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AUTOMATIC MATHEMATICAL MODEL DEVELOPMENT FOR VEHICLE ROUTING PROBLEM VARIANTS

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Abstract

The present thesis proposes for the first time a mathematical model able to find optimal solutions for a wide variety of Vehicle Routing Problem (VRP) variants. The proposed mathematical formulation is a mixed integer linear programming model. In addition, an algorithm based on the aforementioned model was created at the environment of C++ programming language. The basic Vehicle Routing Problem of the mathematical model and source code is Multi Depot Capacitated Heterogeneous Fleet Delivery VRP. There are five options which can add VRP variants to the basic model. These variants are enabled or disabled by the user. The first option is existence of Time Windows and it is enabled when depots and customers impose time restrictions. At the literature, this kind of VRP is known as Time Windows VRP variant. The second option concerns customers and it is related with their demand not only to be delivered products but also to return cargo back to depots. This variant of VRP is known as Simultaneous Pickup and Delivery. The third option is the allowance of more than one vehicles to visit each customer. This feature results in optimal solutions even when a customer requires vehicles to deliver or pickup a big amount of cargo

from his location. In reference, this variant is known as Split Satisfaction of Demand VRP variant. The fourth option has to do with the existence of specific characteristics of vehicles and customers. Specifically, vehicles can be refrigerated or not, can be permitted or not to enter town centers. Respectively, customers can require products that have to be transferred with refrigerated or non refrigerated vehicles and their location can be at the center of a town or at the suburbs. This case is referred at this thesis as Specific Network Characteristics VRP variant. The fifth option concerns vehicles' permission to start and finish at different depots. This instance is referred as Vehicle Relocation Variant. The functionality of the proposed algorithm was tested running computational experiments for thirty two (32) cases of problems that arise from the combination of selected VRP variants with the basic VRP model. The results show that it can give exact solutions and therefore optimize total travelling consumption in an effective way for reasonable company sizes.

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Chapter 1 Travelling Salesman Problem: An Introduction to Vehicle Routing Problem

1.1 Motivation

The objective of this chapter is to introduce the reader to the problem studied in this thesis, the Vehicle Routing Problem. Firstly, there is a reference to a problem category which constitutes the origin of Vehicle Routing Problem and is known as Travelling Salesman Problem. Also, this chapter contains information about real life applications regarding Travelling Salesman Problem.

1.2 Travelling salesman problem

1.2.1 Problem definition

Travelling salesman problem (TSP) is considered as a simple routing problem, though NP-hard. At this problem, a network of cities is known as well as the distances among them. The main question is to find the shortest tour which visits all the above cities exactly once in a way that the finishing city will be also the starting city of this tour.

1.2.2 History of the problem

In the 18th century a mathematician from Ireland named Sir William Rowam Hamilton and the British mathematician named Thomas Penyngton Kirkman started studying the Travelling Salesman Problem. After years later, in 1930, Kalr Menger studied the problem of finding the shortest path joining all of a finite set of points, whose pair wise distances are known. The problem was later promoted by Hassler, Whitney & Merrill at Princeton.

1.2.3 Variations of TSP

Four variations of TSP are presented. The first three concern distance properties at network of nodes.

- Symmetric TSP: TSP is symmetric if the distance travelling from city i to city j is equal to the travelling distance from city j to city i.
- Asymmetric TSP: TSP is asymmetric if the distance travelling from city i to city j is different from the travelling distance from city j to city i.
- Euclidean TSP: TSP is Euclidean if the distance among cities is Euclidean and not real road distance.
- Multiple Travelling Salesman Problem (m-TSP): m-TSP is a more generalized category of TSP. The main difference of this variant is the existence of more than one salesman. In the m-TSP we are given n cities, m salesmen and one depot or home base. All cities should be visited exactly once on one of m tours, starting and ending at the depot. The tours are not allowed to be empty.

1.2.4 Real life applications of Travelling Salesman Problem

Some of the most important applications of Travelling Salesman Problem in the real world concern drilling of printed circuit boards, overhauling gas turbine engines, X-Ray crystallography, computer wiring, the order-picking problem in warehouses, mask plotting in PCB production, printing press scheduling problem, school bus routing problem, crew scheduling problem, interview scheduling problem, hot rolling scheduling problem, mission planning problem and design of global navigation satellite system surveying networks.

1.2.4.1 Drilling of printed circuit boards

A direct application of the TSP is in the drilling problem of printed circuit boards (PCBs) (Grotschel et al., (1)). To connect a conductor on one layer with a conductor on another layer, or to position the pins of integrated circuits, holes have to be drilled through the board. The holes may be of different sizes. To drill two holes of different diameters consecutively, the head of the machine has to move to a tool box and change the drilling equipment. This is quite time consuming. It is clear that one has to choose some diameter, drill all holes of the same diameter, change the drill, drill the holes of the next diameter, etc. Thus, this drilling problem can be viewed as a series of TSPs, one for each hole diameter, where the 'cities' are the initial position and the set of all holes that can be drilled with one and the same drill. The 'distance' between two cities is given by the time it takes to move the drilling head from one position to the other. The aim is to minimize the travel time for the machine head.

1.2.4.2 Overhauling gas turbine engines

(Plante et al., (2)) reported this application and it occurs when gas turbine engines of aircraft have to be overhauled. To guarantee a uniform gas flow through the turbines there are nozzle-guide vane assemblies located at each turbine stage. Such an assembly basically consists of a number of nozzle guide vanes affixed about its circumference. All these vanes have individual characteristics and the correct placement of the vanes can result in substantial benefits (reducing vibration, increasing uniformity of flow, reducing fuel consumption). The problem of placing the vanes in the best possible way can be modeled as a TSP with a special objective function.

1.2.4.3 X-Ray crystallography

Analysis of the structure of crystals (Bland & Shallcross, (3)) is an important application of the TSP. Here, an X-ray diffractometer is used to obtain information about the structure of crystalline material. To this end a detector measures the intensity of Xray reflections of the crystal from various positions. Whereas the measurement itself can be accomplished quite fast, there is a considerable overhead in positioning time since up to hundreds of thousands positions have to be realized for some experiments. In the two examples that we refer to, the positioning involves moving four motors. The time needed to move from one position to the other can be computed very accurately. The result of the experiment does not depend on the sequence in which the measurements at the various positions are taken. However, the total time needed for the experiment depends on the sequence. Therefore, the problem consists of finding a sequence that minimizes the total positioning time. This leads to a traveling salesman problem.

1.2.4.4 Computer wiring

(Lenstra & Rinnooy Kan, (4)) reported a special case of connecting components on a computer board. Modules are located on a computer board and a given subset of pins has to be connected. In contrast to the usual case where a Steiner tree connection is desired, here the requirement is that no more than two wires are attached to each pin. Hence we have the problem of finding a shortest Hamiltonian path with unspecified starting and terminating points. A similar situation occurs for the so-called testbus wiring. To test the manufactured board one has to realize a connection which enters the board at some specified point, runs through all the modules, and terminates at some specified point. For each module we also have a specified entering and leaving point for this test wiring. This problem also amounts to solving a Hamiltonian path problem with the difference that the distances are not symmetric and that starting and terminating point are specified.

1.2.4.5 The order-picking problem in warehouses

This problem is associated with material handling in a warehouse (Ratliff & Rosenthal, (5)). Assume that at a warehouse an order arrives for a certain subset of the items stored in the warehouse. Some vehicle has to collect all items of this order to ship them to the customer. The relation to the TSP is immediately seen. The storage locations of the items correspond to the nodes of the graph. The distance between two nodes is given by the time needed to move the vehicle from one location to the other. The problem of finding a shortest route for the vehicle with minimum pickup time can now be solved as a TSP. In special cases this problem can be solved easily, see (van Dal, (6)) for an extensive discussion and for references.

1.2.4.6 Mask plotting in PCB production

For the production of each layer of a printed circuit board, as well as for layers of integrated semiconductor devices, a photographic mask has to be produced. In our case for printed circuit boards this is done by a mechanical plotting device. The plotter moves a lens over a photosensitive coated glass plate. The shutter may be opened or closed to expose specific parts of the plate. There are different apertures available to be able to generate different structures on the board. Two types of structures have to be considered. A line is exposed on the plate by moving the closed shutter to one endpoint of the line, then opening the shutter and moving it to the other endpoint of the line. Then the shutter is closed. A point type structure is generated by moving (with the appropriate aperture) to the position of that point then opening the shutter just to make a short flash, and then closing it again. Exact modeling of the plotter control problem leads to a problem more complicated than the TSP and also more complicated than the rural postman problem. A real-world application in the actual production environment is reported in (Grotschel et al., (1)).

1.2.4.7 Printing press scheduling problem

One of the major and primary applications of the m-TSP arises in scheduling a printing press for a periodical with multi-editions. Here, there exist five pairs of cylinders between which the paper rolls and both sides of a page are printed simultaneously. There exist three kind of forms, namely 4-, 6- and 8-page forms, which are used to print the editions. The scheduling problem consists of deciding which form will be on which run and the length of each run. In the m-TSP vocabulary, the plate change costs are the inter-city costs. For more details papers by Gorenstein (7) and Carter & Ragsdale (8) can be referred.

1.2.4.8 School bus routing problem

(Angel et al., (9)) investigated the problem of scheduling buses as a variation of the m-TSP with some side constraints. The objective of the scheduling is to obtain a bus loading pattern such that the number of routes is minimized, the total distance travelled by all buses is kept at minimum, no bus is overloaded and the time required to traverse any route does not exceed a maximum allowed policy.

1.2.4.9 Crew scheduling problem

An application for deposit carrying between different branch banks is reported by (Svestka & Huckfeldt, (10)). Here, deposits need to be picked up at branch banks and returned to the central office by a crew of messengers. The problem is to determine the routes having a total minimum cost. Two similar applications are described by (Lenstra & Rinnooy Kan , (11) and Zhang et al., (12)). Papers can be referred for detailed analysis.

1.2.4.10 Interview scheduling problem

(Gilbert & Hofstra, (13)) found the application of m-TSP, having multi period variations, in scheduling interviews between tour brokers and vendors of the tourism industry. Each broker corresponds to a salesman who must visit a specified set of vendor booths, which are represented by a set of T cities.

1.2.4.11 Hot rolling scheduling problem

In the iron and steel industry, orders are scheduled on the hot rolling mill in such a way that the total set-up cost during the production can be minimized. The details of a recent application of modeling such problem can be read from (Tang et al., (14)). Here, the orders are treated as cities and the distance between two cities is taken as penalty cost for production changeover between two orders. The solution of the model will yield a complete schedule for the hot strip rolling mill.

1.2.4.12 Mission planning problem

The mission planning problem consists of determining an optimal path for each army men (or planner) to accomplish the goals of the mission in the minimum possible time. The mission planner uses a variation of the m-TSP where there are n planners, m goals which must be visited by some planners, and a base city to which all planners must eventually return. The application of the m-TSP in mission planning is reported by (Brummit & Stentz, (15), Brummit & Stentz, (16) and Yu et al., (17)). Similarly, the routing problems arising in the planning of unmanned aerial vehicle applications, investigated by (Ryan et al., (18)), can also be modeled as m-TSP.

1.2.4.13 Design of global navigation satellite system surveying networks

A very recent and interesting application of the m-TSP, as investigated by Saleh & Chelouah, (19) arises in the design of global navigation satellite system (GNSS) surveying networks. A GNSS is a space-based satellite system which provides coverage for all locations worldwide and is quite crucial in real-life applications such as early warning and management for disasters, environment and agriculture monitoring, etc. The goal of surveying is to determine the geographical positions of unknown points on and above the earth using satellite equipment. These points, on which receivers are placed, are coordinated by a series of observation sessions. When there are multiple receivers or multiple working periods, the problem of finding the best order of sessions for the receivers can be formulated as an m-TSP. For technical details refer (Saleh & Chelouah, (19)).

Chapter 2 Vehicle Routing Problem

2.1 Motivation

In this chapter, there is information about Vehicle Routing Problem which concerns transportation and logistics companies. It is considered as one of the most challenging optimization problems in the field of Operation Research. The aim of this problem is the optimal routing of vehicles in a way that all customers are visited. Moreover, literature review is presented regarding classic Vehicle Routing Problem and its variants.

2.2 Vehicle Routing Problem

2.2.1 Introduction

The vehicle routing problem (VRP) is a basic problem regarding distribution management. Thousands of companies and organizations, responsible for delivery and collection of goods or people, face this problem every day. The objectives and constraints of the problems faced differ as conditions are highly variable. By building enough flexibility in optimization systems, one can adapt these to various practical contexts.

2.2.2 Problem definition

The Vehicle Routing Problem (VRP) calls for the determination of the optimal set of routes to be performed by a fleet of vehicles to meet the demands of a given set of customers. It is considered as one of the most popular combinatorial optimization problems. The objective of the problem is to minimize the total travelling cost of vehicles which start and finish their routes at locations named depots. Vehicle routing problem is considered NP hard problem, as the computations to solve the problem increase exponentially as problem size increases.

2.2.3 History of the problem

The Vehicle Routing Problem (VRP) arises in several forms because of the variety of constraints encountered in practice. For over fifty (50) years, the VRP has attracted the attention of a large part of the operational research community. This is due partly to the economic importance of the problem, but also to the methodological challenges it poses. For example, the traveling salesman problem (TSP), which is a special case of the VRP, can now be solved for thousands and even tens of thousands of vertices. In contrast, the VRP is much more difficult to solve. For example, in the relatively simple case where only capacity constraints are present (called the capacitated VRP, CVRP), it is still difficult to solve instances with one or two hundred customers by means of exact algorithms. In recent years, much of the research effort has turned to the development of powerful metaheuristics.

The capacitated VRP was formally introduced in 1959 by Dantzig and Ramser (1959) (20). These authors proposed a simple matching-based heuristic for its solution and illustrated it on a toy-sized example. The following years saw the emergence of several heuristics based on a variety of principles including savings, geographical proximity, customer matching, as well as intra-route and inter-route improvement steps. Perhaps the most famous heuristic of this category is the Clarke and Wright (1964) (21) savings heuristic, which has withstood the test of time because of its speed, simplicity and reasonably good accuracy. The development of exact algorithms for the VRP took off in 1981 with the publication of two papers by Christofides, Mingozzi and Toth in Networks (22), and Mathematical Programming. The first of these papers proposed an algorithm based on dynamic programming with state-space relaxation whereas the second proposed two mathematical formulations making use of q-paths and k-shortest spanning trees. A few years later, Laporte, Desrochers and Nobert (23) proposed the first cutting plane approach for a VRP based on the solution of linear relaxation of an integer model. These seminal concepts have made their way into some of the more recent algorithms.

Since then, a variety of exact algorithms based on mathematical programming formulations have been proposed. Some formulations contain vehicle flow or commodity flow variables and are often solved by branch-and-cut. The VRP can also be formulated as a set partitioning problem to which some valid inequalities are added. Some of the most successful implementations by Fukasawa et al,2016 (24) and by Baldacci et al.,2008 (25), are based on this methodology.

The development of modern heuristics for the VRP really started in the 1990s with the advent of metaheuristics. It is fair to say that the study of the VRP has stimulated the growth and understanding of several metaheuristic concepts we now know. The early research in this area was quite fragmented, with a notable bias towards tabu search-based approaches and some of the algorithms were over engineered, but some rationalization has started to take place in recent years. The best metaheuristics are those that simultaneously perform a wide and deep search of the solution space and can solve several variants of the problem. They generally either apply several operators, as in adaptive large neighbourhood search, or combine genetic search with local search, as in the hybrid genetic algorithm recently proposed.

2.2.4 Variations of Vehicle Routing Problem

VRP has many variations to simulate real life conditions. These variations include limitations which are imposed by customers, depots and vehicles or the problem itself. The aforementioned limitations are expressed as additional constraints to the basic VRP problem.

Customers can require products not only to be delivered to them but also to be picked up by their locations. Moreover, they can define the time horizon they want to be visited as personnel, responsible for receiving and sending products, is not available every time during a day. Additionally, they can allow to be visited by more than one vehicles.

In real life, depots can be more than one in case of big companies. Their location can be near to warehouses or cross-docking facilities.

Vehicles can have different characteristics in terms of capacity, ability to refrigerate products, permission to enter town centers. Moreover, in VRP problem with many depots, it depends on the problem studied if a vehicle is allowed to start from a depot and return to a different depot.

Another parameter that defines a VRP variation is the randomness. Number of customers or their demands can be unknown at the time vehicles start their routes and can be revealed later as vehicles visit customers.

All the above constraints define the variations of VRP that are known. The most important variations are:

- Capacitated VRP (CVRP): In a CVRP, we are given a depot, a set of customers, a set of vehicles and a distance measure. Every vehicle has a finite capacity and every customer has a specific demand. The task in the CVRP is to construct vehicle routes such that all customers are visited exactly once and such that the capacities of the vehicles are obeyed. This should be done while minimizing the total distance traveled.
- Multi Depot VRP (MDVRP): In a MDVRP, there are more than one depots. Each vehicle start from a specific depot to service customers. There are two categories regarding the finishing depot depending on the problem studied. The fist category is when Vehicles return to the starting depot, whereas the second category is when vehicles return to any depot available on the graph. The objective of MDVRP is to minimize the total cost and total number of vehicles used ensuring that all customers are serviced.
- Heterogeneous Fleet VRP (HFVRP): In a HFVRP, all vehicles have different characteristics regarding capacity, type of cargo, average consumption or ability to enter town centers. The objective of HFVRP is to minimize total cost ensuring that all limitations are met.
- VRP with Time Windows (VRPTW): In a VRPTW, each customer specifies a time interval $[a_i, b_i]$ called time window, within which he permits vehicles to unload and load cargo at his facilities. There is also another time called service time which is referred to the time needed for the whole process of unloading and loading. These constraints restrict the time horizon during which a customer is available to be serviced by a vehicle. A vehicle is allowed to arrive before a_i and wait until the customer becomes available. However, arrivals after b_i are prohibited. The objective of VRPTW is to minimize total cost and total number of used vehicles ensuring that all customers are visited within time windows.
- Split Delivery VRP (SDVRP): In a SDVRP, each customer can be visited by more than one vehicles which service a fraction of its demand. SDVRP is necessary in the cases where the products delivered to a customer exceed vehicles capacity. Also, it has

been shown that this relaxation could result in substantial savings on the total traveled distance.

- VRP with Pickups and Deliveries (VRPPD): In a VRPPD, vehicles transport goods from depots to customers and also from customers to depots. This is achieved as vehicles both unload products to customers and load products from them. The objective of VRPPD is to minimize total cost and total number of used vehicles ensuring that products don't exceed each vehicle's capacity. Another VRP variant which includes pickups and deliveries is VRP with Backhauls. The main difference is that vehicles firstly unload products to customers and afterwards collect those which have to be returned to the depots.

- Periodic VRP (PVRP): In a PVRP, the planning period has M days and there is a frequency of each customer that shows how many times he requires to be visited by vehicles within this planning period. A solution to the PVRP consists of M sets of routes that jointly satisfy the demand constraints and the frequency constraints. The objective is to minimize the sum of the costs of all routes over the planning horizon.

- Stochastic VRP (SVRP): In a SVRP, some parameters of the VRP are random. Common examples are uncertain customers or demands, and stochastic travel times or service times.

- Dynamic VRP (DVRP): Another extension to standard VRP, which is also common in many real world applications, is when service requests of customers are not completely known before the start of service, but they arrive during the distribution process. Since new orders arrive dynamically, the routes have to be planned again at run time in order to include them.

- Open VRP (OVRP): In an OVRP, each vehicle is not required to return to the central depot after visiting the final customer. This formulation may be appropriate for a situation where vehicles are hired from a third party and the cost of the hire is based on the distance traveled while the vehicles are loaded.

2.2.5 Real life applications of Vehicle Routing Problem

Vehicle Routing Problem has many real life applications in the fields of oil, gas and fuel supply, retail, waste collection and management, mail and small package delivery and food distribution.

2.2.5.1 Oil, gas and fuel supply

The first category of problems surveyed covers applications related to the supply of oil, gas and fuel to houses, gas stations and companies. These problems present a number of specific features such as vehicles with capacitated compartments and sometimes the presence of flow meters to control the delivered quantity. The latter feature implies that sometimes the content of the same compartment can be used to satisfy the demands of several customers, whereas when there is no flow meter, the compartment must be completely emptied in a single customer tank. Cleaning operations may be needed between the loading of different products using the same compartment. These problems are generally solved over long-term planning horizons and incorporate mixed inventory and routing decisions (26), (27).

Campbell et al. (28) worked with Praxair, an industrial gases company with about 60 production facilities and more than 10,000 customers across North America. The problem, modeled as an inventory-routing problem, was solved by means of a two-phase heuristic that first assigns delivery days to customers, and then creates vehicle routes. It was applied to instances in which facilities can have between 50 and 87 customers.

Chiang and Russell (29) integrated purchasing and routing decisions for a propane gas supply chain using set partitioning and tabu search techniques. They modeled the problem as a general multi-depot VRP with time windows in which a tanker starts from a depot, travels through a number of terminals (for pickups) and plants (for deliveries), and returns to the same depot. They reported results for the Illinois and Michigan dispatch areas. In the case of Michigan, they reduced the number of tankers used from 130 to 102 over a one-week planning period.

Avella et al. (30) studied the case of a company supplying three types of fuel to a set of gas stations located in an urban area. They considered that each tank in the delivery vehicles must be either completely full or completely empty. They generated all feasible routes having at most four clients and solved the resulting set partitioning problem by branch-and-price. They solved a one-week instance with 60 clients, and a fleet of six heterogeneous trucks was used to serve about 25 clients per day.

Cornillier et al. (31) studied a problem similar to that of Avella et al. (30) but also considered the loading of tanker trucks divided into compartments, which is of primary importance since there are several small gas stations throughout Eastern Quebec, the area of application, and because trucks are not equipped with flow meters.

Song and Savelsbergh (32) worked with Praxair on another variant of the inventory-routing problem. They developed bounds on the volume delivered per mile, which was used to determine customer-plant assignment.

Ng et al. (33) designed a decision support system combining heuristic and optimal routing for a tanker assignment and routing problem for petroleum products in Hong Kong. They reported an increase in the volume delivered as well as better route designs.

Cornillier et al. (34) studied a richer petrol station replenishment problem with time windows and obtained a distance reduction of about 22% over the solution obtained by the company dispatcher. On a 42-station instance, they reduced the number of routes from 26 to 23.

Day et al. (35) studied the inventory replenishment of a company in Indiana, which distributes carbon dioxide to over 900 customer sites. They developed a heuristic capable of reducing driver labor cost by about 30%.

2.2.5.2 Retail

Retail involves the sales of goods and the provision of services to end-users. This section lists applications dealing with a number of final products and in various sectors such as supermarkets and consultancy services. These applications generally involve time windows and loading constraints.

