)-(5.29), pertaining to the unit of the strategic producer. One can easily verify that these constraints determine the *smp* value correctly even when more than one units are simultaneously marginal according to Rule 2, both in the case that one of them is the strategic unit as well as in the case that it is not.

In order to express Rule 3, we additionally introduce a binary variable $a_{j,t}$ for each unit $j > 1$ and time period t, which takes the value 1 if and only if both $q_{j,t} = M_j$ (i.e., $v_{j,t} = 0$) and $p_{j,t} > p_{1,t}$ (i.e., $u_{j,t} = 1$) hold, and 0 otherwise, as well as a binary variable $h_{j,t}$, which takes the value 1 if and only if both $q_{1,t} = M_1$ (i.e., $v_{1,t} = 0$) and $p_{j,t} \leq p_{1,t}$ (i.e., $u_{j,t} = 0$) hold, and 0 otherwise. Correct values for variables $a_{j,t}$ and $h_{j,t}$ are ensured through the following six constraints:

$$a_{j,t} \geq u_{j,t} - v_{j,t} \tag{5.30}$$

$$a_{j,t} \leq u_{j,t} \tag{5.31}$$

$$a_{j,t} \leq 1 - v_{j,t} \tag{5.32}$$

$$h_{j,t} \geq 1 - v_{1,t} - u_{j,t} \tag{5.33}$$

$$h_{j,t} \leq 1 - v_{1,t} \tag{5.34}$$

$$h_{j,t} \leq 1 - u_{j,t} \tag{5.35}$$

Rule 3 is then expressed through the introduction of the set of constraints (5.36)-(5.39) that follows. Constraints (5.36)-(5.37) are introduced only once as they pertain to unit 1, while constraints (5.38)-(5.39) are introduced once for each unit $i > 1$.

$$\lambda_t \leq p_{1,t} + Kv_{1,t} + \sum_{j>1} K(z_{j,t} + v_{j,t} - 1) + \sum_{j>1} K\alpha_{j,t} \tag{5.36}$$

$$\lambda_t \geq p_{1,t} - Kv_{1,t} - \sum_{j>1} K(z_{j,t} + v_{j,t} - 1) - \sum_{j>1} Ka_{j,t} \quad (5.37)$$

$$\lambda_t \leq p_{i,t} + Kv_{i,t} + \sum_{j \neq i} K(z_{j,t} + v_{j,t} - 1) + \sum_{j>1, j \neq i, p_j > p_i} K(1 - v_{j,t}) + Kh_{i,t} \quad (5.38)$$

$$\lambda_t \geq p_{i,t} - Kv_{i,t} - \sum_{j \neq i} K(z_{j,t} + v_{j,t} - 1) - \sum_{j>1, j \neq i, p_j > p_i} K(1 - v_{j,t}) - Kh_{i,t} \quad (5.39)$$

Constraint (5.38) sets λ_t less than or equal to $p_{i,t}$ plus a summation of non-negative terms, each of which involves a multiplication with the sufficiently large number K. Similarly, constraint (5.39) sets λ_t greater than or equal to $p_{i,t}$ minus the summation of the same exact terms. Thus, if all these terms are equal to 0, then λ_t is set equal to $p_{i,t}$ by these two constraints; otherwise, they are both redundant. The term $v_{i,t}$ is equal to 0 if unit i produces at its technical maximum in time period t; otherwise, it is equal to 1. The term ($z_{j,t}$ + $v_{j,t}$ - 1) is equal to 1 if the energy quantity of unit j in time period t is positive but strictly smaller than its technical maximum; otherwise, it is equal to 0. The term (1 - $v_{j,t}$) is equal to 1 if unit j produces at its technical maximum in time period t; otherwise, it is equal to 0. Finally, the term $h_{i,t}$ is equal to 1 if unit 1 has greater or equal price-offer than unit i and produces at its technical maximum in time period t; otherwise, it is equal to 0. Thus, these two constraints set λ_t equal to the price-offer of unit i in time period t if unit i produces at its technical maximum, and in addition there is no other unit whose energy dispatch is positive but strictly smaller than its technical maximum, and no other unit with larger price-offer (for $j > 1$) or larger or equal price-offer (for $i = 1$) producing at its technical maximum; in any other case, they are redundant. Constraints (5.36)-(5.37) are similar to constraints (5.38)-(5.39), pertaining to the unit of the strategic producer. One can easily verify in this case, too, that these constraints determine the *smp* value correctly even when more than one units are simultaneously marginal according to Rule 3, both in the case that one of them is the strategic unit as well as in the case that it is not.

With the introduction of the above constraints, the complete model for relaxing bilevel feasibility in the case of a uniform pricing clearing scheme is formulated as follows:

$$\underset{p_{1,t}}{\text{Max}}\ F_1 = \sum_{t=1}^{T} (\lambda_t - c_1)\, q_{1,t}$$

s.t. $c_1 \le p_{1,t} \le C$, $t = 1,\ldots,T$

$$\sum_{i \in I} q_{i,t} = d_t,\ t = 1,\ldots,T$$

$m_{i,t}\, z_{i,t} \le q_{i,t} \le M_{i,t}\, z_{i,t}$, $i \in I$, $t = 1,\ldots,T$

$y_{i,t} \ge z_{i,t} - z_{i,t-1}$, $i \in I$, $t = 1,\ldots,T$

constraints (5.10)-(5.15), $i \in I$, $t = 1,\ldots,T$

constraints (5.16)-(5.25), (5.28)-(5.35), (5.38)-(5.39), $i \in I$, $i > 1$, $t = 1,\ldots,T$

constraints (5.26)-(5.27), (5.36)-(5.37), $t = 1,\ldots,T$

$p_{1,t} \in Z^+$, $q_{i,t} \in Z^+$, $i \in I$, $t = 1,\ldots,T$

$y_{i,t}$, $z_{i,t}$, $w_{i,t}$, $v_{i,t}$ binary, $i \in I$, $t = 1,\ldots,T$

$u_{i,t}$, $b_{i,t}$, $g_{i,t}$, $a_{i,t}$, $h_{i,t}$ binary, $i \in I$, $i > 1$, $t = 1,\ldots,T$

5.3.4 The exact solution algorithm

Having elucidated the various actions the proposed exact solution algorithm involves, we are now in a position to present it in a step-by-step basis using pseudocode for the reader's convenience.

Exact Solution Algorithm

Step 0 (Initialization)

Relax bilevel feasibility and solve the resulting single-level model formulation.

Step 1 (Iteration)

While the current optimal solution is bilevel infeasible for the original problem formulation do {

Find the truly optimal ISO solution for the current set of strategic producer price-offers.

Find the maximum simultaneous increase and the maximum simultaneous decrease on the strategic producer price-offers for which this solution remains optimal.

Add a valid inequality excluding the bilevel infeasible solution from further consideration.

Solve the model again.

} end while

Step 2 (Report of final solution)

Return the current set of strategic producer price-offers and the corresponding ISO optimal solution as the optimal solution. □

5.4 Computational results

We have implemented the proposed solution methodologies using C/C++ source code. In this section, we illustrate their application on a small case study, and we present extensive experimental results evaluating their relative computational performance. All tests were performed on a 6-Core @ 3.5 GHz 64-bit AMD Processor with 8 GB system memory, while the commercial optimization software LINGO 13.0 (2011)[42] was internally utilized for the solution of the encountered optimization models.

5.4.1 A small case study

For illustration purposes, we consider first a small case study with 3 production units and a 4-period planning horizon. The technical characteristics and startup costs of the generation units, as well as the price-offers and the demand for energy in each time period are shown in Table 5-1. The technical minima and maxima are given in MW, the startup costs in €, the price-offers for energy in €/MWh, and the energy demand in MWh. The unit variable production cost of the strategic producer (generation unit 1) is 50 €/MWh, while the price cap is 100 €/MWh. We assume that all units are OFF at the beginning of the planning horizon, i.e., that $z_{i,0} = 0$ for $i = 1,\ldots,3$.

Table 5-1 Case study data

				$p_{i,t}$			
Unit (i)	m_i	M_i	s_i	$t = 1$	$t = 2$	$t = 3$	$t = 4$
1	200	500	1300	-	-	-	-
2	240	480	1500	57	58	65	67
3	100	470	1600	64	60	58	62
			d_t	900	950	800	850

First, we apply the proposed heuristic solution approach under a pay-as-bid clearing scheme. Initially, we set the price-offer of the strategic producer in time period t ($t = 1,\ldots,4$) equal to the minimum price-offer of any other producer in the same time period, i.e., 57, 58, 58 and 62, respectively. In the first iteration, the algorithm optimizes the value of $p_{1,1}$, while keeping the values of $p_{1,2}$, $p_{1,3}$ and $p_{1,4}$ fixed. Table 5-2 presents the ISO optimal solution when $p_{1,1}$ is set equal to 50, in which the strategic producer is rewarded with the maximum possible dispatch in each time period for his low price-offers. His total profit upon clearing of the market is equal to 500(50-50) + 500(58-50) + 500(58-50) + 500(62-50) = 14,000, while the optimal ISO objective is equal to 206,400. Note that the strategic producer's profit in the first period of the planning horizon is equal to 0, despite the fact that his corresponding energy dispatch is equal to his technical maximum.

