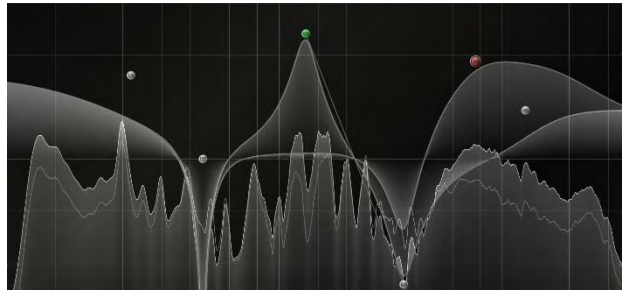




**UNIVERSITY OF THESSALY
DEPARTMENT OF ELECTRICAL & COMPUTER
ENGINEERING**



**Σχεδίαση και Μελέτη Ενεργών Φίλτρων
Design and Study of Active Filters**

Diploma Project

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Περίληψη

Σκοπός αυτής της μελέτης είναι η παρουσίαση στον αναγνώστη, των σημαντικότερων εννοιών γύρω από τα ενεργά φίλτρα, μαζί με ανάλυση κυκλωμάτων και σύγκριση μεταξύ των διαφορετικών τοπολογιών ενεργών φίλτρων. Μπορεί να χρησιμοποιηθεί από αρχάριους, που ενδιαφέρονται για την σχεδίαση ενεργών φίλτρων, σαν σημείο αναφοράς ώστε να μυηθούν στα ενεργά φίλτρα. Το πρώτο κεφάλαιο παρουσιάζει σημαντική ορολογία και βασικές γνώσεις γύρω από τα φίλτρα, όπως, δημοφιλείς τύπους και προσεγγίσεις, καθώς και τεχνικές επίτευξης φίλτρων μεγάλου βαθμού, ανάλογα με τις απαιτήσεις της εφαρμογής τους. Το δεύτερο και τρίτο κεφάλαιο παρουσιάζουν αντίστοιχα, την Sallen-Key (SK) και την Multiple Feedback (MFB) τοπολογία, με λεπτομερή ανάλυση των κυκλωμάτων των χαμηλοπερατών, υψιπερατών και ζωνοπερατών περιπτώσεων, και παρουσιάζονται οι εξισώσεις που τα διέπουν. Παρομοίως, το τέταρτο κεφάλαιο επικεντρώνεται στην State Variable Filter (SVF) τοπολογία, καθώς και στο Biquad φίλτρο, το οποίο είναι μια παραλλαγή του SVF. Το τελευταίο κεφάλαιο είναι μια προσπάθεια προσέγγισης του ιδανικού φίλτρου αποκοπής συχνοτήτων ανάλογα με τις συνθήκες που πρέπει να ικανοποιηθούν. Στο κεφάλαιο αυτό παρουσιάζεται το Twin-T φίλτρο αποκοπής και το φίλτρο Bainter.

Preface

The purpose of this study is to introduce to the reader, the most essential concepts of active filters, along with circuit analysis and comparison between the different active filter topologies. It can be used by beginners, interested in active filter design, as a reference point to grasp the essence of active filtering. The first chapter presents important terminology and basic filter knowledge, such as, popular types and approximations, as well as the technique of cascading filter stages to achieve high order designs and various applications. The second and third chapter present the Sallen-Key (SK) and Multiple Feedback (MFB) topologies respectively. Both chapters provide thorough circuit analysis of the lowpass, highpass and bandpass scenarios, and present the equations that govern each case. In the same sense, the fourth chapter focuses on the State Variable Filter (SVF) topology, as well as, the Biquad filter, which is a variation of the SVF. The last chapter is an effort to approach the ideal notch filter topology, presenting different realizations, such as the Twin-T notch and the Bainter notch, depending on the requirements of the application.

Contents

Περίληψη.....	3
Preface.....	4
1. Introduction.....	7
1.1 Filters and Signals.....	7
1.2 Filter Terminology	9
1.3 Basic Filter Types	11
1.4 Filter Types	15
1.4.1 Butterworth Filters	15
1.4.2 Chebyshev Filters.....	16
1.4.3 Bessel-Thomson Filters.....	17
1.4.4 Elliptic or Cauer Filters.....	18
1.5 Cascading Filter Stages	19
1.6 Applications of Active Filters	20
2. Sallen-Key Filter Topology	22
2.1 General Sallen-Key scheme	22
2.1.1 The Voltage-Controlled Voltage Source (VCVS)	23
2.2 First and second-order Sallen-Key filters	24
2.3 Wide-band Band-pass/reject Filters.....	31
3. Multiple Feedback Filter Topology	32
3.1 Low pass filters	34
3.2 High pass filters	36
3.3 Band Pass MFB	37
4. State Variable Filter Topology	41
4.1 State variable filter general scheme.....	41
4.2 Low Pass SVF.....	43
4.3 High Pass SVF.....	44
4.4 Band Pass SVF.....	45
4.5 Other SVF Realizations	47
4.6 The Biquad Filter.....	48
5. The Perfect Notch Filter	50
5.1 Twin-T Notch	50

5.2 Bainter Notch	52
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1. Introduction

1.1 Filters and Signals

The term filter can have a large number of different meanings. In general, it can be seen as a way to select certain elements with desired properties from a larger set. In circuit analysis a filter is a frequency-selective network that favors certain frequencies of input signal at the expense of others, depending on the application. A frequent task is to attenuate for example, noise or harmonic distortion (Figure 1.1). The fundamental idea upon which filter behavior is relied, is the frequency dependent nature of capacitors and inductors. Capacitors more easily pass high frequencies, while inductors handle better, lower frequencies. The first filter topologies had been designed using solely combinations of passive components, thus we refer to them as passive filters. Nowadays, a modern approach for designing filters is by using operational amplifiers (op-amps) along with resistors and capacitors. These filter networks are called active filters.

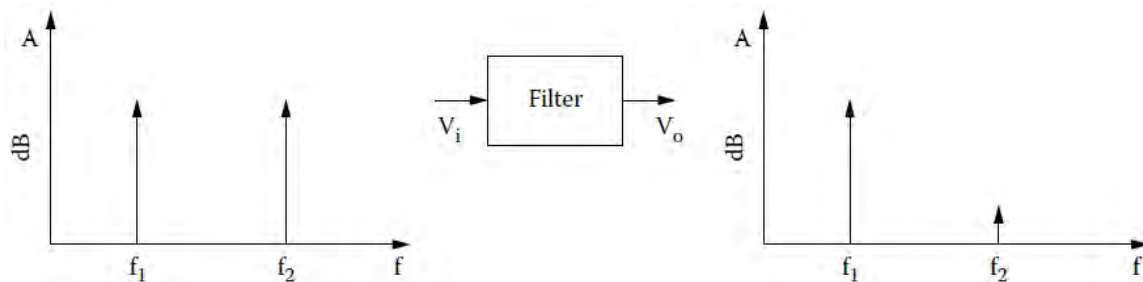


Figure 1.1 using a filter to reduce the effect of an undesired signal

Active filtering outweighs passive one for many reasons:

- The first and most evident difference between the two is that active filters do not need large and expensive inductors, thus enabling small-scale low-cost manufacturing, especially when it comes to low frequency systems, where inductors are bulky and heavy.
- The existence of op-amps in active filters makes their response independent of source and load impedances, due to their ability to set a high input and a low output impedance.

- Another attribute that originates from the isolation of active filters is the ability to cascade filter stages in order to design higher order circuits and satisfy more complex problems.
- Active filters can provide gain or loss as needed to suit filter requirements. Current gain is almost always provided, while voltage gain is an option.
- Many active filters can be easily tuned over a wide range without changing their response shape. Tuning can be done manually, electronically or by voltage control. Tuning ranges can go much higher than is usually possible with passive circuits.

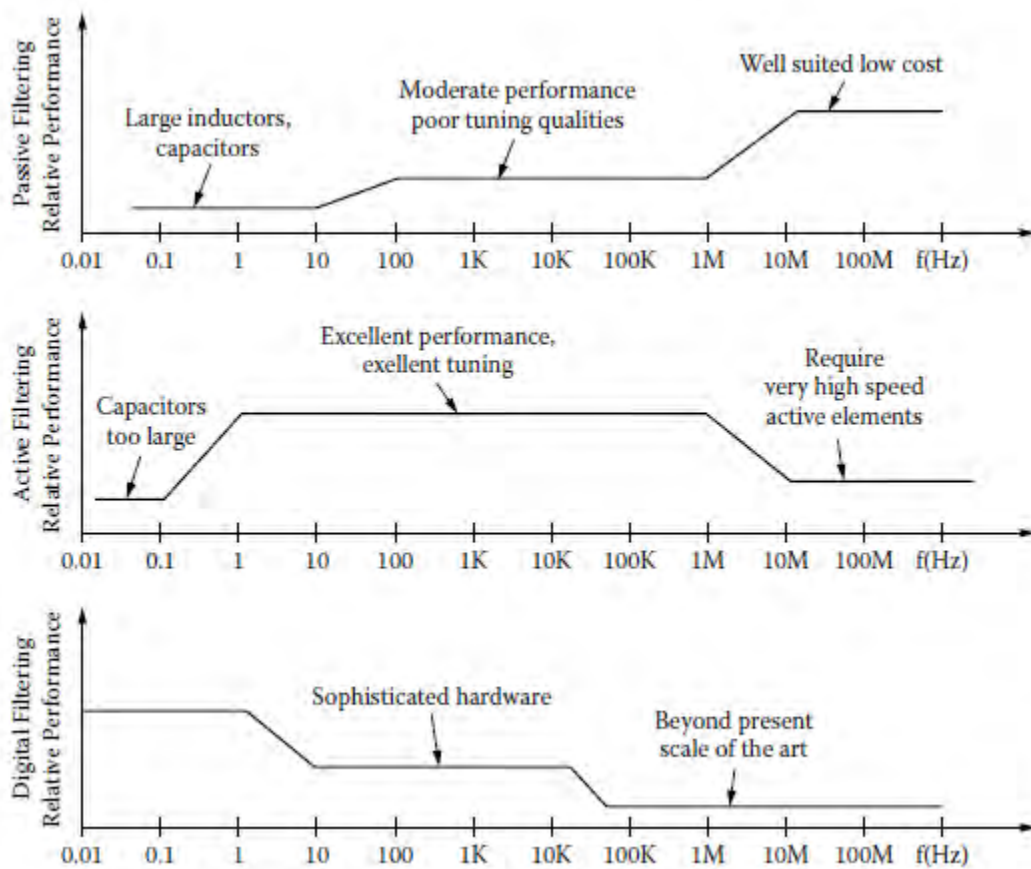


Figure 1.2 a performance comparison of the available filtering technologies

Of course all those advantages come along with some limitations that are op-amp-related and have to do with the signal limits set by the op-amp's characteristics and sensitivity issues due to component and op-amp tolerance, especially in high frequency applications.

In figure 1.2 a comparison is presented among the filter approaches used today. As regards with the active filters, they are limited by the gain-bandwidth product of the op-amp. The gain-bandwidth product is an indicative of the op-amp's open-loop gain at any frequency or vice versa and it must be large for all frequencies of interest, including frequencies in the stopband as well. Though, within the op-amp's operating frequency range, active filters can achieve – depending on the task - ease of tuning, minimum component number, high Q factor or a fixed bandwidth. Some topologies that can achieve these objectives and will be examined later are the Sallen-Key, the multiple feedback (MFB), the state-variable filter (SVF) and the biquad. On the contrary, passive filtering incorporates the use of inductors, and excludes the use of amplifying elements, therefore its applications are not restricted by the bandwidth limitations making them suitable for high frequency systems.

1.2 Filter Terminology

Some essential filter concepts should be introduced for future reference:

- The **cutoff frequency** is the point at which the filter response drops 3dB or to 0.707 of its peak value.
- The **center frequency** is an attribute of bandpass filters. In wide-band bandpass filters, where the bandwidth is greater than an octave, center frequency is computed by

$$f_o = \sqrt{f_1 f_2}$$

Where f_1 is the lower cutoff and f_2 is the upper cutoff frequency.

In narrow-band bandpass filters, center frequency is calculated as the average of the cutoff frequencies:

$$f_o = \frac{f_1 + f_2}{2}$$

- **Damping** or ζ is an index of a filters tendency towards oscillation. Practical damping values range from 2 to 0, with zero being the value of an oscillator, and a value of 1 being a critical value that gives maximum flatness without overshoot (figure 1.3).