Prins (36) studied the case of a French furniture manufacturer and modeled it as a heterogeneous VRP in which each vehicle can perform several trips. To solve it, he adapted several well-known VRP algorithms and developed a tabu search algorithm. On a one-week data set containing 775 stores, his results showed a reduction of 11.7% in distribution costs.

Poot et al. (37) described a savings-based heuristic implemented within Short-rec Distriplanner, a commercial virtual reality system sold by ORTEC Consultants, a Dutch software provider. The authors dealt with several types of constraints such as consistently assigning the same customers to drivers, grouping customers that should be visited first (or last) in a route, and forbidding some product combinations. Results were reported for four anonymous companies.

Gaur and Fisher (38) solved a periodic inventory- routing problem for Albert Heijn, a supermarket chain in the Netherlands. They reported transportation savings of about 4% in the first year of implementation.

Gendreau et al. (39) studied the case of an Italian company manufacturing bedroom furniture. The problem was modeled as a capacitated VRP with three-dimensional loading constraints and was solved by tabu search. Solutions were obtained on five instances involving up to 64 customers, 181 products and four vehicles.

Kant et al. (40) reported the implementation of the ORTEC vehicle routing software (see Poot et al. (37)) for Coca-Cola. The authors considered a problem involving about 10,000 trucks daily, and reported an annual cost savings of about $45 million, as well as major improvements in customer service.

Belfiore and Yoshizaki (41) worked with a Brazilian retail group composed of 519 stores present in 11 Brazilian states. They modeled the problem as a heterogeneous VRP with time windows and split deliveries, and proposed a scatter search heuristic to solve it. Using one week of data, they reported a cost reduction of about 7.5% which could translate into a yearly saving of one million dollars.

Chang et al. (42) described a stochastic dynamic TSP with time windows which was applied to FamilyMart, the second-largest convenience store in Taiwan, with more than 1,500 sales points. They proposed an algorithm combining a shortest n-path algorithm with a convolution-propagation heuristic. They performed their experiments on a 12-customer instance which was said to be representative of a typical route.

Wen et al. (43) solved a VRP with cross-docking for the Danish consultancy Transvision. In this application identical vehicles are used to transport orders from suppliers to customers through a cross-dock. The authors developed a tabu search heuristic embedded within an adaptive memory search to solve an instance containing up to 200 pairs of nodes.

They obtained within a few minutes, solutions that were less than 5% away from optimality.

We note that many retail companies must face the challenges of last-mile distribution and that the modeling and algorithmic know-how provided by operations research can significantly improve their operations. However, while algorithmic design is a rich terrain for academics, many companies rely on commercial black boxes to determine their routing. When they collaborate with academics, often data are not allowed to be publicly used, which may explain in part the low number of real applications described in the scientific literature.

2.2.5.3 Waste collection and management

Waste collection is essential to the proper functioning of any collectivity. Ghiani et al. (44) presented a survey of the strategic and tactical issues related to the application of operations research in solid waste management. A variant of the problem deals with hazardous waste management in which collection, transportation, treatment and disposal of hazardous materials are involved. These problems are characterized by loading and unloading constraints, time windows, and inter-arrival time constraints at customer points.

Tung and Pinnoi (45) studied the waste collection of households and streets garbage cans in five districts of Hanoi. The service is provided by Urenco, a private company paid by the municipal government based on the volume collected. The authors reported a reduction of 4.6% in operating cost and showed that they could reduce their fleet size by 20% or, conversely, increase the volume of waste collected with the current fleet by 20%.

Shih and Chang (46) modeled the routing and scheduling of medical waste from a set of hospitals and clinics as a periodic VRP. The system, tested in central Taiwan, uses dynamic programming to partition customers into routes and a simple 2-opt heuristic to improve each route individually. The authors solved an instance with 346 clinics over six days, with two or three routes scheduled per day with up to 47 visited clinics.

Baptista et al. (47) extended the algorithm of Christofides and Beasley (48) for the periodic VRP to the collection of recycling paper containers in Almada, Portugal. In this application, a single vehicle must perform a route in the morning and another in the afternoon to collect 59 containers. The problem was solved over a one-month horizon.

Still in Portugal, Teixeira et al. (49) studied an urban recyclable waste problem where three types of products (glass, paper and plastic/metal) must be collected separately. They modeled the problem as a periodic VRP which was solved through a three-phase heuristic. Their algorithm yielded a distance reduction of about 29% over historic distances travelled.

A similar problem with different types of waste was studied by Nuortio et al. (50) in Eastern Finland. These authors developed a scheduler and an optimizer system based on a guided variable neighborhood thresholding metaheuristic and reported an average distance improvement of 12%, and a reduction of 44% on a specific instance.

Sahoo et al. (51) worked with Waste Management Inc., a provider of waste-management services based in Houston, which services nearly 20 million residential customers and two million commercial customers throughout the Unites States and Canada. They developed a complete route-management system, deployed over 36 markets areas, and yielding 984 fewer routes and $18 million in savings after one year. In the long run, the number of routes was expected to be reduced by 10%.

Li et al. (52) developed a prototype decision support system (DSS) for the solid waste collection services in Porto Alegre, Brazil. They analyzed the impact of disruptions in trips and the strategy to use when unexpected events occur. In the context of hazardous waste disposal,

Alumur and Kara (53) proposed a model that determines where to open treatment centers, which technologies to employ, how to assign different types of hazardous waste to compatible treatment technologies, and how to route waste residues to disposal centers. The system was applied in the Central Anatolian region of Turkey.

Repoussis et al. (54) developed a complete DSS to manage waste lube oils collection and recycling operations for a multinational Greek company. They modeled this problem as an open VRP with time windows and solved it by means of a list-based threshold accepting metaheuristic. Unit cost reductions of up to 30% were achieved.

Coene et al. (55) studied the problem of a Belgian company collecting waste at slaughterhouses, butcher stores, and supermarkets. Waste products were divided into two categories – high-risk and low-risk – and different vehicles were used for each type. This led to two distinct periodic VRPs, one with 48 low-risk customers and three trucks over a planning period of one week, and one with 262 high-risk customers and three trucks over a two-week planning horizon. Since planning occurs over a time period of several days, the problem was solved as a periodic VRP using a two-phase heuristic in which customers are first assigned to days, and VRPs are solved for each day in the second phase.

Hauge et al. (56) dealt with the transportation of bulky waste containers. This roll-on/roll-off routing problem arises in the collection of industrial waste. It was formulated as a generalized set partitioning problem and solved by means of a hybrid column generation and a tabu search procedure.

Hemmelmayr et al. (57) studied the problem of designing a collection system for general waste in Italy. They considered the bin configuration and sizing problem at each collection site, as well as the service frequency over a given horizon. They analyzed the resulting trade-offs between the bin investment cost and the routing cost. They proposed a hierarchical solution procedure in which the bin location problem was first solved and was followed by the solution of the VRP. They tested both a sequential and an integrated approach.

Battarra et al. (58) solved an urban garbage collection in Italy as a clustered VRP in which 456 large street bins are located at 385 collection points.

Aksen et al. (59) studied the case of a biodiesel production facility in Istanbul, which collects used vegetable oil from restaurants, catering companies and hotels. The resulting selective and periodic inventory-routing problem was solved by means of an adaptive large neighborhood search algorithm.

Huang and Lin (60) studied the problem of efficiently routing and scheduling collectors for municipal waste collection in Taiwan where it is required that residents personally bring their waste to collection vehicles. They proposed a bilevel optimization model that first selects collection points by solving a set covering problem and then solves a VRP with pickup delivery by means of an ant colony optimization heuristic. They used two instances from a sub network of Kaohsiung City in Taiwan, involving 262 and 611 nodes.

In contrast to other applications, waste collection is in great part the responsibility of municipal governments, which usually do not have the expertise or possibility to invest in expensive and intricate specialized software. Also, since privacy or security issues are less critical than in the public sector, they are more prone to collaborate with researchers. Finally we observe that if the waste collection of containers and bins is managed as a vehicle routing problem, most of the home waste management collection problems are modeled as arc routing problems (61).

2.2.5.4 Mail and small package delivery

This section reviews applications ranging from mail delivery to the delivery of Internet orders, touching many variants of the classical VRP such as those involving time windows and pickups and deliveries.

Larsen et al. (62) modeled and solved the routing problem of an overnight mail service provider as an a priori dynamic TSP with time windows. The objective was to minimize the lateness of deliveries. They worked with United Parcel Service (UPS), using 10 days of data for each of four selected areas.

Hollis et al. (63) used a vehicle routing and crew scheduling algorithm based on set covering with column generation to solve the Melbourne metropolitan mail distribution at Australia Post. They worked with instances containing up to 339 locations and five depots and reported a potential cost saving of about 10%.

Cohn (64) et al. studied the load-matching and routing problem with equipment balancing for small package carriers. In this problem, all packages of a given commodity move through the same sequence of intermediate sorting facilities, and the commodities are grouped by common destination to fill trailers more efficiently. The authors used data from a regional subnetwork from UPS, with 263 nodes and more than 2,000 requests, and reported cost reductions of about 5%.

Groer et al. (65) solved a consistent VRP in a context where the objective is to plan the routes in order to have customers consistently visited by the same driver over time, so as to develop good working relationships. They solved an instance with 3,715 customers locations based on five weeks of real customer data provided by a company in the small package shipping industry.

Sungur et al. (66) studied a VRP with time windows in which customers appear probabilistically and have uncertain service times. They worked on two data sets provided by UPS having up to 5,178 potential customers and more than 25,000 service requests. They reported improvements of up to 20% over a weighted objective function value.

Pignac-Robitaille et al. (67) solved a pickup and delivery company specialized in transportation of biomedical samples in Quebec City. They worked on a data set containing 946 requests in which up to 30% of them were known one day in advance. By using the company's strategy which neglects this information, they reduced the number of routes from 54 to 50. Using information about known requests allowed cutting off one additional route and reducing the total distance by an additional 1.3%.

Mail and package delivery is a very important industry. While traditional mail delivery activities are in decline, parcel delivery is growing as a result of Internet trade. This industry operates in a highly competitive environment which forces companies to use state-of-the-art systems and software.

2.2.5.5 Food distribution

Food distribution has its own characteristics, constraints and challenges such as product quality, health and safety (68). The products often have a limited shelf-life, so that distribution operations must take into account temperature, humidity and time-in-transit considerations, as well as many other product-related constraints.

The review of Akkerman et al. (68) focuses on the challenges of food safety, quality and sustainability. These authors outline practical contributions related to strategic network design, tactical network planning and operational transportation planning.

Ahumada and Villalobos (69)studied the particular agri-food supply chain and reviewed the main contribution in the specific field of production and distribution planning for agri-foods based on agricultural crops.

Tarantilis and Kiranoudis (70) dealt with the distribution of fresh milk for one of the largest dairy companies in Greece. The problem was formulated as a heterogeneous fixed fleet VRP and solved through a backtracking adaptive threshold accepting algorithm. The authors solved an instance containing 299 supermarkets located in Athens, with a heterogeneous fleet of 29 vehicles, reducing the total distance by 28% in comparison with the solution used by the company.

Cheong et al. (71) studied a soft drink distribution problem arising in several districts of Singapore. They reduced both the average and maximum number of vehicles needed over a 23-day period.

Tarantilis and Kiranoudis (72) modeled the distribution of fresh meat from depots to 174 butcher shops in Athens as an open multi-depot VRP and solved it by means of a threshold accepting-based metaheuristic. They reported reducing the total traveled distance by 17%.

Prindezis et al. (73) developed a solution system that was applied to the Greater Athens area for the benefit of Athens Central Food Market enterprises. The system, based on a tabu search metaheuristic, is used by nearly 150 Central Market enterprises for planning their daily routes.

Faulın (74),(75) solved a logistics problem for Alimentos Congelados, S.A., a canning company located in Navarra, Spain. Heuristics were used for the initial solution which was improved by linear programming. Results obtained over 11 days showed a 4.6% average distance reduction.

Pamuk et al. (76) improved distribution operations for a major beer producer having about 4,000 customers in Ankara. They used a workload balancing and partitioning model to assign customers to workdays, followed by a simple nearest-neighbor routing heuristic.

Ruiz et al. (77) worked with Nanta S.A., a leading Iberian feed compounder in Spain, offering pig, poultry, ruminants, rabbits and other livestock feeding, and developed a complete DSS. The authors partitioned the customers into regions and created routes with few clients, generally less than six. They reported distance reductions ranging from 7% to 12% and cost reductions of 9% to 11%.

Faulın et al. (78) worked with the Frilac company in Pamplona, northern Spain, which delivers frozen goods such as ice cream, vegetables, precooked dishes, seafood and meat. They developed a complete DSS with database and visualization capabilities based on a savings algorithm. They reported reductions of 13.5% in distance and 10.8% in cost seven months after the implementation.

Belenguer et al. (79) presented a computer program developed to design delivery routes for a medium- sized meat company in Valencia. They used seven days of data to plan the routes of a fleet of seven vehicles serving between 94 and 148 orders per day. They considerably reduced the total lateness and the routes lengths by 8.96%.

Ioannou (80) studied the supply chain of the Hellenic sugar industry in Greece. They handled the transportation part by means of the Map-Route system created by Ioannou et al. (81). This system was developed for a wholesaler and logistics service provider supplying packaged goods and beverages to supermarkets and retail outlets in the Central Athens area. Their objective was to minimize long-term average inventory and routing costs.

Prive et al. (82) studied the distribution of soft drinks and collection of recyclable containers for Distribution Jacques Dubois, a Quebec-based distributor in Canada. They considered vehicle routing costs and the revenue generated by the collection of recyclable containers for 164 customers ordering 125 different products over a one-week planning period. They reported a distance reduction of about 23% with respect the manually designed routes of the company.

Cetinkaya et al. (83) improved the operations of Frito-Lay North America by modeling them as a large-scale, integrated multiproduct inventory lot-sizing and VRP. They solved the model using CPLEX by decomposing it into two sub-problems involving complementary inventory and routing components. They also used some classical TSP heuristics such as savings and cheapest insertions to improve the routes. Their results yield higher vehicle utilization and indicate that financial benefits could be achieved in inventory and delivery management.

Hu et al. (84) studied a food distribution problem for the Northern Grocery company in Beijing. Routes were constructed over a circular transportation network, leading to special characteristics, which helped the generation process.

Battarra et al. (85) studied the distribution of three different types of foods to supermarkets (vegetables, fresh food and non-perishable), which were incompatible in the sense that they could not be delivered simultaneous in the same vehicle. The problem was modeled as a multi-trip VRP with time windows. Six days of data were used with an average of 422 customers per day.

Incompatibility constraints were also considered by Caramia and Guerriero (86) for a milk collection problem where some small farms are inaccessible by large trucks. Since farmers produce different milk types, they used multi- compartments trucks. They worked with ASSO. LA. C., which collects milk from 158 farmers in four towns in Calabria, in southern Italy. They were able to obtain a reduction of about 14.4% in the total distance traveled and they also increased the filling ratio of the tank trucks from 85% to 95%.

Zachariadis et al. (87) worked with a frozen food distribution company operating in Athens. This company uses 27 types of boxes and a homogeneous fleet of eight-pallet trucks. Thus, the problem was modeled as a pallet-picking VRP with three-dimensional rectangular boxes. The authors developed a tabu search algorithm in which pallet-packing is solved with a packing heuristic.

Martınez and Amaya (88) worked with a home delivery service company that produces and delivers Spanish paella. The food is cooked in paella pans which are then delivered to the customers. This problem was solved as a multi-trip VRP with time windows and a loading component for these circulars items. On a set of 19 real instances they reported that their tabu search heuristic could reduce the total trip time by 25.5% on average.

Cattaruzza et al. (89) developed an iterated local search heuristic for the milk collection problem of Battarra et al. (85). Lahrichi et al. (90) worked with the Federation des producteurs de lait du Quebec which is responsible for negotiating the transportation cost on behalf of the dairy producers' of the province of Quebec. They studied two examples having up to 226 farms, four depots and eight vehicles. When optimizing only the collection sequence, they reported small improvements of about 0.5% demonstrating that the current plan was very good. When they allowed the reassignment of farms and plants to vehicles, they obtained up to 4% in distance reduction, which corresponds to savings of a few hundred thousand dollars yearly.

Demir et al. (91) worked with Nabuurs B.V., a Netherlands-based logistics service provider specialized in refrigerated, frozen and ambient food products including beverages. They analyzed the shift from a single-depot planning to a centralized multi-depot planning process. They used a SHORTREC-based simulation model which includes many routing construction and improvement algorithms. They discussed the managerial implications as well as the implementation of the SHORTREC as a tactical planning tool.

Lahyani et al. (92) studied the olive oil collection process in Tunisia. Since olive oil comes in three different grades, it must be transported in multi-compartment vehicles. Cleaning operations may be needed if a compartment must be reused for a different oil grade. On a set of instances having up to seven producers and 39 requests they reported an average reduction of about 11.7% in the total distance traveled.

Food distribution in an industry characterized by both many specific constraints and low return margins, which make this industry less suited for generic optimization software. This specific environment has led to several fruitful research collaborations between academia and industry.

2.3 Literature Review

A first attempt to optimally route a fleet of gasoline delivery trucks by creating a linear programming formulation was performed by G.B Dantzig et al (20). They tried to find a way to assign stations to trucks in such a manner that station demands are satisfied and total mileage covered by the fleet is minimum.

Other authors studied Capacitated VRP (CVRP). Specifically, Baldacci et al. (93) described the recent exact methods for the CVRP and reported a comparison of their computational performances. Cordeau et al. (94) presented a comprehensive overview of the available exact and heuristic algorithms for the VRP, most of which have been adapted to solve other variants.

Solomon & Desrosiers (95) surveyed the significant advances made for some classes of routing problems with time windows as the single and multiple traveling salesman problem, the shortest path problem, the minimum spanning tree problem, the generic vehicle routing problem, the pickup and delivery problem including the dial-a-ride problem, the multi-period vehicle routing problem and the shoreline problem.

Archetti & Speranza (96) presented a survey of the state-of-the-art on the Split Delivery VRP (SDVRP), in which the restriction that each customer has to be visited exactly once is removed.

Regarding Heterogeneous Fleet VRP Problem (HFVRP) Choi and Tcha (97) presented a set covering formulation, solved the Linear Programming relaxation of the formulation by the Column Generation and used a Branch and Bound algorithm to derive integer solution. Liu et al. (98) developed an effective Genetic Algorithm, in which a new chromosome evaluation procedure was introduced, a new single parent crossover operator was proposed and local search was used as mutation.

Emmanouil E. Zachariadis et al (99) examined a Vehicle Routing Problem (VRP) of major practical importance which is referred to as the Load-Dependent VRP (LDVRP) with its pick-up and delivery extension.

There is also reference about Multi-Depot VRP Problem (MDVRP) in which the depots from where vehicles start are more than one. E. Demir et al (91) described the shift from single-depot planning to multi-depot planning for Nabuurs B.V., a large Benelux logistics service provider that implemented a centralized , automated multi-depot planning process throughout its organization.

Chapter 3 Mathematical Formulation of Vehicle Routing Problem

3.1 Motivation

In this chapter, a mathematical formulation of the Vehicle Routing Problem (VRP) is presented. The aim of this formulation is to minimize the total distribution cost of products which have to be delivered from depots to customers and products which have to be picked up from customers in order to return to depots.

This formulation is used to create an algorithm using C++ Programming Language. The algorithm can result in fast and optimal solutions for many VRP categories.

3.2 Mathematical Model

The mathematical model was created to include the following categories of VRP:

- Multi Depot (MD): There can be one or more starting-finishing locations of vehicles which transfer products.
- Capacitated (C): Each vehicle has a specific amount of available space for transferring products.
- Heterogeneous Fleet (HF): All vehicles are not identical. They have different capacity and average consumption. Also, they differ regarding their ability to refrigerate products and enter town centers.
- Time Windows (TW): Each customer allows vehicles to unload and load products during specific time horizon. Each depot also impose time restrictions regarding starting and finishing time of vehicles.
- Simultaneous Pick-ups and Deliveries (PD): Vehicles visit customers in order to unload and load products and meet their pick-up and delivery demand.
- Split satisfaction of pick-up and delivery demand (SD): The total pick-up and delivery demand of a customer can be satisfied by one or more vehicles which visit his location.
- Specific network characteristics (SNC): Customers can require refrigerated or non-refrigerated vehicles to transfer their products. Also, customer's location plays a role in the selection of a vehicle. Customers located at town centers can only be serviced by

small vehicles whereas customers located at suburbs can be serviced by all available vehicles.

- Vehicle Relocation (VR): Vehicles can start from a depot and are permitted to return to the same or to a different depot.

