



Develop software in Python for performing Independent t-test and One-way ANOVA with post hoc test considering the Bonferroni's adjustment.

by

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Abstract

The independent t-test, also called the two sample t-test, is a statistical test that determines whether there is a statistically significant difference between the means in two unrelated groups. The independent-samples t test is commonly referred to as a between-groups design, and can also be used to analyze a control and experimental group. With an independent-samples t test, each case must have values on two variables, the grouping (independent) variable and the test (dependent) variable. The grouping variable divides cases into two mutually exclusive groups or categories, such as boys or girls for the grouping variable gender, while the test variable describes each case on some quantitative dimension such as test performance. The t test evaluates whether the mean value of the test variable (e.g., test performance) for one group (e.g., boys) differs significantly from the mean value of the test variable for the second group (e.g., girls).

The one-way analysis of variance (ANOVA) is used to determine whether there are any significant differences between the means of two or more independent (unrelated) groups. For example, you could use a one-way ANOVA to understand whether exam performance differed based on test anxiety levels amongst students, dividing students into three independent groups (e.g., low, medium and high-stressed students). Also, it is important to realize that the one-way ANOVA is an *omnibus* test statistic and cannot tell you which specific groups were significantly different from each other; it only tells you that at least two groups were different. Since you may have three, four, five or more groups in your study design, determining which of these groups differ from each other is important. You can do this using a post-hoc test.

Post hoc tests are designed for situations in which the researcher has already obtained a significant omnibus F-test with a factor that consists of three or more means and additional exploration of the differences among means is needed to provide specific information on which means are significantly different from each other. We do individual comparisons between groups, e.g.: To compare a group with the group b, using the t-test and adjust the sig values with a Bonferroni adjustment. For Bonferroni adjustment the p value (which must be achieved for significance) is divided by the number of paired comparisons.

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Chapter 1 – Introduction

Hypothesis for the independent t-test:

The null hypothesis for the independent t-test is that the population means from the two unrelated groups are equal:

- $H_0: \mu_1 = \mu_2$

In most cases, we are looking to see if we can show that we can reject the null hypothesis and accept the alternative hypothesis, which is that the population means are not equal:

- $H_A: \mu_1 \neq \mu_2$

To do this, we need to set a significance level alpha that allows us to either reject or accept the alternative hypothesis. Most commonly, this value is set at 0.05.

Assume that we have two completely different (independent) groups of subjects that we want to compare and to determine if they are significantly different from one another: a between-groups design. A one sample t-test allows us to test whether a sample mean (of a normally distributed interval variable) significantly differs from a hypothesized value or population mean. An independent samples t-test is used when you want to compare the means of a normally distributed interval dependent variable for two independent groups. The classic example of this is when you have a sample and you randomly assign half of your subjects to the control condition and the other half to the experimental treatment condition. In this situation, we wish to compare the means of the two conditions/groups. We can no longer assume that we know a population mean / a hypothesized value and we must develop a new sampling distribution.

The hypotheses of interest in an ANOVA are as follows:

- $H_0: \mu_1 = \mu_2 = \mu_3 \dots = \mu_k$ (H_0 is the null hypothesis and k is the number of conditions).
- H_1 : Means are not all equal.

where k = the number of independent comparison groups.

The null hypothesis tested by ANOVA is that the population means for all conditions are the same.

Analysis of variance is a method for testing differences among means by analyzing variance. The test is based on two estimates of the population variance (σ^2). One estimate is called the mean square error (MSE) and is based on differences among scores within the groups. MSE estimates σ^2 regardless of whether the null hypothesis is true (the population means are equal). The second estimate is called the mean square between (MSB) and is based on differences among the sample means. MSB only estimates σ^2 if the population means are equal. If the population means are not equal, then MSB estimates a quantity larger than σ^2 . Therefore, if the MSB is much larger than the MSE, then the population means are unlikely to be equal. On the other hand, if the MSB is about the same as MSE, then the data are consistent with the null hypothesis that the population means are equal.