Table 5-2 Optimal ISO solution when $p_{1,1}$ is set equal to 50

Unit (i)	$t = 1$	$t = 2$	$t = 3$	$t = 4$
1	500	500	500	500
2	400	450	0	0
3	0	0	300	350

Table 5-3 presents the ISO optimal solution when $p_{1,1}$ is set equal to 100. In this case, the strategic producer does not participate in the market in the first time period, as a result of his particularly high price-offer; his total profit upon clearing of the market is equal to 0(100-

50) + 500(58-50) + 500(58-50) + 500(62-50) = 14,000, same as before, while the optimal ISO objective is equal to 213,040.

Table 5-3 Optimal ISO solution when $p_{1,1}$ is set equal to 100

Unit (i)	t = 1	t = 2	t = 3	t = 4
1	0	500	500	500
2	480	350	0	0
3	420	100	300	350

The parametric analysis outlined in Section 4.1 identifies the 3 distinct lower-level optimal solutions and corresponding $p_{1,1}$ interval values depicted in Table 5-4. In the first two of these solutions, the optimal value of $p_{1,1}$ is equal to the associated interval's right end-point. In the third solution, on the other hand, any $p_{1,1}$ value in the associated interval is optimal, since the strategic producer's optimal dispatch in the first period is equal to 0. As shown in this table, the maximum profit that the strategic producer can attain is equal to 19,880, realized for $p_{1,1}$ = 64. The corresponding ISO optimal objective is equal to 212,840. The subsequent solutions visited by the algorithm in the next iterations are presented in Table 5-5.

Table 5-4 ISO optimal solutions for $p_{1,2}$ = 58, $p_{1,3}$ = 58, $p_{1,4}$ = 62

$p_{1,1}$ range	$(Q_{1,1}, Q_{1,2}, Q_{1,3}, Q_{1,4})$	$p_{1,1}^*$	f^*	F_1^*
[50, 57]	(500, 500, 500, 500)	57	181,400 + 500$p_{1,1}$	17,500
[58, 64]	(420, 500, 500, 500)	64	185,960 + 420$p_{1,1}$	19,880
[65, 100]	(0, 500, 500, 500)	65-100	213,040	14,000

Table 5-5 Solutions visited by the heuristic under a pay-as-bid clearing scheme

Iteration	$(p_{1,1}, p_{1,2}, p_{1,3}, p_{1,4})$	f^*	F_1
0	(57, 58, 58, 62)	209,900	17,500
1	(64, 58, 58, 62)	212,840	19,880
2	(64, 60, 58, 62)	213,780	20,580
3	(64, 60, 65, 62)	216,090	21,530
4	(64, 60, 65, 62)	216,090	21,530
5	(64, 60, 65, 62)	216,090	21,530
6	(64, 60, 65, 62)	216,090	21,530
7	(64, 60, 65, 62)	216,090	21,530

As shown in Table 5-5, the algorithm identifies the 4 price-offers (64, 60, 65, 62) upon termination, each of which is optimal for the current values of the other three. This is realized at the end of the 7th iteration, which flags the completion of a full cycle (4 iterations) in which the values of the 4 price-offers and the corresponding strategic producer profit remain unchanged. The strategic producer profit of the solution returned by the algorithm is equal to 21,530, while the optimal ISO objective is equal to 216,090.

We also applied the heuristic algorithm using two different sets of initial price-offer values, i.e., (64, 60, 65, 67), which corresponds to selecting the maximum price-offer of any producer in each time period, and (64, 58, 65, 62), which corresponds to some random selection from the other producers' price offers in the same time period. The collective results comparing the three corresponding final solutions are presented in Table 5-6. All three solutions are pretty close in terms of the optimal ISO objective, with the two solutions obtained with the first and the third set of price-offers coinciding and providing the same strategic producer profit, which is larger than that of the second one.

Table 5-6 Heuristic algorithm results for three different sets of initial price-offers (pay-as-bid pricing)

i	**initial** $(p_{1,1}, p_{1,2}, p_{1,3}, p_{1,4})$	**final** $(p_{1,1}, p_{1,2}, p_{1,3}, p_{1,4})$	f^*	F_1
1	(57, 58, 58, 62)	(64, 60, 65, 62)	216,090	21,530
2	(64, 60, 65, 67)	(60, 60, 65, 67)	216,310	20,310
3	(64, 58, 65, 62)	(64, 60, 65, 62)	216,090	21,530

Table 5-7 presents results similar as those of Table 5-6 for the case that a uniform pricing clearing scheme is adopted. The final solution that the algorithm returns for the second set of initial price-offers is the same with the one returned under the pay-as-bid clearing scheme. On the other hand, the first and the third solution coincide and qualify as the best, with an associated ISO optimal objective equal to 210,090 and corresponding strategic producer profit equal to 21,530. Note that, for this particular example, the profit of the strategic producer in the solution that the algorithm returned for each of the three sets of initial price-offers is the same under both clearing schemes.

Table 5-7 Heuristic algorithm results for three different sets of initial price-offers (uniform pricing)

i	initial ($p_{1,1}$, $p_{1,2}$, $p_{1,3}$, $p_{1,4}$)	final ($p_{1,1}$, $p_{1,2}$, $p_{1,3}$, $p_{1,4}$)	f^*	F_1
1	(57, 58, 58, 62)	(64, 60, 65, 50)	210,090	21,530
2	(64, 60, 65, 67)	(60, 60, 65, 67)	216,310	20,310
3	(64, 58, 65, 62)	(64, 60, 65, 50)	210,090	21,530

Next, we apply the exact solution algorithm under a pay-as-bid clearing scheme first. Solving the optimization problem that results after the lower-level objective is suppressed, we get the following price-offers and energy quantities, respectively, for the strategic producer: $p_{1,t}$ = (100, 100, 100, 100), $q_{1,t}$ = (500, 500, 500, 500). It is easy to confirm that this solution is not bilevel feasible, since solving the ISO's problem for $p_{1,1} = p_{1,2} = p_{1,3} = p_{1,4} = 100$, we get an ISO optimal solution in which $q_{1,t}$ = (0, 0, 0, 0). Using integer parametric programming theory we find that the maximum simultaneous decrease on the 4 price bids of the strategic producer for which this solution remains unchanged is equal to 35. This implies that when $p_{1,1}\in[65, 100]$ and $p_{1,2}\in[65, 100]$ and $p_{1,3}\in[65, 100]$ and $p_{1,4}\in[65, 100]$ then $q_{1,1} = q_{1,2} = q_{1,3} = q_{1,4} = 0$ and the profit of the strategic producer is equal to 0. To express this restriction mathematically, we add binary variables $k_{1,t}$ for $t = 1,\ldots,4$, such that $k_{1,t}$ is equal to 1 if $p_{1,t} \geq 65$, and 0 otherwise. This is expressed as follows mathematically: $k_{1,t} \geq (p_{1,t} - 64)/36$ and $k_{1,t} \leq (p_{1,t} - 50)/15$. Then, the cut in question is expressed by adding the following inequalities: $z_{1,t} \leq (4 - k_{1,1} - k_{1,2} - k_{1,3} - k_{1,4})$ for $t = 1,\ldots,4$. This excludes the previous bilevel infeasible solution from further consideration. Continuing adding similar cuts for each bilevel infeasible solution identified, the algorithm eventually reaches the exact optimal solution, which is $p_{1,t}^*$ = (64, 60, 58, 69), $q_{1,t}^*$ = (420, 470, 500, 380), with f^* = 216,440 and F_1^* = 21,800.

Under a uniform pricing clearing scheme, the algorithm identifies that the exact optimal solution is $p_{1,t}^* = (64, 60, 50, 70)$, $q_{1,t}^* = (420, 470, 500, 380)$, with $smp_t^* = (64, 60, 58, 70)$, $f^* = 212{,}820$ and $F_1^* = 22{,}180$. Thus, although being pretty close, the two optimal solutions under the two clearing schemes are not identical. In the uniform pricing case, the optimal ISO cost is lower, while the strategic producer's optimal profit is slightly higher. For this small case study, the best solution identified by the heuristic algorithm in the case of a pay-as-bid clearing scheme approximates the truly optimal one with a percentage difference of 1.2 %, while the corresponding approximate difference in the case of a uniform pricing clearing scheme is equal to 2.9 %. Of course, the quality of the solutions returned by the heuristic algorithm can potentially be improved through further execution attempts with additional initial solutions. For each of the two clearing schemes, Table 5-8 presents the total execution time, the total number of times (runs) the ISO optimization problem was solved, and the total number of valid inequalities (cuts) added. Note that the latter two figures do not coincide, due to the fact that the identification of a bilevel infeasible solution sometimes leads to the update of an existing cut instead of the introduction of a new one.