The mathematical model is consisted of the objective function and main constraints, used to describe Multi-Depot (MD), Capacitated (C), Heterogeneous Fleet (HF) and Delivery VRP. Despite basic model, there are additive constraints to optionally include other categories of VRP. These categories are:

1. Time Windows (TW)
2. Simultaneous Pick-up and Delivery (PD)
3. Split Satisfaction of Demand (SD)
4. Specific Network Characteristics (SNC)
5. Vehicle Relocation (VR)

All the above categories can be combined with the basic model to create an algorithm able to solve 32 VRP problem categories:

1. Multi-Depot, Capacitated, Heterogeneous Fleet and Delivery VRP (MDCHFVRP)
2. Multi-Depot, Capacitated, Heterogeneous Fleet, Delivery, Time Windows VRP (MDCHFTWVRP)
3. Multi-Depot, Capacitated, Heterogeneous Fleet, Simultaneous Pickup and Delivery VRP (MDCHFPDVRP)
4. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Simultaneous Pickup and Delivery VRP (MDCHFTWPDVRP)
5. Multi-Depot, Capacitated, Heterogeneous Fleet, Split Delivery VRP (MDCHFSDVRP)
6. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Split Delivery VRP (MDCHFTWSDVRP)
7. Multi-Depot, Capacitated, Heterogeneous Fleet, Simultaneous Pickup and Delivery, Split Satisfaction of Demand VRP (MDCHFPDSDVRP)
8. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Simultaneous Pickup and Delivery, Split Satisfaction of Demand VRP (MDCHFTWPDSDVRP)

9. Multi-Depot, Capacitated, Heterogeneous Fleet, Specific Network Characteristics VRP (MDCHFSNCVRP)
10. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Specific Network Characteristics VRP (MDCHFTWSNCVRP)
11. Multi-Depot, Capacitated, Heterogeneous Fleet, Simultaneous Pickup and Delivery, Specific Network Characteristics VRP (MDCHFPDSNCVRP)
12. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Simultaneous Pickup and Delivery, Specific Network Characteristics VRP (MDCHFTWPDSNCVRP)
13. Multi-Depot, Capacitated, Heterogeneous Fleet, Split Delivery, Specific Network Characteristics VRP (MDCHFSDSNCVRP)
14. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Split Delivery, Specific Network Characteristics VRP (MDCHFTWSDSNCVRP)
15. Multi-Depot, Capacitated, Heterogeneous Fleet, Simultaneous Pickup and Delivery, Split Satisfaction of Demand, Specific Network Characteristics VRP (MDCHFPDSDSNCVRP)
16. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Simultaneous Pickup and Delivery, Split Satisfaction of Demand, Specific Network Characteristics VRP (MDCHFTWPDSDSNCVRP)
17. Multi-Depot, Capacitated, Heterogeneous Fleet, Delivery, Vehicle Relocation VRP (MDCHFVRVRP)
18. Multi-Depot, Capacitated, Heterogeneous Fleet, Delivery, Time Windows, Vehicle Relocation VRP (MDCHFTWVRVRP)
19. Multi-Depot, Capacitated, Heterogeneous Fleet, Simultaneous Pickup and Delivery, Vehicle Relocation VRP (MDCHFPDVRVRP)
20. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Simultaneous Pickup and Delivery, Vehicle Relocation VRP (MDCHFTWPDVRVRP)
21. Multi-Depot, Capacitated, Heterogeneous Fleet, Split Delivery, Vehicle Relocation VRP (MDCHFSDVRVRP)
22. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Split Delivery, Vehicle Relocation VRP (MDCHFTWSDVRVRP)
23. Multi-Depot, Capacitated, Heterogeneous Fleet, Simultaneous Pickup and Delivery, Split Satisfaction of Demand, Vehicle Relocation VRP (MDCHFPDSDVRVRP)

24. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Simultaneous Pickup and Delivery, Split Satisfaction of Demand, Vehicle Relocation VRP (MDCHFTWPDSDVRVRP)
25. Multi-Depot, Capacitated, Heterogeneous Fleet, Specific Network Characteristics, Vehicle Relocation VRP (MDCHFSNCVRVRP)
26. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Specific Network Characteristics, Vehicle Relocation VRP (MDCHFTWSNCVRVRP)
27. Multi-Depot, Capacitated, Heterogeneous Fleet, Simultaneous Pickup and Delivery, Specific Network Characteristics, Vehicle Relocation VRP (MDCHFPDSNCVRVRP)
28. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Simultaneous Pickup and Delivery, Specific Network Characteristics, Vehicle Relocation VRP (MDCHFTWPDSNCVRVRP)
29. Multi-Depot, Capacitated, Heterogeneous Fleet, Split Delivery, Specific Network Characteristics, Vehicle Relocation VRP (MDCHFSDSNCVRVRP)
30. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Split Delivery, Specific Network Characteristics, Vehicle Relocation VRP (MDCHFTWSDSNCVRVRP)
31. Multi-Depot, Capacitated, Heterogeneous Fleet, Simultaneous Pickup and Delivery, Split Satisfaction of Demand, Specific Network Characteristics, Vehicle Relocation VRP (MDCHFPDSDSNCVRVRP)
32. Multi-Depot, Capacitated, Heterogeneous Fleet, Time Windows, Simultaneous Pickup and Delivery, Split Satisfaction of Demand, Specific Network Characteristics, Vehicle Relocation VRP MDCHFTWPDSDSNCVRVRP

This mathematical model is a mixed integer-linear programming model as both binary and continuous variables are used.

Firstly pointers, sets, decision variables and input parameters are being presented followed by the equations which describe the objective function and constraints of VRP.

3.2.1 Illustration of Pointers, Sets, Decision Variables and Input Parameters

Pointers

i,j: Pointers of vertices which represent locations of customers and depots

k: Pointer of vehicles

Sets

$M = \{0, 1, \dots, m-1\}$: set of depots (starting and finishing points of vehicles). There are m depots.

$N = \{m, m+1, \dots., m+n\}$: set of customers that have to be serviced (Vehicles pick-up or deliver products). There are n customers.

$K = \{0, 1, \dots, l-1\}$: set of available vehicles. There are l vehicles.

$V = NUM$: set of vertices of customers and depots.

A: Set of available edges (i, j), starting from vertex $i \in V$ and finishing at vertex $j \in V$ $(i \neq j)$

$G(V, A)$: Network of all vertices and all edges (available routes of moving between vertices).

Decision Variables

x_{ijk} : Binary variable. It takes value 1 if vehicle k moves along edge $(i, j) \in A$, 0 otherwise.

T_{ik} : Time variable which shows the moment that vehicle $k \in$ K starts servicing customer $i \in N$. For depots $i \in M$ this variable shows the moment when vehicle $k \in$ K start its route.

D_{ijk} : Variable that shows the load (in Kg) of vehicle $k \in$ K that has to be delivered during movement at edge $(i, j) \in A$.

P_{ijk} : Variable that shows the load (in Kg) of vehicle k that has been picked-up during movement at edge $(i, j) \in A$.

yd_{ik} : Amount of delivery demand at customer i that is delivered from vehicle k.

yp_{ik} : Amount of pick-up demand at customer i that is picked up from vehicle k.

Input Parameters (Data)

m: Total number of depots.

n: Total number of customers.

l: Total number of vehicles.

d_i: Delivery Demand of customer located at vertex $i \in N$.

p_i: Pick-up Demand of customer located at vertex $i \in N$.

a_i: Earliest time of a time window during which a customer $i \in N$ can be serviced and a depot $i \in M$ let vehicles start their routes.

b_i: Latest time of a time window during which a customer $i \in N$ can be serviced and a depot $i \in M$ let vehicles finish their routes.

s_i: Total time of servicing needed at a depot or a customer located at vertex $i \in V$.

dr_i: Customer's $i \in N$ demand of refrigerator required for products being delivered or picked-up (1 if refrigerator is needed, 0 otherwise).

cl_i: This parameter declares if customer $i \in N$ is located at the town center (1 if this happens, 0 otherwise).

C_k: Capacity of vehicle k.

r_k: Vehicle $k \in$ K with or without refrigerator (1 if vehicle k has a refrigerator, 0 otherwise).

vp_k: This parameter declares if vehicle $k \in$ K is permitted to enter the town center (1 if this happens, 0 otherwise).

sl_{ik}: Input parameter which shows that the starting point of vehicle k is Depot i.

d_{ij}: Distance of movement along edge $(i, j) \in A$.

c_{ijk}: Cost of moving along edge $(i, j) \in A$ using vehicle k.

t_{ijk}: Time of moving along edge $(i, j) \in A$ using vehicle k.

ac_k: Average consumption of vehicle $k \in$ K (litres/Km).

mrt: Total travel time in minutes for each vehicle (Maximum driver's shift).

M: Large constant number.

3.2.2 Objective Function and Constraints using mathematical symbols

Basic Model Multi Depot, Capacitated, Heterogeneous Fleet VRP with Deliveries (MDCHFVRP)

Objective Function

$$minimize\ z = \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} c_{ijk} * x_{ijk} \qquad (3.2.2.1)$$

Constraints (subject to)

$$\sum_{k \in K} \sum_{j \in V} x_{ijk} = 1 \qquad \forall\ \mathrm{i} \in N \qquad (3.2.2.2)$$

$$\sum_{j \in N} x_{ijk} \leq sl_{ik} \qquad \forall \mathrm{i} \in M, \mathrm{k} \in K \qquad (3.2.2.3)$$

$$\sum_{j \in V} x_{ijk} - \sum_{j \in V} x_{jik} = 0 \qquad \forall\ \mathrm{i} \in V, k \in K \qquad (3.2.2.4)$$

$$\sum_{k \in K} \sum_{j \in V} D_{jik} - \sum_{k \in K} \sum_{j \in V} D_{ijk} = d_i \qquad \forall\ \mathrm{i} \in N \qquad (3.2.2.5)$$

$$D_{ijk} \leq C_k * x_{ijk} \qquad \forall\ \mathrm{i}, j \in V, k \in \mathrm{K} \qquad (3.2.2.6)$$

$$\sum_{i \in N} \sum_{j \in M} D_{ijk} = 0 \qquad \forall\ k \in \mathrm{K} \qquad (3.2.2.7)$$

$$\sum_{i \in V} \sum_{j \in V} (x_{ijk} * t_{ijk}) \leq mrt \qquad \forall\ k \in \mathrm{K} \qquad (3.2.2.8)$$

$$x_{ijk}\ binary \qquad \forall\ (i, j) \in A, k \in K \qquad (3.2.2.9)$$

$$D_{ijk} \geq 0 \qquad \forall\ (\mathrm{i}, \mathrm{j}) \in \mathrm{A}, \mathrm{k} \in \mathrm{K} \qquad (3.2.2.10)$$

VRP categories can be optionally included in the main VRP problem. This is formulated by adding and removing constraints.

Time Windows VRP Add

$$T_{ik} + s_i + t_{ijk} - M(1 - x_{ijk}) - T_{jk} \leq 0 \qquad \forall\ \text{i} \in V, j \in N, k \in \text{K} \qquad (3.2.2.11)$$

$$T_{ik} + s_i + t_{ijk} - M(1 - x_{ijk}) - b_j \leq 0 \qquad \forall\ \text{i} \in N, j \in M, k \in \text{K} \qquad (3.2.2.12)$$

$$a_i \leq T_{ik} \leq b_i \qquad \forall\ \text{i} \in V, k \in \text{K} \qquad (3.2.2.13)$$

$$T_{ik} \geq 0 \qquad \forall\ \text{i} \in V, \text{k} \in \text{K} \qquad (3.2.2.14)$$

Simultaneous Pick-up and Delivery VRP Add

$$\sum_{k \in K} \sum_{j \in V} P_{ijk} - \sum_{k \in K} \sum_{j \in V} P_{jik} = p_i \qquad \forall\ \text{i} \in N \qquad (3.2.2.15)$$

$$P_{ijk} + D_{jik} \leq C_k * x_{ijk} \qquad \forall\ \text{i}, j \in V, k \in \text{K} \qquad (3.2.2.16)$$

$$\sum_{i \in M} \sum_{j \in N} P_{ijk} = 0 \qquad \forall\ k \in \text{K} \qquad (3.2.2.17)$$

$$P_{ijk} \geq 0 \qquad \forall\ i, j \in V, k \in K \qquad (3.2.2.18)$$

Inequality (3.2.2.6) of basic model is removed -(3.2.2.6)

Split Satisfaction of demand VRP Add

a. Split Delivery VRP

$$\sum_{k \in K} \sum_{j \in V} x_{ijk} \geq 1 \qquad \forall \text{ i} \in N \qquad (3.2.2.19)$$

$$\sum_{j \in V} D_{jik} - \sum_{j \in V} D_{ijk} = yd_{ik} \qquad \forall \text{ i} \in N, k \in K \qquad (3.2.2.20)$$

$$\sum_{k \in \mathrm{K}} yd_{ik} = d_i \qquad \forall \text{ i} \in N \qquad (3.2.2.21)$$

$$yd_{ik} \leq d_i * \sum_{j \in V} x_{ijk} \qquad \forall i \in N, k \in K \qquad (3.2.2.22)$$

$$yd_{ik} \geq 0 \qquad \forall \text{ i} \in N, k \in K \qquad (3.2.2.23)$$

Inequalities (3.2.2.2),(3.2.2.5) of basic model are removed -(3.2.2.2), -(3.2.2.5)

b. Split Simultaneous Pick-up and Delivery VRP [Inequalities are added to (a) Split Delivery VRP]

$$\sum_{j \in V} P_{ijk} - \sum_{j \in V} P_{jik} = yp_{ik} \qquad \forall \text{ i} \in \mathrm{N}, \mathrm{k} \in \mathrm{K} \qquad (3.2.2.24)$$

$$\sum_{k \in \mathrm{K}} yp_{ik} = p_i \qquad \forall \text{ i} \in N \qquad (3.2.2.25)$$

$$yp_{ik} \leq p_i * \sum_{j \in V} x_{ijk} \qquad \forall \text{ i} \in \mathrm{N}, \mathrm{k} \in \mathrm{K} \qquad (3.2.2.26)$$

$$yp_{ik} \geq 0 \qquad \forall \text{ i} \in \mathrm{N}, \mathrm{k} \in \mathrm{K} \qquad (3.2.2.27)$$

Inequality (3.2.2.15) of Simultaneous Pick-up and Delivery VRP Add is removed -(3.2.2.15)

Specific characteristics of Network VRP Add

$$\sum_{j\in V} x_{ijk} \leq 1 - df_{ik} \qquad \forall\, i \in N, k \in K \qquad (3.2.2.28)$$

where $df_{ik} = |dr_i - r_k|$

$$cl_i * \sum_{j\in V} x_{ijk} \leq vp_k \qquad \forall\, i \in N, k \in K \qquad (3.2.2.29)$$

Vehicle Relocation VRP Add

$$\sum_{j\in V} x_{ijk} - \sum_{j\in V} x_{jik} = 0 \qquad \forall\, \mathrm{i} \in N, k \in K \qquad (3.2.2.30)$$

$$\sum_{i\in M}\sum_{j\in N} x_{ijk} - \sum_{i\in M}\sum_{j\in N} x_{jik} = 0 \qquad \forall\, \mathrm{k} \in \mathrm{K} \qquad (3.2.2.31)$$

Inequality (3.2.2.4) of basic model is removed -(3.2.2.4)

3.2.3 Explanation of Objective Function and Constraints

The objective function (3.2.2.1) aims to minimize the total cost required for vehicles to distribute products to all customers and cover total demand. The mathematical type of cost c_{ijk} is $c_{ijk} = a_k * d_{ij}$, where a_k is the average consumption of vehicle k (in liters per Kilometers) and d_{ij} is the distance between two nodes i and j (in kilometers). It is clear that the cost depends on the vehicles that are chosen to transfer products and on the route which they are going to follow.

Constraints (3.2.2.2) are used to express that each costumer is serviced only one time by one vehicle.

Constraints (3.2.2.3) ensure that each vehicle k is permitted to start visiting customers from a depot i if it is initially located at this depot i.

Constraints (3.2.2.4) guarantee the flow conservation at vertices of depots and customers. Specifically, if a vehicle arrives at a customer's vertex, it leaves the same vertex. The same happens to vertices of depots. If a vehicle starts from a specific depot and is chosen to follow a route to cover customers' demand, it returns to the same depot. In mathematical terms, the sum of all possible transitions of a vehicle k from any other vertex to a vertex i will be equal to all possible transitions from this vertex i to any other vertex.

Constraints (3.2.2.5) ensure that delivery products which are loaded to vehicle k and are transported to a customer i are equal to the products which are delivered to customer i plus the delivery products that are moved from customer i to any other vertex with vehicle k continuing their transportation.

Constraints (3.2.2.6) ensure that each vehicle is not able to carry more products than its capacity.

Constraints (3.2.2.7) are used to ensure that each vehicle returns to a depot without any load after delivering all products to customers.

Constraints (3.2.2.8) guarantee that the travel time of each vehicle should not exceed the maximum allowed travel time. This stems from the fact that a driver is not permitted to drive a vehicle more than a specific amount of hours continuously. The maximum allowed travel time can be defined by the user and to our examples is considered eight (8) hours per day.

Constraints (3.2.2.9) and (3.2.2.10) declare that decision variables x_{ijk} are binary and decision variables D_{ijk} are positive respectively.

Constraints (3.2.2.11) define the time when a vehicle k can arrive at a customer j, given the time it starts from vertex i (customer or depot), the service time needed there (vertex i) and the travelling time required for the transition from i to j.

Similarly, constraints (3.2.2.12) define the time when a vehicle can return to a depot j given the time a vehicle is located to a customer i, the service time needed there (vertex i) and the travelling time required for the transition from i to j.

Constraints (3.2.2.13) are used to express the time frames depots and customers impose. These time frames show when activities can take place for both customers and depots as personnel responsible for loading and unloading vehicles is not available every time.

Constraints (3.2.2.14) declare that decision variables T_{ik} are positive.

Constraints (3.2.2.15) ensure that picked up products which are loaded to vehicle k and are transported from a customer i to any other vertex are equal to the products, which the vehicle k has already picked up, and transports from any other vertex to customer i plus the products which are picked up from the customer i.

Constraints (3.2.2.16) are used to express that the sum of products carried by vehicle k in order to be delivered to following customers and products already picked up by previous customers are not allowed to exceed vehicle's capacity.

Constraints (3.2.2.17) are used to ensure that each vehicle starts from a depot and its load is only composed of products that have to be delivered to customers. Specifically, its pick up load which represents products picked up from customers is equal to zero.

Constraints (3.2.2.18) declare that decision variables P_{ijk} are positive.

Constraints (3.2.2.19) are used to express that each costumer can be visited by more than one vehicle. This happens as customer's demand can be met partially by a vehicle.

Constraints (3.2.2.20) ensure that products to be delivered, which are loaded to vehicle k and are transported to a customer i, are equal to the products which are delivered to customer i to cover a part of or his total delivery demand plus the products that are moved from customer i to any other vertex in order to be delivered with vehicle k.

Constraints (3.2.2.21) ensure that all vehicles, which visit a customer i, cover total customer's delivery demand.

Constraints (3.2.2.22) guarantee that amount of delivery demand at customer i that is delivered by vehicle k is less than or equal to total delivery demand of customer i.

Constraints (3.2.2.23) declare that decision variables yd_{ik} are positive. Constraints (3.2.2.22) in combination with (3.2.2.23) enforce the amount of delivery demand to be 0 if vehicle k does not visit customer i.

Constraints (3.2.2.24) ensure that picked up products which are loaded to vehicle k and are transported from a customer i to any other vertex are equal to the products which the vehicle k transports from any other vertex to customer i plus the products which are picked up from the customer i to cover a part of or his total pick up demand.

Constraints (3.2.2.25) ensure that all vehicles, which visit a customer i, cover total customer's pick up demand.

Constraints (3.2.2.26) guarantee that amount of pick up demand at customer i that is delivered by vehicle k is less than or equal to total pick up demand of customer i.

Constraints (3.2.2.27) declare that decision variables yp_{ik} are positive. Constraints (3.2.2.26) in combination with (3.2.2.27) enforce the amount of pick up demand to be 0 if vehicle k does not visit customer i.

Constraints (3.2.2.28) guarantee that customers who demand to be delivered or be picked up products that need to be refrigerated will be satisfied by vehicles with refrigerator. Those who be delivered or be picked up products that do not demand refrigeration will be satisfied by vehicles without refrigerator.

Constraints (3.2.2.29) ensure that customers whose location is at the center of the town will be satisfied by vehicles that have the ability to enter the center. Those whose location is not at the town center will be satisfied by all kinds of vehicles.

Both (3.2.2.28) and (3.2.2.29) constraints are used to simulate real life applications. Many customers at real world want to be delivered refrigerated products. Their demand is satisfied using refrigerated vehicles. Also, town centers have narrow roads and only small trucks are permitted so customers who are located near town centers are serviced by small vehicles.

Constraints (3.2.2.30) guarantee the flow conservation at vertices of customers when relocation of vehicles is permitted. Specifically, if a vehicle arrives at a customer's vertex, it leaves the same vertex. In mathematical terms, the sum of all possible transitions of a vehicle k from any other vertex to a customer's vertex i will be equal to all possible transitions from this vertex i to any other vertex.

Constraints (3.2.2.31) guarantee the flow conservation at vertices of depots when relocation of vehicles is permitted. Specifically, if a vehicle starts from a specific depot and is chosen to follow a route to cover customers' demand, it can return to any depot the company has. In mathematical terms, the sum of all possible transitions of a vehicle k from any depot to any customer when it starts its route, will be equal to all possible transitions from any customer to any depot when it finishes picking up and delivering products.

Chapter 4 Application of Vehicle Routing Problem Mathematical Model to C++ Algorithm

4.1 Motivation

In this chapter, there is information about the algorithm which was created based on the mathematical model of Chapter 3. Furthermore, simulation results are presented for each VRP case.

4.2 Structure of Algorithm

The algorithm uses text files (.txt extension) to input data in order to solve VRP problems. These text files contain data regarding depots, customers, vehicles and information important to define VRP problem category.

Data of depots contains information about coordinates of each depot (latitude and longitude), lower and upper time frames within which loading and unloading can take place, and service time.

Data of customers contains information about coordinates of each customer (latitude and longitude), lower and upper time frames within which loading and unloading can take place, service time needed, pick-up and delivery demand, demand of refrigerated vehicles and location information (town center or suburb).

Data of vehicles contains information about every available vehicle regarding its capacity, its ability to refrigerate products, its ability to enter town centers, its starting location and its average consumption.

Data of VRP category defines if a VRP problem includes Time Windows, Simultaneous Pickup and Delivery, Split Satisfaction of customers' demand, Specific Network Characteristics, vehicles' ability to relocate depots.

The computation of Euclidean distance between locations is achieved through algorithm. This distance is multiplied by the average consumption of each vehicle in order to create data regarding vehicle's movement cost.

4.3 Software and Hardware Characteristics

The software which was used for algorithm creation has the following characteristics:

- Windows Edition: Windows 10 64 bit
- Programming Language: C++
- Environment: Microsoft Visual Studio 2010
- Solver: IBM CPLEX Optimization Studio v12.4

The Characteristics of Personal Computer which was used to simulate data are:

- Processor: Intel(R) Core i5-5250U CPU 1.6GHz
- Installed Memory (RAM): 8.00GHz

4.4 Data used for simulation

Data of a Greek Supermarket company was used to test the source code. This data is consisted of supermarket stores (customers) located at the cities of Larisa, Volos, Katerini, Trikala and Karditsa, depot facilities (depots) located near cities of Katerini, Larisa and Trikala and a number of heterogeneous vehicles. It is noted that for some VRP cases a part of the aforementioned points was used in order to make results depict all parameters that are taken into consideration.

The images below show the geographical sites of all depots and customers. The first image refers to the whole network whereas the following four (4) images focus on the depots and customers of prefectures of Larisa, Volos, Katerini, Trikala and Karditsa.