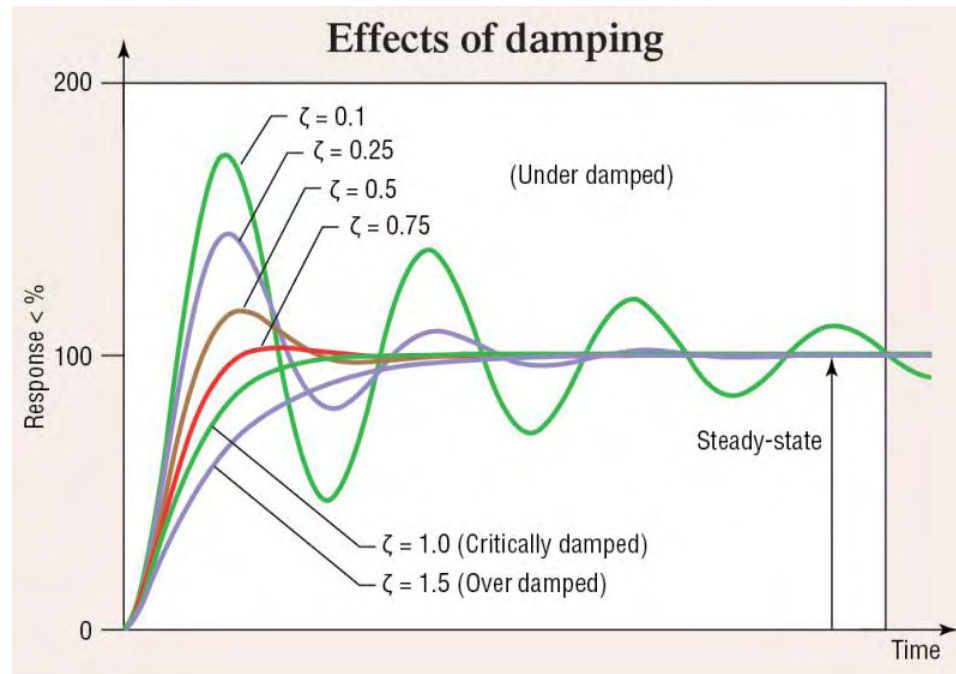


Figure 1.3 overshoot and ringing for different damping values

- **Q Factor** is simply the inverse of damping, $Q = 1/2\zeta$, and is used to measure the peaking of 2nd order filters. The higher the Q, the higher the peak of the response at the cutoff frequency (figure 1.4). Q factor along with the center frequency are indicative of a filter's bandwidth (BW):

$$BW = \frac{\omega_n}{Q}$$

- **Filter order** determines the slope of its rolloff – the transition region between stopband and passband – with frequency. The higher the order, the steeper the rolloff slope.
- **Decibel (dB)** is a logarithmic unit that indicates ratio or gain. It is used to indicate the level of acoustic waves and electronic signals. Decibels are defined as $20 \log_{10} V_o/V_{in}$. A rolloff slope of -6 dB/octave or -20 dB/decade, simply means that the amplitude of a transfer function decreases by 6 dB at each doubling of the frequency, or by 20 dB at each multiplication of the frequency by 10.

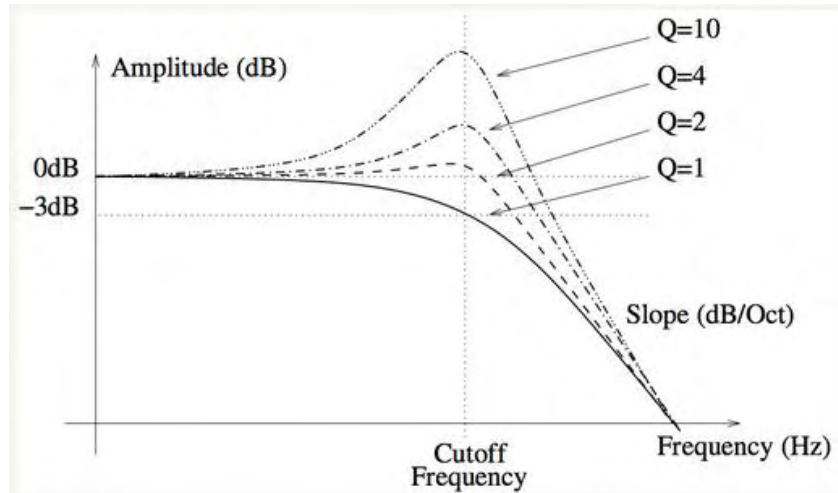


Figure 1.4 peak behavior of a filter for different Q factors

1.3 Basic Filter Types

The frequency-domain behavior of a filter is described mathematically in terms of its transfer function. This is the ratio of the Laplace transforms of its output and input signals. The voltage transfer function of a filter can therefore be written as

$$H(s) = \frac{V_o(s)}{V_i(s)} \quad (1.1)$$

where

$$s = \sigma + j\omega$$

Where σ is the Neper frequency in nepers per second (NP/s) and ω is the angular frequency in radians per second (rad/s).

The transfer function defines the filter's response to any arbitrary input signals. The transfer function magnitude versus frequency is called the amplitude response or sometimes, especially in audio applications, the frequency response.

$$|H(j\omega)| = \left| \frac{V_o(j\omega)}{V_i(j\omega)} \right| \quad (1.2)$$

or

$$A = 20 \log |H(j\omega)| \quad (1.3)$$

and the phase is

$$\arg H(j\omega) = \arg \frac{V_o(j\omega)}{V_i(j\omega)} \quad (1.4)$$

The general form of a filters transfer equation is

$$H(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0} \quad (1.5)$$

The degree of the denominator is the order of the filter. Solving for the roots of the equation determines the poles and zeros of the circuit. Each pole provides a -6 dB/octave or -20 dB/decade response, while each zero provides a +6 dB/octave or +20 dB/decade response.

The various filter types can be defined according to the following classification:

1. **Lowpass filters** (LPF) pass low-frequency signals, and reject signals at frequencies above the filter's cutoff frequency (Figure 1.5). Its transfer function is:

$$H_{LP}(s) = \frac{H_0}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

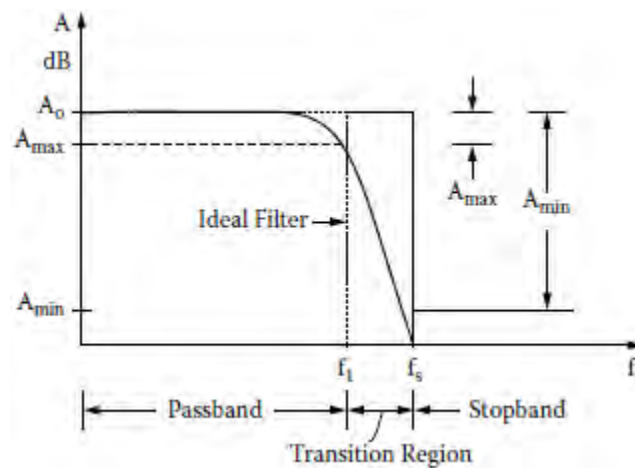


Figure 1.5 lowpass frequency response.

2. A **high-pass filter** (HPF) is the inverse of the low-pass filter and rejects signals below its cutoff frequency (Figure 1.6). Its transfer function is:

$$H_{HP}(s) = \frac{H_0 s^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

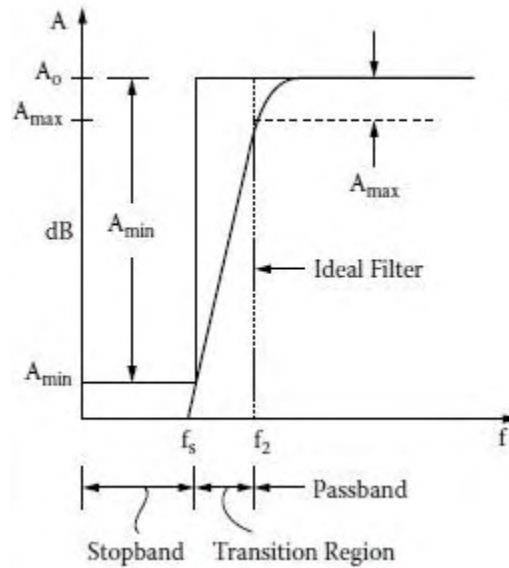


Figure 1.6 highpass frequency response

3. **Band-pass filters** (BPF) pass frequencies within a specified band and reject components outside the band (Figure 1.7). When a narrow bandwidth bandpass filter is amplified, it is called a **resonator filter**. Its transfer function is:

$$H_{BP}(s) = \frac{H_0 \omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$$H_0 = H/Q$$

Q has a particular meaning for the bandpass response. It is the selectivity of the filter.

4. **Band-reject filters** are filters with effectively the opposite function of the bandpass filter (Figure 1.8). A subcategory of band-reject filters are the **notch filters**, which attenuate frequencies in a very narrow bandwidth around the cutoff frequency. Its transfer function is:

$$H_{BR}(s) = \frac{H_0(s^2 + \omega_0^2)}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

5. **All-pass or phase-shift filters** leave the frequency response intact. Their function is to change the phase of the signal without affecting its amplitude.

$$H_{AP}(s) = \frac{s^2 - \frac{\omega_0}{Q}s + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

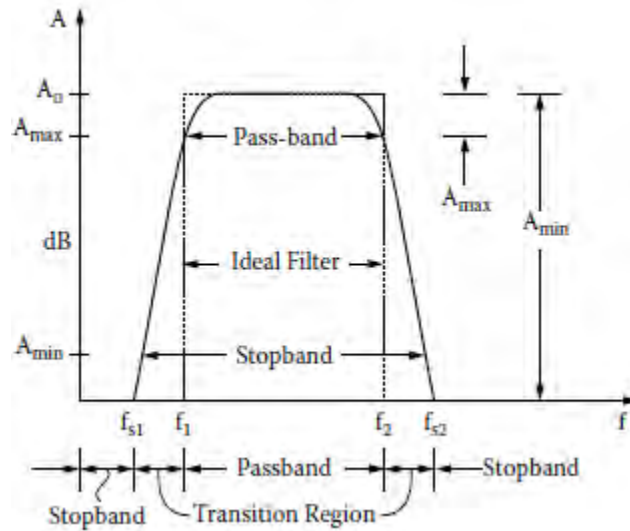


Figure 1.7 bandpass frequency response.

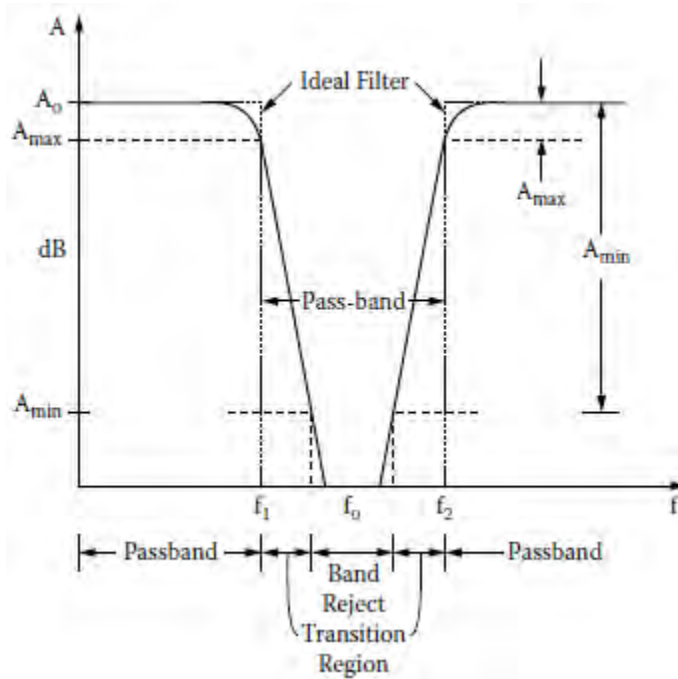


Figure 1.8 band-reject frequency response.

1.4 Filter Types

A great deal of work has already been done and a number of standard filter characteristics have already been defined. These usually provide sufficient flexibility to solve the majority of filtering problems. The classic filter functions were developed by mathematicians (most bear their inventors' names), and each was designed to optimize some filter property. The most widely-used of these are presented below.

