University of Thessaly Department of Electrical and Computer Engineering

Diploma thesis by Palantas Panagiotis



"Layered coding of correlated sources for the multiple access fading channel"

"Πολυεπίπεδη κωδικοποίηση συσχετισμένων πηγών για το κανάλι πολλαπλής πρόσβασης με διαλείψεις"

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Abstract

In this thesis, we study the optimum transmission of correlated Gaussian sources over a quasi-static Rayleigh fading channel. We analyze two different source and channel coding strategies in terms of overall expected distortion (ED) combined with Distributed Source Coding (DSC). Our goal is to minimize the expected distortion and study the qualitative behavior of our communication strategies for higher transmit SNR.

Περίληψη

Στη παρούσα διπλωματική εργασία μελετάμε την βέλτιστη μετάδοση πληροφορίας η οποία παράγεται από Γκαουσιανές πηγές και μεταδίδετε μέσω καναλιού με διαλείψεις. Μελετάμε δύο διαφορετικές στρατηγικές κωδικοποίησης πηγής και καναλιού συνδυάζοντάς τες με κατανεμημένη κωδικοποίηση πηγής και τις αναλύουμε ως προς την αναμενόμενη συνολική αλλοίωση που θα δεχτεί η πληροφορία που μεταδίδουμε. Ο στόχος μας είναι να ελαχιστοποιήσουμε την αναμενόμενη συνολική που μεταδίδεται και να μελετήσουμε τα ποιοτικά χαρακτηριστικά των στρατηγικών που προτείνουμε με την αύξηση του SNR.

0.1 Introduction

Slow channel variations result in a non-ergodic channel. For this scenario, Shannon's source-channel separation theorem does not apply and a joint optimization of source and channel coding strategies is necessary. In this work we consider a more advanced network that consists of two terminals communications with a common destination, i.e., a multiple access channel (MAC).

For a point-to-point channel it is known that increasing the coding rate of a source results in decreased distortion; however, this causes the outage probability to increase. This trade-off indicates that for each given SNR value there is an optimal operating rate for the average distortion sense [1]. Alternatively, instead of transmitting the compressed signal at a single rate, one could compress the source into multiple layers and transmit them at different rates. It has been proven that this type of variable rate transmission enables one to adapt to channel variations without having to provide the transmitter (CSIT) with channel state information. In addition, this method considerably improves the expected distortion [2].

However, for the MAC the problem of lossy communication is more challenging. One technique which is used so as to reduce overall expected distortion is DSC [3–6], which refers to the compression of multiple correlated sources that do not communicate with each other. The compressed information of each source is sent to a common receiver for joint decoding. Furthermore, it has been shown that allowing the transmitted signals to interfere can improve the end-to-end distortion [7] by using a Wyner-Ziv type setup, which considers the compressed version of one of the sources as a remote side information for the compression of the other source.

For multiple fading states, the minimum expected distortion is formulated as the solution of a convex optimization problem with linearly many variables and constraints in [1]. The distortion exponent has been studied for Gaussian Source Coding fading over a time-varying fading MIMO channel in the presence of time-varying correlated side information at the receiver and two upper bounds have been derived [8]. The seminal paper, Cover, El Gamal and Salehi [9] provided sufficient conditions for transmitting losslessly correlated observations over a multiple access channel (MAC). However, single letter characterization of the capacity region is still unknown. Indeed Duek [10] proved that the conditions given in [9] are only sufficient and may not be necessary. Joint source channel coding with discrete sources, channels and side information is discussed in [11]. In [12] the authors extend the side information scenario and also consider continuous alphabet channels. This covers the important special case of the Gaussian MAC

In this thesis we investigate the optimum transmission of correlated Gaussian sources by using progressive transmission and broadcast strategy combined with DSC. More precisely, we consider two strategies that utilize a layered source coder. The first source-channel coding scheme is based on the compression of a source in layers, where each layer is a refinement of the previous ones. Next, the layers are transmitted successively in time using channel codes at different rates. We call this scheme layered source coding with progressive transmission (denoted as LS). The second source-channel coding scheme is called broadcast strategy with layered sources (denoted as BS). Similar to LS, information is communicated in layers, where each layer consists of the successive refinement information for the previous layers. The layers are channel-coded and then superimposed by assigning different power levels while they are communicated throughout the entire transmission block. We consider successive decoding at the receiver, where the layers are decoded starting from the layer with the highest power. The decoded codewords are subtracted from the received signal. Our approach offers a viable alternative for the transmission of correlated data in wireless fading channels. The reason is that we combine two widely-used low complexity techniques.