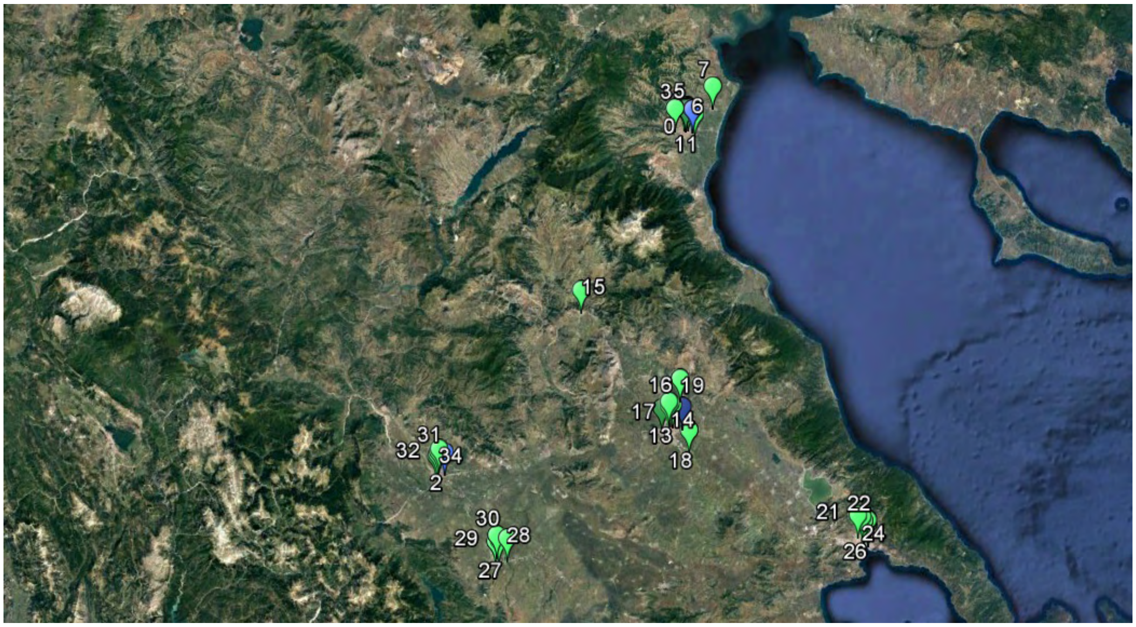

Figure 1 Locations of all depots and customers

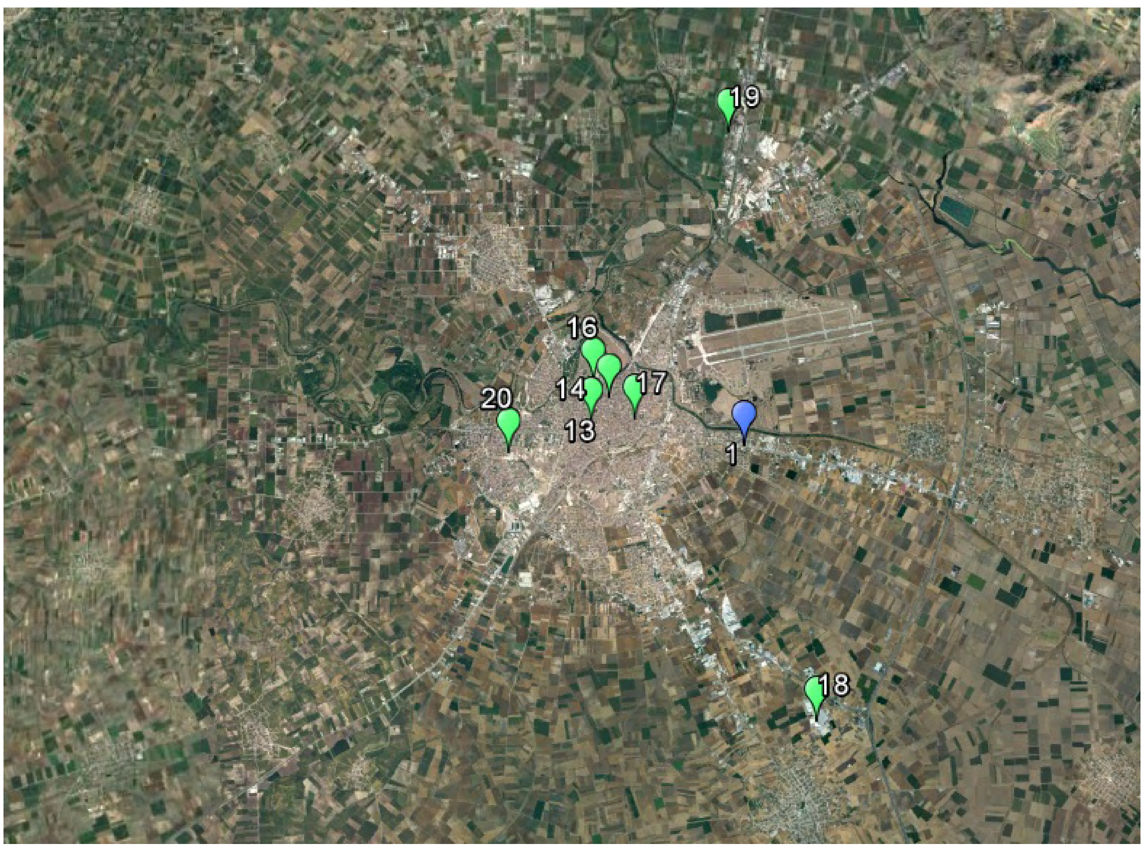

Figure 2 Locations of depots and customers at prefecture of Larisa

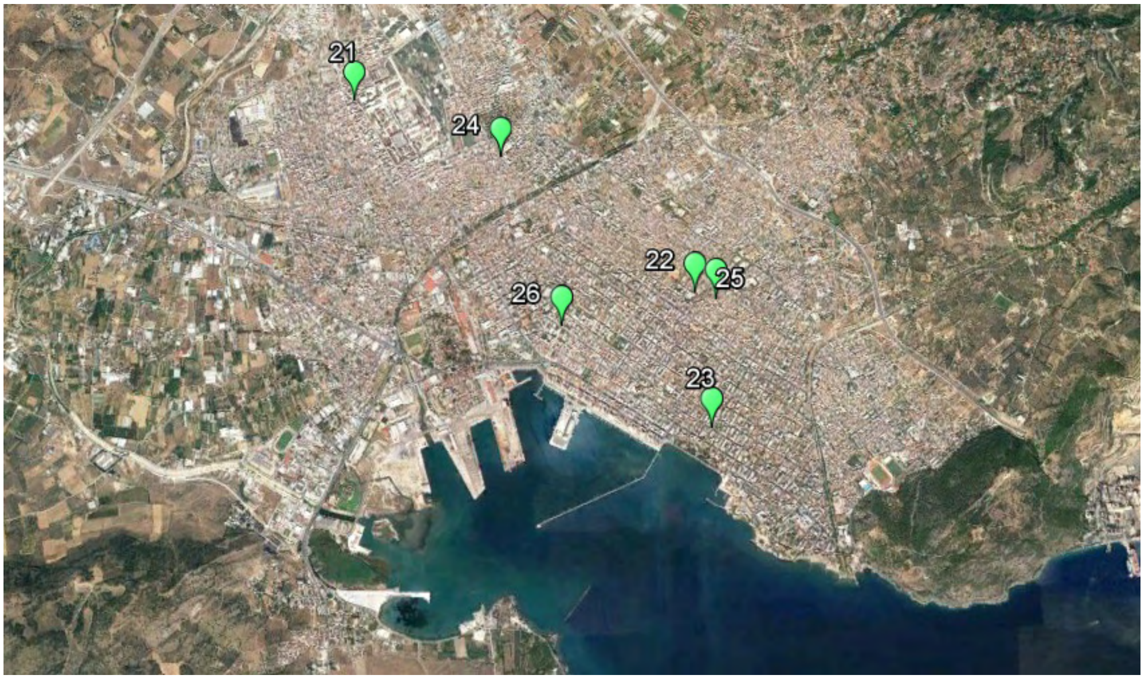

Figure 3 Locations of customers at prefecture of Volos

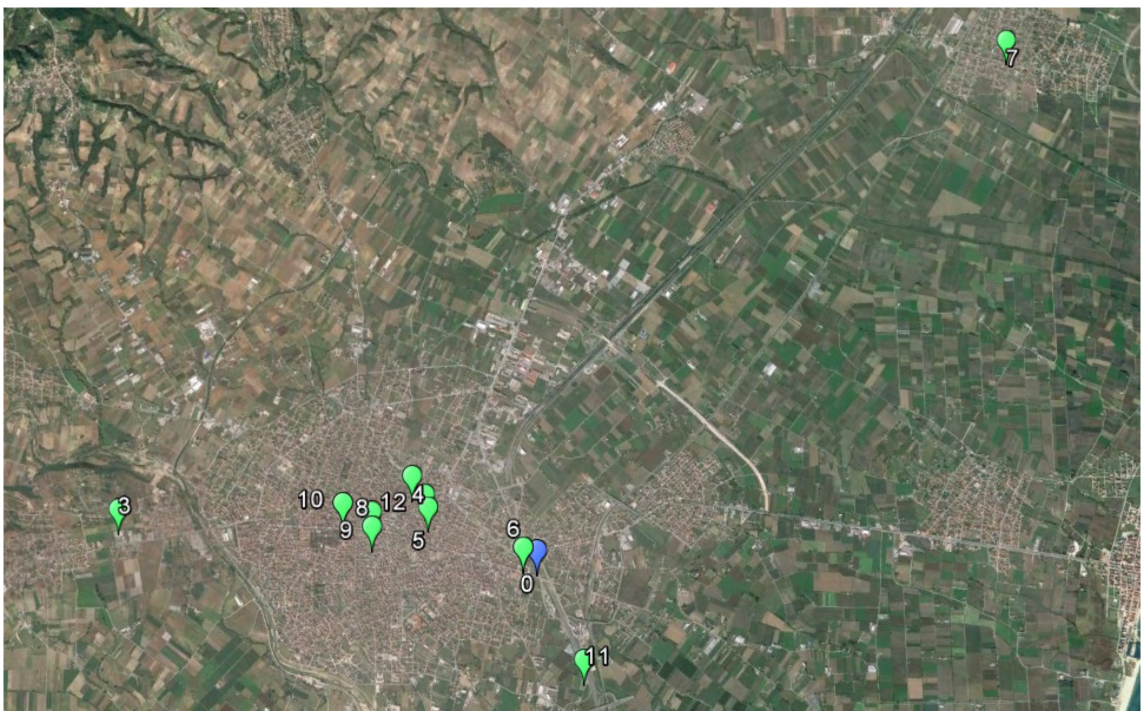

Figure 4 Locations of depots and customers at prefecture of Katerini

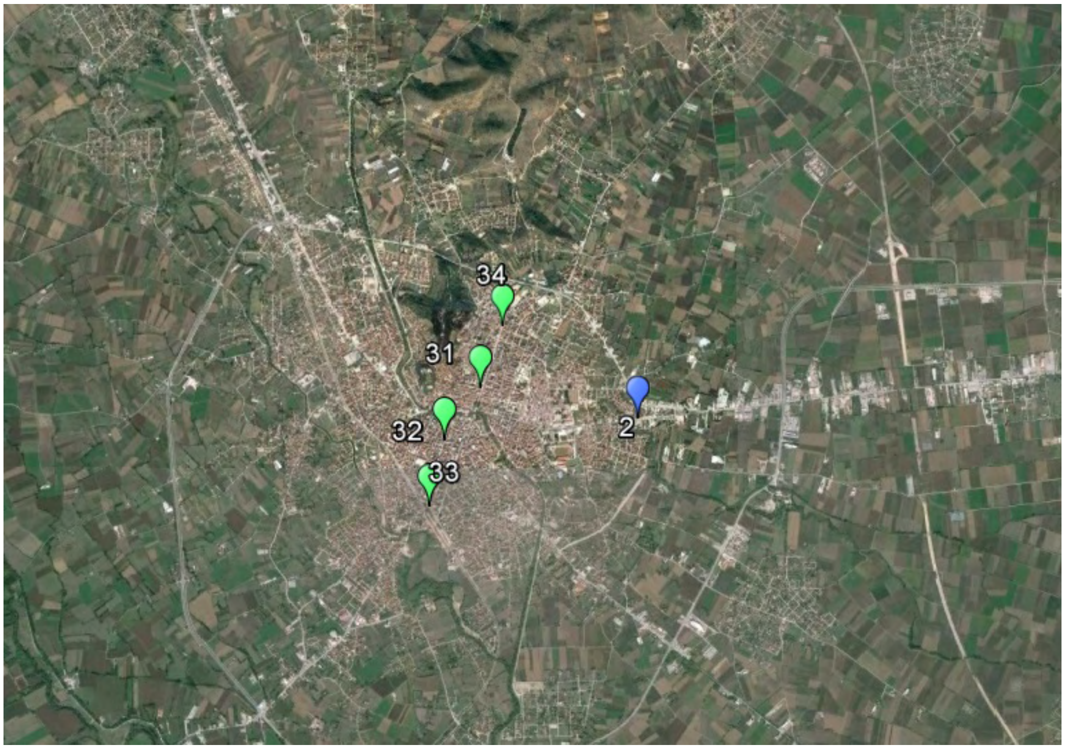

Figure 5 Locations of depots and customers at prefecture of Trikala

Figure 6 Locations of customers at prefecture of Karditsa

4.5 Simulation Results

In order to confirm algorithm's functionality, computational experiments were run for all VRP cases. Data used for each case differ regarding the number of depots, customers and vehicles. The table below summarizes the results after running the algorithm for all VRP instances.

A/A	Instance	Instance Number	m	n	l	Running Time	Solution Value Total consumption (litres)
1	MDCHFVRP	00000	3	32	3	42.47	76.7195
2	MDCHFVRVRP	00001	3	18	5	156.56	29.2232
3	MDCHFSNCVRP	00010	3	18	4	0.62	39.7444
4	MDCHFSNCVRVRP	00011	3	18	5	0.55	33.4074
5	MDCHFSDVRP	00100	3	32	4	274.83	76.7581
6	MDCHFSDVRVRP	00101	3	18	5	44.44	29.9362
7	MDCHFSDSNCVRP	00110	3	18	5	1.84	39.2137
8	MDCHFSDSNCVRVRP	00111	3	18	5	0.86	34.0318
9	MDCHFPDVRP	01000	3	32	3	199.55	76.7195
10	MDCHFPDVRVRP	01001	3	32	3	126.94	76.7195
11	MDCHFPDSNCVRP	01010	3	18	4	2.02	39.7444
12	MDCHFPDSNCVRVRP	01011	3	18	5	4.48	33.4074
13	MDCHFPDSDVRP	01100	3	18	4	600.59	33.7694
14	MDCHFPDSDVRVRP	01101	3	18	5	641.89	32.4654
15	MDCHFPDSDSNCVRP	01110	3	18	5	10.78	73.5243
16	MDCHFPDSDSNCVRVRP	01111	3	18	5	1.89	51.9066
17	MDCHFTWVRP	10000	3	32	3	52.17	76.7195
18	MDCHFTWVRVRP	10001	3	18	5	181.78	29.2232
19	MDCHFTWSNCVRP	10010	3	18	4	2.27	39.7444
20	MDCHFTWSNCVRVRP	10011	3	18	5	1.92	33.4074
21	MDCHFTWSDVRP	10100	3	24	4	80.64	58.8033
22	MDCHFTWSDVRVRP	10101	3	18	5	61.73	29.9362
23	MDCHFTWSDSNCVRP	10110	3	18	5	1.39	39.2137
24	MDCHFTWSDSNCVRVRP	10111	3	18	5	0.78	34.0318

A/A	Instance	Instance Number	m	n	l	Running Time	Solution Value Total consumption (litres)
25	MDCHFTWPDVRP	11000	3	32	3	824.69	76.7195
26	MDCHFTWPDVRVRP	11001	3	18	5	632.42	29.2232
27	MDCHFTWPDSNCVRP	11010	3	18	4	3.60	39.7444
28	MDCHFTWPDSNCVRVRP	11011	3	18	5	2.58	33.4074
29	MDCHFTWPDSDVRP	11100	3	18	4	708.03	33.7694
30	MDCHFTWPDSDVRVRP	11101	3	18	5	652.92	32.4654
31	MDCHFTWPDSDSNCVRP	11110	3	18	5	8.67	40.087
32	MDCHFTWPDSDSNCVRVRP	11111	3	18	5	4.58	51.9066

m: number of depots, n: number of customers, l: number of vehicles used

Table 1 Algorithm's benchmark instances

4.5.1 Multi Depot Capacitated Heterogeneous Fleet VRP (MDCHFVRP)

The first case of VRP which is examined to test the efficiency of the algorithm is Multi Depot Capacitated Heterogeneous Fleet VRP Problem. In this case, there are three (3) depots, thirty two (32) customers and three (3) vehicles, each of which is initially located at one of the three depots. The results are consisted by the route of each vehicle and its load during movement. The results are represented by the values of decision variables D_{ijk}, P_{ijk}, T_{ik}, yd_{ik}, yp_{ik} which show:

D_{ijk} : This variable shows the load (in Kg) of vehicle $k \in$ K that has to be delivered during movement at edge $(i, j) \in A$.

P_{ijk} : This variable shows the load (in Kg) of vehicle k that has been picked-up during movement at edge $(i, j) \in A$.

T_{ik} : Shows the time that vehicle $k \in$ K starts servicing vertex $i \in V$.

yd_{ik} : Amount of delivery demand at customer i that is delivered from vehicle k.

yp_{ik} : Amount of pick-up demand at customer i that is delivered from vehicle k.

Total amount of vehicle load can be computed by the sum of decision variables D_{ijk} and P_{ijk}.

The Table 2 below summarizes the results for Multi Depot Capacitated Heterogeneous Fleet VRP Problem.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
00000	0	0	6	213				
		6	5	197				
		5	12	180				
		12	4	163				
		4	9	145				
		9	8	128				
		8	10	106				
		10	3	82				
		3	7	51				
		7	11	32				
		11	0	0				
	1	1	17	311				
		17	14	285				
		14	16	271				
		16	13	257				
		13	20	242				
		20	15	223				
		15	19	205				
		19	21	172				
		21	24	154				
		24	22	129				
		22	25	102				
		25	23	86				
		23	26	58				
		26	18	34				
		18	1	0				
	2	2	34	159				
		34	31	140				
		31	32	112				
		32	33	94				
		33	29	80				
		29	27	58				

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		27	28	35				
		28	30	17				
		30	2	0				

Table 2 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet VRP.

4.5.2 Multi Depot Capacitated Heterogeneous Fleet Vehicle Relocation VRP (MDCHFVRVRP)

At the case of Multi Depot Capacitated Heterogeneous Fleet Vehicle Relocation VRP Problem, we examine a network consisted of three (3) depots, eighteen (18) customers and five (5) vehicles. Three (3) of the vehicles are located at the depot near Katerini and the other two (2) are at the depot near Larisa. However, each vehicle has the ability to return to other depot after transferring all products to customers.

As we can see at the Table 3, only two (2) vehicles are selected to service the customers. Furthermore, vehicle 2 relocates to minimize the consumption and optimize the Problem's solution.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
00001	0	0	6	89				
		6	8	73				
		8	9	51				
		9	12	34				
		12	5	17				
		5	0	0				
	2	0	11	297				
		11	7	265				
		7	4	246				
		4	10	228				
		10	3	204				
		3	15	173				
		15	19	155				
		19	16	122				

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		16	20	108				
		20	13	89				
		13	14	74				
		14	17	60				
		17	18	34				
		18	1	0				

Table 3 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Vehicle Relocation VRP.

4.5.3 Multi Depot Capacitated Heterogeneous Fleet Specific Network Characteristics VRP (MDCHFSNCVRP)

Another instance that is examined to confirm the functionality of the algorithm is Multi Depot Capacitated Heterogeneous Fleet Specific Network Characteristics VRP Problem. This case differs from the others because it includes some characteristics of vehicles which are the existence of refrigerator and the size of vehicles. At this time the network is consisted of three (3) depots, eighteen (18) customers and four (4) vehicles. Customers can demand to be delivered with refrigerated products or with products which do not require refrigeration during their transfer.

The table below shows that four (4) vehicles are used to cover total demand. Furthermore it presents the routes of all vehicles and load evolution.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
00010	0	0	11	115				
		11	7	83				
		7	4	64				
		4	10	46				
		10	8	22				
		8	0	0				
	1	0	6	98				
		6	5	82				
		5	9	65				
		9	3	48				

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		3	12	17				
		12	0	0				
	2	1	18	88				
		18	17	54				
		17	14	28				
		14	16	14				
		16	1	0				
	3	1	19	85				
		19	15	52				
		15	20	34				
		20	13	15				
		13	1	0				

Table 4 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Specific Network Characteristics VRP.

4.5.4 Multi Depot Capacitated Heterogeneous Fleet Specific Network Characteristics Vehicle Relocation VRP (MDCHFSNCVRVRP)

The case of Multi Depot Capacitated Heterogeneous Fleet Specific Network Characteristics Vehicle Relocation VRP Problem combines the basic model with two other features, vehicle relocation and additional vehicle characteristics. At this instance, there are three (3) depots, eighteen (18) customers and five (5) vehicles.

The following table shows that vehicles 1, 2 and 3 are chosen to transfer the products. Also we can see that Vehicle 2 starts from depot 0 and finishes at depot 1.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
00011	1	0	6	98				
		6	5	82				
		5	9	65				
		9	3	48				
		3	12	17				
		12	0	0				

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
	2	0	11	200				
		11	7	168				
		7	4	149				
		4	8	131				
		8	10	109				
		10	15	85				
		15	19	67				
		19	20	34				
		20	13	15				
		13	1	0				
	3	1	18	88				
		18	17	54				
		17	14	28				
		14	16	14				
		16	1	0				

Table 5 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Specific Network Characteristics Vehicle Relocation VRP.

4.5.5 Multi Depot Capacitated Heterogeneous Fleet Split Satisfaction of Demand VRP (MDCHFSDVRP)

Another interesting case of VRP is Multi Depot Capacitated Heterogeneous Fleet Split Satisfaction of Demand VRP Problem. At this case we examine a network of three (3) depots, thirty two (32) customers and four (4) vehicles. The demand of products of customer 11 is high and equal to the capacity of the biggest vehicle.

The Table 6 below shows that both vehicles 0 and 3 servicing customer 11 in order to meet his demands. Moreover, the table presents the routes of all vehicles, their load (Dijk) and the amount of products transferred to each customer (ydik).

Instance	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
00100	0	0	11	450				
		11	0	0			450	
	1	1	17	315				

Instance	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		17	14	289			26	
		14	16	275			14	
		16	13	261			14	
		13	20	246			15	
		20	15	227			19	
		15	19	209			18	
		19	21	176			33	
		21	24	154			22	
		24	22	129			25	
		22	25	102			27	
		25	23	86			16	
		23	26	58			28	
		26	18	34			24	
		18	1	0			34	
	2	2	30	159				
		30	28	142			17	
		28	27	124			18	
		27	29	101			23	
		29	33	79			22	
		33	32	65			14	
		32	31	47			18	
		31	34	19			28	
		34	2	0			19	
	3	0	6	231				
		6	5	215			16	
		5	12	198			17	
		12	4	181			17	
		4	9	163			18	
		9	8	146			17	
		8	10	124			22	
		10	3	100			24	
		3	7	69			31	
		7	11	50			19	
		11	0	0			50	

Table 6 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Split Satisfaction of Demand VRP.

4.5.6 Multi Depot Capacitated Heterogeneous Fleet Split Satisfaction of Demand Vehicle Relocation VRP (MDCHFSDVRP)

Multi Depot Capacitated Heterogeneous Fleet Split Satisfaction of Demand Vehicle Relocation VRP is one more case of VRP problem which can be optimized using the algorithm. The example which was used is composed of three (3) depots, eighteen (18) customers and five vehicles (5) vehicles.