Chapter 2 – Methods (Theory)

Independent t Test

Sampling Distribution of the Difference between the Means

To test for the potential statistical significance of a true difference between sample means, we need a sampling distribution of the difference between sample means (**Difference (D) = Mean 1 – Mean 2**). This would be a sampling distribution that will provide us with the probability that the difference between our two sample means differs from the null hypothesis population of sample mean differences: a population in which there is no difference between samples or, restated, the independent variable has no effect. The sampling distribution of the difference between the means can be created by taking all possible sample sizes of n_1 and n_2 , calculating the sample means, and then taking the difference of those means. If you do this repeatedly for all of the possible combinations of your sample sizes, then you end up with a family of distributions of differences between the two means when they are randomly drawn from the same null hypothesis population.

- But, how confident are we that the difference between the means (D) deviates (is far way) from zero, i.e. is it significant? As we can see, in the following math formula, the t value, except from the difference between means, depends on the variability and the size of the trial $n=n_1+n_2$ patients (i.e. the error)

Where n_1 is the sample size of participants in the first group and n_2 in the second group. - Error has two factors: variability and size of the trials.

We must consider the difference between the two means (D) in conjunction with the error of this Difference (SE), i.e. the overall variability (the SD of the two statements) and the size of the trial (n).

Then, we could test statistically whether the difference between the two means deviates from zero (i.e. is significant) using the t-test:

$$t = \frac{(\text{mean 1}) - (\text{mean 2})}{SE} = \frac{\bar{x}_1 - \bar{x}_2}{SE}$$

$$\text{where } SE = \sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} \text{ and } s^2 = \frac{\sum_{j=1}^{n_1} (x_j - \bar{x}_1)^2 + \sum_{i=1}^{n_2} (x_i - \bar{x}_2)^2}{(n_1 - 1) + (n_2 - 1)}$$

In fact, what we do is estimate the true population variability (or variance, s^2) by taking the average variance of our samples but weighted by their respective sample sizes. Sample size or degrees of freedom affects the accuracy of our variance estimates, so an estimate from a sample with a large sample size would be more accurate than an estimated variance from a smaller sample. So we need to weight our average variance by the respective sample sizes of each sample. In using this approach, we are going to make a new assumption—that the sample variances are estimating the same underlying population variance, the variance of the null hypothesis population.

Significance of the difference, P-value

- We have to answer the following question: How confident are we that the value t is different from zero, i.e. significant; alternatively, what is the error probability (i.e. the P-value, or the probability of false-positive result) for claiming that the t is significant?

If t is different from zero then, we claim the difference between means (taking into account the variability of the data and the size of the trial) is significant (i.e. different from zero).

We could answer the question by comparing the value t (the sign is ignored) with the value 5% point of the t -distribution with n_1+n_2-2 df (see Table of t -distribution)