This thesis is organized as follows: We introduce the system model in Section 0.2, we review the effect of BS and LS on end-to-end distortion in Section 0.3, we use BS and LS strategies by allowing interference and we compare it with DSC in terms of overall expected distortion in Section 0.4, we combine BS and LS with DSC in Section 0.5, and we analyze the results of our proposed strategies in Section 0.6. Finally, we conclude the thesis in Section 0.7.

0.2 System model

0.2.1 Point to point communication

We consider a Gaussian source T_x which transmits information over a quasistatic flat Rayleigh fading channel. The additive noise is modelled as complex Gaussian with variance σ^2 , while there is an average power constraint of P_x . Furthermore, we denote the instantaneous fading level by h_x and the average received signal to noise ratio by $SNR_x = \frac{P}{\sigma^2}$. The fading levels are accurately measured at the receivers, while the transmitters are only aware of the statistics.

During each transmission block, which corresponds to N channel uses, a sequence of K source samples are compressed and sent over the channel. This corresponds to a bandwidth expansion ratio of $b = \frac{N}{K}$.

0.2.2 Two sources and a common receiver

Considering a system where T_x and T_y are two Gaussian correlated sources with a common destination, each link has flat Rayleigh fading with instantaneous fading levels h_x and h_y , and average received signal to noise ratios SNR_x and SNR_y . Once again, the fading levels are accurately measured at the receivers, while the transmitters are only aware of the statistics. We define a channel frame as a block of N channel uses and assume the fading is constant for multiple channel frames. The sources are zero-mean jointly Gaussian with the covariance matrix (1) and they programmed to transmit information to the destination with minimal expected distortion in squared error sense.

$$K_{XY} = \begin{bmatrix} \sigma_x^2 & \rho \sigma_x \sigma_y \\ \rho \sigma_x \sigma_y & \sigma_y^2 \end{bmatrix}$$
(1)

where ρ is the correlation coefficient.

In this system model we allow T_x and T_y to transmit at the same time, and as a result the signals are superimposed. We will consider successive decoding at the receiver, where the signals are decoded in order from the signal with the highest transmission power to the signal with the lowest transmission power and the decoded information is subtracted from the received signal.

0.3 Layered source

In this section we discuss the transmission of the source into multiple layers so as to reduce the overall point to point expected distortion. The first strategy we consider is the progressive transmission(LS) [2] for two layers in which the base layer is transmitted at a channel rate of R_1 bits per channel use (bpcu) and αN proportion of channel uses ($0 \le \alpha \le 1$) is used. In the second portion, the enhancement layer consisting of the successive refinement bits [13] of the source at a rate of R_2 bpcu is transmitted.

In the case that the base and the enhancement layer are received, the achieved rate is $\alpha R_1 b + (1 - \alpha) R_2 b$ bits per source sample. Moreover, in the case of an outage event in the enhancement layer, the achieved rate is $\alpha R_1 b$ bits per source sample. Using the successive refinability property, these correspond to distortions of $D(\alpha R_1 b + (1 - \alpha) R_2 b)$ and $D(\alpha R_1 b)$, respectively, where D(R) is the distortion rate function of the Gaussian source which is equal to $\sigma^2 2^{-2R}$. In case of an outage of the base layer, the achieved distortion is D(0). Then the expected distortion expression for 2-level LS can be written as [2]:

$$E(R_{1}, R_{2}, SNR) = (1 - P_{out}^{R_{2}})D(\alpha R_{1}b + (1 - \alpha)R_{2}b) + (P_{out}^{R_{2}} - P_{out}^{R_{1}})D(\alpha R_{1}b) + P_{out}^{R_{1}}$$
(2)

The probability of outage at rate R_1 is denoted by $P_{out}^{R_1}$ and it can be calculated as follows:

$$P_{out}^{R_1} = \Pr\{\frac{1}{2}\log_2(1+|h^2|SNR) < R_1\}$$
(3)

The probability of outage at rate R_2 is denoted by $P_{out}^{R_2}$ and it can be calculated as follows:

$$P_{out}^{R_2} = \Pr\{\frac{1}{2}\log_2(1+|h^2|SNR) < R_2\}$$
(4)