The following table presents the results after simulating the VRP model. Vehicle 2 is the only vehicle that changes depot as it starts from depot 0 and returns to depot 1. The minimization of vehicles' consumption is achieved using three (3) of the five(5) available vehicles.

Instance	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
00101	0	0	11	447				
		11	0	0			447	
	1	0	6	107				
		6	8	91			16	
		8	9	69			22	
		9	4	52			17	
		4	12	34			18	
		12	5	17			17	
		5	0	0			17	
	2	0	11	300				
		11	7	247			53	
		7	10	228			19	
		10	3	204			24	
		3	15	173			31	
		15	19	155			18	
		19	16	122			33	
		16	20	108			14	
		20	13	89			19	
		13	14	74			15	
		14	17	60			14	

Instance	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		17	18	34			26	
		18	1	0			34	

Table 7 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Split Satisfaction of Demand Vehicle Relocation VRP.

4.5.7 Multi Depot Capacitated Heterogeneous Fleet Split Satisfaction of Demand Specific Network Characteristics VRP (MDCHFSDSNCVRP)

Multi Depot Capacitated Heterogeneous Fleet Split Satisfaction of Demand Specific Network Characteristics VRP is also examined and results are summarized at the following table. At this case, there are (3) depots, eighteen (18) customers and five vehicles (5) vehicles. Customer 11 requires a big amount of products and is visted by vehicles 0 and 2.

Instance	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
00110	0	0	11	450				
		11	0	0			450	
	1	0	6	98				
		6	5	82			16	
		5	9	65			17	
		9	3	48			17	
		3	12	17			31	
		12	0	0			17	
	2	0	8	133				
		8	10	111			22	
		10	4	87			24	
		4	7	69			18	
		7	11	50			19	
		11	0	0			50	
	3	1	16	88				
		16	14	74			14	
		14	17	60			14	
		17	18	34			26	
		18	1	0			34	

Instance	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
	4	1	13	85				
		13	20	70			15	
		20	15	51			19	
		15	19	33			18	
		19	1	0			33	

Table 8 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Split Satisfaction of Demand Specific Network Characteristics VRP.

4.5.8 Multi Depot Capacitated Heterogeneous Fleet Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP (MDCHFSDSNCVRVRP)

Next case which is examined is Multi Depot Capacitated Heterogeneous Fleet Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP which combines the basic model with the model created for partially satisfying customers' demand and adding specific vehicles' characteristics.

The table below states the routes which optimize total consumption of chosen vehicles.

Instance	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
00111	0	0	11	368			368	
		11	0	0				
	1	0	12	98				
		12	3	81			17	
		3	9	50			31	
		9	5	33			17	
		5	6	16			17	
		6	0	0			16	
	2	0	11	300				
		11	7	168			132	
		7	4	149			19	
		4	8	131			18	

Instance	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		8	10	109			22	
		10	15	85			24	
		15	19	67			18	
		19	20	34			33	
		20	13	15			19	
		13	1	0			15	
	3	1	18	88				
		18	17	54			34	
		17	14	28			26	
		14	16	14			14	
		16	1	0			14	

Table 9 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP.

4.5.9 Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery VRP (MDCHFPDVRP)

At the case of Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery VRP Problem vehicles both unload and load products to meet customers' demands. The total amount of products which are transferred between two nodes is the sum of decision variables Dijk and Pijk. In this case the network has (3) depots, thirty two (32) customers and three (3) vehicles.

Table 10 depicts the routes of vehicles and the load that has to be delivered and have been picked up at every edge of network.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
01000	0	0	11	213	0			
		11	7	181	23			
		7	3	162	31			
		3	10	131	46			
		10	8	107	54			
		8	9	85	70			
		9	4	68	90			
		4	12	50	112			

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		12	5	33	133			
		5	6	16	142			
		6	0	0	159			
	1	1	18	315	0			
		18	26	281	21			
		26	23	257	27			
		23	25	229	49			
		25	22	213	70			
		22	24	186	86			
		24	21	161	103			
		21	19	139	128			
		19	15	106	139			
		15	20	88	152			
		20	13	69	158			
		13	16	54	177			
		16	14	40	195			
		14	17	26	216			
		17	1	0	231			
	2	2	34	159	0			
		34	31	140	22			
		31	32	112	39			
		32	33	94	55			
		33	29	80	62			
		29	27	58	88			
		27	28	35	99			
		28	30	17	108			
		30	2	0	125			

Table 10 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery VRP

4.5.10 Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Vehicle Relocation VRP (MDCHFPDVRVRP)

The Problem of Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Vehicle Relocation VRP differs from case 4.5.9 only because vehicles have the ability to relocate during their routes. The number of depots, customers and vehicles remain the same. The optimized solution does not have difference in the objective value as relocation is not used.

The results are summarized at the Table 11. It is remarked that the direction of routes at this case is opposite comparing to the case 4.5.9.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
01001	0	0	6	213	0			
		6	5	197	17			
		5	12	180	26			
		12	4	163	47			
		4	9	145	69			
		9	8	128	89			
		8	10	106	105			
		10	3	82	113			
		3	7	51	128			
		7	11	32	136			
		11	0	0	159			
	1	1	17	315	0			
		17	14	289	15			
		14	16	275	36			
		16	13	261	54			
		13	20	246	73			
		20	15	227	79			
		15	19	209	92			
		19	21	173	103			
		21	24	154	128			
		24	22	129	145			
		22	25	102	161			
		25	23	86	182			
		23	26	58	204			
		26	18	34	210			
		18	1	0	231			
	2	2	30	159	0			
		30	28	142	17			
		28	27	124	26			
		27	29	101	37			
		29	33	79	63			

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		33	32	65	70			
		32	31	47	86			
		31	34	19	103			
		34	2	0	125			

Table 11 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Vehicle Relocation VRP.

4.5.11 Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Specific Network Characteristics VRP (MDCHFPDSNCVRP)

The Table 12 shows the solution of VRP Problem which has Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Specific Network Characteristics. The size of the problem examined is three (3) depots, eighteen (18) customers and four (4) vehicles. All the vehicles are used to cover demands of customers.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
01010	0	0	8	115	0			
		8	10	93	16			
		10	4	69	24			
		4	7	51	46			
		7	11	32	54			
		11	0	0	69			
	1	0	12	98	0			
		12	3	81	21			
		3	9	50	36			
		9	5	33	56			
		5	6	16	65			
		6	0	0	82			
	2	1	18	88	0			
		18	17	54	21			
		17	14	28	36			
		14	16	14	57			
		16	1	0	75			

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
	3	1	13	85	0			
		13	20	70	19			
		20	15	51	25			
		15	19	33	38			
		19	1	0	49			

Table 12 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Specific Network Characteristics VRP.

4.5.12 Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Specific Network Characteristics Vehicle Relocation VRP (MDCHFPDSNCVRVRP)

Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Specific Network Characteristics Vehicle Relocation VRP Problem is examined using a network of three (3) depots, eighteen (18) customers and five (5) vehicles. Only three (3) vehicles are chosen to visit all customers. The evolution of their load at every edge is illustrated at Table 13.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
01011	1	0	6	98	0			
		6	5	82	17			
		5	9	65	26			
		9	3	48	46			
		3	12	17	61			
		12	0	0	82			
	2	0	11	200	0			
		11	7	168	15			
		7	4	149	23			
		4	8	131	45			
		8	10	109	61			
		10	15	85	69			
		15	19	67	82			
		19	20	34	93			
		20	13	15	99			

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		13	1	0	118			
	3	1	16	88	0			
		16	14	74	18			
		14	17	60	39			
		17	18	34	54			
		18	1	0	75			

Table 13 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Specific Network Characteristics Vehicle Relocation VRP.

4.5.13 Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Split Satisfaction of Demand VRP (MDCHFPDSDVRP)

Another case which is studied is Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Split Satisfaction of Demand VRP Problem at a network of three (3) depots, eighteen (18) customers and four (4) vehicles.

Whereas Customer 3 demands from vehicles to pick up 480 kg of products, Customer 11 demands to be delivered with products whose weight is 500 kg. Table 14 shows that both customers 3 and 11 are visited by two vehicles, 0 and 3.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
01100	0	0	11	331	0			
		11	3	38	0		293	0
		3	8	38	246		0	246
		8	6	16	262		22	16
		6	0	0	279		16	17
	1	1	19	173	0			
		19	15	140	11		33	11
		15	20	122	24		18	13
		20	13	103	30		19	6
		13	16	88	49		15	19
		16	14	74	67		14	18
		14	17	60	88		14	21

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		17	18	34	103		26	15
		18	1	0	124		34	21
	3	0	11	350	0			
		11	7	143	15		207	15
		7	3	124	23		19	8
		3	10	93	257		31	234
		10	9	69	265		24	8
		9	4	52	285		17	20
		4	12	34	307		18	22
		12	5	17	328		17	21
		5	0	0	337		17	9

Table 14 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Split Satisfaction of Demand VRP.

4.5.14 Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Split Satisfaction of Demand Vehicle Relocation VRP (MDCHFPDSDVRVRP)

Another case that is examined is Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Split Satisfaction of Demand Vehicle Relocation VRP Problem. At this case Customer 3 demands vehicles to pick up from his location 480 kg of products and Customer 11 demands to be transferred with 500 kg products. The table below summarizes the results and it is clear that vehicles 0 and 2 transfer products to satisfy customer 3 and vehicles 0 and 1 transfer products to satisfy customer 11. Furthermore, vehicle 2 starts from depot 0 and returns to depot 1.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
01101	0	0	11	450	0			
		11	3	70	0		380	0
		3	8	39	411		31	411
		8	12	17	427		22	16
		12	0	0	448		17	21

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
	1	0	11	153	0			
		11	5	33	15		120	15
		5	6	16	24		17	9
		6	0	0	41		16	17
	2	0	7	251	0			
		7	4	232	8		19	8
		4	9	214	30		18	22
		9	10	197	50		17	20
		10	3	173	58		24	8
		3	15	173	127		0	69
		15	19	155	140		18	13
		19	16	122	151		33	11
		16	20	108	169		14	18
		20	13	89	175		19	6
		13	14	74	194		15	19
		14	17	60	215		14	21
		17	18	34	230		26	15
		18	1	0	251		34	21

Table 15 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Split Satisfaction of Demand Vehicle Relocation VRP.

4.5.15 Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Split Satisfaction of Demand Specific Network Characteristics VRP (MDCHFPDSDSNCVRP)

Algorithm is used to simulate another instance of VRP Problem, Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Split Satisfaction of Demand Specific Network Characteristics VRP.

At this instance Customer 3 is partially serviced by refrigerated vehicles 1 and 3 and Customer 11 is partially serviced by non-refrigerated vehicles 0 and 2. The results in total are depicted at the following table.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
01110	0	0	11	283	0			
		11	0	0	0			
	1	0	12	98	0			
		12	3	81	21		17	21
		3	9	50	297		31	276
		9	5	33	317		17	20
		5	6	16	326		17	9
		6	0	0	343		16	17
	2	0	11	300	0			
		11	7	83	15		217	15
		7	4	64	23		19	8
		4	10	46	45		18	22
		10	8	22	53		24	8
		8	0	0	69		22	16
	3	1	18	88	0			
		18	17	54	21		34	21
		17	14	28	36		26	15
		14	16	14	57		14	21
		16	3	0	75		14	18
		3	1	0	279		0	204
	4	1	13	85	0			
		13	20	70	19		15	19
		20	15	51	25		19	6
		15	19	33	38		18	13
		19	1	0	49		33	11

Table 16 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Split Satisfaction of Demand Specific Network Characteristics VRP.

4.5.16 Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP (MDCHFPDSDSNCVRVRP)

The case Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP Problem differs from the case 4.5.15 due to vehicles' ability to relocate. The number of depots, customers and vehicles available remain the same as the previous case.

At the Table 17 we can see the results after optimizing the problem using the algorithm. We can observe that vehicles 2 and 3 relocate after visiting the customers and this contributes to minimization of total consumption.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
01111	0	0	11	368	0			
		11	0	0	15		368	15
	1	0	5	81	0			
		5	9	64	9		17	9
		9	3	47	29		17	20
		3	6	16	333		31	304
		6	0	0	350		16	17
	2	0	11	300	0			
		11	7	168	0		132	0
		7	4	149	8		19	8
		4	8	131	30		18	22
		8	10	109	46		22	16
		10	15	85	54		24	8
		15	19	67	67		18	13
		19	20	34	78		33	11
		20	13	15	84		19	6
		13	1	0	103		15	19
	3	1	18	105	0			
		18	17	71	21		34	21
		17	14	45	36		26	15
		14	16	31	57		14	21
		16	3	17	75		14	18

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		3	12	17	251		0	176
		12	0	0	272		17	21

Table 17 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pick-up and Delivery Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP.

4.5.17 Multi Depot Capacitated Heterogeneous Fleet Time Windows VRP (MDCHFTWVRP)

At the case of Multi Depot Capacitated Heterogeneous Fleet Time Windows VRP the algorithm takes into consideration the time restrictions depots and customers impose.

At the Table 18 the results of a network with three (3) depots, thirty two (32) customers and three (3) vehicles are presented. At this table we can see the moments when a vehicle starts from depot and arrives at customers.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
10000	0	0	6	213		8h00		
		6	5	197		9h00		
		5	12	180		9h16		
		12	4	163		9h31		
		4	9	145		9h46		
		9	8	128		10h01		
		8	10	106		10h16		
		10	3	82		10h31		
		3	7	51		10h48		
		7	11	32		11h16		
		11	0	0		11h39		
	1	1	18	315		8h00		
		18	26	281		9h00		
		26	23	257		10h12		
		23	25	229		10h28		
		25	22	213		10h43		
		22	24	186		10h58		
		24	21	161		11h14		

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		21	19	139		11h30		
		19	15	106		12h48		
		15	20	88		13h39		
		20	13	69		15h58		
		13	16	54		16h15		
		16	14	40		16h30		
		14	17	26		16h45		
		17	1	0		17h00		
	2	2	34	159		8h00		
		34	31	140		9h59		
		31	32	112		10h14		
		32	33	94		10h29		
		33	29	80		10h44		
		29	27	58		11h26		
		27	28	35		11h42		
		28	30	17		11h59		
		30	2	0		12h17		

Table 18 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows VRP.

4.5.18 Multi Depot Capacitated Heterogeneous Fleet Time Windows Vehicle Relocation VRP (MDCHFTWVRVRP)

The case of Multi Depot Capacitated Heterogeneous Fleet Time Windows Vehicle Relocation VRP Problem is also examined. The data used are consisted of three (3) depots, eighteen (18) customers and five (5) vehicles. Only two (2) vehicles are chosen and vehicle 2 starts from depot 0 and returns to depot 1 to optimize problem's solution as stated at Table 19.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
10001	0	0	5	89		8h00		
		5	12	72		12h00		
		12	9	55		16h13		
		9	8	38		16h28		
		8	6	16		16h43		
		6	0	0		17h00		

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
	2	0	11	297		8h00		
		11	7	265		9h00		
		7	4	246		9h23		
		4	10	228		9h47		
		10	3	204		10h02		
		3	15	173		10h19		
		15	19	155		11h27		
		19	16	122		12h18		
		16	20	108		12h39		
		20	13	89		12h56		
		13	14	74		13h13		
		14	17	60		13h28		
		17	18	34		13h43		
		18	1	0		17h00		

Table 19 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Vehicle Relocation VRP.

4.5.19 Multi Depot Capacitated Heterogeneous Fleet Time Windows Specific Network Characteristics VRP (MDCHFTWSNCVRP)

One other VRP category that simulates real life Problems is Multi Depot Capacitated Heterogeneous Fleet Time Windows Specific Network Characteristics VRP. At the case examined, there are three (3) depots, eighteen (18) customers and four (4) vehicles.

Table 20 which follows shows the optimized routes of all vehicles and the moments vehicles visit each customer’s node. Also we can see the time each vehicle start from its depot.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
10010	0	0	8	115		8h00		
		8	10	93		9h00		
		10	4	69		9h15		
		4	7	51		9h30		
		7	11	32		9h54		
		11	0	0		10h17		

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
10010	1	0	6	98		8h00		
		6	5	82		9h00		
		5	9	65		9h16		
		9	3	48		9h31		
		3	12	17		9h49		
		12	0	0		10h08		
10010	2	1	16	88		8h00		
		16	14	74		9h00		
		14	17	60		9h15		
		17	18	34		9h30		
		18	1	0		9h52		
10010	3	1	13	85		8h00		
		13	20	70		9h00		
		20	15	51		9h17		
		15	19	33		10h09		
		19	1	0		11h00		

Table 20 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Specific Network Characteristics VRP.

4.5.20 Multi Depot Capacitated Heterogeneous Fleet Time Windows Specific Network Characteristics Vehicle Relocation VRP (MDCHFTWSNCVRVRP)

At the instance of Multi Depot Capacitated Heterogeneous Fleet Time Windows Specific Network Characteristics Vehicle Relocation VRP Problem a network of three (3) depots, eighteen (18) customers and five (5) vehicles is examined.

Table 21 presents the routes of vehicles with the corresponding moments of visiting each customer. It is observed that vehicle 2 relocates to optimize total consumption of vehicles.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
10011	1	0	6	98		8h00		
		6	5	82		9h00		

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		5	9	65		9h16		
		9	3	48		9h31		
		3	12	17		9h49		
		12	0	0		17h00		
	2	0	11	200		8h00		
		11	7	168		9h00		
		7	4	149		9h23		
		4	8	131		9h47		
		8	10	109		10h02		
		10	15	85		10h17		
		15	19	67		11h26		
		19	20	34		12h17		
		20	13	15		12h41		
		13	1	0		17h00		
	3	1	16	88		8h00		
		16	14	74		9h00		
		14	17	60		9h15		
		17	18	34		9h30		
		18	1	0		9h52		

Table 21 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Specific Network Characteristics Vehicle Relocation VRP.

4.5.21 Multi Depot Capacitated Heterogeneous Fleet Time Windows Split Satisfaction of Demand VRP (MDCHFTWSDVRP)

At the case of Multi Depot Capacitated Heterogeneous Fleet Time Windows Split Satisfaction of Demand VRP Problem more than one vehicle can visit a customer and the problem takes into consideration time restrictions customers and depots impose. At the example examined there are three (3) depots, twenty four (24) customers and four (4) vehicles.

As it can be seen at the Table 22 vehicle 0 and vehicle 3 both visit Customer 11 to meet his demands. The Table presents also the optimized routes of all vehicles.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
10100	0	0	11	450		8h00		
		11	0	0		9h00	450	
	1	1	17	315		8h00		
		17	14	289		9h00	26	
		14	16	275		9h15	14	
		16	13	261		9h30	14	
		13	20	246		9h45	15	
		20	15	227		10h02	19	
		15	19	209		10h54	18	
		19	21	176		11h45	33	
		21	24	154		13h03	22	
		24	22	129		13h19	25	
		22	25	102		13h35	27	
		25	23	86		13h50	16	
		23	26	58		14h05	28	
		26	18	34		14h21	24	
		18	1	0		15h33	34	
	3	0	6	231		8h00		
		6	5	215		9h00	16	
		5	12	198		9h16	17	
		12	4	181		9h31	17	
		4	9	163		9h46	18	
		9	8	146		10h01	17	
		8	10	124		11h00	22	
		10	3	100		11h15	24	
		3	7	69		11h32	31	
		7	11	50		12h00	19	
		11	0	0		12h23	50	

Table 22 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Split Satisfaction of Demand VRP.

4.5.22 Multi Depot Capacitated Heterogeneous Fleet Time Windows Split Satisfaction of Demand Vehicle Relocation VRP (MDCHFTWSDVRVRP)

Multi Depot Capacitated Heterogeneous Fleet Time Windows Split Satisfaction of Demand Vehicle Relocation VRP is similar to the previous case 4.5.21. At this time vehicles can start and finish at a different depot.

The table below presents the results after running the algorithm for a case with (3) depots, eighteen (18) customers and five (5) vehicles.. Customer 11 is visited by vehicles 0 and 2. Only three (3) of five (5) available vehicles are selected. Vehicle 2 starts from depot 0 and returns to depot 1.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
10101	0	0	11	450		8h00		
		11	0	0		9h00	450	
10101	1	0	6	107		8h00		
		6	8	91		9h00	16	
		8	9	69		9h17	22	
		9	4	52		11h15	17	
		4	12	34		11h30	18	
		12	5	17		11h45	17	
		5	0	0		12h00	17	
10101	2	0	11	297		8h00		
		11	7	247		9h00	50	
		7	10	228		9h23	19	
		10	3	204		9h48	24	
		3	15	173		10h05	31	
		15	19	155		11h13	18	
		19	16	122		12h04	33	
		16	20	108		12h25	14	
		20	13	89		12h42	19	
		13	14	74		12h59	15	
		14	17	60		13h14	14	
		17	18	34		13h29	26	
		18	1	0		17h00	34	

Table 23 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Split Satisfaction of Demand Vehicle Relocation VRP.

4.5.23 Multi Depot Capacitated Heterogeneous Fleet Time Windows Split Satisfaction of Demand Specific Network Characteristics VRP (MDCHFTWSDSNCVRP)

At the case of Multi Depot Capacitated Heterogeneous Fleet Time Windows Split Satisfaction of Demand Specific Network Characteristics VRP Problem, a network of three (3) depots, eighteen (18) customers and five (5) available vehicles is studied.

The results are summarized at Table 24. Customer 11 is serviced by vehicles 0 and 2 as both of them do not have refrigeration. In this case, all available vehicles are used as there are specific network characteristics.

Routes of all vehicles and arrival times to customers are presented below.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
10110	0	0	11	283		8h00		
		11	0	0		9h00	283	
	1	0	6	98		8h00		
		6	5	82		9h00	16	
		5	9	65		9h16	17	
		9	3	48		9h31	17	
		3	12	17		9h49	31	
		12	0	0		10h08	17	
	2	0	11	300		8h00		
		11	7	83		9h00	217	
		7	4	64		9h23	19	
		4	10	46		9h47	18	
		10	8	22		10h02	24	
		8	0	0		10h17	22	
	3	1	16	88		8h00		
		16	14	74		9h00	14	
		14	17	60		9h15	14	
		17	18	34		9h30	26	
		18	1	0		9h52	34	
	4	1	19	85		8h00		

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		19	15	52		9h00	33	
		15	20	34		9h51	18	
		20	13	15		10h43	19	
		13	1	0		11h00	15	

Table 24 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Split Satisfaction of Demand Specific Network Characteristics VRP.

4.5.24 Multi Depot Capacitated Heterogeneous Fleet Time Windows Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP (MDCHFTWSDSNCVRVRP)

Multi Depot Capacitated Heterogeneous Fleet Time Windows Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP Problem is also used to test algorithm functionality.