T Distribution Critical Values Table

α (1 tail)	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
α (2 tail)	0.1	0.05	0.02	0.01	0.005	0.002	0.001
df							
1	6.3138	12.7060	31.8211	63.6573	127.3447	318.4691	636.3450
2	3.9890	6.9580	17.0007	17.0007	19.0000	23.0000	27.0000
3	3.1824	5.8409	14.2670	14.2670	15.9914	19.2451	22.3183
4	2.7764	5.1915	12.7060	12.7060	14.0130	17.0468	19.9471
5	2.5758	4.7794	11.9176	11.9176	13.1527	15.8565	18.5925
6	2.4477	4.5593	11.4588	11.4588	12.7903	15.3329	17.9993
7	2.3646	4.3881	11.0941	11.0941	12.5008	14.9456	17.4588
8	2.3060	4.2914	10.8032	10.8032	12.2851	14.6028	17.0245
9	2.2622	4.2141	10.5964	10.5964	12.1008	14.3479	16.6942
10	2.2281	4.1531	10.4171	10.4171	11.9308	14.1267	16.4689
11	2.2010	4.1039	10.2639	10.2639	11.7759	13.9280	16.2576
12	2.1798	4.0623	10.1279	10.1279	11.6327	13.7469	16.0581
13	2.1627	4.0271	10.0081	10.0081	11.4994	13.5811	15.8691
14	2.1483	3.9976	9.8942	9.8942	11.3759	13.4291	15.6901
15	2.1358	3.9726	9.7865	9.7865	11.2614	13.2891	15.5211
16	2.1247	3.9501	9.6839	9.6839	11.1559	13.1591	15.3611
17	2.1147	3.9291	9.5861	9.5861	11.0581	13.0381	15.2091
18	2.1056	3.9091	9.4921	9.4921	10.9661	12.9241	15.0641
19	2.0972	3.8901	9.4011	9.4011	10.8791	12.8161	14.9261
20	2.0895	3.8721	9.3121	9.3121	10.7961	12.7131	14.7941
21	2.0824	3.8551	9.2251	9.2251	10.7171	12.6151	14.6671
22	2.0758	3.8391	9.1401	9.1401	10.6411	12.5211	14.5451
23	2.0696	3.8241	9.0561	9.0561	10.5681	12.4311	14.4271
24	2.0638	3.8091	8.9731	8.9731	10.4971	12.3441	14.3131
25	2.0583	3.7941	8.8911	8.8911	10.4281	12.2601	14.2021
26	2.0531	3.7791	8.8101	8.8101	10.3611	12.1781	14.0941
27	2.0481	3.7641	8.7301	8.7301	10.2961	12.0981	13.9891
28	2.0433	3.7491	8.6511	8.6511	10.2321	12.0201	13.8861
29	2.0387	3.7341	8.5731	8.5731	10.1691	11.9441	13.7851
30	2.0343	3.7191	8.4961	8.4961	10.1071	11.8691	13.6861
31	2.0299	3.7041	8.4201	8.4201	10.0461	11.7951	13.5891
32	2.0257	3.6891	8.3451	8.3451	9.9861	11.7221	13.4931
33	2.0216	3.6741	8.2711	8.2711	9.9271	11.6501	13.3981
34	2.0176	3.6591	8.1981	8.1981	9.8691	11.5791	13.3041
35	2.0137	3.6441	8.1251	8.1251	9.8121	11.5091	13.2111
36	2.0098	3.6291	8.0531	8.0531	9.7561	11.4391	13.1191
37	2.0060	3.6141	7.9811	7.9811	9.7011	11.3701	13.0281
38	2.0023	3.5991	7.9101	7.9101	9.6461	11.3021	12.9381
39	2.0000	3.5841	7.8391	7.8391	9.5921	11.2351	12.8491
40	1.9978	3.5691	7.7691	7.7691	9.5391	11.1691	12.7611
41	1.9957	3.5541	7.7001	7.7001	9.4861	11.1041	12.6741
42	1.9937	3.5391	7.6311	7.6311	9.4341	11.0391	12.5881
43	1.9917	3.5241	7.5631	7.5631	9.3821	10.9751	12.5031
44	1.9897	3.5091	7.4951	7.4951	9.3311	10.9121	12.4191
45	1.9878	3.4941	7.4281	7.4281	9.2801	10.8501	12.3361
46	1.9859	3.4791	7.3611	7.3611	9.2291	10.7881	12.2531
47	1.9840	3.4641	7.2951	7.2951	9.1791	10.7271	12.1711
48	1.9822	3.4491	7.2291	7.2291	9.1301	10.6661	12.0901
49	1.9804	3.4341	7.1641	7.1641	9.0811	10.6061	12.0091
50	1.9786	3.4191	7.1001	7.1001	9.0321	10.5461	11.9291