1.4.1 Butterworth Filters

Butterworth or maximally flat response is one of the most popular filters, designed to have a frequency response as flat as possible in the passband. The rolloff is smooth and monotonic with a rate of 20 dB/decade or 6 dB/octave for every pole.

The general equation for a Butterworth filter's amplitude response is

$$|H(j\omega)| = \frac{K}{\left[1 + \left(\frac{\omega}{\omega_1}\right)^{2n}\right]^{\frac{1}{2}}} \quad (1.6)$$

where n is the order of the filter, ω_1 is the -3 dB frequency of the filter, and K is the gain of the filter.

We see that $|H(0)| = K$ and $|H(j\omega)|$ is monotonically decreasing with ω . In addition, the -3 dB cutoff frequency is at $\omega = 1$ for all n

$$|H(j\omega)| = \frac{K}{\sqrt{2}} \quad \text{for all } n \quad (1.7)$$

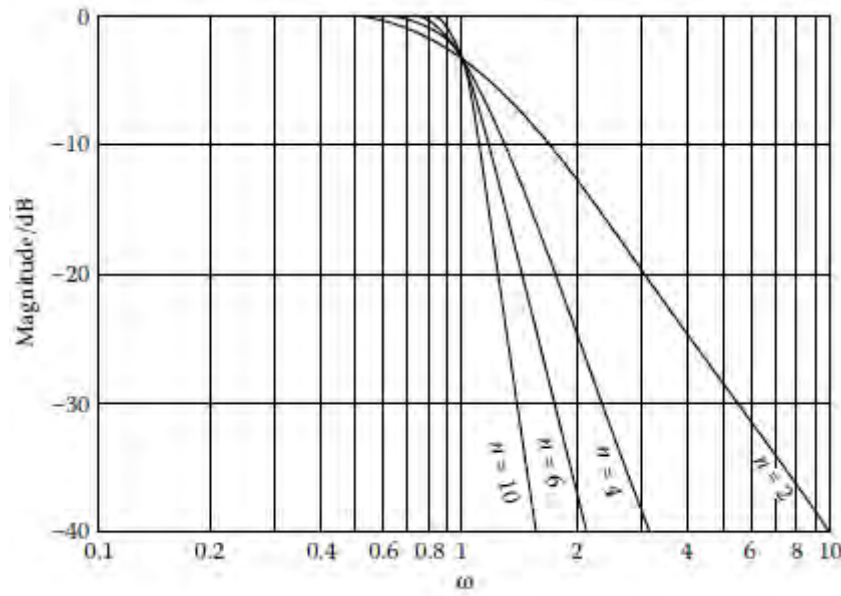


Figure 1.9 amplitude response curves for Butterworth filters of various orders ($K=1$)

The cutoff frequency is thus seen to be $\omega = 1$. The parameter n controls the closeness of approximation in both the band and the stop-band.

For $\omega \gg 1$, the amplitude response of a Butterworth function can be written as (with $K = 1$)

$$|H(j\omega)| \cong \frac{1}{\omega^n} \quad \omega \gg 1 \quad (1.8)$$

Hence the slope is obtained as

$$A = 20 \log |H(j\omega)| = 20 \log \left| \frac{1}{\omega^n} \right| = -20n \log(\omega) \quad (1.9)$$

Consequently, the amplitude response falls at a rate of $-20n$ dB/decade or $-6n$ dB/octave.

1.4.2 Chebyshev Filters

Another approximation to the ideal filter is the Chebyshev or equal ripple response. This sort of filter will have ripple in the passband amplitude response. As the ripple increases, the rolloff becomes sharper. The amount of passband ripple is one of the parameters used

in specifying a Chebyshev filter. The Chebyshev characteristic has a steeper rolloff near the cutoff frequency when compared to the Butterworth, but at the expense of monotonicity in the passband and poorer transient response. The ripple height, or distance between maximum and minimum in the pass-band is given as

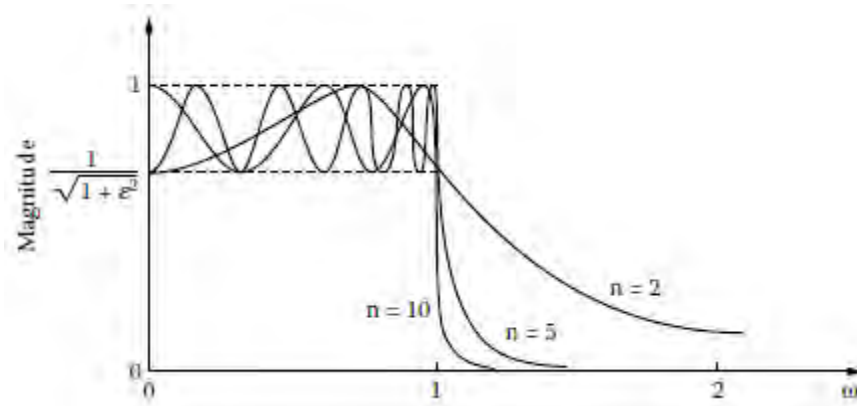


Figure 1.10 amplitude response curves for Chebyshev filters of various orders

$$Ripple = 1 - \frac{1}{(1 + \varepsilon^2)^{\frac{1}{2}}}$$

In figure 1.10 we see the amplitude response for various Chebyshev filter orders. An important thing is that for a filter of order n we will have $n-1$ peaks or dips in the pass-band response. The most important attribute of this filter realization is that it sacrifices passband amplitude stability for a steeper cut-off slope.

1.4.3 Bessel-Thomson Filters

Another filter approximation is the Bessel-Thompson. It is useful when linear phase shift with frequency is needed to avoid overshoot or ringing on the output of a filtering stage. The higher the filter order, the more linear the phase response. If a square wave is to be passed through a Butterworth or Chebyshev filter overshoot or ringing will appear on the output. In that case a Bessel-Thompson filter can be used. Figure 1.11 depicts the difference for a square wave through a 4th order Butterworth and a 4th order Bessel filter. It is obvious that the signal passing through the Butterworth filter causes ringing and

overshoot while the Bessel filter will introduce linear phase shift with respect to frequency (figure 1.12). Hence it will act as a delay line and will just round off the input square wave at the places where high frequency harmonic components are present in the input waveform.

1.4.4 Elliptic or Cauer Filters

Elliptic or Cauer filter has the sharpest rolloff of all previous mentioned types but has ripples in both the passband and the stopband, as illustrated in figure 1.13. The phase response is very non-linear as well. Despite the ripple effect, if frequency selectivity is of great concern, the elliptic filter is the design to be used.

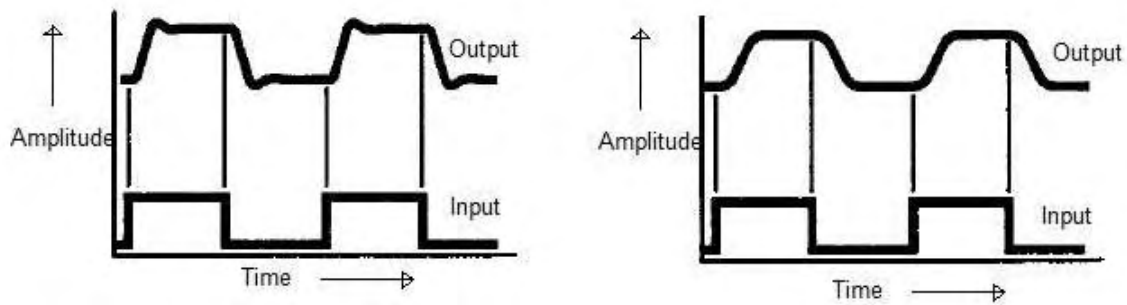


Figure 1.11 a square wave input passing through a 4th order Butterworth filter on the left and a 4th order Bessel filter on the right.

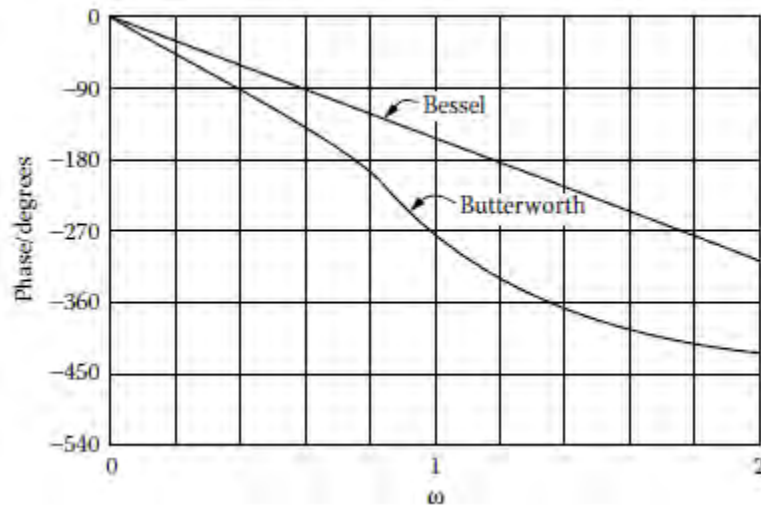


Figure 1.12 phase response comparison between a Butterworth and a Bessel filter with respect to frequency.

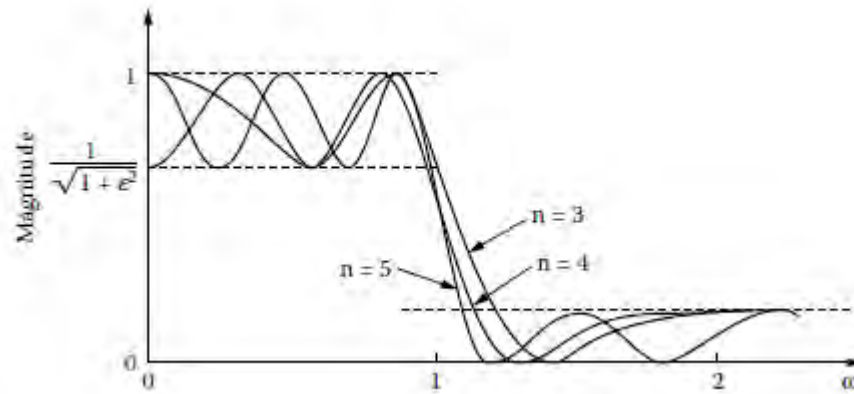


Figure 1.13 response curve of an elliptic filter of various orders.

1.5 Cascading Filter Stages

First and second order filters perform much better when they are combined by suitable cascading to build higher-order filters, in order to achieve a greater stopband attenuation or a broad passband with some special transmission characteristic. Because of the use of op-amps which theoretically provide an infinite input and zero output impedance, a simple cascade of first and second order filters can be made without interactions between the individual stages. The system's transfer function is the product of each stage's transfer function and each filter can be tuned separately.

The order of the filter is given by the highest power of frequency that appears under the transfer function. As the order of the filter increases, the ultimate response falloff versus frequency gets better. The rate is $6N$ dB/octave, where N is the order.

Figure 1.14 shows how to construct even and odd-order filters. The filter is broken into complex-conjugate-pole pairs that can be realized by either Sallen-Key, or MFB circuits. To implement an n -order filter, $n/2$ stages are required. Adding a first-order real pole at the beginning of the system an odd-order filter can be constructed, as shown in 3.b. The stages are normally arranged with the lowest Q near the input and the highest Q near the output.

Each stage is carefully chosen to be a factor of the overall response shape desired, the stages cascaded are rarely identical and the response shape of the individual stages normally appears wildly different from the final response shape. Further, the cutoff

frequencies of each individual stage may also be very much different from the final overall cutoff frequency desired.