Table 5-8 Case study execution time, number of runs and number of cuts for the exact solution algorithm

Clearing scheme	Time (minutes)	# of runs	# of cuts
pay-as-bid	62	78	71
uniform	129	75	49

5.4.2 Randomly generated problems

In this subsection, we test the performance of the proposed solution algorithms on randomly generated problems. For the heuristic algorithm, we generated random problem instances with the following sizes expressed as AxB where A = number of generation units and B = number of time periods: 3x4, 4x4, 5x24, 6x24, 7x24, 8x24, 9x24, and 10x24. The emphasis on the number 24 for B is mainly due to the fact that realistic day-ahead electricity markets are typically solved over a planning horizon consisted of 24 hourly time periods. Each of the remaining units, besides the one pertaining to the strategic producer, was assigned the technical characteristics (technical minimum/maximum, start-up cost and price bids) of a factual unit participating in the Greek electricity market, according to the data provided by Andrianesis et al., 2013b[2]. The particular unit with which the association was made was selected randomly. The data pertaining to the strategic producer's unit were then generated as follows: The technical minimum (m_1) was an integer selected randomly in the interval $[\min_{i>1} m_i, \max_{i>1} m_i]$, the technical maximum (M_1) was set equal to m_1 + *range*, where *range* was an integer selected randomly in the interval $[\min_{i>1}(M_i - m_i), \max_{i>1}(M_i - m_i)]$, the start-up cost ($s_1$) was an integer selected randomly in the interval $[\min_{i>1} s_i, \max_{i>1} s_i]$, the unit variable production cost (c_1) was set equal to $0.9\min_{i>1,t} p_{it}$ rounded to the nearest integer, and the price-cap (C_1) was set equal to $1.1\max_{i>1,t} p_{it}$, rounded to the nearest integer. Finally, the demand for energy in each time period t was an integer distributed uniformly in the interval $[\sum_i (M_i - m_i) + \min_i m_i, \sum_{i>1} M_i + m_i]$.

The heuristic algorithm was applied three times on each problem instance, each time with a different set of initial price-offers for the strategic producer. In the first case, the price-offer of each time period t was set equal to the minimum price-offer of any other producer in the same time period, in the second case it was set equal to the maximum price-offer of any other producer in the same time period, and finally, in the third case, it was set equal to some of the other bids in the same time period, with the selection being made randomly. As the total number of distinct problem instances solved for each problem size was equal to 20, the total number of times the heuristic was applied on each problem size was equal to 3x20 = 60. For each clearing scheme, the following table presents the average and maximum computational times for each problem size.

Table 5-9 Computational times (in seconds) of the heuristic algorithm on random problem instances

	pay-as-bid		uniform	
size	**avg**	**max**	**avg**	**max**
3x4	4.37	6.53	4.39	6.73
4x4	13.36	42.73	14.24	48.61
5x24	113.68	440.04	124.66	508.27
6x24	215.91	998.90	218.13	1,028.66
7x24	282.26	1,583.76	226.14	1,146.01
8x24	394.13	1,811.94	251.91	1,274.35
9x24	462.76	2,181.66	275.24	1,325.85
10x24	494.10	2,775.76	283.93	1,547.34

As the above results demonstrate, the computational times of the heuristic algorithm are quite reasonable, enabling the solution of problems whose size approaches that of realistic problems encountered in practical environments of the greek electricity market. The variance of the computational times appears to be significant but not excessive. This is acceptable, considering that the optimization problem under consideration is highly non-convex and combinatorial. An interesting observation that can be made based on the results of Table 5-9 regards the fact that the computational times increase much faster with problem size in the case of a pay-as-bid clearing scheme than in the case of a uniform clearing scheme. This can be possibly explained by the fact that under a pay-as-bid clearing scheme there is a much larger set of alternative solutions that the algorithm must comparatively evaluate, due to the fact that the strategic producer's profit is dependent on his exact price-offer even in those time periods in which he is not marginal.

For the needs of the present dissertation, we applied the exact solution algorithm on problem instances with sizes 3x4 and 4x4, which were the same as those in the case of the heuristic algorithm. For each of the two clearing schemes, Table 5-10 presents the average and maximum computational times, the average and maximum number of runs, and the average and maximum number of cuts utilized. These results demonstrate the excessive computational requirements of the algorithm. In particular, the computational times are considerably large, partially due to the significant number of runs and cuts that these two relatively small problem sizes necessitated. The computational resource requirements of the algorithm are undoubtedly substantial. Consequently, it can be applied on particularly small sized problems only, whereas its application on realistic problem instances seems implausible at the moment. The computational requirements appear much higher under a uniform clearing scheme, the most reasonable explanation for this being the significantly more complicated model formulation due to the necessity for the explicit system marginal price representation.

Table 5-10 Computational results for the application of the exact algorithm on random problems

	pay-as-bid						uniform					
	times (secs)		# iterations		# valid inequalities		times (secs)		# iterations		# valid inequalities	
size	avg	max	avg	max	avg	max	avg	max	avg	max	avg	max
3x4	962	3,714	39.80	121	20.45	71	4,339	8,585	51.15	95	24.35	60
4x4	2,253	9,122	63.95	167	26.5	142	5,897	12,011	57.85	117	38.60	78

Table 5-11 presents results regarding the quality of the solutions returned by the heuristic solution algorithm. More specifically, for each clearing scheme and each of the two problem sizes, 3x4 and 4x4, this table presents the average and maximum percentage difference between the strategic producer’s profit in the solution provided by the heuristic, and that in the optimal solution identified by the exact solution algorithm. These results demonstrate that the heuristic algorithm provides high quality solutions at least for these two particular problem sizes. The maximum percentage difference in the strategic producer’s profit is less than 3.5 % in the worst case under both clearing schemes. Additionally, there does not seem to be significant difference in the heuristic algorithm’s effectiveness between the two clearing schemes. These results seem quite promising, leaving open the possibility that the heuristic algorithm may be capable of providing high quality solutions for problems of realistic size, too.

Table 5-11 Results regarding the quality of the solutions provided by the heuristic algorithm

	pay-as-bid		uniform	
	percentage difference (heuristic vs. exact)		percentage difference (heuristic vs. exact)	
size	**avg**	**max**	**avg**	**max**
3x4	0.58	2.71	0.48	2.70
4x4	0.55	3.23	0.68	3.44

5.5 Summary

In this chapter, we considered the problem of finding the optimal bidding strategy of an energy producer that participates in a multi-period day-ahead electricity market. The problem is formulated as an integer bilevel optimization model with perfect knowledge of the market's parameters, the technical characteristics and the bidding offers of the remaining producers. Due to the absence of generic solution methodologies for integer bilevel programming, we elaborated on several interesting theoretical properties and we utilized them to develop both a heuristic as well as an exact algorithmic solution methodology, for both clearing schemes.

The heuristic solution methodology is straightforward and can be applied to similar strategic bidding optimization problems, even when they incorporate additional restrictions modeled by expressions involving integer decision variables, as long as the lower-level problem remains linear. Next, we demonstrated how the related theoretical groundwork can be modified to enable the generation of valid inequalities. The significance of these inequalities lies in that they can be embedded within a cutting plane algorithmic procedure for identifying the exact optimal solution of the problem. We implemented this solution methodology, and we illustrated its applicability as well as that of the heuristic solution algorithm on a small numerical example.

We concluded with experimental results demonstrating the computational capabilities of the proposed solution algorithms and evaluating their relative performance. More effective between the two turns out to be the heuristic approach, which is not surprising. The heuristic solution algorithm provides high quality solutions and its computational requirements are very moderate, enabling the solution of realistic problem instances in reasonable times. The exact solution algorithm, on the other hand, exhibits significantly higher computational requirements, which prohibit its application on realistic problem instances at the moment. As a consequence, the practical application of the exact solution algorithm necessitates further algorithmic enhancements to overcome the significant computational obstacles that the current implementation exhibits. The problem formulation can be made more realistic through the incorporation of additional problem aspects, such as minimum uptimes/downtimes and ramp-up/ramp down constraints. While this increases the problem complexity, at the same time it makes it more challenging.