180	1.8740	1.8738	-1.2000	3.4473	4.0447	2.1833	0.0000	187	1.8531	1.8527	2.3483	2.9024	2.9407	3.1244	3.3433
181	1.8734	1.8736	-1.2000	3.4480	4.0450	2.1830	0.0000								
182	1.8728	1.8738	-1.2000	3.4487	4.0453	2.1827	0.0000	188	1.8526	1.8527	2.3483	2.9023	2.9406	3.1243	3.3430
183	1.8722	1.8737	-1.2000	3.4490	4.0450	2.1823	0.0000								
184	1.8716	1.8734	-1.2000	3.4493	4.0447	2.1820	0.0000	189	1.8520	1.8526	2.3483	2.9021	2.9403	3.1239	3.3426
185	1.8710	1.8731	-1.2000	3.4496	4.0444	2.1817	0.0000								
186	1.8704	1.8726	-1.2000	3.4499	4.0441	2.1814	0.0000	190	1.8515	1.8525	2.3481	2.9019	2.9402	3.1237	3.3425
187	1.8698	1.8720	-1.2000	3.4502	4.0438	2.1811	0.0000								
188	1.8692	1.8714	-1.2000	3.4505	4.0435	2.1808	0.0000	191	1.8510	1.8525	2.3483	2.9018	2.9400	3.1234	3.3421
189	1.8686	1.8708	-1.2000	3.4508	4.0432	2.1805	0.0000								
190	1.8680	1.8702	-1.2000	3.4511	4.0429	2.1802	0.0000	192	1.8508	1.8524	2.3489	2.9017	2.9398	3.1232	3.3419
191	1.8674	1.8696	-1.2000	3.4514	4.0426	2.1799	0.0000								
192	1.8668	1.8690	-1.2000	3.4517	4.0423	2.1796	0.0000	193	1.8508	1.8523	2.3486	2.9015	2.9397	3.1230	3.3417
193	1.8662	1.8684	-1.2000	3.4520	4.0420	2.1793	0.0000								
194	1.8656	1.8678	-1.2000	3.4523	4.0417	2.1790	0.0000	194	1.8508	1.8523	2.3487	2.9014	2.9395	3.1228	3.3414
195	1.8650	1.8672	-1.2000	3.4526	4.0414	2.1787	0.0000								
196	1.8644	1.8666	-1.2000	3.4529	4.0411	2.1784	0.0000	195	1.8507	1.8522	2.3486	2.9013	2.9393	3.1226	3.3411
197	1.8638	1.8660	-1.2000	3.4532	4.0408	2.1781	0.0000								
198	1.8632	1.8654	-1.2000	3.4535	4.0405	2.1778	0.0000	196	1.8507	1.8521	2.3483	2.9012	2.9392	3.1225	3.3409
199	1.8626	1.8648	-1.2000	3.4538	4.0402	2.1775	0.0000								
200	1.8620	1.8642	-1.2000	3.4541	4.0399	2.1772	0.0000	197	1.8506	1.8521	2.3484	2.9010	2.9390	3.1222	3.3406
201	1.8614	1.8636	-1.2000	3.4544	4.0396	2.1769	0.0000								
202	1.8608	1.8630	-1.2000	3.4547	4.0393	2.1766	0.0000	198	1.8506	1.8520	2.3483	2.9009	2.9388	3.1219	3.3403
203	1.8602	1.8624	-1.2000	3.4550	4.0390	2.1763	0.0000								
204	1.8596	1.8618	-1.2000	3.4553	4.0387	2.1760	0.0000	199	1.8505	1.8520	2.3482	2.9008	2.9387	3.1217	3.3401
205	1.8590	1.8612	-1.2000	3.4556	4.0384	2.1757	0.0000								
206	1.8584	1.8606	-1.2000	3.4559	4.0381	2.1754	0.0000	200	1.8505	1.8518	2.3481	2.9007	2.9385	3.1215	3.3398
207	1.8578	1.8600	-1.2000	3.4562	4.0378	2.1751	0.0000								
208	1.8572	1.8594	-1.2000	3.4565	4.0375	2.1748	0.0000								
209	1.8566	1.8588	-1.2000	3.4568	4.0372	2.1745	0.0000								
210	1.8560	1.8582	-1.2000	3.4571	4.0369	2.1742	0.0000								
211	1.8554	1.8576	-1.2000	3.4574	4.0366	2.1739	0.0000								
212	1.8548	1.8570	-1.2000	3.4577	4.0363	2.1736	0.0000								
213	1.8542	1.8564	-1.2000	3.4580	4.0360	2.1733	0.0000								
214	1.8536	1.8558	-1.2000	3.4583	4.0357	2.1730	0.0000								
215	1.8530	1.8552	-1.2000	3.4586	4.0354	2.1727	0.0000								
216	1.8524	1.8546	-1.2000	3.4589	4.0351	2.1724	0.0000								
217	1.8518	1.8540	-1.2000	3.4592	4.0348	2.1721	0.0000								
218	1.8512	1.8534	-1.2000	3.4595	4.0345	2.1718	0.0000								
219	1.8506	1.8528	-1.2000	3.4598	4.0342	2.1715	0.0000								
220	1.8500	1.8522	-1.2000	3.4601	4.0339	2.1712	0.0000								
221	1.8494	1.8516	-1.2000	3.4604	4.0336	2.1709	0.0000								
222	1.8488	1.8510	-1.2000	3.4607	4.0333	2.1706	0.0000								