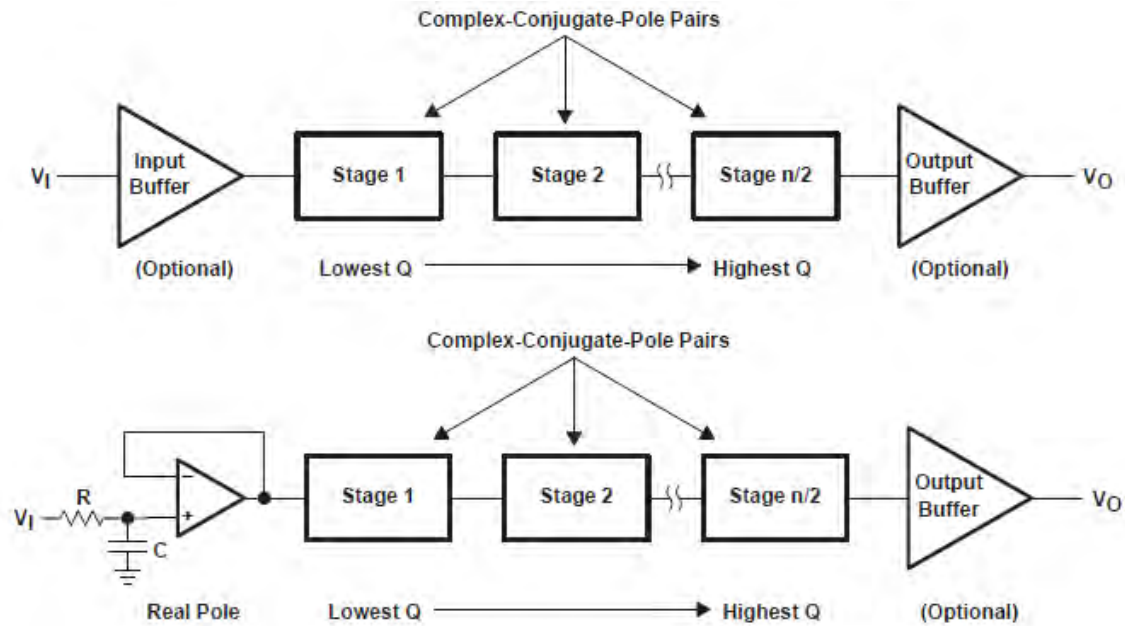


Figure 1.14 cascading filter stages for odd or even order filter construction.

1.6 Applications of Active Filters

Active filtering plays an important role in audio processing. An active filter added to a guitar preamplifier can dramatically change the sound of the instrument by selectively emphasizing portions of the acoustic spectrum of the guitar. Synthesizers with many real instrument options are realized with the aid of active filtering. Their function depend on the control of the harmonics of a sinusoidal, sawtooth or square wave input signal and the envelope or the amplitude, as seen in figure 1.15. Audio equalizers cut or boost certain audio bands by setting up an arrangement of active bandpass filters. A general schematic of an audio equalizer is presented in figure 1.16.

Another field active filters are used is brainwave research, where a method similar to that of an audio equalizer is used to segregate the four commonly recognized brainwaves, delta, theta, alpha and beta.

Oscillators and signal sources can be created by providing enough feedback to a filter. Moreover, tone creation or detection and noise rejection heavily rely upon active filters

and their characteristics. Such as in modems, biomedical instrumentation, communications, lowpass noise filtering for general instrumentation and anti-alias filtering for data acquisition systems. Active filters excel in controlled frequency filtering (vcf) as well, applied in spectrum analysis.

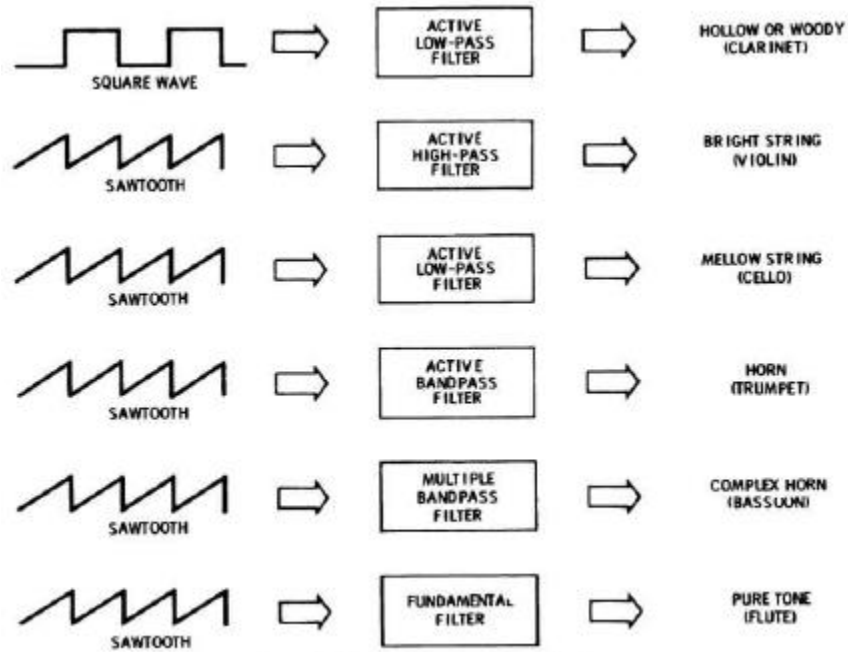


Fig. 10-4. Active formant filter basics.

Figure 1.15 input manipulation through filtering for simulating instrument tones.

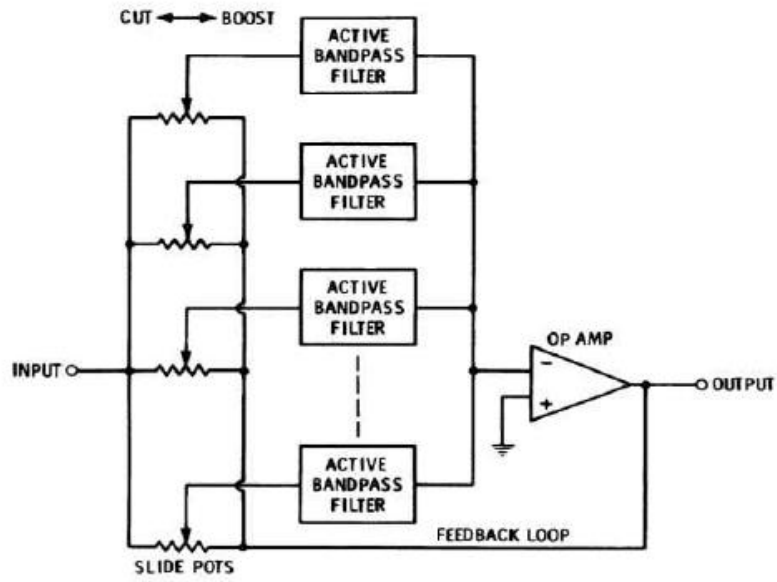


Figure 1.16 general schematic for a sound equalizer.

2. Sallen-Key Filter Topology

2.1 General Sallen-Key scheme

One of the most widely used filter topologies is that of Sallen and Key, also known as a voltage control-controlled voltage source (VCCVS). It was first introduced in 1955 by R.P. Sallen and E. L. Key of MIT's Lincoln Labs. It utilizes an op-amp and a voltage divider configuring an amplifier and as such it offers good stability, requires minimum number of elements and has low impedance, which is important for cascading filters with four or more poles.

The popularity of this topology is attributed to the fact that it shows the least dependence of filter performance on the performance of the op-amp. That is because the op-amp is configured as an amplifier whose gain bandwidth product does not limit the performance of the filter, in contrast to an integrator configuration, allowing the design of a higher frequency filter for a given op-amp.

Another advantage is the small component spread which is good for manufacturability. Plus, frequency and Q value are somewhat independent though very sensitive to gain parameter which renders this topology unsuitable for high Q applications.

2.1.1 The Voltage-Controlled Voltage Source (VCCVS)

The need for buffering a high impedance source to a filter is satisfied with a non-inverting VCCVS op-amp. Typical input impedances are greater than tens of mega ohms and output impedances are typically less than a few ohms, depending on the amplifier being used.

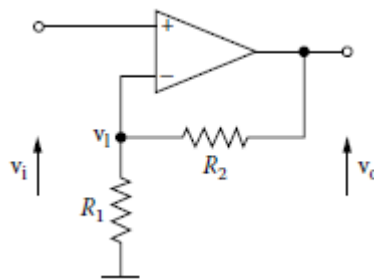


Figure 2.1 A non-inverting VCCVS op-amp with resistive feedback

From figure 1.16, we have:

node v_1

$$G_1 V_1 + G_2 (V_1 - V_0) = 0$$

$$V_0 = \left(\frac{G_1 + G_2}{G_2} \right) V_1$$

but,

$$V_1 = V_- = V_+ = V_i \quad (\text{ideal opamp})$$

therefore,

$$V_0 = \left(1 + \frac{G_1}{G_2} \right) V_i = \left(1 + \frac{R_2}{R_1} \right) V_i$$

$$K = \frac{V_0}{V_i} = 1 + \frac{R_2}{R_1} \quad (2.1)$$

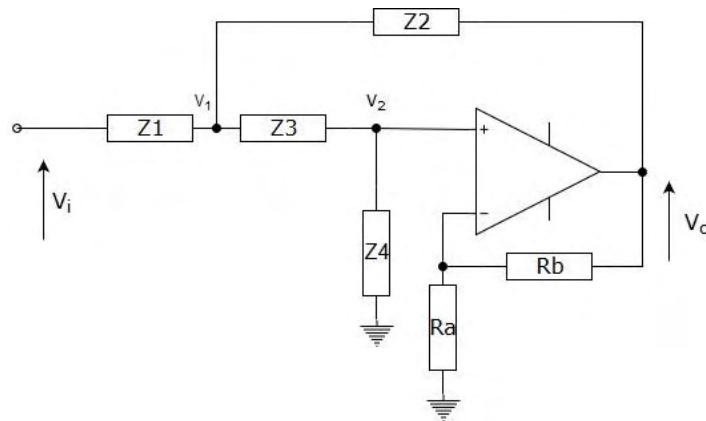


Figure 2.2 2nd-Order General Sallen-Key topology

2.2 First and second-order Sallen-Key filters

In figure 2.2 we have the general second-order Sallen-Key circuit from which we can obtain the first-order one by omitting Z2 and Z3 impedances. For example let's analyze the first-order LPF circuit in figure 2.3:

For node V_1 :

$$\frac{1}{R}(V_1 - V_i) + sCV_1 = 0 \rightarrow V_1 = \frac{1}{1 + sRC}V_i$$

Also

$$V_0 = KV_i \text{ and } K = 1 + \frac{R_b}{R_a}$$

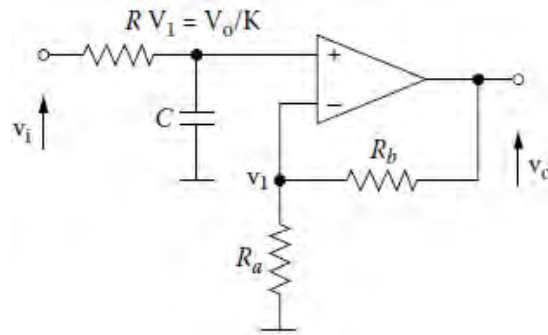


FIGURE 2.3 First-order LPF

$$V_0(s) = \frac{K}{1 + sRC}V_i(s)$$

$$H(s) = \frac{V_0(s)}{V_i(s)} = \frac{K}{1 + \frac{s}{\omega_1}} \quad (2.4)$$

Where

$$s = j\omega \text{ and } \omega_1 = \frac{1}{RC}$$

$\omega_1 = 2\pi f_1$, f_1 is the cutoff frequency of the filter.

From the transfer function we have:

1. For $s/\omega_1 \ll 1$ we have:

$$|H(j\omega)| \cong K$$

$$A = 20 \log |H(j\omega)| = 20 \log K \text{ dB}$$

Hence the slope is 0 dB/decade.

2. For $s/\omega_1 \gg 1$ we have:

$$|H(j\omega)| = \frac{K}{\frac{\omega}{\omega_1}} = K \left(\frac{\omega}{\omega_1} \right)^{-1}$$

$$A = 20 \log |H(j\omega)| = 20 \log K + 20 \log \left(\frac{\omega}{\omega_1} \right)^{-1} = 20 \log K - 20 \log \left(\frac{\omega}{\omega_1} \right) \text{ dB}$$

Hence the slope for $\omega/\omega_1 = 10$ is -20 dB/decade

3. For $s/\omega_1 = 1$:

$$|H(j\omega)| = \frac{K}{\sqrt{2}}$$

$$A = 20 \log \frac{K}{\sqrt{2}} = 20 \log K - 20 \log \sqrt{2} = 20 \log K - 3 \text{ dB}$$

Which is the -3dB cutoff frequency point.

Figure 2.4 shows the frequency response of the filter.