Chapter 6 Enhanced exact solution algorithm for the multi-period variant of the problem

In this chapter, we develop an improved version of the exact solution algorithm presented in the previous chapter for optimal price-bidding of energy producers in day-ahead electricity markets with multi-period planning horizons. We embed special optimality conditions into the model reformulation in which bilevel feasibility has been relaxed, which ensure that the energy quantity distribution in each time period of the planning horizon will be optimal for the corresponding set of producers that will be identified as active in that time period. Consequently, solving the original problem to global optimality becomes equivalent to identifying the optimal set of active producers ($z_{i,t} = 1$) in each time period. This also results in a small modification of the cuts utilized for excluding bilevel infeasible solutions; the main difference lies in that these cuts enforce particular unit commitments and not energy quantities as before. A simple procedure for extending the intervals within which these inequalities are valid is also devised. This constitutes another significant improvement, because it enables the elimination of an increased number of bilevel infeasible solutions from further consideration.

We illustrate the application of the proposed methodology on a small case study, and we present extensive computational results demonstrating its performance and behavior on randomly generated problems. These results reveal that it is capable of handling small to medium sized problems efficiently, which is utterly important considering the inevitable lack of generic solution methodologies for the treatment of such problems, as well as the fact that the applicability of specialized solution methodologies which have been previously proposed in the related literature appears rather limited on realistic size problems.

6.1 Single period optimality conditions

In this subsection, we focus our analysis on the treatment of a particular time period of the planning horizon; consequently, we drop the subscript denoting the time period as redundant for simplicity. As will become apparent next, the analysis that follows is applicable to any time period, after all decision variables are suitably augmented through the inclusion of the second subscript denoting the time period. Having made this clarification, assume that, in the multi-period variant of the problem, the optimal unit commitments in some time period of the planning horizon (z_i^*, $i \in I$) have been identified. In this case, the identification of the optimal solution for this time period can be straightforwardly accomplished by solving the following optimization problem, in which we additionally impose the optimistic approach assumption, so that the strategic producer is always favored in case of multiple optima.

$$\text{Min} \sum_{i \in I:\, z_i^* = 1} p_i q_i \tag{6.1}$$

$$\text{s.t.} \sum_{i \in I:\, z_i^* = 1} q_i = d \tag{6.2}$$

$$m_i \le q_i \le M_i \,,\; i \in I\text{: } z_i^* = 1 \tag{6.3}$$

$$q_i = 0,\; i \in I\text{: } z_i^* = 0 \tag{6.4}$$

$$q_i \in Z^+,\; i \in I\text{: } z_i^* = 1 \tag{6.5}$$

The energy quantity of each unit i such that $z_i^* = 0$ is fixed to 0-value in this formulation. Thus, the problem reduces to finding the energy distribution that minimizes the total variable production cost while also respecting the technical minima/maxima of the generation units in that time period. Consider now the problem of examining whether a particular given energy distribution is optimal. The feasibility of this distribution can be trivially checked; therefore, the optimality check reduces to examining whether there is another feasible distribution among the same generation units that results in lower production cost. The following important theoretical result can be utilized to provide a confirmative response to this question:

Proposition 6.1 *An energy distribution in a particular time period that is feasible with respect to constraints* (*6.2*)-(*6.5*) *is also optimal with respect to* (*6.1*) *if and only if there do not exist two distinct production units i and j, such that* $z_i^* = z_j^* = 1$, $p_i < p_j$, $q_i < M_i$ *and* $q_j > m_i$.

Proof We prove the forward part first. Suppose that there exist two distinct units *i* and *j*, such that $z_i^* = z_j^* = 1$, $p_i < p_j$, $q_i < M_i$ *and* $q_j > m_i$. Decreasing the output quantity of unit *j* by one and increasing the output quantity of unit *i* by one results in an alternative feasible distribution which has lower cost, due to the fact that $p_i < p_j$. This contradicts the optimality of the initial distribution. Consider the reverse part now. Suppose that for a feasible distribution no such unit pair exists. The objective function of the problem is the weighted sum of the unit quantities, with the corresponding price-offers utilized as weights. Since the total sum of the quantities is fixed, this objective can only be decreased if at least one unit of energy is multiplied by a smaller weight in this sum. The fact that no unit pair for making such an exchange exists proves that the current distribution is optimal. □

Based on Proposition 6.1, a simple algorithmic procedure can be carried out for finding the optimal solution to Problem (6.1)-(6.5). This procedure initializes the energy dispatch of each active generation unit *i* to m_i, and then allocates additional energy quantities to the active generation units in non-decreasing order of their price-offers. Each time the next generation unit is selected, it is allocated the minimum between the largest additional quantity it can accommodate, which is equal to $M_i - m_i$, and the residual energy demand. If the former of these two quantities is smaller, a proper update takes place and the procedure proceeds to the next active generation unit for allocation. If the latter quantity is smaller instead, the procedure terminates with the current solution being optimal.

The important theoretical result of Proposition 6.1 enables us to impose suitable constraints for each time period of the planning horizon to the multi-period variant of the problem, imposing the optimality conditions this proposition prescribes. In conjunction with the constraint set of the lower-level problem, these conditions ensure that the energy distribution identified in each time period will be lower-level optimal for the associated set of active and inactive energy producers.

Note that, naturally, these conditions should only pertain to units which are both active in the same time period. This can be accommodated suitably, using the binary variables z_i denoting the status of unit i. When none of the two units involved pertains to the strategic producer, the utilization of auxiliary binary decision variables w_i and v_i introduced in Section 5.3.3 is required to this end. With the help of these variables, the optimality conditions for any two generation units i ($i > 1$) and j ($j > 1$), such that and $p_i > p_j$, are expressed as follows:

$$3 - w_i - v_j \geq z_i + z_j,\ i \in I: i > 1 \tag{6.6}$$

If $z_i = z_j = 1$, then this constraint imposes the restriction that $1 \geq w_i + v_j$, which implies that either $w_i = 0$ (equivalently $q_i = m_i$), or $v_j = 0$ (equivalently $q_j = M_j$). In any other case, constraint (6.6) is redundant. Thus, a solution in which $q_i > m_i$ and $q_j < M_j$ is eliminated. When the strategic producer is involved, on the other hand, things become a little more elaborate, due to the fact that his actual price-offer is subject to optimization and thus not known in advance. In order to address this difficulty, we introduce the following decision variable for each $i > 1$ in this particular time period:

x_i binary decision variable that takes the value 1 if and only if $p_i < p_1$, and 0 otherwise.

Correct values for variables x_i are ensured through the following two constraints:

$$p_1 \leq (C_1 - p_i)\, x_i + p_i,\ i \in I: i > 1 \tag{6.7}$$

$$p_1 \geq c_1 + (p_i + 1 - c_1)\, x_i,\ i \in I: i > 1 \tag{6.8}$$

If $x_i = 0$, then constraints (6.7)-(6.8) impose the restriction $c_1 \leq p_1 \leq p_i$; otherwise, they impose the restriction $p_i + 1 \leq p_1 \leq C_1$. Utilizing variable x_i, we express the optimality conditions as follows in case the strategic producer is involved:

$$4 - w_1 - v_i \geq z_1 + z_i + x_i,\ i \in I: i > 1 \tag{6.9}$$

$$3 - v_1 - w_i \geq z_1 + z_i - x_i\ ,\ i \in I: i > 1 \tag{6.10}$$

If $z_1 = z_i = x_i = 1$, then constraint (6.9) imposes the restriction $1 \geq w_1 + v_i$, which means that either $w_1 = 0$ (equivalently $q_1 = m_1$), or $v_i = 0$ (equivalently $q_i = M_i$); otherwise, this constraint is redundant. Thus, a solution in which $q_1 > m_1$ and $q_i < M_i$ is eliminated in this case. Similarly, if $z_1 = z_i = 1$ and $x_i = 0$, then constraint (6.10) imposes the restriction $1 \geq v_1 + w_i$, which means that either $v_1 = 0$ (equivalently $q_1 = M_1$), or $w_i = 0$ (equivalently $q_i = m_i$); otherwise, this constraint is redundant. Thus, a solution in which $q_i > m_i$ and $q_1 < M_1$ is eliminated in this case.