This case differs from case 4.23 only because vehicles are able to change location after servicing customers. The Optimization results show that at this case one (1) less vehicle is used and total demand is covered. Also it can be noticed that vehicle 2 relocates and Customer 11 is visited by vehicles 0 and 2. Both these vehicles transfer non refrigerated products to this customer. Table 25 below depicts the aforementioned results.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
10111	0	0	11	368		8h00		
		11	0	0		9h00	368	
	1	0	6	98		8h00		
		6	5	82		9h00	16	
		5	9	65		9h16	17	
		9	3	48		9h31	17	
		3	12	17		9h49	31	
		12	0	0		10h08	17	
	2	0	11	300		8h00		
		11	7	168		9h00	132	
		7	4	149		9h23	19	

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		4	8	131		9h47	18	
		8	10	109		10h02	22	
		10	15	85		10h17	24	
		15	19	67		11h26	18	
		19	20	34		12h17	33	
		20	13	15		12h41	19	
		13	1	0		12h58	15	
	3	1	18	88		8h00		
		18	17	54		9h00	34	
		17	14	28		9h22	26	
		14	16	14		9h37	14	
		16	1	0		17h00	14	

Table 25 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP.

4.5.25 Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery VRP (MDCHFTWPDVRP)

At the case of Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery VRP Problem a network of three (3) depots, thirty two (32) customers and three (3) vehicles is examined.

At Table 26 the routes of the three vehicles, the load to be delivered, the picked up load and time of arrivals are presented. It is noted that all vehicles are used to optimize the total consumption.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
11000	0	0	11	213	0	8h00		
		11	7	181	23	9h00		
		7	3	162	31	9h23		
		3	10	131	46	9h51		
		10	8	107	54	10h08		
		8	9	85	70	10h23		
		9	4	68	90	10h38		

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		4	12	50	112	10h53		
		12	5	33	133	11h08		
		5	6	16	142	11h23		
		6	0	0	159	11h39		
	1	1	18	315	0	8h00		
		18	26	281	21	9h00		
		26	23	257	27	10h12		
		23	25	229	49	10h28		
		25	22	213	70	10h43		
		22	24	186	86	10h58		
		24	21	161	103	11h14		
		21	19	139	128	11h30		
		19	15	106	139	12h48		
		15	20	88	152	13h38		
		20	13	69	158	14h30		
		13	16	54	177	14h47		
		16	14	40	195	15h02		
		14	17	26	216	15h17		
		17	1	0	231	15h32		
	2	2	34	159	0	8h00		
		34	31	140	22	14h41		
		31	32	112	39	14h56		
		32	33	94	55	15h12		
		33	29	80	62	15h27		
		29	27	58	88	16h09		
		27	28	35	99	16h25		
		28	30	17	108	16h42		
		30	2	0	125	17h00		

Table 26 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery VRP.

4.5.26 Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Vehicle Relocation VRP (MDCHFTWPDVRVRP)

One other case the algorithm can optimize is Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Vehicle Relocation VRP Problems.

At our instance there is a network of (3) depots, eighteen (18) customers and five (5) vehicles.

The results which are summarized at Table 27 show that only two (2) vehicles are chosen. At this table there is information regarding routes, load and arrival moments of vehicles.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
11001	0	0	5	89	0	8h00		
		5	12	72	9	12h00		
		12	9	55	30	16h13		
		9	8	38	50	16h28		
		8	6	16	66	16h43		
		6	0	0	83	17h00		
	2	0	11	297	0	8h00		
		11	7	265	15	9h00		
		7	4	246	23	9h23		
		4	10	228	45	9h47		
		10	3	204	53	10h02		
		3	15	173	68	10h19		
		15	19	155	81	14h22		
		19	16	122	92	15h13		
		16	20	108	110	15h34		
		20	13	89	116	15h51		
		13	14	74	135	16h08		
		14	17	60	156	16h23		
		17	18	34	171	16h38		
		18	1	0	192	17h00		

Table 27 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Vehicle Relocation VRP.

4.5.27 Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Specific Network Characteristics VRP (MDCHFTWPDSNCVRP)

The case of VRP Problem named Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Specific Network Characteristics VRP is also studied. The network examined is consisted of (3) depots, eighteen (18) customers and four (4) vehicles.

Focusing on the results which are shown at Table 28, it is noted that all four (4) available vehicles are used.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
11010	0	0	8	115	0	8h00		
		8	10	93	16	9h00		
		10	4	69	24	9h15		
		4	7	51	46	9h30		
		7	11	32	54	9h54		
		11	0	0	69	10h17		
	1	0	6	98	0	8h00		
		6	5	82	17	9h00		
		5	9	65	26	9h16		
		9	3	48	46	9h31		
		3	12	17	61	9h49		
		12	0	0	82	10h08		
	2	1	16	88	0	8h00		
		16	14	74	18	9h00		
		14	17	60	39	9h15		
		17	18	34	54	9h30		
		18	1	0	75	9h52		
	3	1	13	85	0	8h00		
		13	20	70	19	9h00		
		20	15	51	25	9h17		
		15	19	33	38	16h09		

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		19	1	0	49	17h00		

Table 28 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Specific Network Characteristics VRP.

4.5.28 Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Specific Network Characteristics Vehicle Relocation VRP (MDCHFTWPDSNCVRVRP)

In order to test Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Specific Network Characteristics Vehicle Relocation VRP Problem a network consisting of (3) depots, eighteen (18) customers and five (5) vehicles is simulated.

The following table shows that vehicle 1 visits five (5) customers (12, 3, 9, 5, 6), vehicle 2 visits nine (9) customers (11, 7, 4, 8, 10, 15, 19, 20, 13) and relocates and vehicle 3 visits the rest four (4) customers. More detailed information is presented at the aforementioned table.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
11011	1	0	12	98	0	8h00		
		12	3	81	21	9h00		
		3	9	50	36	9h19		
		9	5	33	56	9h37		
		5	6	16	65	12h00		
		6	0	0	82	12h16		
	2	0	11	200	0	8h00		
		11	7	168	15	9h00		
		7	4	149	23	9h23		
		4	8	131	45	9h47		
		8	10	109	61	10h02		
		10	15	85	69	10h17		
		15	19	67	82	11h26		
		19	20	34	93	12h17		
		20	13	15	99	12h41		

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		13	1	0	118	12h58		
	3	1	18	88	0	8h00		
		18	17	54	21	9h00		
		17	14	28	36	9h22		
		14	16	14	57	9h37		
		16	1	0	75	9h52		

Table 29 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Specific Network Characteristics Vehicle Relocation VRP.

4.5.29 Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Split Satisfaction of Demand VRP (MDCHFTWPDSDVRP)

Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Split Satisfaction of Demand VRP is also tested using the algorithm. The case studied is consisted of (3) depots, eighteen (18) customers and four (4) vehicles. Customer 3 demands to return to depot amount of products equal to 480 Kg. Customer 11 demands 500 kg of products to be delivered to his location.

Solution results show that Customers 3 and 11 are visited by vehicles 0 and 3 to satisfy their demands.

Table 30 presents results and values of decision variables in detail.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
11100	0	0	11	450	0	8h00		
		11	3	69	0	9h00	381	0
		3	8	38	246	9h21	31	246
		8	6	16	262	16h43	22	16
		6	0	0	279	17h00	16	17
	1	1	19	173	0	8h00		
		19	15	140	11	9h00	33	11
		15	20	122	24	9h51	18	13
		20	13	103	30	10h43	19	6

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		13	16	88	49	11h00	15	19
		16	14	74	67	11h14	14	18
		14	17	60	88	11h30	14	21
		17	18	34	103	11h45	26	15
		18	1	0	124	12h07	34	21
	3	0	11	231	0	8h00		
		11	7	112	15	9h51	119	15
		7	3	93	23	10h14	19	8
		3	10	93	257	10h42	0	234
		10	9	69	265	10h59	24	8
		9	4	52	285	11h14	17	20
		4	12	34	307	11h29	18	22
		12	5	17	328	11h44	17	21
		5	0	0	337	12h00	17	9

Table 30 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Split Satisfaction of Demand VRP.

4.5.30 Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Split Satisfaction of Demand Vehicle Relocation VRP (MDCHFTWPDSDVRVRP)

Another case of a VRP Problem is Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Split Satisfaction of Demand Vehicle Relocation VRP.

The example of this case that is examined is consisted of (3) depots, eighteen (18) customers and five (5) vehicles.

The optimized results show that only vehicle 2 relocates as it starts from depot 0 and returns to depot 1. Furthermore, whereas Customer 11 is visited by vehicles 0 and 1, Customer 3 is visited by vehicles 0 and 2.

Table 31 which follows summarizes the results.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
11101	0	0	11	450	0	8h00		
		11	3	70	0	9h00	380	0
		3	8	39	411	9h21	31	411
		8	12	17	427	9h39	22	16
		12	0	0	448	17h00	17	21
	1	0	11	153	0	8h00		
		11	5	33	15	9h00	120	15
		5	6	16	24	12h00	17	9
		6	0	0	41	17h00	16	17
	2	0	7	251	0	8h00		
		7	4	232	8	9h00	19	8
		4	9	214	30	9h24	18	22
		9	10	197	50	12h42	17	20
		10	3	173	58	12h57	24	8
		3	15	173	127	13h14	0	69
		15	19	155	140	14h22	18	13
		19	16	122	151	15h13	33	11
		16	20	108	169	15h34	14	18
		20	13	89	175	15h51	19	6
		13	14	74	194	16h08	15	19
		14	17	60	215	16h23	14	21
		17	18	34	230	16h38	26	15
		18	1	0	251	17h00	34	21

Table 31 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Split Satisfaction of Demand Vehicle Relocation VRP.

4.5.31 Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Split Satisfaction of Demand Specific Network Characteristics VRP (MDCHFTWPDSDSNCVRP)

At the case of Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Split Satisfaction of Demand Specific Network

Characteristics VRP Problem, there are (3) depots, eighteen (18) customers and five (5) vehicles.

Table 32 shows the values of all decision variables of the problem. Also it shows the chosen routes, the load, arrival times and products delivered and picked up. As it can be seen both five (5) vehicles are used to optimize the total consumption. Also customers 4 and 11 are visited by vehicles 0 and 2 which are non refrigerated vehicles. All other customers are visited only once.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
11110	0	0	11	450	0	8h00		
		11	4	0	0	9h00	450	0
		4	0	0	273	17h00	0	273
	1	0	12	98	0	8h00		
		12	3	81	21	9h12	17	21
		3	9	50	36	9h31	31	15
		9	5	33	56	9h49	17	20
		5	6	16	65	10h04	17	9
		6	0	0	82	10h20	16	17
	2	0	8	133	0	8h00		
		8	10	111	16	11h06	22	16
		10	4	87	24	11h21	24	8
		4	7	69	231	11h36	18	207
		7	11	50	239	12h00	19	8
		11	0	0	254	12h23	50	15
	3	1	18	88	0	8h00		
		18	17	54	21	9h00	34	21
		17	14	28	36	9h22	26	15
		14	16	14	57	9h37	14	21
		16	1	0	75	9h52	14	18
	4	1	19	85	0	8h00		
		19	15	52	11	9h00	33	11
		15	20	34	24	9h51	18	13
		20	13	15	30	16h43	19	6

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		13	1	0	49	17h00	15	19

Table 32 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Split Satisfaction of Demand Specific Network Characteristics VRP.

4.5.32 Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP (MDCHFTWPDSDSNCVRVRP)

The last case studied is Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP Problem. In our case there are (3) depots, eighteen (18) customers and five (5) vehicles.

Table 33 presents the optimized routes, load, arrival times and products delivered and picked up. It is noted that four (4) of the five (5) vehicles are used. Furthermore, vehicles 2 and 3 relocate as vehicle 2 starts from depot 0 and returns to depot 1 and vehicle 3 starts from depot 1 and returns to depot 0. Vehicles 0 and 2 transfer non refrigerated products and vehicles 1 and 3 transfer refrigerated products to satisfy customers' demands. Customer 11 is visited by two vehicles 0 and 2 in order to be delivered all products. The same happens for customer 3 as both vehicles 1 and 3 visit his location to cover his increased pick up demand.

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
11111	0	0	11	368	0	8h00		
		11	0	0	15	9h00	368	15
	1	0	6	50	0	8h00		
		6	3	34	17	9h00	16	17
		3	9	34	313	9h20	0	296
		9	5	17	333	9h38	17	20
		5	0	0	342	9h53	17	9
	2	0	11	300	0	8h00		

Instance Number	Vehicle	From point i	To point j	Dijk	Pijk	Tik	ydik	ypik
		11	7	168	0	9h00	132	0
		7	4	149	8	12h00	19	8
		4	8	131	30	12h24	18	22
		8	10	109	46	12h39	22	16
		10	15	85	54	12h54	24	8
		15	19	67	67	14h03	18	13
		19	20	34	78	14h54	33	11
		20	13	15	84	15h18	19	6
		13	1	0	103	15h35	15	19
	3	1	18	136	0			
		18	17	102	21	9h00	34	21
		17	14	76	36	9h22	26	15
		14	16	62	57	9h37	14	21
		16	3	48	75	9h52	14	18
		3	12	17	259	11h17	31	184
		12	0	0	280	11h35	17	21

Table 33 Route Selection of each vehicle for Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pick-up and Delivery Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP.

Chapter 5 Summary of Vehicle Routing Problem Mathematical Model and its algorithmic application

In this master thesis, we introduce for the first time a mathematical model and create an algorithm able to solve a wide variety of Vehicle Routing Problems.

This algorithm can be used by companies especially logistics to help them plan the routes of their available vehicles to transfer products to their customers. The aim of the implementation of this algorithm is the minimization of vehicles' movement cost and therefore the reduction of the emissions of carbon dioxide (CO_2) to the environment.

The algorithm has several parameters of real life in order to adapt to the needs of the companies which will decide to use it. It takes into consideration the availability of many depots, time restrictions depots and customers impose, the demand of simultaneous pick-up and delivery of products, the ability of split satisfaction of demand of vehicles, specific characteristics of network such as products' need of refrigeration, location of customers and ability of vehicles to change starting and finishing location.

The proposed algorithm was tested on constructed data sets which represent thirty two (32) cases of Vehicle Routing Problems in order to evaluate its performance. The results show that it can give exact solutions and therefore optimize total travel consumption in an effective way for reasonable company sizes.

In terms of future research directions, the mathematical model of MDCHFTWPDSDSNCVR (Multi Depot, Capacitated, Heterogeneous Fleet, Time Windows, Simultaneous Pick-ups and Delivery, Split satisfaction of pick-up and delivery demand, Specific network characteristics, Vehicle Relocation VRP Problem) can be modified to include the case of the Periodic Vehicle Routing Problem. At the case of Periodic VRP the planning horizon of routing vehicles is extended to several days where customers do not generally require delivery or pick-up every day but rather according to one of a limited number of possible combinations of visit days.

Bibliography

1. Grotschel M, Holland O. Solution of Large-scale Symmetric Traveling Salesman Problems. Mathematical Programming. 1991;51:141-202.

2. R.D. Plante, Lowe TJ, Chandrasekaran R. The Product Matrix Traveling Salesman Problem: An Application and Solution Heuristics. Operations Research. 1987;35:772-83.

3. Bland RE, Shallcross DE. Large traveling salesman problem arising from experiments in X-ray crystallography: a preliminary report on computation. Operations Research Letters. 1989;8(3):125-8.

4. Lenstra JK, Kan AHGR. Some Simple Applications of the Travelling Salesman Problem. Amsterdam: Stichting Mathematisch Centrum; 1974.

5. H.D. Ratliff, Rosenthal AS. Order-Picking in a Rectangular Warehouse: A Solvable Case for the Travelling Salesman Problem. Operations Research. 1983;31:507-21.

6. Dal Rv. Special Cases of the Traveling Salesman Problem. Groningen: Wolters-Noordhoff; 1992.

7. Gorenstein S. Printing press scheduling for multi-edition periodicals. Management Science. 1970;16(6):B373–83.

8. Carter AE, Ragsdale CT. Scheduling pre-printed newspaper advertising inserts using genetic algorithms. Omega. 2002;30:415–21.

9. Angel RD, Caudle WL, Noonan R, Whinston A. Computer assisted school bus scheduling. Management Science. 1972;18:279–88.

10. Svestka JA, Huckfeldt VE. Computational experience with an m-salesman traveling salesman algorithm. Management Science. 1973;19(7):790–9.

11. Lenstra JK, Kan AHGR. Some simple applications of the traveling salesman problem. Operational Research Quarterly. 1975;26:717–33.

12. Zhang T, Gruver WA, Smith MH. Team scheduling by genetic search. Proceedings of the second international conference on intelligent processing and manufacturing of materials1999. p. 839–44.

13. Gilbert KC, Hofstra RB. A new multiperiod multiple traveling salesman problem with heuristic and application to a scheduling problem. Decision Sciences. 1992;23:250–9.

14. Tang L, Liu J, Rong A, Yang Z. A multiple traveling salesman problem model for hot rolling scheduling in Shangai Baoshan Iron & Steel Complex. European Journal of Operational Research. 2000;124:267–82.

15. Brummit B, Stentz A, editors. Dynamic mission planning for multiple mobile robots. Proceedings of the IEEE international conference on robotics and automation; 1996.

16. Brummit B, Stentz A, editors. GRAMMPS: a generalized mission planner for multiple mobile robots. Proceedings of the IEEE international conference on robotics and automation; 1998.

17. Yu Z, Jinhai L, Guochang G, Rubo Z, Haiyan Y, editors. An implementation of evolutionary computation for path planning of cooperative mobile robots. Proceedings of the fourth world congress on intelligent control and automation; 2002.

18. Ryan JL, Bailey TG, Moore JT, Carlton WB, editors. Reactive Tabu search in unmanned aerial reconnaissance simulations. Proceedings of the 1998 wintersimulation conference; 1998.

19. Saleh HA, Chelouah R. The design of the global navigation satellite system surveying networks using genetic algorithms. Engineering Applications of Artificial Intelligence. 2004;17:111–22.

20. Dantzig GB, Ramser JH. The Truck Dispatching Problem Management Science. 1959;6:80-91.

21. Clarke G, Wright JW. Scheduling of Vehicles from a Central Depot to a Number of Delivery Points. Operations Research. 1964 12(4):568 - 81

22. Christofides N, Mingozzi A, Toth P. State-space relaxation procedures for the computation of bounds to routing problems. Networks. 1981;11:145–64.

23. Laporte G, Desrochers M, Nobert Y. Two exact algorithms for the distance-constrained vehicle routing problem. Networks. 1984;14(1):161–72

24. Fukasawa R, Longo H, Lysgaard J, Aragão MPd, Reis M, Uchoa E, et al. Robust Branch-and-Cut-and-Price for the Capacitated Vehicle Routing Problem. Mathematical Programming. 2006;106(3):491–511.

25. Baldacci R, Christofides N, Mingozzi A. An exact algorithm for the vehicle routing problem based on the set partitioning formulation with additional cuts. Mathematical Programming. 2008;115(2):351–85.

26. Andersson H, Hoff A, Christiansen M, Hasle G, Løkketangen A. Industrial aspects and literature survey: Combined inventory management and routing. Computers & Operations Research. 2010;37(9):1515–36.

27. Coelho LC, Cordeau J-F, Laporte G. Thirty years of inventory-routing. Transportation Science. 2014;48(1):1–19.

28. Campbell AM, Clarke LW, Savelsbergh MWP. The Vehicle Routing Problem. Philadelphia: SIAM Monographs on Discrete Mathematics and Applications; 2002.

29. Chiang W-C, Russell RA. Integrating purchasing and routing in a propane gas supply chain. European Journal of Operational Research. 2004;154(3):710–29.

30. Avella P, Boccia M, Sforza A. Solving a fuel delivery problem by heuristic and exact approaches. European Journal of Operational Research. 2004;152(1):170–9.

31. Cornillier F, Boctor FF, Laporte G, Renaud J. An exact algorithm for the petrol station replenishment problem. Journal of the Operational Research Society. 2008;59(5):607–15.

32. Song J-H, Savelsbergh MWP. Performance measurement for inventory routing. Transportation Science. 2007;41(1):44–54.

33. Ng WL, Leung SCH, Lam JKP, Pan SW. Petrol delivery tanker assignment and routing: a case study in Hong Kong. Journal of the Operational Research Society. 2008;59(9):1191–200.

34. Cornillier F, Laporte G, Boctor FF, Renaud J. The petrol station replenishment problem with time windows. Computers & Operations Research. 2009;36(3):919–35.

35. Day JM, Wright PD, Schoenherr T, Venkataramanan M, Gaudette K. Improving routing and scheduling decisions at a distributor of industrial gases. Omega. 2009;37(1):227–37.

36. Prins C. Efficient heuristics for the heterogeneous fleet multitrip VRP with application to a large-scale real case. Journal of Mathematical Modelling and Algorithms. 2002;1(2):135–50.

37. Poot A, Kant G, Wagelmans APM. A savings based method for real-life vehicle routing problems. Journal of the Operational Research Society. 2002;53:57–68.

38. Gaur V, Fisher ML. A periodic inventory routing problem at a supermarket chain. Operations Research. 2004;52(6):813–22.

39. Gendreau M, Iori M, Laporte G, Martello S. A tabu search algorithm for a routing and container loading problem. Transportation Science. 2006;40(3):342–50.

40. Kant G, Jacks M, Aantjes C. Coca-cola enterprises optimizes vehicle routes for efficient product delivery. Interfaces. 2008;38(1):40–50.

41. Belfiore P, Yoshizaki HTY. Scatter search for a real-life heterogeneous fleet vehicle routing problem with time windows and split deliveries in Brazil. European Journal of Operational Research. 2009;199(3):750–8.

42. Chang T-S, Wan Y-W, Ooi WT. A stochastic dynamic traveling salesman problem with hard time windows. European Journal of Operational Research. 2009;198(3):748–59.

43. Wen M, Larsen J, Clausen J, Cordeau J-F, Laporte G. Vehicle routing with crossdocking. Journal of the Operational Research Society. 2009;60(12):1708–18.

44. Ghiani G, Lagana D, Manni E, Musmanno R, Vigo D. Operations research in solid waste management: A survey of strategic and tactical issues. Computers & Operations Research. 2014;44(22–32).

45. Tung DV, Pinnoi A. Vehicle routing-scheduling for waste collection in Hanoi. European Journal of Operational Research. 2000;125(3):449–68.

46. Shih L-H, Chang H-C. A routing and scheduling system for infectious waste collection. Environmental Modeling & Assessment. 2001;6(4):261–9.

47. Baptista S, Oliveira RC, Zuquete E. A period vehicle routing case study. European Journal of Operational Research. 2002;139(2):220–9.

48. Christofides N, Beasley JE. The periodic routing problem. Networks. 1984;14(2):237–56.

49. Teixeira J, Antunes AP, Sousa JPd. Recyclable waste collection planning a case study. European Journal of Operational Research. 2004;158(3):543–54.