The second strategy we developed for transmitting the layered source is the broadcast strategy (BS) [14] for two layers. The base layer is transmitted using βP power at a channel rate of R_1 bits per channel use (bpcu) and the enhancement layer using $(1 - \beta)P$ power at a channel rate of R_2 bits per channel use (bpcu). Also, the power assignment rule is denoted by β ($0 \le \beta \le 1$). The receiver attempts to decode the base layer first, as it reads the enchantment layer as noise. In the case where the receiver fails to decode the base layer successfully, the achieved rate is equal to zero and the distortion is D(0). If there is a successful decoding of the base layer, it is subtracted from the received signal and the receiver attempts to decode the enhancement layer. In the case of an outage the achieved rate is equal to bR_1 bits per source sample and the distortion is $D(bR_1)$. Otherwise the achieved rate is equal to $bR_1 + bR_2$ bits per source sample and the distortion is $D(bR_1 + bR_2)$. Then, the expected distortion expression for 2-level BS can be written as [2]:

$$E(R_{1}, R_{2}, \beta, SNR) = (1 - P_{out}^{R_{2}})D(R_{1}b + R_{2}b) + (P_{out}^{R_{2}} - P_{out}^{R_{1}})D(R_{1}b) + P_{out}^{R_{1}}$$
(5)

The probability of outage at rate R_1 is denoted by $P_{out}^{R_1}$ and it can be calculated as follows:

$$P_{out}^{R_1} = \Pr\{\frac{1}{2}\log_2(1 + \frac{|h^2|\beta SNR}{1 + |h^2|(1 - \beta)SNR}) < R_1\}$$
(6)

The probability of outage at rate R_2 is denoted by $P_{out}^{R_2}$ and it can be calculated as follows:

$$P_{out}^{R_2} = \Pr\{\frac{1}{2}\log_2(1+|h^2|(1-\beta)SNR) < R_2\}$$
(7)

We compare both of the above strategies with the case of one layer which is equal to the direct transmission and with the uncoded transmission(UT) which is optimal for an additive white Gaussian channel and a source with squared-error distortion metric with b = 1 [15].

The encoder-decoder pair required for LS is simpler than the ones required for BS, because BS requires SNR dependent power allocation among layers, superimposition of codewords and sequential decoding.

0.4 Distributed source coding

Firstly, we review the rate-distortion function for Gaussian sources when compressed side information may be absent and then we compare layered sources vs DSC in terms of overall expected distortion.

We focus on the asymmetric scenario where Y is compressed separately and X is compressed with respect to Y.Without loss of generality, we can write, Y = aX + Z when X and Y are jointly Gaussian with correlation



Figure 1: Expected distortion vs. SNR plots. The topmost curve one layer corresponds to direct transmission without layering.

matrix in (1). Here $Z \sim N(0, \sigma_z^2)$ is independent of X with $\sigma_z^2 = \sigma_y^2 - a^2 \sigma_x^2$ and $a = \rho \frac{\sigma_y}{\sigma_x}$. Suppose Y is not available at X's encoder, and may or may not be available at the X's decoder. Let D_1 denote the squared error distortion achieved when Y is present at the destination, D_2 denote the distortion achieved when Y is absent. The rate distortion function of X when lossy side information may be absent, can be expressed as [16]:

 $R(D_1, D_2)$

$$= \begin{cases} \frac{1}{2} \ln(\frac{\sigma_x^2(\sigma_z^2 + \sigma_w^2)}{D_1(a^2 D_2 + \sigma_z^2 + \sigma_w^2)}) & \text{if } D_1 \le \sigma_2^2, D_2 \le \sigma_x^2 \\ \frac{1}{2} \ln(\frac{\sigma_x^2}{D_2}) & \text{if } D_1 \ge \sigma_2^2, D_2 \le \sigma_x^2 \\ \frac{1}{2} \ln(\frac{\sigma_x^2(\sigma_z^2 + \sigma_w^2)}{D_1(a^2 \sigma_x^2 + \sigma_z^2 + \sigma_w^2)}) & \text{if } D_1 \le \sigma_2^2, D_2 > \sigma_x^2 \\ 0 & \text{if } D_1 \ge \sigma_2^2, D_2 > \sigma_x^2 \end{cases}$$
(8)