Note that we deliberately do not utilize decision variables u_i, which were utilized in Section 5.3.3 for imposing a correct smp definition under a uniform clearing scheme, in constraints (6.9) and (6.10) for the following reason. In case the *smp* is determined both by the strategic producer and by some other generation *i* due to a tie in their price-offers ($p_1 = p_i$), it is indifferent which of the two constraints $smp = p_1$ or $smp = p_i$ will be imposed in the problem formulation. On the other hand, if there is a tie in these two price-offers, we want to prioritize the energy allocation to the strategic producer so that in case of multiple lower optima the optimistic approach is respected. Variables u_i treat the case $p_1 = p_i$ the same way they treat the case $p_1 > p_i$, whereas variables x_i treat it the same way they treat the case $p_1 < p_i$. Although transferring a unit of energy between units 1 and i does not change the value of the objective function when $p_1 = p_i$, if variables u_i were used instead of variables x_i, generation unit i would have priority over the strategic producer in such a case, thus violating the optimistic approach assumption. Consequently, decision variables x_i are utilized to ensure that a solution in which $p_1 = p_i$, $q_1 < M_1$ and $q_i > m_i$ is also excluded.

6.2 Valid inequalities modification

Of course, imposing the optimality conditions in question does not ensure that the set of units that will be identified as active in each time period will be optimal. If it happens to be, the identified price-offers of the strategic producer will pertain to the exact optimal solution of the problem. If not, this implies that the unit commitments ($z_{i,t}$) for some time period t in this solution will not be optimal for the corresponding set of price-offers of the strategic producer. In order to pursue our search for the optimal solution, we need to exclude this solution from further consideration. To perform this, we employ a suitable modification of the original procedure utilized in the previous chapter in order to generate valid-cuts for excluding bilevel infeasible solutions as explained next.

First, we solve the lower-level problem for the given set of strategic producer price-offers in order to identify the truly optimal generation unit commitment. Next, we identify the largest interval range for these price-offers within which this unit commitment remains unchanged. In doing so, we extend these intervals even if some quantities change, as long as the corresponding unit commitments remain the same. This is a significant difference with respect to the original approach, in which even a change in a single quantity signified the end of the corresponding interval. After identifying these interval ranges, we employ typical integer programming modeling techniques to generate a valid inequality imposing the truly optimal lower-level unit commitment and eliminate the identified bilevel infeasible solution from further consideration.

Once the maximum simultaneous increase/decrease on the strategic price-offers that does not alter the optimal unit commitment has been identified, it is often the case that the current unit commitment remains optimal when the values of some (not all) price-offers of the strategic producer are increased/decreased further. Therefore, at that point, we investigate whether there is a proper subset of these price-offers that can be further increased/decreased beyond this value without altering the optimal unit commitment. This procedure is pursued repeatedly, until the point where it is not possible to increase/decrease the value of a single price-offer by one unit without altering the optimal unit commitment. This implies that the length of the final interval that will be identified may be different for any two distinct price-offers, depending on the exact order in which these subsets are investigated. Of course, one can investigate all possible combinations and choose the bounds that maximize the cumulative length of all these intervals, but we do not pursue this since it exhibits a combinatorial nature and may lead to performance degradation. Instead, we choose to investigate these subsets randomly, and adopt any path that actually increases the cumulative interval length without altering our intermediate decisions.

To give a particular example, note that, for the small case study of Section 5.4.1, we identified that, starting from an initial value 100 for all 4 price-offers of the strategic producer, their maximum simultaneous decrease for which the solution $q_{1,t}$ = (0, 0, 0, 0) remains optimal to the ISO problem is equal to 35. As it turns out, however, the same solution remains optimal when $p_{1,1}$ is decreased to 63, $p_{1,2}$ is decreased to 62 and $p_{1,3}$ is decreased to 64. This is not the only possible path that can be pursued for extending the initially identified intervals of length 35, but was identified using a randomized neighborhood search. However, it is a path that besides extending the initial intervals it also makes them *tight*, in the sense that decreasing any of the 4 price-offers further beyond the values 63, 62, 64 and 65, respectively, by even one unit alters the optimal solution. Of course, this constitutes a significant enhancement, since it results in the identification of more value-combinations for the strategic producer price-offers, thus succeeding in eliminating more bilevel infeasible solutions. After these improved intervals have been identified, we utilize one of the two following constraints in order to impose a particular unit commitment, 0 or 1, respectively, for a production unit *i* in period *t*.

$$z_{i,t} \leq \sum_{t} (2 - W_l^t - W_r^t) \tag{6.11}$$

$$z_{i,t} \geq 1 - \sum_{t} (2 - W_l^t - W_r^t) \tag{6.12}$$

Constraints (6.11) and (6.12) are analogous to constraints (5.7) and (5.8), but impose a particular unit commitment instead of energy quantity. Variables W_l^t and W_l^t are used in the same exact way for signifying whether price-offer $p_{1,t}$ belongs to its associated interval. If all these binary variables for t = 1,…,T are equal to 1, then the two summations in these constraints are eliminated; thus, $z_{i,t}$ is set equal to 0 if constraint (6.11) is used for generation unit *i*, or it is set equal to 1 if constraint (6.12) is used instead. Of course, if at least one of these variables is equal to 0, then both these constraints become redundant. Using suitably one of these two constraints for each generation unit, one can impose a particular unit commitment, thus eliminating a bilevel infeasible solution. After incorporating the proposed valid inequality into the model formulation, the problem is re-solved again to identify the next candidate solution for optimality, exactly as in the original solution approach. The procedure continues similarly, eventually terminating as soon as the first bilevel feasible solution is encountered, which naturally comprises the problem's exact optimal solution.

6.3 Computational results

We have implemented the improved version of the exact solution algorithm using C/C++ source code. In this section, we present updated results for the small case study presented in the previous chapter, and we present experimental results evaluating the computational performance of the enhanced solution algorithm. All tests were performed on a 6-Core @ 3.5 GHz 64-bit AMD Processor with 8 GB system memory, while the commercial optimization software LINGO 13.0 (2011)[42] was internally utilized for the solution of the encountered optimization models.

6.3.1 A small case study

For comparison purposes, we consider the small case study with 3 production units and a 4-period planning horizon presented in the previous chapter. The technical characteristics and startup costs of the generation units, as well as the price-offers and the demand for energy in each time period are shown in Table 5-1 of Subsection 5.4.1. For each of the two clearing schemes, Table 6-1 presents the total execution time, the total number of times (runs) the ISO optimization problem was solved, and the total number of valid inequalities (cuts) added.

Table 6-1 Execution time, number of runs and number of cuts for the case study (enhanced exact solution algorithm)

Clearing scheme	Time (minutes)	# of runs	# of cuts
pay-as-bid	41	35	24
uniform	53	6	5

Table 6-1 confirms that, for this small case study, there are clear computational savings from the application of the enhanced exact solution algorithm as compared to the previous one, under both clearing schemes. For the pay-as-bid clearing scheme, the percentage difference in time is 33.87 %, while the corresponding percentage difference in the case of the uniform pricing clearing scheme is equal to 58.91 %.

6.3.2 Randomly generated problems

In this subsection, we test the performance of the proposed solution algorithm on randomly generated problems. For comparative purposes, we consider the same random problem instances as those of the previous chapter. For each of the two clearing schemes, Table 6-2 presents the average and maximum computational times, the average and maximum number of runs, and the average and maximum number of cuts utilized. These results demonstrate the reduced computational requirements of the algorithm, confirming that its effectiveness is enhanced. In particular, the computational times are considerably lower than those of the exact algorithm of the previous chapter, mainly due to the significantly smaller number of runs and cuts.

Table 6-2 Computational results for the application of the exact algorithm on random problems

	pay-as-bid						uniform					
	times (secs)		# iterations		# valid inequalities		times (secs)		# iterations		# valid inequalities	
size	**avg**	**max**	**avg**	**max**	**avg**	**max**	**avg**	**max**	**avg**	**max**	**avg**	**max**
3x4	494	2,431	5.4	35	3.05	24	1,407	3,426	3.80	10	2.5	5
4x4	1,245	4,559	10.85	65	2.25	4	2,935	4,961	7.85	20	2.30	5

Table 6-3 presents specific results comparing the computational time required by the two algorithmic versions. More specifically, for each clearing scheme and each of the two problem sizes, this table presents the average and maximum percentage difference in computational time between the original and the enhanced algorithmic version. The maximum percentage difference in average time is 91.7 % and 94.3 % for each of the two clearing schemes, respectively. The average percentage difference in computational time is slightly less than 50% for the pay-as-bid clearing scheme and even more for the uniform clearing scheme. The time savings are considerable for both clearing schemes, especially for the uniform clearing scheme which involves a considerably more complex and resource demanding model formulation.

Table 6-3 Comparison of the computational times of the two algorithmic versions

	pay-as-bid		uniform	
	percentage improvement (enhanced vs. orig.)		percentage improvement (enhanced vs. orig.)	
size	avg	max	avg	max
3x4	48.7	91.7	67.6	93.4
4x4	44.7	91.1	50.2	94.3

6.4 Summary

In this chapter, we addressed an improved version of the exact solution algorithm presented in the previous chapter for the multi-period variant of the problem. This improved methodology utilizes special optimality conditions embedded into the model reformulation, which ensure that the energy quantity distribution in each time period of the planning horizon is optimal for the corresponding set of producers that are identified as active in that time period. Consequently, solving the original problem to global optimality becomes equivalent to identifying the optimal set of active producers in each time period.