50. Nuortio T, Kytojoki J, Niska H, Braysy O. Improved route planning and scheduling of waste collection and transport. Expert Systems with Applications. 2006;30(2):223–32.

51. Sahoo S, Kim S, Kim B-I, Kraas B, Jr AP. Routing optimization for waste management. Interfaces. 2005;35(1):24–36.

52. Li F, Golden BL, Wasil EA. A record-to-record travel algorithm for solving the heterogeneous fleet vehicle routing problem. Computers & Operations Research. 2007;34(9):2734–42.

53. Alumur S, Kara BY. A new model for the hazardous waste location-routing problem. Computers & Operations Research. 2007;34(5):1406–23.

54. Repoussis PP, Paraskevopoulos DC, Zobolas G, Tarantilis CD, Ioannou G. A web-based decision support system for waste lube oils collection and recycling. European Journal of Operational Research. 2009;195(3):676–700.

55. Coene S, Arnout A, Spieksma FCR. On a periodic vehicle routing problem. Journal of the Operational Research Society. 2010;61(12):1719–28.

56. Hauge K, Larsen J, Lusby RM, Krapper E. A hybrid column generation approach for an industrial waste collection routing problem. Computers & Industrial Engineering. 2014;71:10–20.

57. Hemmelmayr VC, Doerner KF, Hartl RF, Vigo D. Models and algorithms for the integrated planning of bin allocation and vehicle routing in solid waste management. Transportation Science. 2013;48(1):103–20.

58. Battarra M, Erdogan G, Vigo D. Exact algorithms for the clustered vehicle routing problem. Operations Research. 2014;62(1):58–71.

59. Aksen D, Kaya O, Salman FS, Tuncel O. An adaptive large neighborhood search algorithm for a selective and periodic inventory routing problem. European Journal of Operational Research. 2014;239(2):413–26.

60. Huang S-H, Lin P-C. Vehicle routing–scheduling for municipal waste collection system under the “keep trash off the ground” policy. Omega. 2015;55:24–37.

61. Ghiani G, Mourao C, Pinto L, Vigo D. Routing in waste collection. In A. Cor- beran and G. Laporte, editors, Arc Routing: Problems, Methods, and Applications, volume 20 of Monographs on Discrete Mathematics and Applications. Philadelphia: MOS-SIAM Series on Optimization; 2014.

62. Larsen A, Madsen OBG, Solomon MM. The a priori dynamic traveling salesman problem with time windows. Transportation Science. 2004;38(4):459–72.

63. Hollis BL, Forbes MA, Douglas BE. Vehicle routing and crew scheduling for metropolitan mail distribution at Australia Post. European Journal of Operational Research. 2006;173(1):133–50.

64. Cohn A, Root S, Wang A, Mohr D. Integration of the load-matching and routing problem with equipment balancing for small package carriers. Transportation Science. 2007;41(2):238–52.

65. Groer C, Golden BL, Wasil EA. The consistent vehicle routing problem. Manufacturing & Service Operations Management. 2009;11(4):630–43.

66. Sungur I, Ren Y, Ordonez F, Dessouky M, Zhong H. A model and algorithm for the courier delivery problem with uncertainty. Transportation Science. 2010;44(2):193–205.

67. Pignac-Robitaille O, Simard S, Ruiz A, Renaud J. Planification des tournees dans le domaine de la messagerie rapide / vehicle routing for fast courier services. INFOR: Information Systems and Operational Research. 2014;52(1):20–7.

68. Akkerman R, Farahani P, Grunow M. Quality, safety and sustainability in food distribution: a review of quantitative operations management approaches and challenges. OR Spectrum. 2010;32(4):863–904.

69. Ahumada O, Villalobos JR. Application of planning models in the agri-food supply chain: A review. European Journal of Operational Research. 2009;196(1):1–20.

70. Tarantilis CD, Kiranoudis CT. A meta-heuristic algorithm for the efficient distribution of perishable foods. Journal of Food Engineering. 2001;50(1):1–9.

71. Cheong YM, Leong HL, Huang HC. Modeling the vehicle routine problem for a soft drink machine distribution company. Asia-Pacific Journal of Operational Research. 2002;19(1):17–34.

72. Tarantilis CD, Kiranoudis CT. Distribution of fresh meat. Journal of Food Engineering. 2002;51(1):85–91.

73. Prindezis N, Kiranoudis CT, Marinos-Kouris D. A business-to-business fleet management service provider for central food market enterprises. Journal of Food Engineering. 2003;60(2):203–10.

74. Faulın J. Combining linear programming and heuristics to solve a transportation problem for a canning company in Spain. International Journal of Logistics. 2003;6(1-2):17–27.

75. Faulın J. Applying MIXALG procedure in a routing problem to optimize food product delivery. Omega. 2003;31(5):387–95.

76. Pamuk S, Koksalan M, Gullu R. Analysis and improvement of the product delivery system of a beer producer in Ankara. Journal of the Operational Research Society. 2004;55(11):1137–44.

77. Ruiz R, Maroto C, Alcaraz J. A decision support system for a real vehicle routing problem. European Journal of Operational Research. 2004;153(3):593–606.

78. Faulın J, Sarobe P, Simal J. The DSS LOGDIS optimizes delivery routes for FRILAC's frozen products. Interfaces. 2005;35(3):202–14.

79. Belenguer JM, Benavent E, Martinez MC. RutaRep: a computer package to design dispatching routes in the meat industry. Journal of Food Engineering. 2005;70(3):435–45.

80. Ioannou G. Streamlining the supply chain of the hellenic sugar industry. Journal of Food Engineering. 2005;70(3):323–32.

81. Ioannou G, Kritikos MN, Prastacos GP. Map-Route: a GIS-based decision support system for intra-city vehicle routing with time windows. Journal of the Operational Research Society. 2002;53:842–54.

82. J. Prive, Renaud J, Boctor FF, Laporte G. a vehicle-routing problem arising in soft-drink distribution. Journal of the Operational Research Society. 2006;57(9):1045–52.

83. Cetinkaya S, Uster H, Easwaran G, Keskin BB. An integrated outbound lo- gistics model for Frito-Lay: Coordinating aggregate-level production and distribution decisions. Interfaces. 2009;39(5):460–75.

84. Hu X, Wang Z, Huang M, Zeng AZ. A computer-enabled solution procedure for food wholesalers’ distribution decision in cities with a circular transportation infrastructure. Computers & Operations Research. 2009;36(7):2201–9.

85. Battarra M, Monaci M, Vigo D. An adaptive guidance approach for the heuristic solution of a minimum multiple trip vehicle routing problem. Computers & Operations Research. 2009;36(11):3041–50.

86. Caramia M, Guerriero F. A milk collection problem with incompatibility constraints. Interfaces. 2010;40(2):130–43.

87. Zachariadis EE, Tarantilis CD, Kiranoudis CT. The pallet-packing vehicle routing problem. Transportation Science. 2012;46(3):341–58.

88. Martınez L, Amaya C-A. A vehicle routing problem with multi-trips and time windows for circular items. Journal of the Operational Research Society. 2012;64(11):1630–43.

89. Cattaruzza D, Absi N, Feillet D, Vigo D. An iterated local search for the multi-commodity multi-trip vehicle routing problem with time windows. Computers & Operations Research. 2014;51:257–67.

90. Lahrichi N, Crainic TG, Gendreau M, W. Rei, Rousseau L-M. Strategic analysis of the dairy transportation problem. Journal of the Operational Research Society. 2014;66(1):44–56.

91. Demir E, Woensel TV, Kok Td. Multidepot distribution planning at logistics service provider Nabuurs B.V. Interfaces. 2014;44(6):591–604.

92. Lahyani R, Coelho LC, Khemakhem M, Laporte G, Semet F. A multi-compartment vehicle routing problem arising in the collection of olive oil in tunisia. Omega. 2015;51(1):1–10.

93. Baldacci R, Toth P, Vigo D. Recent advances in vehicle routing exact algorithms. 4OR. 2007;5(4):269–98.

94. Cordeau J-F, Laporte G, Savelsbergh MWP, Vigo D. Vehicle Routing. Management Science. 2007;14:367–428.

95. Solomon MM, Desrosiers J. Time Window Constrained Routing and Scheduling Problems Transportation Science. 1988;22(1):1-13.

96. Archetti C, Speranza MG. The Split Delivery Vehicle Routing Problem: A Survey. Operations Research/Computer Science Interfaces. 2008;43:103-22.

97. Choi E, Tcha D-W. A column generation approach to the heterogeneous fleet vehicle routing problem. Computers & Operations Research. 2007;34(7):2080–95.

98. Liu S, Huang W, Ma H. An effective genetic algorithm for the fleet size and mix vehicle routing problems. Transportation Research Part E: Logistics and Transportation Review. 2009;45(3):434–45.

99. Zachariadis EE, Tarantilis CD, Kiranoudis CT. The load-dependent vehicle routing problem and its pick-up

and delivery extension. Transportation Research. 2015;2(71):158-81.

Annex

A.1 Data for each Vehicle Routing Problem Instance

Coordinates and Time Windows of Depots for all Vehicle Routing instances. Time Windows are used only if VRP instance includes Time Windows.

Number	Latitude	Longitude	Earliest time		Latest time		Service Time* (minutes)
			hours	minutes	hours	minutes	
i	x[i]	y[i]	a[i]		b[i]		s[i]
1	40.267319	22.524785	07	00	19	00	0
2	39.630292	22.454417	07	00	19	00	0
3	39.555049	21.785850	07	00	19	00	0

Coordinates of Customers for all Vehicle Routing instances.

Number	Latitude	Longitude
i	x[i]	y[i]
1	40.271350	22.474764
2	40.274192	22.509921
3	40.271310	22.511823
4	40.267590	22.523156
5	40.315162	22.584259
6	40.269608	22.505105
7	40.270951	22.505092
8	40.271762	22.501614
9	40.257465	22.530175
10	40.272479	22.511452
11	39.634905	22.417347
12	39.639409	22.421810
13	39.890399	22.187992
14	39.642578	22.417967
15	39.635404	22.427961
16	39.579057	22.470248
17	39.690648	22.452158
18	39.629226	22.397275
19	39.378711	22.927446
20	39.366394	22.954074
21	39.358204	22.955030
22	39.375021	22.939066

Number	Latitude	Longitude
23	39.365996	22.955695
24	39.364469	22.943652
25	39.361888	21.927413
26	39.364924	21.950151
27	39.359076	21.917542
28	39.372823	21.921150
29	39.557704	21.768363
30	39.553202	21.764490
31	39.547442	21.762816
32	39.563015	21.770842

- **Multi Depot Capacitated Heterogeneous Fleet VRP Problem (00000)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	32	3

Customers' additional data

Number	Delivery Demand
i	d[i]
1	31
2	18
3	17
4	16
5	19
6	22
7	17
8	24
9	32
10	17
11	15
12	14
13	18
14	14
15	26
16	34
17	33
18	19
19	18

Number	Delivery
20	27
21	28
22	25
23	16
24	24
25	23
26	18
27	22
28	17
29	28
30	18
31	14
32	19

Data of Vehicles

Number	Capacity	Starting depot	Average Consumption (litres/Km)
k	C[k]	sltemp[k]	ac[k]
1	450	1	0.25
2	500	2	0.25
3	450	3	0.25

- **Multi Depot Capacitated Heterogeneous Fleet Vehicle Relocation VRP Problem (00001)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number	Delivery Demand
i	d[i]
1	31
2	18

Number	Delivery
3	17
4	16
5	19
6	22
7	17
8	24
9	32
10	17
11	15
12	14
13	18
14	14
15	26
16	34
17	33
18	19

Data of Vehicles

Number	Capacity	Starting depot	Average Consumption (litres/Km)
k	C[k]	sltemp[k]	ac[k]
1	450	1	0.25
2	350	1	0.26
3	300	1	0.20
4	450	2	0.25
5	600	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Specific Network Characteristics VRP Problem (00010)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	4

Customers’ additional data

Number i	Delivery Demand d[i]	Demand of Refrigerator dr[i]	Center Location cl[i]
1	31	1	0
2	18	0	1
3	17	1	1
4	16	1	1
5	19	0	1
6	22	0	1
7	17	1	1
8	24	0	1
9	32	0	0
10	17	1	1
11	15	0	1
12	14	1	1
13	18	0	0
14	14	1	0
15	26	1	1
16	34	1	0
17	33	0	0
18	19	0	1

Data of Vehicles

Number k	Capacity C[k]	Refrigerated Vehicle r[k]	Vehicle permission to enter town centers vp[k]	Starting depot sltemp[k]	Average Consumption (litres/Km) ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	450	1	1	2	0.25
4	600	0	1	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Specific Network Characteristics Vehicle Relocation VRP Problem (00011)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number	Delivery Demand	Demand of Refrigerator	Center Location
i	d[i]	dr[i]	cl[i]
1	31	1	0
2	18	0	1
3	17	1	1
4	16	1	1
5	19	0	1
6	22	0	1
7	17	1	1
8	24	0	1
9	32	0	0
10	17	1	1
11	15	0	1
12	14	1	1
13	18	0	0
14	14	1	0
15	26	1	1
16	34	1	0
17	33	0	0
18	19	0	1

Data of Vehicles

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
k	C[k]	r[k]	vp[k]	sltemp[k]	ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	300	0	1	1	0.20
4	450	1	1	2	0.25

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
5	600	0	1	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Split Delivery VRP Problem (00100)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	32	4

Customers' additional data

Number	Delivery Demand
i	d[i]
1	31
2	18
3	17
4	16
5	19
6	22
7	17
8	24
9	500
10	17
11	15
12	14
13	18
14	14
15	26
16	34
17	33
18	19
19	22
20	27
21	28
22	25
23	16
24	24

Number	Delivery
25	23
26	18
27	22
28	17
29	28
30	18
31	14
32	19

Data of Vehicles

Number	Capacity	Starting depot	Average Consumption (litres/Km)
k	C[k]	sltemp[k]	ac[k]
1	450	1	0.25
2	500	2	0.25
3	450	3	0.25
4	350	1	0.23

- **Multi Depot Capacitated Heterogeneous Fleet Split Delivery Vehicle Relocation VRP Problem (00101)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number	Delivery Demand
i	d[i]
1	31
2	18
3	17
4	16
5	19
6	22

Number	Delivery
7	17
8	24
9	500
10	17
11	15
12	14
13	18
14	14
15	26
16	34
17	33
18	19

Data of Vehicles

Number	Capacity	Starting depot	Average Consumption (litres/Km)
k	C[k]	sltemp[k]	ac[k]
1	450	1	0.25
2	350	1	0.26
3	300	1	0.20
4	450	2	0.25
5	600	2	0.30

- **<u>Multi Depot Capacitated Heterogeneous Fleet Split Delivery Specific Network Characteristics VRP Problem (00110)</u>**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number i	Delivery Demand d[i]	Demand of Refrigerator dr[i]	Center Location cl[i]
1	31	1	0
2	18	0	1
3	17	1	1
4	16	1	1
5	19	0	1
6	22	0	1
7	17	1	1
8	24	0	1
9	500	0	0
10	17	1	1
11	15	0	1
12	14	1	1
13	18	0	0
14	14	1	0
15	26	1	1
16	34	1	0
17	33	0	0
18	19	0	1

Data of Vehicles

Number k	Capacity C[k]	Refrigerated Vehicle r[k]	Vehicle permission to enter town centers vp[k]	Starting depot sltemp[k]	Average Consumption (litres/Km) ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	300	0	1	1	0.20
4	450	1	1	2	0.25
5	600	0	1	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Split Delivery Specific Network Characteristics Vehicle Relocation VRP Problem (00111)**

Size of Problem

Number of depots m	Number of customers n	Number of vehicles l
3	18	5

Customers' additional data

Number i	Delivery Demand d[i]	Demand of Refrigerator dr[i]	Center Location cl[i]
1	31	1	0
2	18	0	1
3	17	1	1
4	16	1	1
5	19	0	1
6	22	0	1
7	17	1	1
8	24	0	1
9	500	0	0
10	17	1	1
11	15	0	1
12	14	1	1
13	18	0	0
14	14	1	0
15	26	1	1
16	34	1	0
17	33	0	0
18	19	0	1

Data of Vehicles

Number k	Capacity C[k]	Refrigerated Vehicle r[k]	Vehicle permission to enter town centers vp[k]	Starting depot sltemp[k]	Average Consumption (litres/Km) ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	300	0	1	1	0.20
4	450	1	1	2	0.25

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
5	600	0	1	2	0.30

- **<u>Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pickup and Delivery VRP Problem (01000)</u>**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	32	3

Customers' additional data

Number	Delivery Demand	Pick-up Demand
i	d[i]	p[i]
1	31	15
2	18	22
3	17	9
4	16	17
5	19	8
6	22	16
7	17	20
8	24	8
9	32	23
10	17	21
11	15	19
12	14	21
13	18	13
14	14	18
15	26	15
16	34	21
17	33	11
18	19	6
19	22	25
20	27	16
21	28	22
22	25	17

Number	Delivery	Pick-up
23	16	21
24	24	6
25	23	11
26	18	9
27	22	26
28	17	17
29	28	17
30	18	16
31	14	7
32	19	22

Data of Vehicles

Number k	Capacity C[k]	Starting depot sltemp[k]	Average Consumption (litres/Km) ac[k]
1	450	1	0.25
2	500	2	0.25
3	450	3	0.25

- **<u>Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pickup and Delivery Vehicle Relocation VRP Problem (01001)</u>**

Size of Problem

Number of depots m	Number of customers n	Number of vehicles l
3	32	5

Customers' additional data

Number i	Delivery Demand d[i]	Pick-up Demand p[i]
1	31	15
2	18	22
3	17	9
4	16	17
5	19	8

Number	Delivery	Pick-up
6	22	16
7	17	20
8	24	8
9	32	15
10	17	21
11	15	19
12	14	21
13	18	13
14	14	18
15	26	15
16	34	21
17	33	11
18	19	6
19	22	25
20	27	16
21	28	22
22	25	17
23	16	21
24	24	6
25	23	11
26	18	9
27	22	26
28	17	17
29	28	17
30	18	16
31	14	7
32	19	22

Data of Vehicles

Number	Capacity	Starting depot	Average Consumption (litres/Km)
k	C[k]	sltemp[k]	ac[k]
1	450	1	0.25
2	350	1	0.26
3	300	1	0.20
4	450	2	0.25
5	600	2	0.30

- **<u>Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pickup and Delivery Specific Network Characteristics VRP Problem (01010)</u>**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	4

Customers' additional data

Number	Delivery Demand	Pick-up Demand	Demand of Refrigerator	Center Location
i	d[i]	p[i]	dr[i]	cl[i]
1	31	15	1	0
2	18	22	0	1
3	17	9	1	1
4	16	17	1	1
5	19	8	0	1
6	22	16	0	1
7	17	20	1	1
8	24	8	0	1
9	32	15	0	0
10	17	21	1	1
11	15	19	0	1
12	14	21	1	1
13	18	13	0	0
14	14	18	1	0
15	26	15	1	1
16	34	21	1	0
17	33	11	0	0
18	19	6	0	1

Data of Vehicles

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
k	C[k]	r[k]	vp[k]	sltemp[k]	ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	450	1	1	2	0.25
4	600	0	1	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pickup and Delivery Specific Network Characteristics Vehicle Relocation VRP Problem (01011)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number	Delivery Demand	Pick-up Demand	Demand of Refrigerator	Center Location
i	d[i]	p[i]	dr[i]	cl[i]
1	31	15	1	0
2	18	22	0	1
3	17	9	1	1
4	16	17	1	1
5	19	8	0	1
6	22	16	0	1
7	17	20	1	1
8	24	8	0	1
9	32	15	0	0
10	17	21	1	1
11	15	19	0	1
12	14	21	1	1
13	18	13	0	0
14	14	18	1	0
15	26	15	1	1
16	34	21	1	0
17	33	11	0	0
18	19	6	0	1

Data of Vehicles

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
k	C[k]	r[k]	vp[k]	sltemp[k]	ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
3	300	0	1	1	0.20
4	450	1	1	2	0.25
5	600	0	1	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pickup and Delivery Split Satisfaction of Demand VRP Problem (01100)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	4

Customers' additional data

Number	Delivery Demand	Pick-up Demand
i	d[i]	p[i]
1	31	480
2	18	22
3	17	9
4	16	17
5	19	8
6	22	16
7	17	20
8	24	8
9	500	15
10	17	21
11	15	19
12	14	21
13	18	13
14	14	18
15	26	15
16	34	21
17	33	11
18	19	6

Data of Vehicles

Number k	Capacity C[k]	Starting depot sltemp[k]	Average Consumption (litres/Km) ac[k]
1	450	1	0.25
2	500	2	0.24
3	450	3	0.25
4	350	1	0.23

- **<u>Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pickup and Delivery Split Satisfaction of Demand Vehicle Relocation VRP Problem (01101)</u>**

Size of Problem

Number of depots m	Number of customers n	Number of vehicles l
3	18	5

Customers' additional data

Number i	Delivery Demand d[i]	Pick-up Demand p[i]
1	31	480
2	18	22
3	17	9
4	16	17
5	19	8
6	22	16
7	17	20
8	24	8
9	500	15
10	17	21
11	15	19
12	14	21
13	18	13
14	14	18
15	26	15
16	34	21
17	33	11
18	19	6

Data of Vehicles

Number k	Capacity C[k]	Starting depot sltemp[k]	Average Consumption (litres/Km) ac[k]
1	450	1	0.25
2	350	1	0.26
3	300	1	0.20
4	450	2	0.25
5	600	2	0.30

- **<u>Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pickup and Delivery Split Satisfaction of Demand Specific Network Characteristics VRP Problem (01110)</u>**

Size of Problem

Number of depots m	Number of customers n	Number of vehicles l
3	18	5

Customers' additional data

Number i	Delivery Demand d[i]	Pick-up Demand p[i]	Demand of Refrigerator dr[i]	Center Location cl[i]
1	31	480	1	0
2	18	22	0	1
3	17	9	1	1
4	16	17	1	1
5	19	8	0	1
6	22	16	0	1
7	17	20	1	1
8	24	8	0	1
9	500	15	0	0
10	17	21	1	1
11	15	19	0	1
12	14	21	1	1
13	18	13	0	0
14	14	18	1	0

Number	Delivery	Pick-up	Demand of	Center
15	26	15	1	1
16	34	21	1	0
17	33	11	0	0
18	19	6	0	1

Data of Vehicles

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
k	C[k]	r[k]	vp[k]	sltemp[k]	ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	300	0	1	1	0.20
4	450	1	1	2	0.25
5	600	0	1	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Simultaneous Pickup and Delivery Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP Problem (01111)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number	Delivery Demand	Pick-up Demand	Demand of Refrigerator	Center Location
i	d[i]	p[i]	dr[i]	cl[i]
1	31	480	1	0
2	18	22	0	1
3	17	9	1	1
4	16	17	1	1
5	19	8	0	1
6	22	16	0	1
7	17	20	1	1