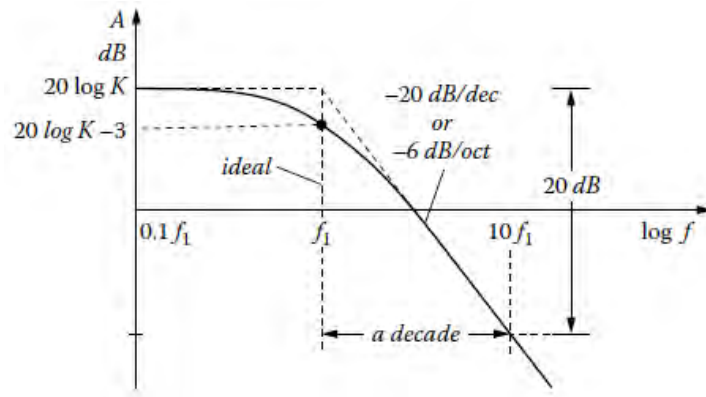


Figure 2.4 First-order LPF frequency response

We're going to analyze the general second-order Sallen-Key topology. From figure 2.2 we have:

For node V_1 :

$$\frac{1}{Z_1}(V_1 - V_i) + \frac{1}{Z_2}(V_1 - V_0) + \frac{1}{Z_3}(V_1 - V_2) = 0 \quad (2.2)$$

For node V_2 :

$$\frac{1}{Z_3}(V_2 - V_1) + \frac{1}{Z_4}V_2 = 0 \rightarrow$$

$$V_1 = V_2 \left(1 + \frac{Z_3}{Z_4} \right) \quad (2.3)$$

Also

$$V_2 = \frac{V_0}{K} \quad (2.4)$$

Combining equations (2.3) and (2.4) we obtain:

$$V_1 = \frac{V_0}{K} \left(1 + \frac{Z_3}{Z_4} \right) \quad (2.5)$$

And by substituting (2.5) in (2.2):

$$\frac{V_0}{V_i} = K \frac{Z_2 Z_4}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3 + Z_2 Z_4 + Z_1 Z_4 (1 - K)} \quad (2.6)$$

To extract the equations that define a second-order low-pass Sallen-Key filter we substitute $Z_1 \rightarrow R_1$, $Z_3 \rightarrow R_2$, $Z_2 \rightarrow 1/sC_1$ and $Z_4 \rightarrow 1/sC_2$. Therefore:

$$H(s) = \frac{K \frac{1}{sC_1} \frac{1}{sC_2}}{R_1 \frac{1}{sC_1} + R_1 R_2 + R_2 \frac{1}{sC_1} + \frac{1}{sC_1} \frac{1}{sC_2} + R_1 \frac{1}{sC_2} (1 - K)} \rightarrow$$

$$H(s) = \frac{K}{s^2 R_1 R_2 C_1 C_2 + s [C_2 (R_1 + R_2) + C_1 R_1 (1 - K)] + 1} \rightarrow$$

$$H(s) = \frac{K \frac{1}{R_1 R_2 C_1 C_2}}{s^2 + s \frac{[C_2 (R_1 + R_2) + C_1 R_1 (1 - K)]}{R_1 R_2 C_1 C_2} + \frac{1}{R_1 R_2 C_1 C_2}} \quad (2.7)$$

From the standard second-order form:

$$H(s) = \frac{K \omega_n^2}{s^2 + \frac{\omega_n}{Q} s + \omega_n^2} \quad (2.8)$$

Therefore we can calculate the natural frequency, and the Q factor as follows:

$$\omega_n = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}} \quad (2.9)$$

$$\frac{\omega_n}{Q} = \frac{C_2 (R_1 + R_2) + C_1 R_1 (1 - K)}{R_1 R_2 C_1 C_2} \rightarrow$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2(R_1 + R_2) + C_1 R_1(1 - K)} \quad (2.10)$$

In order to observe the **frequency response** of the filter, from equation (2.8):

$$H(s) = \frac{K}{\left(\frac{s}{\omega_n}\right)^2 + \left(\frac{s}{\omega_n}\right)Q + 1}$$

1. For $s/\omega_n \ll 1$

$$H(j\omega) \cong K$$

$$A = 20 \log|H(j\omega)| = 20 \log K \text{ dB}$$

2. For $s/\omega_n \gg 1$

$$H(j\omega) \cong \frac{K}{\left(\frac{\omega}{\omega_n}\right)^2} = K \left(\frac{\omega}{\omega_n}\right)^{-2}$$

$$A = 20 \log|H(j\omega)| = 20 \log K \left(\frac{\omega}{\omega_n}\right)^{-2} = 20 \log K - 40 \log\left(\frac{\omega}{\omega_n}\right) \text{ dB}$$

For $\omega/\omega_n = 10$

$$A = 20 \log K - 40 \text{ dB}$$

The slope is -40 dB/decade

3. For $s/\omega_n = 1$

$$|H(j\omega)| \cong \left| \frac{K}{1+j} \right| = \frac{K}{\sqrt{2}}$$

$$A = 20 \log \frac{K}{\sqrt{2}} = 20 \log K - 20 \log \sqrt{2} = 20 \log K - 3 \text{ dB}$$

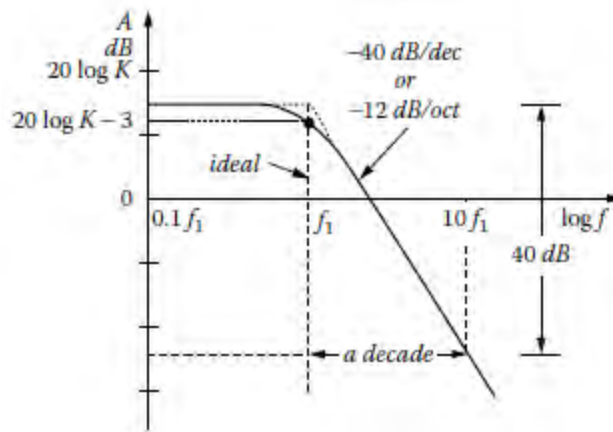


Figure 2.5 2nd SK LP frequency response

We see that the slope is two times steeper than the slope of the first order one, due to the order of the filter.

Likewise for the second-order high-pass filter:

$Z_1 \rightarrow 1/sC_1$, $Z_3 \rightarrow 1/sC_2$, $Z_2 \rightarrow R_1$ and $Z_4 \rightarrow R_2$. Therefore:

$$H(s) = \frac{KR_1R_2}{\frac{1}{sC_1}R_1 + \frac{1}{sC_1}\frac{1}{sC_2} + R_1\frac{1}{sC_2} + R_1R_2 + \frac{1}{sC_1}R_2(1-K)} \rightarrow$$

$$H(s) = \frac{Ks^2R_1R_2C_1C_2}{s^2R_1R_2C_1C_2 + s[R_1(C_1 + C_2) + R_2C_2(1-K)] + 1} \rightarrow$$

$$H(s) = \frac{Ks^2}{s^2 + s\frac{[R_1(C_1 + C_2) + R_2C_2(1-K)]}{R_1R_2C_1C_2} + \frac{1}{R_1R_2C_1C_2}}$$

From the standard form of high-pass filter:

$$H(s) = \frac{Ks^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

We evaluate:

$$\omega_n = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}}$$

$$\frac{\omega_n}{Q} = \frac{R_1(C_1 + C_2) + C_2 R_2(1 - K)}{R_1 R_2 C_1 C_2} \rightarrow$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1(C_1 + C_2) + C_2 R_2(1 - K)}$$

2.3 Wide-band Band-pass/reject Filters

Wide-band bandpass filters with relatively constant gain can be realized simply by clarifying the specifications into lowpass and highpass filter stages and cascading them. A wide-band approach is valid when the separation between cutoffs is at least an octave so that minimum interaction occurs in the passband.

In the same manner, a wide-band band-reject filter can be constructed according to figure 2.6. The first step is to parallel the filter stages and sum both outputs with the aid of an inverting amplifier. The amplifier can also provide K gain to the circuit.

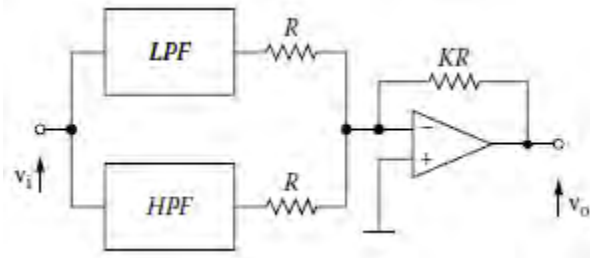


Figure 2.6 wide-band band-reject building block

Sallen-Key filters exhibit serious disadvantages in forming notch filters. The resonant/notch frequency cannot be adjusted easily due to component interaction. Moreover, the gain is fixed by other design parameters and there is a wide spread in component values. Since there are easier to use circuits, Sallen-Key topology is avoided for notch filter construction.

3. Multiple Feedback Filter Topology

Another very popular configuration is the multiple feedback filter (MFB) which uses a single op-amp as an integrator, providing RC feedback around it. A general configuration for the 2nd order case is shown in figure 3.1. The MFB configuration exhibits greater dependence of the transfer function on the op-amp parameters in contrast to the Sallen-Key, responsible for which, is the integrator configuration. It is difficult to generate high Q, high frequency sections due to the limitations of the open-loop gain of the op-amp. It is called a **multiple feedback infinite gain amplifier filter** and it inverts the phase of the signal. Another thing to mention is that the component spread is higher than that of the Sallen-Key.

With this configuration we can have a low-pass, a high-pass, and narrow-band band-pass characteristic.

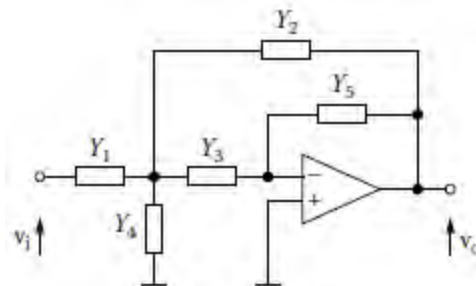


Figure 3.1 General multiple feedback filter topology

For node V_1 :

$$\frac{1}{Z_1}(V_1 - V_i) + \frac{1}{Z_2}(V_1 - V_0) + \frac{1}{Z_4}V_1 + \frac{1}{Z_3}(V_1 - V_2) = 0 \quad (3.1)$$

For node V_2 :

$$\frac{1}{Z_3}(V_2 - V_1) + \frac{1}{Z_5}(V_2 - V_0) = 0 \quad (3.2)$$

Also

$$V_2 = V_- = V_+ = 0 \quad (3.3)$$

By replacing equation (3.3) into (3.2) we get

$$V_1 = -\frac{Z_3}{Z_5} V_0 \quad (3.4)$$

Utilizing equations (3.3) and (3.4) into (3.1) we get

$$-V_0 \left(\frac{Z_3}{Z_5 Z_1} + \frac{Z_3}{Z_2 Z_5} + \frac{1}{Z_5} + \frac{Z_3}{Z_4 Z_5} + \frac{1}{Z_2} \right) = \frac{1}{Z_1} V_i \rightarrow$$

$$H(s) = \frac{V_0}{V_i} = -\frac{Z_2 Z_4 Z_5}{Z_2 Z_3 Z_4 + Z_1 Z_3 Z_4 + Z_1 Z_2 Z_4 + Z_1 Z_2 Z_3 + Z_1 Z_4 Z_5} \quad (3.5)$$

3.1 Low pass filters

We're going to calculate the frequency response of the second-order low-pass MFB circuit in Figure 3.1. We substitute $Z_1 \rightarrow R_1$, $Z_2 \rightarrow R_2$, $Z_3 \rightarrow R_3$, $Z_4 \rightarrow 1/sC_1$ and $Z_5 \rightarrow 1/sC_2$. From equation (3.5) and for these impedance values we get:

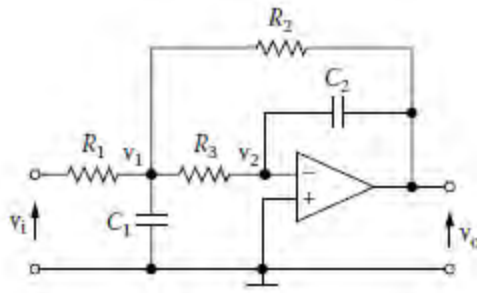


Figure 3.2 2nd order low pass MFB

$$H(s) = -\frac{R_2 \frac{1}{sC_1} \frac{1}{sC_2}}{R_2 R_3 \frac{1}{sC_1} + R_1 R_3 \frac{1}{sC_1} + R_1 R_2 \frac{1}{sC_1} + R_1 R_2 R_3 + R_1 \frac{1}{sC_1} \frac{1}{sC_2}} \rightarrow$$

$$H(s) = -\frac{\frac{1}{R_1 R_3 C_1 C_2}}{s^2 + s \frac{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}{C_1} + \frac{1}{R_2 R_3 C_1 C_2}} \quad (3.6)$$