223	1.8482	1.8504	-1.2000	3.4610	4.0330	2.1703	0.0000								
224	1.8476	1.8500	-1.2000	3.4613	4.0327	2.1700	0.0000								
225	1.8470	1.8492	-1.2000	3.4616	4.0324	2.1697	0.0000								
226	1.8464	1.8486	-1.2000	3.4619	4.0321	2.1694	0.0000								
227	1.8458	1.8480	-1.2000	3.4622	4.0318	2.1691	0.0000								
228	1.8452	1.8474	-1.2000	3.4625	4.0315	2.1688	0.0000								
229	1.8446	1.8468	-1.2000	3.4628	4.0312	2.1685	0.0000								
230	1.8440	1.8462	-1.2000	3.4631	4.0309	2.1682	0.0000								
231	1.8434	1.8456	-1.2000	3.4634	4.0306	2.1679	0.0000								
232	1.8428	1.8450	-1.2000	3.4637	4.0303	2.1676	0.0000								
233	1.8422	1.8444	-1.2000	3.4640	4.0300	2.1673	0.0000								
234	1.8416	1.8438	-1.2000	3.4643	4.0297	2.1670	0.0000								
235	1.8410	1.8432	-1.2000	3.4646	4.0294	2.1667	0.0000								
236	1.8404	1.8426	-1.2000	3.4649	4.0291	2.1664	0.0000								
237	1.8398	1.8420	-1.2000	3.4652	4.0288	2.1661	0.0000								
238	1.8392	1.8414	-1.2000	3.4655	4.0285	2.1658	0.0000								
239	1.8386	1.8408	-1.2000	3.4658	4.0282	2.1655	0.0000								
240	1.8380	1.8402	-1.2000	3.4661	4.0279	2.1652	0.0000								
241	1.8374	1.8396	-1.2000	3.4664	4.0276	2.1649	0.0000								
242	1.8368	1.8390	-1.2000	3.4667	4.0273	2.1646	0.0000								
243	1.8362	1.8384	-1.2000	3.4670	4.0270	2.1643	0.0000								
244	1.8356	1.8378	-1.2000	3.4673	4.0267	2.1640	0.0000								
245	1.8350	1.8372	-1.2000	3.4676	4.0264	2.1637	0.0000								
246	1.8344	1.8366	-1.2000	3.4679	4.0261	2.1634	0.0000								
247	1.8338	1.8360	-1.2000	3.4682	4.0258	2.1631	0.0000								
248	1.8332	1.8354	-1.2000	3.4685	4.0255	2.1628	0.0000								
249	1.8326	1.8348	-1.2000	3.4688	4.0252	2.1625	0.0000								
250	1.8320	1.8342	-1.2000	3.4691	4.0249	2.1622	0.0000								
251	1.8314	1.8336	-1.2000	3.4694	4.0246	2.1619	0.0000								
252	1.8308	1.8330	-1.2000	3.4697	4.0243	2.1616	0.0000								
253	1.8302	1.8324	-1.2000	3.4700	4.0240	2.1613	0.0000								
254	1.8296	1.8318	-1.2000	3.4703	4.0237	2.1610	0.0000								
255	1.8290	1.8312	-1.2000	3.4706	4.0234	2.1607	0.0000								
256	1.8284	1.8306	-1.2000	3.4709	4.0231	2.1604	0.0000								
257	1.8278	1.8300	-1.2000	3.4712	4.0228	2.1601	0.0000								
258	1.8272	1.8294	-1.2000	3.4715	4.0225	2.1598	0.0000								
259	1.8266	1.8288	-1.2000	3.4718	4.0222	2.1595	0.0000								
260	1.8260	1.8282	-1.2000	3.4721	4.0219	2.1592	0.0000								
261	1.8254	1.8276	-1.2000	3.4724	4.0216	2.1589	0.0000								
262	1.8248	1.8270	-1.2000	3.4727	4.0213	2.1586	0.0000								
263	1.8242	1.8264	-1.2000	3.4730	4.0210	2.1583	0.0000								
264	1.8236	1.8258	-1.2000	3.4733	4.0207	2.1580	0.0000								
265	1.8230	1.8252	-1.2000	3.4736	4.0204	2.1577	0.0000								
266	1.8224	1.8246	-1.2000	3.4739	4.0201	2.1574	0.0000								
267	1.8218	1.8240	-1.2000	3.4742	4.0198	2.1571	0.0000								
268	1.8212	1.8234	-1.2000	3.4745	4.0195	2.1568	0.0000								
269	1.8206	1.8228	-1.2000	3.4748	4.0192	2.1565	0.0000								
270	1.8200	1.8222	-1.2000	3.4751	4.0189	2.1562	0.0000								
271	1.8194	1.8216	-1.2000	3.4754	4.0186	2.1559	0.0000								
272	1.8188	1.8210	-1.2000	3.4757	4.0183	2.1556	0.0000								
273	1.8182	1.8204	-1.2000	3.4760	4.0180	2.1553	0.0000								
274	1.8176	1.8198	-1.2000	3.4763	4.0177	2.1550	0.0000								
275	1.8170	1.8192	-1.2000	3.4766	4.0174	2.1547	0.0000								
276	1.8164	1													