In order to exclude from consideration those solutions for which these sets are not optimal, the algorithm employs a special type of cuts based on integer parametric programming theory. The main difference regarding these cuts lies in that they no longer enforce energy quantities but particular unit commitments instead. We also devised an enhanced procedure for extending the intervals within which these inequalities are valid. This constitutes another significant improvement, because it enables the elimination of an increased number of bilevel infeasible solutions from further consideration.

We illustrated the application of the enhanced exact solution algorithm on a small case study, and we presented computational results demonstrating its behavior and performance on randomly generated problems for both clearing schemes. These results show that the proposed methodology is capable of handling medium sized problems without necessitating excessive computational resources. This is very important considering the absence of generic solution methodologies for the treatment of such problems, as well as the fact that the applicability of specialized solution methodologies which have been previously proposed in the related literature appears rather limited on realistic size problems.

Chapter 7 Summary, Conclusions and Future Research

In this dissertation, we addressed the problem of optimal strategic bidding of energy producers in day-ahead electricity markets with indivisibilities. We developed several optimization models for key variants of this problem, all of which fall within the class of bilevel programming. The utilization of binary variables for the modeling of the commitment of the electricity generation units, in conjunction with the imposition of a lower bound on the energy quantity that each unit will provide should it enter the market, prohibit the application of typical methodologies for solving these models, such as the substitution of the lower-level problem by its first-order KKT optimality conditions. Instead, we utilized the special structure of these models combined with key results from the theory of integer parametric programming in order to develop specialized solution methodologies for tackling them.

First, we considered the single-period variant of the problem and we developed an exact solution algorithm for obtaining its global optimum. Our computational results demonstrate the high efficiency of this algorithm, even for large scale problem instances. Next, we considered the multi-period variant of the problem and we utilized its theoretical properties to develop a heuristic solution algorithm, which works in successive iterations by focusing on single time periods. We also demonstrated how the related theoretical groundwork can be modified to enable the generation of valid inequalities to a suitable relaxation of the problem in which the bilevel feasibility of the obtained solution is not guaranteed. The significance of these inequalities lies in that they can be embedded within a cutting plane algorithmic procedure for identifying the exact optimal solution of the problem. We implemented this solution methodology, and we illustrated its applicability as well as that of the heuristic solution algorithm on a small numerical example. We concluded with experimental results demonstrating the computational capabilities of the two proposed solution algorithms, as well as evaluating their relative performance. These results demonstrate that the computational requirements of the heuristic solution algorithm are very moderate, enabling the solution of problems whose size approaches that of realistic ones in reasonable times. The exact solution algorithm, on the other hand, exhibits significantly higher computational requirements, which prohibit its application on realistic problem instances at the moment.

We concluded the dissertation with the development of an enhanced version of the exact solution algorithm for the multi-period variant of the problem. This became possible through the incorporation of special optimality conditions ensuring an optimal energy quantity distribution in each time period of the planning horizon, for any feasible set of active generation units in that period. As a consequence of this enhancement, the identification of the problem's global optimal solution becomes equivalent to identifying the optimal set of active producers in each time period of the planning horizon. We illustrated the application of the enhanced solution algorithm on a case study, and we presented extensive computational results demonstrating its performance and behavior on randomly generated problems. These results reveal that the developed enhancements improve considerably the performance of the proposed exact solution algorithm.

Several model extensions stem out as possible directions for future research. These include the incorporation of additional restrictions that may be present in practice, and the study of markets that operate under different operational rules and assumptions. As far as the model formulation is concerned, the future incorporation of additional problem characteristics that are present in realistic applications, such as minimum uptimes and downtimes, ramp-up/ramp-down constraints, and step-wise priceoffers appears quite promising. A related meaningful extension in this direction is to consider different remuneration schemes than those considered in this work, as well as to include additionally cost-recovery mechanisms (e.g., see Andrianesis et al., 2013a[1]; 2013b[2]) and rules for the offered bids of the participating producers. A related research direction that also seems interesting is the development of a model that includes demand-side bidding to model the demand elasticity.

Tasks such as the above are expected to increase the model formulation complexity, making the problem more challenging and, at the same time, more realistic. In particular, although the minimum uptimes/downtimes and the ramp-up/ramp-down constraints increase the complexity of the optimization model, modifying the proposed solution methodologies to incorporate them can be carried out rather straightforwardly. Due to the fact that they are both typically modeled with linear constraints involving the existing decision variables $z_{i,t}$ and $q_{i,t}$ respectively, the optimization model remains mixed-integer bilinear after their inclusion, which implies that the validity of the integer parametric programming property utilized in both methodologies is retained. Of course, the question of how much the computational performance of the two methodologies or the quality of the proposed heuristic will be affected by the inclusion of these problem characteristics remains open and stems as an interesting direction for future research.

Another interesting direction for future research is the further refinement of the proposed heuristic algorithm’s involved decisions, in order to expedite its computational performance. The question of how initial solutions (seeds) can be wisely selected for enhancing solution quality remains open, too. As far as the exact solution algorithm is concerned, its practical application seems to necessitate further algorithmic enhancements. In that direction, the question of whether special valid-cuts, independent of the particular interval each strategic price-offer belongs to, can be devised for excluding bilevel infeasible solutions appears quite promising. The fulfilment of this task stems as a promising way to overcome the significant computational obstacles the current implementation exhibits.

Appendix: List of Dissertation Publications

Parts of the work presented in this dissertation have been published in scientific journals and scientific conference proceedings and have been presented in international conferences as follows:

Journal Papers

[J.1] Kozanidis, G., Kostarelou, E., Andrianesis, P., Liberopoulos, G. 2013. Mixed integer parametric bilevel programming for optimal strategic bidding of energy producers in day-ahead electricity markets with indivisibilities. *Optimization*, **62**(8) 1045-1068.

[J.2] Kostarelou, E., Kozanidis, G. 2020. Bilevel programming solution algorithms for optimal price-bidding of energy producers in multi-period day-ahead electricity markets with non-convexities. *Optimization and Engineering*, in press.

[J.3] Kozanidis, G., Kostarelou, E. 2020. An exact cutting plane bilevel programming solution algorithm for optimal price-bidding of energy producers in electricity markets, *in preparation*.

Papers in International Conferences

[C.1] Kozanidis, G., Kostarelou, E., Andrianesis, P., Liberopoulos, G. 2011. Mixed integer bilevel programming for optimal bidding strategies in day-ahead electricity markets with indivisibilities. 1st *International Symposium & 10th Balkan Conference on Operational Research* (BALCOR), Thessaloniki, Greece, September 22-25, 8 pages.

Abstracts and Presentations in International Conferences

[P.1] Kostarelou, E., Kozanidis, G. 2013. Mixed integer bilevel programming with upper level decision variables that appear at the lower objective, but not in any of the lower level constraints. 2nd *International Symposium and 24th National Conference on Operational Research*, Athens, Greece, September 26-28.

[P.2] Kostarelou, E., Kozanidis, G. 2014. Mixed integer bilevel programming with upper level decision variables that appear at the lower objective, but not in any of the lower level constraints. 20th *Conference of the IFORS*, Barcelona, Spain, July 13-18.

[P.3] Kostarelou, E., Kozanidis, G. 2018. Exact and heuristic bilevel programming algorithms for optimal price bidding of energy producers in multi-period day-ahead electricity markets. 5th *International Conference on "Energy, Sustainability and Climate Change"* (ESCC 2018), Mykonos, Greece, June 4-6.

[P.4] Kozanidis G., Kostarelou, E. 2020. An exact cutting plane bilevel programming solution algorithm for optimal price-bidding of energy producers in electricity markets. 7th *International Conference on "Energy, Sustainability and Climate Change"* (ESCC 2020), Skiathos, Greece, August 24-26.

Papers, Abstracts and Presentations in National Conferences

[Π.1] Κοζανίδης, Γ., Κωσταρέλου, Ε., Ανδριανέσης, Π., Λυμπερόπουλος, Γ. 2011. Μεικτός ακέραιος διεπίπεδος προγραμματισμός για βέλτιστη υποβολή προσφορών σε αγορές ημερήσιου προγραμματισμού ηλεκτρικής ενέργειας με αδιαιρετότητες. Πρακτικά, *1^{o} Εθνικό Συνέδριο Ελληνικής Μαθηματικής Εταιρείας και Ελληνικής Εταιρείας Επιχειρησιακών Ερευνών*, Αθήνα, Ελλάδα, 24-25 Ιουνίου, 12 σελίδες.