We can calculate the gain K by setting $s = 0$:

$$K = H(0) = -\frac{R_2}{R_1} \quad (3.7)$$

And rewrite the equation (3.6) as:

$$H(s) = \frac{K \frac{1}{R_2 R_3 C_1 C_2}}{s^2 + s \frac{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}{C_1} + \frac{1}{R_2 R_3 C_1 C_2}} \quad (3.8)$$

$$H(s) = \frac{K \omega_n^2}{s^2 + \frac{\omega_n}{Q} s + \omega_n^2} \quad (3.9)$$

Where

$$\omega_n = \sqrt{\frac{1}{R_2 R_3 C_1 C_2}} \quad (3.10)$$

$$Q = \frac{\omega_n C_1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \rightarrow$$

$$Q = \frac{\sqrt{R_2 R_3 C_1 C_2} R_1}{C_2 (R_1 R_2 + R_1 R_3 + R_2 R_3)} \rightarrow$$

$$Q = \frac{\sqrt{R_2 R_3 C_1 C_2}}{(R_2 + (1 - K) R_3) C_2} \quad (3.11)$$

3.2 High pass filters

For the high-pass circuit we substitute $Z_1 \rightarrow 1/sC_1$, $Z_2 \rightarrow 1/sC_2$, $Z_3 \rightarrow 1/sC_3$, $Z_4 \rightarrow R_1$ and $Z_5 \rightarrow R_2$

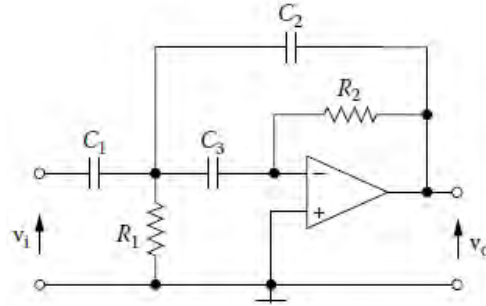


Figure 3.3 HP MFB

$$H(s) = -\frac{\frac{1}{sC_2} R_1 R_2}{\frac{1}{sC_2} \frac{1}{sC_3} R_1 + \frac{1}{sC_1} \frac{1}{sC_3} R_1 + \frac{1}{sC_1} \frac{1}{sC_2} R_1 + \frac{1}{sC_1} \frac{1}{sC_2} \frac{1}{sC_3} + \frac{1}{sC_1} R_1 R_2} \rightarrow$$

$$H(s) = -\frac{s^2 \frac{C_1}{C_2}}{s^2 + s \frac{(C_1 + C_2 + C_3)}{R_2 C_2 C_3} + \frac{1}{R_1 R_2 C_2 C_3}}$$

$$K = \lim_{s \rightarrow \infty} H(s) = -\frac{C_1}{C_2}$$

$$H(s) = \frac{K s^2}{s^2 + \frac{\omega_n}{Q} s + \omega_n^2}$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_2 C_3}}$$

$$\frac{\omega_n}{Q} = \frac{(C_1 + C_2 + C_3)}{R_2 C_2 C_3} \rightarrow$$

$$Q = \frac{\omega_n R_2 C_2 C_3}{C_1 + C_2 + C_3} \rightarrow$$

$$Q = \frac{\sqrt{R_1 R_2 C_2 C_3}}{R_1 (C_3 + (1 - K) C_2)}$$

3.3 Band Pass MFB

When the separation between the upper and lower frequencies exceeds a ratio of approximately 2, the bandpass filter is considered a wide-band type of filter. The specification is then separated into individual lowpass and highpass requirements and met by cascade of active lowpass and highpass filters.

Though, with the MFB topology we can design a 2nd order narrow-band bandpass filter as shown in figure 3.4.

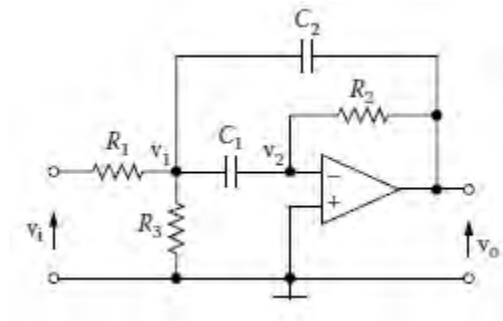


Figure 3.4 2nd order bandpass MFB filter.

From this circuit, working in the same sense as above:

node v1:

$$-\frac{V_i}{R_1} + \left(\frac{1}{R_1} + \frac{1}{R_3} + sC_1 + sC_2 \right) V_1 - sC_1 V_2 - sC_1 V_o = 0$$

Node v2:

$$-sC_1 V_1 + \left(sC_1 + \frac{1}{R_2} \right) V_2 - \frac{1}{R_2} V_o = 0$$

Again $V_2 = 0$, hence

$$V_1 = -\frac{1}{sC_1 R_2} V_o$$

$$H(s) = \frac{V_o}{V_i} = -\frac{s \frac{C_1}{R_1}}{s^2 C_1 C_2 + s(C_1 + C_2) \frac{1}{R_2} + \left(\frac{1}{R_1} + \frac{1}{R_3}\right) \frac{1}{R_2}}$$

Transforming it to the general form

$$H(s) = -\frac{s \frac{1}{R_1 C_2}}{s^2 + s \frac{C_1 + C_2}{C_1 C_2} \frac{1}{R_2} + \left(\frac{1}{R_1} + \frac{1}{R_3}\right) \frac{1}{R_2} \frac{1}{C_1 C_2}}$$

Let $C_1 = C_2 = C$, $R_1 = 1/R_2$, $R_3 \rightarrow \infty$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{C}$$

At $\omega = 1$ rad/s,

$$\frac{\omega_n}{Q} = \frac{C_1 + C_2}{C_1 C_2} \frac{1}{R_2} \Rightarrow Q = \frac{1}{2} R_2$$

$$K = H(j\omega_0) = -\frac{1}{2} \frac{R_2}{R_1} = -2Q^2$$

$$R_2 = 2Q, R_1 = \frac{1}{2Q}$$

From the equations above we can deduct that the tuning of the resonant frequency is possible by altering the two capacitors, but keeping their values identical. Damping or its inverse Q factor can be varied by varying the ratio of the two resistors, keeping their product constant. Though the limitation this circuit exhibits is pointed out by the dependency between gain and Q. The gain of this circuit is fixed at $-2Q^2$ and cannot be independently adjusted. Increasing the Q has the result of lowering the input resistance and raising the op-amp feedback resistance. Using the R3 resistor it is possible to lower the circuit gain and raise the input impedance at the same time, without affecting any other attribute of the filter. Overall, this is a good, general-purpose, low-Q filter. The upper Q limit depends on the op-amp and the desired frequency.

From the transfer function we have:

$$H(s) = -\frac{\frac{1}{R_1 C \omega_0} \left(\frac{s}{\omega_0}\right)}{\left(\frac{s}{\omega_0}\right)^2 + \frac{2}{R_2 C \omega_0} \left(\frac{s}{\omega_0}\right) + 1}$$

The general form is:

$$H(s) = -\frac{K_0 \left(\frac{s}{\omega_0}\right)}{\left(\frac{s}{\omega_0}\right)^2 + \frac{1}{Q} \left(\frac{s}{\omega_0}\right) + 1}$$

1. for $s/\omega_0 \ll 1$

$$H(j\omega_0) = -K_0 \left(j \frac{\omega}{\omega_0}\right)$$

$$A = 20 \log K_0 \left(\frac{\omega}{\omega_0}\right)$$

$$A = 20 \log K_0 + 20 \log \frac{\omega}{\omega_0}$$

Hence the slope is 20 dB/dec

2. for $s/\omega_0 \gg 1$

$$H(j\omega_0) = -K_0 \left(j \frac{\omega}{\omega_0}\right)^{-1}$$

$$A = 20 \log K_0 \left(\frac{\omega}{\omega_0}\right)^{-1}$$

$$A = 20 \log K_0 - 20 \log \left(\frac{\omega}{\omega_0}\right)$$

Hence the slope is -20 dB/dec

2. for $s/\omega_0 = 1$

$$H(j\omega_0) = -\frac{jK_0}{-1 + j\frac{1}{Q} + 1} = -QK_0 = K$$

$$K_0 = \frac{K}{Q}$$

Figure 3.5 depicts the frequency response of the narrow-band bandpass filter.

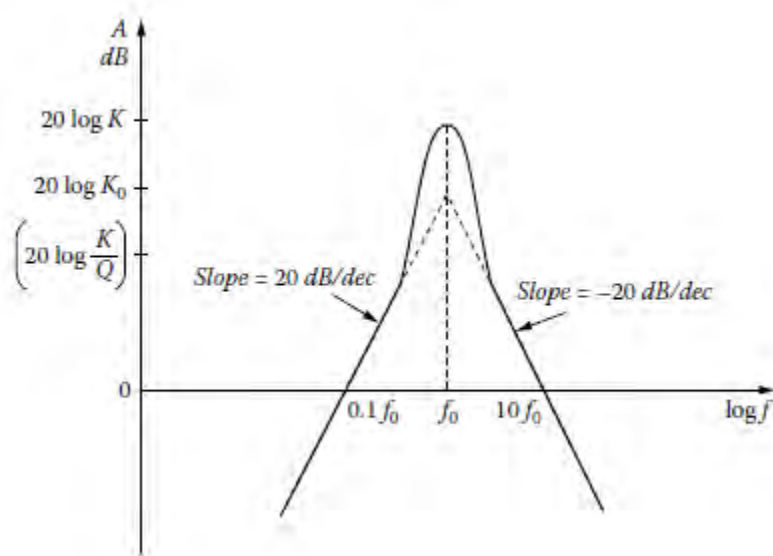


Figure 3.5 narrow-band bandpass filter frequency response

4. State Variable Filter Topology

4.1 State variable filter general scheme

The state variable filter or SVF, is a type of multiple-feedback filter circuit that can produce all three filter responses, low pass, high pass and band pass simultaneously from the same single active filter design. Figure 4.1 shows that it uses three op-amps for its basic operation cascaded together to produce the individual filter outputs and more specifically, a summing amplifier followed by two consecutive integrator stages. The input signal is injected in the summing amplifier along with the feedback networks of the two integrator stages to obtain the high pass response. Subsequently the high pass signal is passed through the first integrator stage. Integrators use a capacitor within their feedback loop and as such, the output voltage is proportional to the integral of the input voltage.

Because of its versatility, this filter is called the universal filter. This filter provides the user with easy control of the gain and Q-factor. It offers several features which are not available with the other simpler filters. By properly summing all three filter types outputs some very interesting responses can be made. Bandpass filters with high Q can be built. The damping and/or critical frequency could be electronically tuned. The low-pass and high-pass outputs are inverted in phase while the band-pass output maintains the phase. The gain of each of the outputs of the filter is also independently variable. With an added amplifier section summing the low-pass and high-pass sections the notch function can also be synthesized. By changing the ratio of the summed sections, low-pass notch, standard notch and high-pass notch functions can be realized.

Beginning with the analysis of the SVF, we start from the first stage which is a summing amplifier. The input signal, the high pass and the low pass outputs are fed back to the negative connector, while a portion of the band pass output is fed back to the positive connector.