where $\sigma_2^2 = \frac{D_2(\sigma_z^2 + \sigma_w^2)}{a^2 D_2 + \sigma_z^2 + \sigma_w^2}$, $\sigma_w^2 = \frac{\sigma_y^2 D_y}{\sigma_y^2 - D_y}$. We are mainly interested in the compression rate of X in the regime $D_1 \leq \sigma_2^2, D_2 \leq \sigma_x^2$. In this region, for a given R_x, D_2 and D_y , the distortion D_1 is equal to

$$D_1(\overline{R}_x, D_2, D_y) = \frac{\sigma_x^2(\sigma_x^2 \sigma_y^2 + a^2 \sigma_x^2 D_y)}{a^2 D_2 \sigma_y^2 - a^2 D_2 D_y + \sigma_z^2 \sigma_y^2 + a^2 \sigma_x^2 D_y} 2^{-2\overline{R}_x}$$

where \overline{R}_x is the compression rate of X in bits per sample.

Finally, the side information Y is compressed using \overline{R}_{y} bits per source sample and is sent directly to the destination with a distortion of:

$$D_y(\overline{R}_y) = \sigma_y^2 2^{-2\overline{R}_y} \tag{9}$$

Assuming that source X transmits at a channel rate of R_x bits per channel use (bpcu) with corresponding compression rates $R_x = bR_x$, and source Y transmits at a channel rate of R_y bits per channel use (bpcu) with corresponding compression rates $R_y = bR_y$ the distortions for our strategy can be expressed in terms of error/success probabilities as [7]:

$$ED_{x} = P^{1} D_{1}(bR_{x}, D_{2}, D_{y}) + P^{2} D_{2} + P^{3} D_{1}(0, D_{2}, D_{y}) + P^{4} \sigma_{x}^{2}$$
(10)

$$ED_y = (P^1 + P^3)D_y(bR_y) + (P^2 + P^4)\sigma_y^2$$
(11)

If Y is lost, the target distortion for X is denoted by D_2 , the probability of receiving successfully both the compressed information of X and Ysources by P^1 , the probability of receiving successfully only the compressed information of X source by P^2 , the probability of receiving successfully only the compressed information of Y source by P^3 , and the probability of failure to receive the compressed information of both X and Y sources by P^4 .

0.5 Layered source combined with distributed source coding

In previous sections we reviewed layered source coding and distributed source coding techniques.

In this section we consider a more complex scenario where Y is compressed separately into multiple layers and is transmitted at different rates in order to adapt to the channel variations without the need of channel state information at the transmitter. Also, X is compressed with respect to Y.

For a given communication mode, average channel SNRs, source correlation and bandwidth ratio, the expected distortion is a function of the source rates and the amount of channel coding. We will assume that a complete frame will be discarded if the channel decoder can not correct all the errors.

Firstly, we study the scenario where the multiple layer strategy which is used for Y is the progressive transmission (LS) for two layers. We assume that source X transmits at a channel rate of R_x bits per channel use (bpcu) with corresponding compression rates $\bar{R}_x = bR_x$, and that source Y transmits the base layer at a channel rate of R_{y1} and the enhancement layer, consisting of the successive refinement bits of the source at a rate of R_{y2} bpcu. The distortions for our strategy can be expressed in terms of error/success probabilities as:

$$ED_{x} = P^{1} D_{1}(\bar{R}_{x}, D_{2}, D_{y}(\alpha \bar{R}_{y1} + (1 - \alpha)\bar{R}_{y2})) + P^{2} D_{1}(\bar{R}_{x}, D'_{2}, D_{y}(\alpha \bar{R}_{y1})) + P^{3} D_{2} + P^{4} D_{1}(0, D_{2}, D_{y}(\alpha \bar{R}_{y1} + (1 - \alpha)\bar{R}_{y2})) + P^{5} D_{1}(0, D'_{2}, D_{y}(\alpha \bar{R}_{y1})) + P^{6} \sigma_{x}^{2}$$

$$(12)$$

$$ED_{y} = (P^{1} + P^{4}) D_{y}(\alpha \bar{R}_{y1} + (1 - \alpha)\bar{R}_{y2}) + (P^{2} + P^{5}) D_{y}(\alpha \bar{R}_{y1}) + (P^{3} + P^{6}) \sigma_{y}^{2}$$
(13)