[Π.2] Κωσταρέλου, Ε., Κοζανίδης, Γ. 2012. Ακριβείς και ευρετικοί αλγόριθμοι μεικτού ακέραιου διεπίπεδου προγραμματισμού για βέλτιστη υποβολή προσφορών σε αγορές ημερήσιου προγραμματισμού ηλεκτρικής ενέργειας. Πρακτικά, 23^{o} *Εθνικό Συνέδριο Ελληνικής Εταιρείας Επιχειρησιακών Ερευνών*, Αθήνα, Ελλάδα, 12-14 Σεπτεμβρίου, 49-54, 5 σελίδες.

In Table A-1, we relate each of the above works to the chapters of this dissertation. For each chapter, the publications are listed in chronological order, with the most recent one at the top.

Table A-1 List of publications and association to dissertation chapters.

Chapter 4	**Chapter 5**	**Chapter 6**
[J.1]: entire chapter	[J.2]: entire chapter	[J.3]: entire chapter
[C.1]: early work	[P.1]: early work	[P.4]: early work
[Π.1]: early work	[P.2]: early work	
	[P.3]: early work	
	[Π.2]: early work	

Refferences

[1] P. Andrianesis, G. Liberopoulos, G. Kozanidis, A. Papalexopoulos (2013a). Recovery mechanisms in day-ahead electricity markets with non-convexities – part I: design and evaluation methodology. *IEEE Trans. Power Syst.*, *Vol. 28, pp. 960–968.*

[2] P. Andrianesis, G. Liberopoulos, G. Kozanidis, A. Papalexopoulos (2013b). Recovery mechanisms in day-ahead electricity markets with non-convexities – part II: implementation and numerical evaluation. *IEEE Trans. Power Syst.*, *Vol. 28, pp. 969–977.*

[3] A. Badri, S. Jadid, M. Rashidinejad, M.P. Moghaddam (2008). Optimal bidding strategies in oligopoly markets considering bilateral contracts and transmission constraints. *Electric Power Systems Research, Vol. 78, pp. 1089–1098.*

[4] P. Bajpai and S.N. Singh (2008). Strategic bidding in network constrained electricity markets using FAPSO. *International Journal of Energy Sector Management Vol. 2 No. 2, pp. 274-296.*

[5] A.G. Bakirtzis, N.P. Ziogos, A.C. Tellidou and G.A. Bakirtzis (2007). Electricity Producer Offering Strategies in Day-Ahead Energy Market With Step-Wise Offers. *IEEE Transactions on Power Systems, Vol. 22, No. 4, pp. 1804-1818.*

[6] J.F. Bard (1998). Practical Bilevel Optimization: Algorithms and Applications. *Kluwer Academic Publishers, Boston, MA, USA.*

[7] J.F. Bard and J.T. Moore (1992). An Algorithm for the Discrete Bilevel Programming Problem. *Naval Research Logistics, Vol. 39, pp. 419-435.*

[8] L.A. Barroso, R.D. Carneiro, S. Granville, M.V. Pereira, and M.H.C. Fampa (2006). Nash Equilibrium in Strategic Bidding: A Binary Expansion Approach. *IEEE Transactions on Power Systems, Vol. 21, No. 2, pp 629–638.*

[9] W. Candler and R. Norton (1977). Multilevel programming and development policy. *Working paper, No. 258, World Bank, Washington DC, USA.*

[10] M. Caramia and R. Mari (2015). Enhanced exact algorithms for discrete bilevel linear problems. *Optimization Letters, Vol. 9, pp. 1447-1468.*

[11] S. Dempe (2002). Foundations of Bilevel Programming. *Kluwer Academic Publishers, New York, NY, USA.*

[12] S. Dempe (2010). Annotated bibliography on bilevel programming and mathematical programs with equilibrium constraints. *Optimization, Vol. 52, pp. 333-359.*

[13] S.T. DeNegre and T.K. Ralphs (2009). A branch-and-cut algorithm for integer bilevel linear programs. J.W. Chinneck, B. Kristjansson, M.J. Saltzman, eds. *Operations Research/Computer Science Interfaces Series*, Springer, *Vol. 47, pp. 65-78.*

[14] L.F. Domínguez and E.N. Pistikopoulos (2010). Multiparametric programming based algorithms for pure integer and mixed-integer bilevel programming problems. *Computers & Chemical Engineering, Vol. 34, pp. 2097–2106.*

[15] N.P. Faísca, V. Dua, B. Rustem, P.M. Saraiva and E.N. Pistikopoulos (2007). Parametric global optimization for bilevel programming. *Journal of Global Optimization, Vol. 38, No. 4, pp. 609–623.*

[16] M. Fampa, L.A. Barroso, D. Candal and L. Simonetti (2008). Bilevel optimization applied to strategic pricing in competitive electricity markets. *Computational Optimization & Applications, Vol. 39, pp. 121–142.*

[17] R. Fernández-Blanco, J.M. Arroyo and N. Alguacil (2017). On the solution of revenue- and network-constrained day-ahead market clearing under marginal pricing - Part I: An exact bilevel programming approach. *IEEE Transactions on Power Systems, Vol.* 32*, No. 1, pp. 208-219*.

[18] M. Fischetti, I. Ljubić, M. Monaci and M. Sinnl (2017). A new general-purpose algorithm for mixed-integer bilevel linear programs. *Operations Research, Vol. 65, No. 6, pp. 1615-1637*.

[19] A.A. Foroud, M. Amirahmadi, M. Bahmanzadeh and A.A. Abdoos (2011). Optimal bidding strategy for all market players in a wholesale power market considering demand response programs. *European Transactions on Electrical Power, Vol. 21, pp. 293–311*.

[20] S.A. Gabriel and F.U. Leuthold (2010). Solving discretely-constrained MPEC problems with applications in electric power markets. *Energy Economics, Vol. 32, pp. 3–14.*

[21] A.M. Geoffrion and R. Nauss (1977). Parametric and postoptimality analysis in integer linear programming. *Management Science, Vol. 23, No. 5, pp. 453–466.*

[22] V.P. Gountis and A.G. Bakirtzis (2004). Bidding strategies for electricity producers in a competitive electricity marketplace. *IEEE Transactions on Power Systems, Vol. 19, No. 1, pp. 356-365*.

[23] G. Gross and D. Finlay (2000). Generation supply bidding in perfectly competitive electricity markets. *Computational & Mathematical Organization Theory*, *Vol. 6, pp. 83–98*.

[24] Z.H. Gümüş and C.A. Floudas (2005). Global optimization of mixed-integer bilevel programming problems. *Computational Management Science, Vol. 2, No. 3, 181-212.*

[25] B.F. Hobbs, C.B. Metzler and J-S. Pang (2000). Strategic Gaming Analysis for Electric Power Systems: An MPEC Approach. *IEEE Transactions on Power Systems, Vol. 15, No. 2, pp.638-645.*

[26] X. Hu and D. Ralph (2007). Using EPECs to model bilevel games in restructured electricity markets with locational prices. *Operations Research, Vol. 55, No. 5, pp. 809–827.*

[27] L. Jenkins (1982). Parametric mixed integer programming: an application to solid waste management. *Management Science, Vol. 28, No. 11, pp. 1270–1284*.

[28] P-M. Kleniati and C.S. Adjiman (2014a). Branch-and-sandwich: A deterministic global optimization algorithm for optimistic bilevel programming problems. Part I: Theoretical development. *Journal of Global Optimization, Vol. 60, pp. 425-458*.

[29] P-M. Kleniati and C.S. Adjiman (2014b). Branch-and-sandwich: A deterministic global optimization algorithm for optimistic bilevel programming problems. Part II: Convergence analysis and numerical results. *Journal of Global Optimization, Vol. 60, pp. 459-481*.

[30] P-M. Kleniati and C.S. Adjiman (2015). A generalization of the branch-and-sandwich algorithm: From continuous to mixed-integer nonlinear bilevel problems. *Computers & Chemical Engineering, Vol. 72, pp. 373-386*.

[31] E. Kostarelou and G. Kozanidis (2013). Mixed integer bilevel programming with upper-level decision variables that appear at the lower objective, but not in any of the lower-

level constraints. 2nd *International Symposium and* 24th *National Conference on Operational Research. Athens, Greece, September 26-28.*

[32] E. Kostarelou and G. Kozanidis (2014). Mixed integer bilevel programming with upper-level decision variables that appear at the lower objective, but not in any of the lower-level constraints. 20th *Conference of the IFORS, Barcelona, Spain, July 13-18.*

[33] E. Kostarelou and G. Kozanidis (2018). Exact and heuristic bilevel programming algorithms for optimal price bidding of energy producers in multi-period day-ahead electricity markets. 5th *International Conference on Energy, Sustainability and Climate Change (ESCC), Mykonos, Greece, June 4-6.*

[34] E. Kostarelou and G. Kozanidis (2020). Bilevel programming solution algorithms for optimal price-bidding of energy producers in multi-period day-ahead electricity markets with non-convexities. *Optimization and Engineering*, in press https://doi.org/10.1007/s11081-020-09521-y.