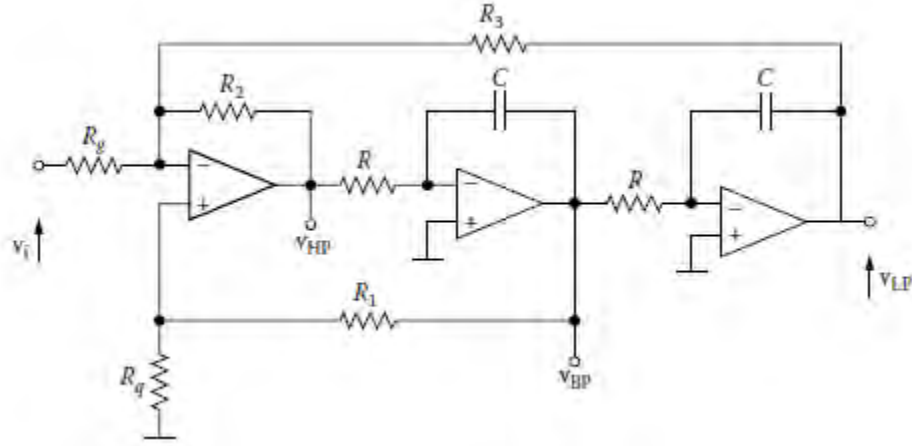


Figure 4.1 state-variable filter general scheme

For the negative connector we have:

$$\frac{V_i - V^-}{R_g} + \frac{V_{HP} - V^-}{R_2} + \frac{V_{LP} - V^-}{R_3} = 0$$

$$V^- \left(\frac{1}{R_g} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{V_i}{R_g} + \frac{V_{HP}}{R_2} + \frac{V_{LP}}{R_3} \quad (4.1)$$

For the positive connector we have:

$$\frac{V_{BP} - V^+}{R_1} = \frac{V^+}{R_q}$$

$$\frac{V_{BP}}{R_1} = V^+ \left(\frac{1}{R_1} + \frac{1}{R_q} \right)$$

$$V^+ = V_{BP} \frac{R_q}{R_q + R_1} \quad (4.2)$$

Though, $V^- = V^+$

Hence equation 4.1 becomes

$$\frac{R_q}{R_q + R_1} \left(\frac{1}{R_g} + \frac{1}{R_2} + \frac{1}{R_3} \right) V_{BP} = \frac{V_i}{R_g} + \frac{V_{HP}}{R_2} + \frac{V_{LP}}{R_3}$$

$$V_{HP} = -\frac{R_2}{R_3}V_{LP} - \frac{R_2}{R_g}V_i + \frac{R_q}{R_q + R_1} \left(1 + \frac{R_2}{R_3} + \frac{R_2}{R_g}\right)V_{BP} \quad (4.3)$$

After the summing amplifier stage, signal V_{HP} is passed through two consecutive identical integrator stages. Integrators use a frequency dependent impedance in the form of a capacitor within their feedback loop. As a capacitor is used the output voltage is proportional to the integral of the input voltage. More thoroughly:

$$\frac{V^- - V_{HP}}{R} + (V^- - V_{BP})sC = 0$$

But, $V^- = V^+ = 0$

$$V_{BP} = -\frac{1}{sRC}V_{HP} \quad (4.4)$$

Which can be re-written in the frequency domain as:

$$V_{BP} = -\frac{1}{2\pi f_c RC}V_{HP}$$

Frequency-wise the output signal is $1/2\pi f_c RC$ times the input signal. $1/2\pi RC$ is a constant, therefore, as frequency grows the magnitude of the output falls. Plus, integrators exhibit a -180 degrees phase lag because the input signal is connected directly to the inverting input terminal of the op-amp.

Likewise, for the following integrator:

$$V_{LP} = -\frac{1}{sRC}V_{BP} \quad (4.5)$$

4.2 Low Pass SVF

From equation 4.4 and 4.5

$$V_{HP} = -sRCV_{BP} \quad (4.6)$$

$$V_{BP} = -sRCV_{LP} \quad (4.7)$$

Hence:

$$V_{HP} = s^2 R^2 C^2 V_{LP} \quad (4.8)$$

From equation 4.3 and 4.7 we have:

$$s^2 R^2 C^2 V_{LP} = -\frac{R_2}{R_3} V_{LP} - \frac{R_2}{R_g} V_i - \frac{R_q}{R_q + R_1} \left(1 + \frac{R_2}{R_3} + \frac{R_2}{R_g}\right) sRC V_{LP}$$

$$\left[s^2 R^2 C^2 + \frac{R_q}{R_q + R_1} \left(1 + \frac{R_2}{R_3} + \frac{R_2}{R_g}\right) sRC + \frac{R_2}{R_3} \right] V_{LP} = -\frac{R_2}{R_g} V_i$$

$$H_{LP} = \frac{V_{LP}}{V_i} = -\frac{\frac{R_2}{R_g}}{s^2 R^2 C^2 + \frac{R_q}{R_q + R_1} \left(1 + \frac{R_2}{R_3} + \frac{R_2}{R_g}\right) sRC + \frac{R_2}{R_3}}$$

The general form of a low pass filter is

$$H(s) = \frac{K \omega_n^2}{s^2 + \frac{\omega_n}{Q} s + \omega_n^2}$$

Hence

$$H_{LP} = \frac{V_{LP}}{V_i} = -\frac{\frac{R_2}{R_g} \frac{1}{R^2 C^2}}{s^2 + \frac{R_q}{R_q + R_1} \left(1 + \frac{R_2}{R_3} + \frac{R_2}{R_g}\right) \frac{1}{RC} s + \frac{R_2}{R_3} \frac{1}{R^2 C^2}} \quad (4.9)$$

We can calculate the gain K by setting $s = 0$:

$$K = H(0) = \frac{R_3}{R_g}$$

4.3 High Pass SVF

From equation 4.4 and 4.5

$$V_{LP} = \frac{1}{s^2 R^2 C^2} V_{HP} \quad (4.10)$$

Equation 4.3 from equations 4.4 and 4.10:

$$V_{HP} = -\frac{R_2}{R_3} \frac{1}{s^2 R^2 C^2} V_{HP} - \frac{R_2}{R_g} V_i - \frac{R_q}{R_q + R_1} \left(1 + \frac{R_2}{R_3} + \frac{R_2}{R_g}\right) \frac{1}{sRC} V_{HP}$$

$$V_{HP} \left[1 + \frac{R_2}{R_3} \frac{1}{s^2 R^2 C^2} + \frac{R_q}{R_q + R_1} \left(1 + \frac{R_2}{R_3} + \frac{R_2}{R_g}\right) \frac{1}{sRC}\right] = -\frac{R_2}{R_g} V_i$$

$$H_{HP} = \frac{V_{HP}}{V_i} = -\frac{\frac{R_2}{R_g}}{1 + \frac{R_2}{R_3} \frac{1}{s^2 R^2 C^2} + \frac{R_q}{R_q + R_1} \left(1 + \frac{R_2}{R_3} + \frac{R_2}{R_g}\right) \frac{1}{sRC}}$$

The general form of a high pass filter is

$$H = \frac{Ks^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

Hence

$$H_{HP} = -\frac{\frac{R_2}{R_g} s^2}{s^2 + \frac{R_q}{R_q + R_1} \left(1 + \frac{R_2}{R_3} + \frac{R_2}{R_g}\right) \frac{1}{RC} s + \frac{R_2}{R_3} \frac{1}{R^2 C^2}}$$

Gain K is

$$K = \lim_{s \rightarrow \infty} H_{HP}(s) = \frac{R_2}{R_g}$$

4.4 Band Pass SVF

Equation 4.3 from equations 4.5 and 4.6:

$$-sRCV_{BP} = -\frac{R_2}{R_3} \left(-\frac{1}{sRC}\right) V_{BP} - \frac{R_2}{R_g} V_i + \frac{R_q}{R_q + R_1} \left(1 + \frac{R_2}{R_3} + \frac{R_2}{R_g}\right) V_{BP}$$

$$H_{BP} = \frac{V_{BP}}{V_i} = \frac{\frac{R_2}{R_g} \frac{1}{RC} s}{s^2 + \frac{R_q}{R_q + R_1} \left(1 + \frac{R_2}{R_3} + \frac{R_2}{R_g}\right) \frac{1}{RC} s + \frac{R_2}{R_3} \frac{1}{R^2 C^2}}$$

$$\omega_n = \sqrt{\frac{R_2}{R_3}} \frac{1}{RC}$$

From the general bandpass equation:

$$H_{BP}(s) = \frac{H_0 \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Hence

$$Q = \frac{R_q + R_1}{R_q \left(1 + \frac{R_2}{R_3} + \frac{R_2}{R_g}\right)} \sqrt{\frac{R_2}{R_3}} \frac{1}{RC}$$

For $s = j\omega$

$$K = H_{BP}(j\omega) = \frac{\frac{R_2}{R_g}}{\frac{R_q}{R_q + R_1} \left(1 + \frac{R_2}{R_3} + \frac{R_2}{R_g}\right)}$$

For $\omega = 1 \text{ rad/s}$

For a normalized filter:

$$R = R_g = R_2 = 1\Omega$$

$$C = 1F$$

Hence

$$R_3 = 1\Omega$$

Substituting in the above equations:

$$Q = \frac{R_q + R_1}{3R_q} \Rightarrow R_1 = (3Q - 1)R_q$$

$$K = \frac{R_q + R_1}{3R_q} = \frac{R_q + (3Q - 1)R_q}{3R_q} = Q$$

As repeated before frequency can be tuned in steps by switching the capacitors or by varying the frequency-determining resistors, independent of Q or gain. Circuit Q and bandwidth are adjusted with a single resistor. As the frequency changes, the Q and

percentage bandwidth stay constant. The absolute bandwidth goes up or down proportionately with the center frequency.

The circuit gain is Q and the op-amp need only have a gain of $3Q$ or so, open-loop, at the resonance frequency. This is a much less severe restriction than the earlier filters, so state-variable techniques are ideal for high Q and high frequency uses.

4.5 Other SVF Realizations

A SVF notch can be constructed by summing the highpass and lowpass outputs along with an appropriate voltage divider network.

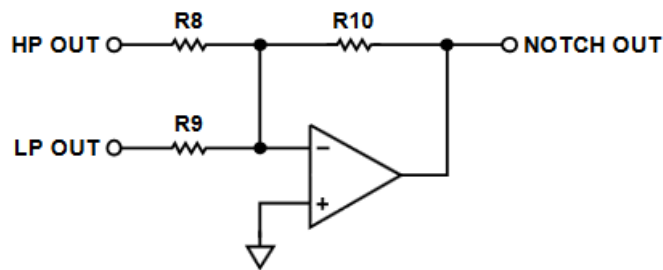


Figure 4.2 state-variable filter notch construction

$$R_8 = R_9$$

$$K = \frac{R_{10}}{R_8}$$

In the same manner an all-pass filter can be realized by summing the input signal along with the bandpass output of the filter.

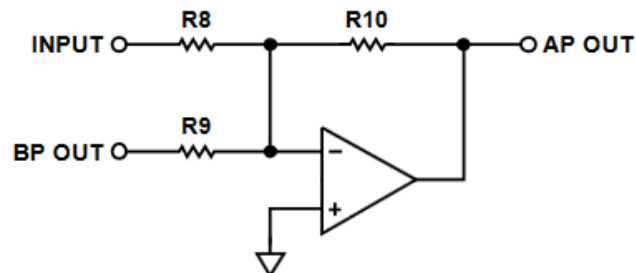


Figure 4.3 state-variable filter all-pass filter construction

$$R_8 = R_{10}$$

$$R_9 = \frac{R_8}{2}$$

4.6 The Biquad Filter

The Biquad filter consists of two integrators and inverter. Loss is introduced into one of the integrators with a damping and Q-setting resistor. The Biquad demonstrates a particular useful characteristic, constant bandwidth. In the Biquad filter, as the frequency changes, the absolute bandwidth remains constant, while in the SVF, the bandwidth narrows at lower and expands at higher frequencies.