Thus, we may argue that the difference between the two means is significant with a probability error $P < 0.05$.

Confidence interval (CI) of the difference between two means.

The significance of the difference between the two means D can also be assessed using the 95% CI. The 95% CI is defined as:

$$(D - t^*SE, D + t^*SE) \quad t \text{ is the 5\% point of the t-distribution for } n_1 + n_2 - 2 \text{ df}$$

If zero is not included in the 95% CI, there is significance difference between the two treatments.

Complete Example in SPSS

To test the effectiveness of treatment of RRMS in two groups of patients and whether it differs, we are checking the annual frequency of relapses with t-test for two independent samples.

	group	improvement
1	1.00	25.00
2	1.00	31.00
3	1.00	-12.00
4	1.00	43.00
5	1.00	-7.00
6	1.00	38.00
7	1.00	49.00
8	1.00	34.00
9	1.00	-11.00
10	2.00	30.00
11	2.00	27.00
12	2.00	42.00
13	2.00	32.00
14	2.00	31.00
15	2.00	28.00
16	2.00	39.00
17	2.00	45.00


```

T-TEST GROUPS=group(1 2)
/MISSING=ANALYSIS
/VARIABLES=improvement
/CRITERIA=CI(.95).

```

T-Test

[DataSet0] C:\Users\Apyycho\Desktop\IME\ergasia1\wemiss.sav

Group Statistics

	group	N	Mean	Std. Deviation	Std. Error Mean
improvement	1.00	9	21.1111	24.39847	8.11262
	2.00	8	34.2500	6.79811	2.40349

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
improvement	Equal variances assumed	15.380	.001	-1.472	15	.162	-13.13889	8.92669	-32.16567	5.88790
	Equal variances not assumed			-1.553	9.383	.154	-13.13889	8.48136	-32.16129	5.88351

Remark:

We see from the table above the 95% CI includes zero and the value of $P = 0.162 > 0.05$ so our initial hypothesis that the two treatments did not differ in their effectiveness is correct.