Number	Delivery	Pick-up	Demand of	Center
8	24	8	0	1
9	500	15	0	0
10	17	21	1	1
11	15	19	0	1
12	14	21	1	1
13	18	13	0	0
14	14	18	1	0
15	26	15	1	1
16	34	21	1	0
17	33	11	0	0
18	19	6	0	1

Data of Vehicles

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
k	C[k]	r[k]	vp[k]	sltemp[k]	ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	300	0	1	1	0.20
4	450	1	1	2	0.25
5	600	0	1	2	0.30

- **<u>Multi Depot Capacitated Heterogeneous Fleet Delivery Time Windows VRP Problem (10000)</u>**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	32	3

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand
	hours	minutes	hours	minutes		
i	a[i]		b[i]		s[i]	d[i]
1	09	00	17	00	15	31

Number	Earliest time		Latest time		Service Time	Delivery
2	09	00	17	00	15	18
3	09	00	12	00	15	17
4	09	00	17	00	15	16
5	09	00	12	00	15	19
6	09	00	17	00	15	22
7	09	00	17	00	15	17
8	09	00	17	00	15	24
9	09	00	17	00	15	32
10	09	00	17	00	15	17
11	09	00	17	00	15	15
12	09	00	17	00	15	14
13	09	00	17	00	15	18
14	09	00	17	00	15	14
15	09	00	17	00	15	26
16	09	00	17	00	15	34
17	09	00	17	00	15	33
18	09	00	17	00	15	19
19	09	00	17	00	15	22
20	09	00	17	00	15	27
21	09	00	17	00	15	28
22	09	00	17	00	15	25
23	09	00	17	00	15	16
24	09	00	17	00	15	24
25	09	00	17	00	15	23
26	09	00	17	00	15	18
27	09	00	17	00	15	22
28	09	00	17	00	15	17
29	10	00	17	00	15	28
30	10	00	17	00	15	18
31	10	00	17	00	15	14
32	10	00	17	00	15	19

Data of Vehicles

Number	Capacity	Starting depot	Average Consumption (litres/Km)
k	C[k]	sltemp[k]	ac[k]
1	450	1	0.25
2	500	2	0.25
3	450	3	0.25

- **Multi Depot Capacitated Heterogeneous Fleet Delivery Time Windows Vehicle Relocation VRP Problem (10001)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand
	hours	minutes	hours	minutes		
i	a[i]		b[i]		s[i]	d[i]
1	09	00	17	00	15	31
2	09	00	17	00	15	18
3	09	00	12	00	15	17
4	09	00	17	00	15	16
5	09	00	12	00	15	19
6	09	00	17	00	15	22
7	09	00	17	00	15	17
8	09	00	17	00	15	24
9	09	00	17	00	15	32
10	09	00	17	00	15	17
11	09	00	17	00	15	15
12	09	00	17	00	15	14
13	09	00	17	00	15	18
14	09	00	17	00	15	14
15	09	00	17	00	15	26
16	09	00	17	00	15	34
17	09	00	17	00	15	33
18	09	00	17	00	15	19

Data of Vehicles

Number	Capacity	Starting depot	Average Consumption (litres/Km)
k	C[k]	sltemp[k]	ac[k]
1	450	1	0.25
2	350	1	0.26
3	300	1	0.20
4	450	2	0.25
5	600	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Delivery Time Windows Specific Network Characteristics VRP Problem (10010)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	4

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand	Demand of Refrigerator	Center Location
	hours	minutes	hours	minutes				
i	a[i]		b[i]		s[i]	d[i]	dr[i]	cl[i]
1	09	00	17	00	15	31	1	0
2	09	00	17	00	15	18	0	1
3	09	00	12	00	15	17	1	1
4	09	00	17	00	15	16	1	1
5	09	00	12	00	15	19	0	1
6	09	00	17	00	15	22	0	1
7	09	00	17	00	15	17	1	1
8	09	00	17	00	15	24	0	1
9	09	00	17	00	15	32	0	0
10	09	00	17	00	15	17	1	1
11	09	00	17	00	15	15	0	1
12	09	00	17	00	15	14	1	1
13	09	00	17	00	15	18	0	0
14	09	00	17	00	15	14	1	0
15	09	00	17	00	15	26	1	1
16	09	00	17	00	15	34	1	0
17	09	00	17	00	15	33	0	0
18	09	00	17	00	15	19	0	1

Data of Vehicles

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
k	C[k]	r[k]	vp[k]	sltemp[k]	ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	450	1	1	2	0.25
4	600	0	1	2	0.30

- **<u>Multi Depot Capacitated Heterogeneous Fleet Delivery Time Windows Specific Network Characteristics Vehicle Relocation VRP Problem (10011)</u>**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' demand

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand	Demand of Refrigerator	Center Location
	hours	minutes	hours	minutes				
i	a[i]		b[i]		s[i]	d[i]	dr[i]	cl[i]
1	09	00	17	00	15	31	1	0
2	09	00	17	00	15	18	0	1
3	09	00	12	00	15	17	1	1
4	09	00	17	00	15	16	1	1
5	09	00	12	00	15	19	0	1
6	09	00	17	00	15	22	0	1
7	09	00	17	00	15	17	1	1
8	09	00	17	00	15	24	0	1
9	09	00	17	00	15	32	0	0
10	09	00	17	00	15	17	1	1
11	09	00	17	00	15	15	0	1
12	09	00	17	00	15	14	1	1
13	09	00	17	00	15	18	0	0
14	09	00	17	00	15	14	1	0
15	09	00	17	00	15	26	1	1

Number	Earliest time		Latest time		Service	Delivery	Demand of	Center
16	09	00	17	00	15	34	1	0
17	09	00	17	00	15	33	0	0
18	09	00	17	00	15	19	0	1

Data of Vehicles

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
k	C[k]	r[k]	vp[k]	sltemp[k]	ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	300	0	1	1	0.20
4	450	1	1	2	0.25
5	600	0	1	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Delivery Time Windows Split Satisfaction of Demand VRP Problem (10100)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	24	4

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand
	hours	minutes	hours	minutes		
i	a[i]		b[i]		s[i]	d[i]
1	09	00	17	00	15	31
2	09	00	17	00	15	18
3	09	00	12	00	15	17
4	09	00	17	00	15	16
5	09	00	12	00	15	19
6	09	00	17	00	15	22
7	09	00	17	00	15	17
8	09	00	17	00	15	24
9	09	00	17	00	15	500

Number	Earliest time		Latest time		Service Time	Delivery
10	09	00	17	00	15	17
11	09	00	17	00	15	15
12	09	00	17	00	15	14
13	09	00	17	00	15	18
14	09	00	17	00	15	14
15	09	00	17	00	15	26
16	09	00	17	00	15	34
17	09	00	17	00	15	33
18	09	00	17	00	15	19
19	09	00	17	00	15	22
20	09	00	17	00	15	27
21	09	00	17	00	15	28
22	09	00	17	00	15	25
23	09	00	17	00	15	16
24	09	00	17	00	15	24

Data of Vehicles

Number k	Capacity C[k]	Starting depot sltemp[k]	Average Consumption (litres/Km) ac[k]
1	450	1	0.25
2	500	2	0.24
3	450	3	0.25
4	350	1	0.23

- **Multi Depot Capacitated Heterogeneous Fleet Delivery Time Windows Split Satisfaction of Demand Vehicle Relocation VRP Problem (10101)**

Size of Problem

Number of depots m	Number of customers n	Number of vehicles l
3	18	5

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand
	hours	minutes	hours	minutes		
i	a[i]		b[i]		s[i]	d[i]
1	09	00	17	00	15	31
2	09	00	17	00	15	18
3	09	00	12	00	15	17
4	09	00	17	00	15	16
5	09	00	12	00	15	19
6	09	00	17	00	15	22
7	09	00	17	00	15	17
8	09	00	17	00	15	24
9	09	00	17	00	15	500
10	09	00	17	00	15	17
11	09	00	17	00	15	15
12	09	00	17	00	15	14
13	09	00	17	00	15	18
14	09	00	17	00	15	14
15	09	00	17	00	15	26
16	09	00	17	00	15	34
17	09	00	17	00	15	33
18	09	00	17	00	15	19

Data of Vehicles

Number	Capacity	Starting depot	Average Consumption (litres/Km)
k	C[k]	sltemp[k]	ac[k]
1	450	1	0.25
2	350	1	0.26
3	300	1	0.20
4	450	2	0.25
5	600	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Delivery Time Windows Split Satisfaction of Demand Specific Network Characteristics VRP Problem (10110)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand	Demand of Refrigerator	Center Location
	hours	minutes	hours	minutes				
i	a[i]		b[i]		s[i]	d[i]	dr[i]	cl[i]
1	09	00	17	00	15	31	1	0
2	09	00	17	00	15	18	0	1
3	09	00	12	00	15	17	1	1
4	09	00	17	00	15	16	1	1
5	09	00	12	00	15	19	0	1
6	09	00	17	00	15	22	0	1
7	09	00	17	00	15	17	1	1
8	09	00	17	00	15	24	0	1
9	09	00	17	00	15	500	0	0
10	09	00	17	00	15	17	1	1
11	09	00	17	00	15	15	0	1
12	09	00	17	00	15	14	1	1
13	09	00	17	00	15	18	0	0
14	09	00	17	00	15	14	1	0
15	09	00	17	00	15	26	1	1
16	09	00	17	00	15	34	1	0
17	09	00	17	00	15	33	0	0
18	09	00	17	00	15	19	0	1

Data of Vehicles

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
k	C[k]	r[k]	vp[k]	sltemp[k]	ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	300	0	1	1	0.20
4	450	1	1	2	0.25

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
5	600	0	1	2	0.30

- **<u>Multi Depot Capacitated Heterogeneous Fleet Delivery Time Windows Split Satisfaction of Demand Specific Network Characteristics Vehicle Relocation VRP Problem (10111)</u>**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand	Demand of Refrigerator	Center Location
	hours	minutes	hours	minutes				
i	a[i]		b[i]		s[i]	d[i]	dr[i]	cl[i]
1	09	00	17	00	15	31	1	0
2	09	00	17	00	15	18	0	1
3	09	00	12	00	15	17	1	1
4	09	00	17	00	15	16	1	1
5	09	00	12	00	15	19	0	1
6	09	00	17	00	15	22	0	1
7	09	00	17	00	15	17	1	1
8	09	00	17	00	15	24	0	1
9	09	00	17	00	15	500	0	0
10	09	00	17	00	15	17	1	1
11	09	00	17	00	15	15	0	1
12	09	00	17	00	15	14	1	1
13	09	00	17	00	15	18	0	0
14	09	00	17	00	15	14	1	0
15	09	00	17	00	15	26	1	1
16	09	00	17	00	15	34	1	0
17	09	00	17	00	15	33	0	0
18	09	00	17	00	15	19	0	1

Data of Vehicles

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
k	C[k]	r[k]	vp[k]	sltemp[k]	ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	300	0	1	1	0.20
4	450	1	1	2	0.25
5	600	0	1	2	0.30

- **<u>Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pickup and Delivery VRP Problem (11000)</u>**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	32	3

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand	Pick-up Demand
	hours	minutes	hours	minutes			
i	a[i]		b[i]		s[i]	d[i]	p[i]
1	09	00	17	00	15	31	15
2	09	00	17	00	15	18	22
3	09	00	12	00	15	17	9
4	09	00	17	00	15	16	17
5	09	00	12	00	15	19	8
6	09	00	17	00	15	22	16
7	09	00	17	00	15	17	20
8	09	00	17	00	15	24	8
9	09	00	17	00	15	32	23
10	09	00	17	00	15	17	21
11	09	00	17	00	15	15	19
12	09	00	17	00	15	14	21
13	09	00	17	00	15	18	13
14	09	00	17	00	15	14	18

Number	Earliest time		Latest time		Service	Delivery	Pick-up
15	09	00	17	00	15	26	15
16	09	00	17	00	15	34	21
17	09	00	17	00	15	33	11
18	09	00	17	00	15	19	6
19	09	00	17	00	15	22	25
20	09	00	17	00	15	27	16
21	09	00	17	00	15	28	22
22	09	00	17	00	15	25	17
23	09	00	17	00	15	16	21
24	09	00	17	00	15	24	6
25	09	00	17	00	15	23	11
26	09	00	17	00	15	18	9
27	09	00	17	00	15	22	26
28	09	00	17	00	15	17	17
29	10	00	17	00	15	28	17
30	10	00	17	00	15	18	16
31	10	00	17	00	15	14	7
32	10	00	17	00	15	19	22

Data of Vehicles

Number	Capacity	Starting depot	Average Consumption (litres/Km)
k	C[k]	sltemp[k]	ac[k]
1	450	1	0.25
2	500	2	0.25
3	450	3	0.25

- **Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pickup and Delivery Vehicle Relocation VRP Problem (11001)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand	Pick-up Demand
	hours	minutes	hours	minutes			
i	a[i]		b[i]		s[i]	d[i]	p[i]
1	09	00	17	00	15	31	15
2	09	00	17	00	15	18	22
3	09	00	12	00	15	17	9
4	09	00	17	00	15	16	17
5	09	00	12	00	15	19	8
6	09	00	17	00	15	22	16
7	09	00	17	00	15	17	20
8	09	00	17	00	15	24	8
9	09	00	17	00	15	32	15
10	09	00	17	00	15	17	21
11	09	00	17	00	15	15	19
12	09	00	17	00	15	14	21
13	09	00	17	00	15	18	13
14	09	00	17	00	15	14	18
15	09	00	17	00	15	26	15
16	09	00	17	00	15	34	21
17	09	00	17	00	15	33	11
18	09	00	17	00	15	19	6

Data of Vehicles

Number	Capacity	Starting depot	Average Consumption (litres/Km)
k	C[k]	sltemp[k]	ac[k]
1	450	1	0.25
2	350	1	0.26
3	300	1	0.20
4	450	2	0.25
5	600	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pickup and Delivery Specific Network Characteristics VRP Problem (11010)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	4

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand	Pick-up Demand
	hours	minutes	hours	minutes			
i	a[i]		b[i]		s[i]	d[i]	p[i]
1	09	00	17	00	15	31	15
2	09	00	17	00	15	18	22
3	09	00	12	00	15	17	9
4	09	00	17	00	15	16	17
5	09	00	12	00	15	19	8
6	09	00	17	00	15	22	16
7	09	00	17	00	15	17	20
8	09	00	17	00	15	24	8
9	09	00	17	00	15	32	15
10	09	00	17	00	15	17	21
11	09	00	17	00	15	15	19
12	09	00	17	00	15	14	21
13	09	00	17	00	15	18	13
14	09	00	17	00	15	14	18
15	09	00	17	00	15	26	15
16	09	00	17	00	15	34	21
17	09	00	17	00	15	33	11
18	09	00	17	00	15	19	6

Number	Demand of Refrigerator	Center Location
i	dr[i]	cl[i]
1	1	0
2	0	1
3	1	1
4	1	1
5	0	1
6	0	1
7	1	1

Number	Demand of	Center
8	0	1
9	0	0
10	1	1
11	0	1
12	1	1
13	0	0
14	1	0
15	1	1
16	1	0
17	0	0
18	0	1

Data of Vehicles

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
k	C[k]	r[k]	vp[k]	sltemp[k]	ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	450	1	1	2	0.25
4	600	0	1	2	0.30

- **<u>Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pickup and Delivery Specific Network Characteristics Vehicle Relocation VRP Problem (11011)</u>**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand	Pick-up Demand
	hours	minutes	hours	minutes			
i	a[i]		b[i]		s[i]	d[i]	p[i]
1	09	00	17	00	15	31	15

Number	Earliest time		Latest time		Service Time	Delivery	Pick-up
2	09	00	17	00	15	18	22
3	09	00	12	00	15	17	9
4	09	00	17	00	15	16	17
5	09	00	12	00	15	19	8
6	09	00	17	00	15	22	16
7	09	00	17	00	15	17	20
8	09	00	17	00	15	24	8
9	09	00	17	00	15	32	15
10	09	00	17	00	15	17	21
11	09	00	17	00	15	15	19
12	09	00	17	00	15	14	21
13	09	00	17	00	15	18	13
14	09	00	17	00	15	14	18
15	09	00	17	00	15	26	15
16	09	00	17	00	15	34	21
17	09	00	17	00	15	33	11
18	09	00	17	00	15	19	6

Number	Demand of Refrigerator	Center Location
i	dr[i]	cl[i]
1	1	0
2	0	1
3	1	1
4	1	1
5	0	1
6	0	1
7	1	1
8	0	1
9	0	0
10	1	1
11	0	1
12	1	1
13	0	0
14	1	0
15	1	1
16	1	0
17	0	0
18	0	1

Data of Vehicles

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
k	C[k]	r[k]	vp[k]	sltemp[k]	ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	300	0	1	1	0.20
4	450	1	1	2	0.25
5	600	0	1	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pickup and Delivery Split Satisfaction of Demand VRP Problem (11100)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	4

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand	Pick-up Demand
	hours	minutes	hours	minutes			
i	a[i]		b[i]		s[i]	d[i]	p[i]
1	09	00	17	00	15	31	480
2	09	00	17	00	15	18	22
3	09	00	12	00	15	17	9
4	09	00	17	00	15	16	17
5	09	00	12	00	15	19	8
6	09	00	17	00	15	22	16
7	09	00	17	00	15	17	20
8	09	00	17	00	15	24	8
9	09	00	17	00	15	500	15
10	09	00	17	00	15	17	21
11	09	00	17	00	15	15	19
12	09	00	17	00	15	14	21
13	09	00	17	00	15	18	13
14	09	00	17	00	15	14	18
15	09	00	17	00	15	26	15
16	09	00	17	00	15	34	21

Number	Earliest time		Latest time		Service Time	Delivery	Pick-up
17	09	00	17	00	15	33	11
18	09	00	17	00	15	19	6

Data of Vehicles

Number	Capacity	Starting depot	Average Consumption (litres/Km)
k	C[k]	sltemp[k]	ac[k]
1	450	1	0.25
2	500	2	0.24
3	450	3	0.25
4	350	1	0.23

- **Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pickup and Delivery Split Satisfaction of Demand Vehicle Relocation VRP Problem (11101)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand	Pick-up Demand
	hours	minutes	hours	minutes			
i	a[i]		b[i]		s[i]	d[i]	p[i]
1	09	00	17	00	15	31	480
2	09	00	17	00	15	18	22
3	09	00	12	00	15	17	9
4	09	00	17	00	15	16	17
5	09	00	12	00	15	19	8
6	09	00	17	00	15	22	16
7	09	00	17	00	15	17	20
8	09	00	17	00	15	24	8
9	09	00	17	00	15	500	15
10	09	00	17	00	15	17	21
11	09	00	17	00	15	15	19

Number	Earliest time		Latest time		Service	Delivery	Pick-up
12	09	00	17	00	15	14	21
13	09	00	17	00	15	18	13
14	09	00	17	00	15	14	18
15	09	00	17	00	15	26	15
16	09	00	17	00	15	34	21
17	09	00	17	00	15	33	11
18	09	00	17	00	15	19	6

Data of Vehicles

Number	Capacity	Starting depot	Average Consumption (litres/Km)
k	C[k]	sltemp[k]	ac[k]
1	450	1	0.25
2	350	1	0.26
3	300	1	0.20
4	450	2	0.25
5	600	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pickup and Delivery Split Satisfaction of Demand Specific Network Characteristics VRP Problem (11110)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand	Pick-up Demand
	hours	minutes	hours	minutes			
i	a[i]		b[i]		s[i]	d[i]	p[i]
1	09	00	17	00	15	31	15
2	09	00	17	00	15	18	480
3	09	00	12	00	15	17	9
4	09	00	17	00	15	16	17

Number	Earliest time		Latest time		Service	Delivery	Pick-up
5	09	00	12	00	15	19	8
6	09	00	17	00	15	22	16
7	09	00	17	00	15	17	20
8	09	00	17	00	15	24	8
9	09	00	17	00	15	500	15
10	09	00	17	00	15	17	21
11	09	00	17	00	15	15	19
12	09	00	17	00	15	14	21
13	09	00	17	00	15	18	13
14	09	00	17	00	15	14	18
15	09	00	17	00	15	26	15
16	09	00	17	00	15	34	21
17	09	00	17	00	15	33	11
18	09	00	17	00	15	19	6

Number i	Demand of Refrigerator dr[i]	Center Location cl[i]
1	1	0
2	0	1
3	1	1
4	1	1
5	0	1
6	0	1
7	1	1
8	0	1
9	0	0
10	1	1
11	0	1
12	1	1
13	0	0
14	1	0
15	1	1
16	1	0
17	0	0
18	0	1

Data of Vehicles

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
k	C[k]	r[k]	vp[k]	sltemp[k]	ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	300	0	1	1	0.20
4	450	1	1	2	0.25
5	600	0	1	2	0.30

- **Multi Depot Capacitated Heterogeneous Fleet Time Windows Simultaneous Pickup and Delivery Split Satisfaction of Demand Specific Network Characteristics VehicleRelocation VRP Problem (11111)**

Size of Problem

Number of depots	Number of customers	Number of vehicles
m	n	l
3	18	5

Customers' additional data

Number	Earliest time		Latest time		Service Time (minutes)	Delivery Demand	Pick-up Demand
	hours	minutes	hours	minutes			
i	a[i]		b[i]		s[i]	d[i]	p[i]
1	09	00	17	00	15	31	15
2	09	00	17	00	15	18	480
3	09	00	12	00	15	17	9
4	09	00	17	00	15	16	17
5	09	00	12	00	15	19	8
6	09	00	17	00	15	22	16
7	09	00	17	00	15	17	20
8	09	00	17	00	15	24	8
9	09	00	17	00	15	500	15
10	09	00	17	00	15	17	21
11	09	00	17	00	15	15	19
12	09	00	17	00	15	14	21
13	09	00	17	00	15	18	13

Number	Earliest time		Latest time		Service	Delivery	Pick-up
14	09	00	17	00	15	14	18
15	09	00	17	00	15	26	15
16	09	00	17	00	15	34	21
17	09	00	17	00	15	33	11
18	09	00	17	00	15	19	6

Number	Demand of Refrigerator	Center Location
i	dr[i]	cl[i]
1	1	0
2	0	1
3	1	1
4	1	1
5	0	1
6	0	1
7	1	1
8	0	1
9	0	0
10	1	1
11	0	1
12	1	1
13	0	0
14	1	0
15	1	1
16	1	0
17	0	0
18	0	1

Data of Vehicles

Number	Capacity	Refrigerated Vehicle	Vehicle permission to enter town centers	Starting depot	Average Consumption (litres/Km)
k	C[k]	r[k]	vp[k]	sltemp[k]	ac[k]
1	450	0	1	1	0.25
2	350	1	1	1	0.26
3	300	0	1	1	0.20
4	450	1	1	2	0.25
5	600	0	1	2	0.30