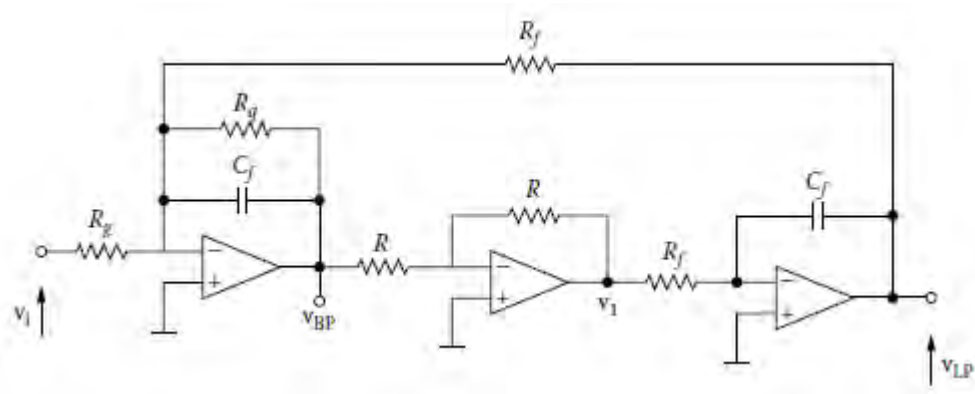


Figure 4.4 biquad filter scheme

$$Y = \frac{1}{R_q} + sC_f = \frac{1 + sR_q C_f}{R_q} \Rightarrow Z = \frac{R_q}{1 + sR_q C_f}$$

$$V_{BP} = -\frac{Z}{R_g} V_i - \frac{Z}{R_f} V_{LP} \quad (4.11)$$

$$V_1 = -V_{BP} \quad (4.12)$$

Equation 4.11 from 4.12

$$V_{BP} = -\frac{Z}{R_g} V_i - \frac{Z}{R_f} \frac{1}{sR_f C_f} V_{BP}$$

$$V_{BP} \left(1 + \frac{Z}{sR_f^2 C_f} \right) = -\frac{Z}{R_g} V_i$$

$$V_{BP} \left(\frac{sR_f^2 C_f + Z}{sR_f^2 C_f} \right) = -\frac{Z}{R_g} V_i$$

$$\frac{V_{BP}}{V_i} = -\frac{sR_f^2 C_f Z}{(sR_f^2 C_f + Z)R_g} = -\frac{sR_f^2 C_f}{R_g \left(1 + \frac{sR_f^2 C_f}{Z} \right)} = -\frac{\frac{sR_f^2 C_f}{R_g}}{1 + sR_f^2 C_f \frac{1 + sR_q C_f}{R_q}}$$

$$H_{BP} = -\frac{\frac{sR_f^2 C_f R_q}{R_g}}{R_q + sR_f^2 C_f + s^2 R_f^2 R_q C_f^2} = -\frac{\frac{1}{R_g C_f} s}{s^2 + \frac{1}{R_q C_f} s + \frac{1}{R_f^2 C_f^2}}$$

$$\omega_n = \frac{1}{R_f C_f}$$

$$BW = \frac{\omega_n}{Q} = \frac{1}{R_q C_f} \Rightarrow Q = \frac{R_q}{C_f} \quad (4.13)$$

$$K = H_{BP}(j\omega_n) = -\frac{R_q}{R_g}$$

The center frequency is easily tuned by merely adjusting the value of R_f . Also, Q may be adjusted by changing the value of R_q , and the gain of the filter may be changed by adjusting the value of R_g . The biquad filter is capable of attaining high values of Q , in the neighborhood of 100, and is a much more stable network than those discussed in the previous bandpass filters.

With a little observation of equation 4.13, it can be seen that bandwidth is independent of frequency tuning resistor R_f . The Q goes up and the percentage bandwidth goes down as frequency is increased and vice versa.

The biquad is handy if you want a group of identical absolute bandwidth channels in a system, such as in telephone applications and spectrum analyzers.

5. The Perfect Notch Filter

The notch filter's significance is found in applications where elimination of undesirable signals is needed, such as, tone-signaling, hearing-aid feedback, and mains rejection systems. Another use is in distortion analyzers for calculating the total harmonic distortion (THD) of an input signal. Variable notch filters are also used in sound engineering for graphic equalizers, synthesizers and electronic crossovers to deal with narrow peaks in the acoustic response of the music.

For example, 50/60 Hz hum or prominent acoustic feedback can be eliminated thanks to the very narrow band-stop region of notch filters. The bandwidth can be as low as around 10-20 Hz, with the unwanted frequency reduced by 40dB or more.

In the previous chapters, the inadequacy of Sallen-Key and MFB topologies to deliver a satisfying notch behavior was pointed out. While, by summing the highpass and lowpass output of a SVF, a satisfying, medium Q notch can be realized, if there is a need for greater precision, higher Q and generally a more independent control over the center frequency and component selection, there are more elaborate designs to choose.

5.1 Twin-T Notch

Twin-T filter (figure 5.1) consists of three resistors and three capacitors. It is widely used as a general-purpose notch circuit. The passive implementation has a Q that is fixed at 0.25. This issue can be rectified with the application of positive feedback to the reference node. The amount of signal feedback, set by the R_4/R_5 ratio, determines the value of Q of the circuit which in turn determines the notch depth. For maximum notch depth R_4 , R_5 and the associated op-amp can be eliminated.

It operates by phase shifting the signals in the different legs and adding them to the output. At the notch frequency, the signals passing through each leg are 180 degrees out of phase and cancel out. In order to achieve such a precise response the components tolerances must be very low.

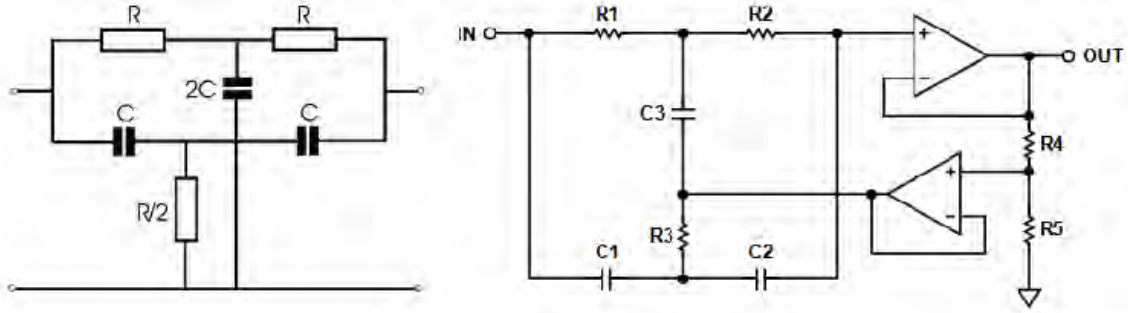


Figure 5.1 Twin-T notch filter. The passive configuration on the left and the active configuration on the right

From the two circuits above:

$$R_1 = R_2 = R, R_3 = R/2$$

$$C_1 = C_2 = C, C_3 = 2C$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{s^2 + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} = \frac{s^2 + \left(\frac{1}{RC}\right)^2}{s^2 + \left(\frac{1}{RC}\right) \left(\frac{4}{1 + \frac{R_5}{R_4}}\right) s + \left(\frac{1}{RC}\right)^2}$$

Center frequency is calculated by

$$f_0 = \frac{1}{2\pi RC}$$

And Q is set by the R5/R4 ratio as such

$$Q = \left(1 + \frac{R_5}{R_4}\right) / 4$$

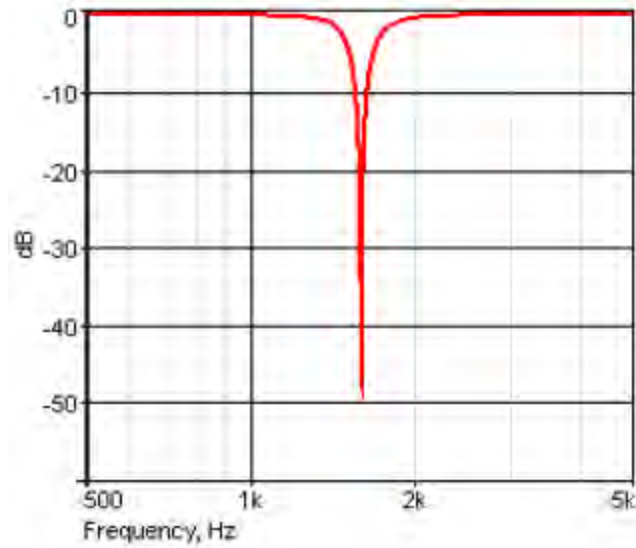


Figure 5.2 2nd order Twin-T notch filter response

Figure 5.2 shows the response of a 2nd order Twin-T notch filter. Such a narrow bandwidth and abrupt attenuation behavior could be very useful for example, in removing the 60 Hz interference component in measuring instruments.

5.2 Bainter Notch

Another notch filter topology that differentiates from the others is the Bainter notch. In contrast to most notch realizations, its Q is not based on component matching, yet on the gain of the amplifiers. Consequently, the notch depth is not affected by temperature drift or circuit aging and it remains relatively constant even though the filter's notch frequency may shift.

As seen in figure 5.3, it is made up of three amplifier stages with two feedback loops. A highpass filter, a lowpass filter and the last amplifier providing the notch output.

The transfer function for the Bainter notch topology is:

$$\frac{V_{OUT}}{V_{IN}} = \frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} = \frac{K_2 \left(s^2 + \frac{K_1}{R_3 R_5 C_1 C_2} \right)}{s^2 + \frac{R_5 + R_6}{R_5 R_6 C_2} s + \frac{K_2}{R_4 R_5 C_1 C_2}}$$

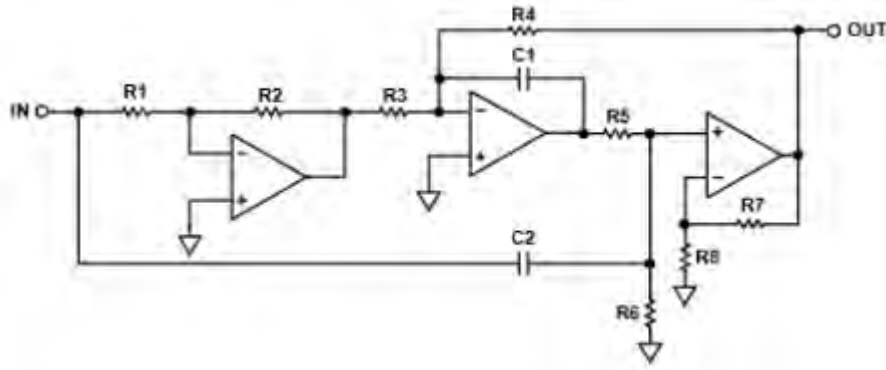


Figure 5.3 Bainter notch filter circuit

R_6 tunes Q , R_1 tunes center frequency, varying R_3 sets the ratio of ω_0/ω_z producing lowpass notch ($R_4 > R_3$), standard notch ($R_4 = R_3$) and highpass notch ($R_4 < R_3$). K_1 and K_2 set the gain above and below the notch frequency. With an appropriate op-amp, a Q factor of 200 and above can be achieved.

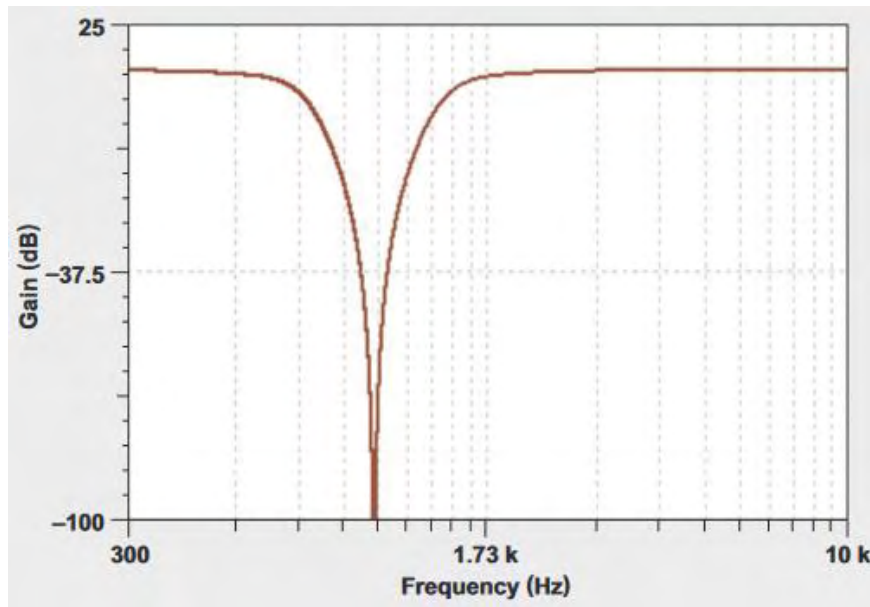


Figure 5.4 6th order Bainter notch filter response.

Figure 5.4 shows the frequency response of a 6th order Bainter notch circuit. With correct component selection and by cascading 2nd order filters, an attenuation less than -100 dB

can be achieved. Also the rolloff alongside the notch frequency is smooth with no peaks, providing a stable behavior.

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