One-way ANOVA

Analysis of Variance is a test that looks at the variance, or ways in which data is different, for more than two groups. One-way ANOVA is used for groups that only have one independent variable. It checks if the scores within each group vary about the mean, which is called within group variance. It also checks if the means between the groups vary, which is called between group variance. When you do an one-way ANOVA and get significant results it means that there is some difference between the groups, you must do further tests to determine where the difference is.

The basic logic behind the ANOVA:

If we have 4 groups, group 1 could differ from groups 2-4, groups 2 and 4 could differ from groups 1 and 3, group 1 and 2 could differ from 3, but not 4, etc.

Since our hypothesis should be as precise as possible (presuming you're researching something that isn't completely new), you will want to determine the precise nature of these differences.

There are two sources of variation here, the *between group* and the *within group variation*. This gives us the basic layout for the ANOVA table.

Source	SS	df	MS	F
Between				
Within				
Total				

SS stands for Sum of Squares. It is the sum of the squares of the deviations from the means. In other words, each number in the SS column is a variation.

df stands for degrees of freedom.

MS stands for Mean Square. It is a kind of "average variation" and is found by dividing the variation by the degrees of freedom. So, each number in the MS column is found by dividing the number in the SS column by the number in the df column and the result is a *variance*.

F stands for an F variable. F was the ratio of two independent chi-squared variables divided by their respective degrees of freedom. So the F column will be found by dividing the two numbers in the MS column.

Filling in the ANOVA's table

Sum of Square = Variations

You can add up the two sources of variation, the *between group* and the *within group*.

df = Degrees of Freedom

Total degrees of freedom, $N-1$, where N is the number of treatments or groups.

If k groups were there in the problem (*we are comparing between the group*) there are $k-1$ degrees of freedom. In general, that is one less than the number of groups, since k represents the number of groups, that would be $k-1$.

This raises the question of how many degrees of freedom there are *within* the groups. Well, if there are $N-1$ degrees of freedom altogether, and $k-1$ of them were between the groups, then $(N-1) - (k-1) = N-1-k+1 = N-k$ of them are within the groups.

Mean Squares = Variances

The variances are found by dividing the variations by the degrees of freedom, so divide the SS(between) by the df(between) to get the MS (between) and divide the SS(within) by the df(within) to get the MS(within) .

There is no total variance. Well, there is, but no one cares what it is, and it isn't put into the table.

F

Once you have the variances, you divide them to find the F test statistic.

So, divide MS(between) by MS(within) to get F .

Ideally we want to maximize MS_{between} or $MS_{\text{treatment}}$, because we're predicting that our treatment will differentially effect our groups.

MS_{within} or MS_{error} = average variance among subjects in the same group

Ideally we want to minimize MS_{error} , because -ideally- our treatment influences everyone equally – everyone improves, and does so at the same rate (i.e. variability is low) .

If $F = MS_{\text{treatment}} / MS_{\text{error}}$, then making $MS_{\text{treatment}}$ large and MS_{error} small will result in a large value of F

Like t , a large value corresponds to small p -values, which makes it more likely to reject H_0

However, before we calculate MS, we need to calculate what are called *sums of squares*, or SS

Example for how we fill the ANOVA's table

Treatment	Measures		
X	1	2	2
Y	5	6	5
Z	2	1	

Step1:

$$\text{Mean}_x = (1+2+2)/3=1.667$$

$$\text{Mean}_y = 5.333$$

$$\text{Mean}_z = 1.5$$

$$\text{Overall mean} = \mu = (1+2+2+5+6+5+2+1)/8=3$$

$$\text{Estimated effects} = \text{Estimated treatment mean} - \text{Estimated overall mean}$$

$$A1 = \text{Mean}_x - \mu = -1.333$$

$$A2 = 2.333$$

$$A3 = -1.5$$

Step 2: The ANOVA table

Cause of the variation	df	SS	MS	F
Treatment
Residuals	
Total		

$$dftreat=3-1=2$$

$$dftot=8-1=7$$

$$dfres=7-2=5$$

SStreat = "sum of squares between treatment groups"