[35] G. Kozanidis and E. Kostarelou (2020). An exact cutting plane bilevel programming solution algorithm for optimal price-bidding of energy producers in electricity markets. 7th *International Conference on Energy, Sustainability and Climate Change (ESCC), Skiathos, Greece, August 24-26.*

[36] G. Kozanidis, E. Kostarelou, P. Andrianesis and G. Liberopoulos (2011). Mixed integer bilevel programming for optimal bidding strategies in day-ahead electricity markets with indivisibilities. *In: 1st International Symposium & 10th Balkan Conference on Operational Research (BALCOR), Sep 22–25; Thessaloniki, Greece. 8 pages.*

[37] G. Kozanidis, E. Kostarelou, P. Andrianesis and G. Liberopoulos (2013). Mixed integer parametric bilevel programming for optimal strategic bidding of energy producers in

day-ahead electricity markets with indivisibilities. *Optimization, Vol. 62, No. 8, pp. 1045-1068.*

[38] M. Köppe, M. Queyranne and C.T. Ryan (2010). Parametric Integer Programming Algorithm for Bilevel Mixed Integer Programs. *Journal of Optimization Theory & Applications, Vol. 146, pp. 137–150.*

[39] R.H. Kwon and D. Frances (2012). Optimization-based bidding in day-ahead electricity auction markets: A review of models for power producers. *In: Sorokin A., Rebennack S., Pardalos P., Iliadis N., Pereira M. (eds) Handbook of Networks in Power Systems I. Energy Systems. Springer, Berlin, Heidelberg, 41-59.*

[40] T. Li and M. Shahidehpour (2005). Strategic Bidding of Transmission-Constrained GENCOs with Incomplete Information. *IEEE Transactions on Power Systems, Vol. 20, pp. 437–447.*

[41] T. Li, M. Shahidehpour and A. Keyhani (2004). Market power analysis in electricity markets using supply function equilibrium model. *IMA Journal of Management Mathematics, Vol. 15, No. 4, pp. 339–354.*

[42] LINGO 13.0, (2011). User's guide, LINDO Systems, Chicago, IL. *Available at: http://www.lindo.com/. Accessed on July 27, 2019.*

[43] P. Loridan and J. Morgan (1996). Weak via strong Stackelberg problem: New results. *Journal of Global Optimization, Vol. 8, pp. 263–287.*

[44] L. Lozano and J.C. Smith (2019). A Value-function-based Exact Approach for the Bilevel Mixed-Integer Programming Problem. *Operations Research, Vol. 65, pp. 768-786.*

[45] Y. Ma, C. Jiang, Z. Hou and C. Wang (2006). The Formulation of the Optimal Strategies for the Electricity Producers Based on the Particle Swarm Optimization Algorithm. *IEEE Transactions on Power Systems, Vol. 21 No. 4, pp. 1663-1671*.

[46] A. Mitsos (2010). Global Solution of Nonlinear Mixed-Integer Bilevel Programs. *Journal of Global Optimization, Vol. 47, pp. 557-582.*

[47] J.T. Moore and J. Bard (1990). The Mixed Integer Linear Bilevel Programming Problem. *Operations Research, Vol. 38, pp. 911-921*.

[48] H. Noltemeier (1970). Sensitivetätsanalyse bei diskreten linearen optimierungsproblemen, in *Lecture Notes in Operations Research and Mathematical Systems*, M. Beckmann and H.P. Kunzi, eds., Springer-Verlag, New York, 310.

[49] R.P. O'Neill, P.M. Sotkiewicz, B.F. Hobbs, M.H. Rothkopf, W.R. Stewart Jr. (2005). Efficient market-clearing prices in markets with nonconvexities. *European Journal of Operational Research, Vol. 164, pp. 269–285.*

[50] M.V. Pereira, S. Granville, M.H.C. Fampa, R. Dix, and L.A. Barroso (2005). Strategic bidding under uncertainty: A binary expansion approach. *IEEE Transactions on Power Systems, Vol. 20, No. 1, pp. 180–188*.

[51] C. Ruiz and A.J. Conejo (2009). Pool Strategy of a Producer With Endogenous Formation of Locational Marginal Prices. *IEEE Transactions on Power Systems, Vol. 24, No. 4, pp.1855-1866.*

[52] G.K. Saharidis and M.G. Ierapetritou (2010). Resolution method for mixed integer bi-level linear problems based on decomposition technique. *Journal of Global Optimization, Vol. 44, No. 1, pp. 29-51*.

[53] F.C. Schweppe, M.C. Caramanis, R.D. Tabors and R.E Bohn (1988). Spot pricing of electricity. *Kluwer, Boston, MA, USA.*

[54] S. Soleymani, A.M. Ranjbar and A.R. Shirani (2008). New approach to bidding strategies of generating companies in day ahead energy market. *Energy Conversion & Management, Vol. 49, pp. 1493–1499.*

[55] A. Tsoukalas, B. Rustem and E.N. Pistikopoulos (2009). A global optimization algorithm for generalized semi-infinite, continuous minimax with coupled constraints and bi-level problems. *Journal of Global Optimization, Vol. 44, pp. 235-250.*

[56] V. Vahidinasab and S. Jadid (2009). Multiobjective environmental/techno-economic approach for strategic bidding in energy markets. *Applied Energy, Vol. 86, pp. 496–504.*

[57] X. Wang and P. Xu (2017). The watermelon algorithm for the bilevel integer linear programming problem. *SIAM Journal on Optimization, Vol. 27, pp. 1403-1430.*

[58] J.D. Weber and T.J. Overbye (2002). An Individual Welfare Maximization Algorithm for Electricity Markets. *IEEE Transactions on Power Systems, Vol. 17, No. 3, pp. 590 – 596.*

[59] U.P. Wen and A.D. Huang (1996). A simple Tabu Search method to solve the mixed-integer linear bilevel programming problem. *European Journal of Operational Research, Vol. 88, pp. 563-571.*

[60] U.P. Wen and Y.H. Yang (1990). Algorithms for Solving the Mixed Integer Linear Programming Problem. *Computers & Operations Research*, *Vol. 17, No. 2, pp. 133-142.*

[61] W. Wiesemann, A. Tsoukalas, P.M. Kleniati and B. Rustem (2013). Pessimistic bilevel optimization. *SIAM Journal on Optimization, Vol. 23, pp. 353-380.*

[62] P. Xu and L. Wang (2014). An exact algorithm for the bilevel mixed integer linear programming problem under three simplifying assumptions. *Computers & Operations Research*, *Vol. 41, No. 3, pp. 309–318*.

[63] D. Yue, J. Gao, B. Zeng and F. You (2019). A projection-based reformulation and decomposition algorithm for global optimization of a class of mixed integer bilevel linear programs. *Journal of Global Optimization*, *Vol. 73, pp. 27–57.*

[64] D. Zhang, Y. Wang and P.B. Luh (2000). Optimization based bidding strategies in the deregulated market. *IEEE Transactions on Power Systems, Vol. 15, pp. 981–986*.

[65] G. Zhang, G. Zhang, Y. Gao and J. Lu (2009). A Bilevel Optimization Model and a PSO-based Algorithm in Day-ahead Electricity Markets. *IEEE International Conference on Systems, Man, and Cybernetics, San Antonio TX, USA, pp. 611-616.*

[66] G. Zhang, G. Zhang, Y. Gao and J. Lu (2011). Competitive Strategic Bidding Optimization in Electricity Markets using Bi-level Programming and Swarm Technique. *IEEE Transactions on Industrial Electronics, Vol. 58, pp. 2138 – 2146.*

[67] F. Zhao, P.B. Luh, J.H. Yan, G.A. Stern and S.C. Chang (2008). Payment Cost Minimization Auction for Deregulated Electricity Markets With Transmission Capacity Constraints. *IEEE Transactions on Power Systems*, *Vol. 23, No. 2, pp.532-544.*

[68] F. Zhao, P.B. Luh, J.H. Yan, G.A. Stern and S.C. Chang (2010). Bid Cost Minimization versus Payment Cost Minimization: A Game Theoretic Study of Electricity Markets. *IEEE Transactions on Power Systems*, *Vol. 25, No. 1, pp. 181 – 194.*