Next, we consider a scenario where Y is compressed and transmitted using broadcast strategy (BS) for two layers. We transmit the base layer using βP power at a channel rate of R_{y1} bits per channel use (bpcu) and the enhancement layer using $(1 - \beta)P$ power at a channel rate of R_{y2} bits per channel use (bpcu). The distortions for our strategy can be expressed in terms of error/success probabilities as:

$$ED_{x} = P^{1} D_{1}(R_{x}, D_{2}, D_{y}(R_{y1} + R_{y2})) + P^{2} D_{1}(\bar{R}_{x}, D'_{2}, D_{y}(\bar{R}_{y1})) + P^{3} D_{2} + P^{4} D_{1}(0, D_{2}, D_{y}(\bar{R}_{y1} + \bar{R}_{y2})) + P^{5} D_{1}(0, D'_{2}, D_{y}(\bar{R}_{y1})) + P^{6} \sigma_{x}^{2}$$

$$(14)$$

$$ED_{y} = (P^{1} + P^{4}) D_{y}(\bar{R_{y1}} + \bar{R_{y2}}) + (P^{2} + P^{5}) D_{y}(\bar{R_{y1}}) + (P^{3} + P^{6}) \sigma_{y}^{2}$$
(15)

If the base layer of Y is lost, we denote the target distortion for X by D_2 , otherwise, if the enhancement layer of Y is lost and the base layer is received successfully, we denote the target distortion for X by D'_2 , the probability of receiving successfully both the compressed information of X and Y sources by P^1 , the probability of receiving successfully the compressed information of X and the base layer of Y sources by P^2 , the probability of receiving successfully only the compressed information of X source by P^3 , the probability of receiving successfully only the compressed information of Y source by P^4 , the probability of receiving successfully only the compressed information of Y source by P^4 , the probability of receiving successfully only the base layer of Y source by P^5 , and the probability of failure to receive the compressed information of both X and Y sources by P^6 .

0.6 Results

To evaluate the performance of the the proposed joint source-channel communication techniques we perform extensive simulations. We plot the expected



Figure 2: Expected distortion vs. SNR plots for $\rho = 0.2$.

distortion vs. SNR for the direct transmission, LS and BS with 2 layers and UT which we are using as an optimal bound Fig. 1 for b = 1. The results are obtained through an exhaustive search over all possible rates, and by normalizing channel and power allocations in unit. It is clear that BS with two layers outperforms LS for a given bandwidth expansion.

To compared the proposed transmission schemes with the literture, for a fixed SNR and correlation coefficient, we vary R_x , R_y , α,β,D_2,D'_2 , we compute the corresponding (ED_x, ED_y) pairs, and we plot the minimum values of their AVERAGE $(\frac{ED_x+ED_y}{2})$ so as to determine an optimal assignment of rates and the distortion. We assume $\sigma_x^2 = 1$, $\sigma_y^2 = 1$ and b = 1. We then consider a symmetric scenario where the terminals are at an equal distance to the destination.



Figure 3: Expected distortion vs. SNR plots for $\rho = 0.8$.

In figure 2 and 3, we carry out the minimization of (10) and (11) numerically, and compare it with the layered sources techniques for increasing SNR values. We illustrate the achievable minimal average distortion with such optimal bit allocation over a wide range of channel SNRs for the usual direct transmission, LS and BS with 2 layers and DSC for different correlation coefficient factors. It is obvious that DSC outperforms the layered sources techniques at low and medium SNR regimes.

In figure 4 and 5, we carry out the minimization of (12),(13),(14) and (15) and compare the expected distortions achieved by our proposed combined techniques with the layered sources techniques and DSC for increasing SNR values. To compare them we have plotted their overall expected distortion vs.SNR. We also observe that distributed source coding combined

with BS provides significant reduction to the end-to-end distortion when the correlation of the two sources is high. Moreover, both DSC with LS and DSC with BS outperform DSC at high SNR regimes.

0.7 Conclusion

In this thesis we proposed compression and transmission strategies that make use of source correlation, layering, and interference. Furthermore, we demonstrate that by combining two commonly used techniques (DSC and layered sources techniques), the expected distortion of correlated Gaussian sources can be minimized for high correlation coefficients.

In addition we argue that our approach offers a viable alternative for the transmission of correlated data in wireless fading channels. The reason for this, is that complexity is shifted to the destination that is responsible for interference cancellation and for power adaptation.

Finally, it should be noted that this thesis only considers layered source strategies with two layers. In the future, this work could be extended by considering layered sources with an arbitrary number of layers.



Figure 4: Expected distortion vs. SNR plots for $\rho = 0.2$.



Figure 5: Expected distortion vs. SNR plots for $\rho = 0.8$.

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