$$= \sum A_i^2 \cdot n_i$$

$$= (-1.33)^2 \cdot 3 + (2.33)^2 \cdot 3 + (1.5)^2 \cdot 2 = 26.17$$

SSres = "sum of squares within treatment groups"

$$= \sum_i \sum_j (y_{ij} - \text{mean}_i)^2 = \sum_i ss_{rowi}$$

$$= (1 - 1.667)^2 + (2 - 1.667)^2 + (2 - 1.667)^2 + [0.667] + [0.5] = 1.83$$

SStot = "Total sum of squares"

$$= \sum_{ij} (y_{ij} - \mu)^2$$

$$= (1 - 3)^2 + (2 - 3)^2 + \dots + (1 - 3)^2 = 28$$

Remark:

The total "SS" is always equal to the sum of the other "SS". $SStot = SStreat + SSres$

MS = SS/df, then:

$$MStreat = SStreat / dftreat = 26.17 / 2 = 13.08$$

$$MSres = SSres / dfres = 1.83 / 5 = 0.37$$

The F-value is just given by: $F = MStreat / MSres = 13.08 / 0.37 = 35.68$

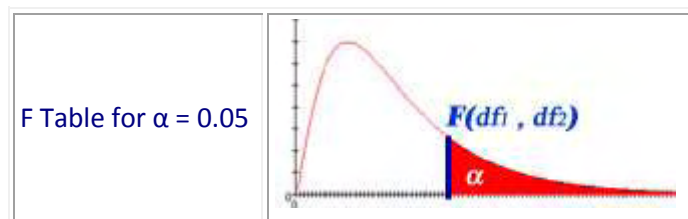
Interpretation:

The F -value says us how far away we are from the hypothesis "we cannot distinguish between error and treatment", i.e. "Treatment is not relevant according to our data"! A big F-value implies that the effect of the treatment is relevant!

The significance of the value F is determined in a similar manner to t-test (ie. Simulate random 10000 times the study, assuming no different on the treatments and calculate the 10000 F-tests, which form the F-distribution, and we find the rate of F-tests that are larger than the F)

If $F > 5\%$ point of F-distribution we can claim that the treatments differ significantly (with a small probability of error $P < 0.05$)

F-distribution



/	df ₁ =1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
df ₂ =1	161.44 76	199.50 00	215.70 73	224.58 32	230.16 19	233.98 60	236.76 84	238.88 27	240.54 33	241.88 17	243.90 60	245.94 99	248.01 31	249.05 18	250.09 51	251.14 32	252.19 57	253.25 29	254.3 144
2	18.512 8	19.000 0	19.164 3	19.246 8	19.296 4	19.329 5	19.353 2	19.371 0	19.384 8	19.395 9	19.412 5	19.429 1	19.445 8	19.454 1	19.462 4	19.470 7	19.479 1	19.487 4	19.49 57
3	10.128 0	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	8.7855	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.5720	8.5494	8.526 4
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	5.9988	5.9644	5.9117	5.8578	5.8025	5.7744	5.7459	5.7170	5.6877	5.6581	5.628 1
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.3985	4.365 0
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990	4.0600	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.668 9
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767	3.6365	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.229 8
8	5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.927 6
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7475	2.706 7
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782	2.9130	2.8450	2.7740	2.7372	2.6996	2.6609	2.6211	2.5801	2.537 9
11	4.844 3	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962	2.8536	2.7876	2.7186	2.6464	2.6090	2.5705	2.5309	2.4901	2.4480	2.4045
12	4.747 2	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.3410	2.2962
13	4.667 2	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.6710	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064
14	4.600 1	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6022	2.5342	2.4630	2.3879	2.3487	2.3082	2.2664	2.2229	2.1778	2.1307
15	4.543 1	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658
16	4.494 0	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096
17	4.451 3	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943	2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.1040	2.0584	2.0107	1.9604
18	4.413 9	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168
19	4.380 7	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779	2.3080	2.2341	2.1555	2.1141	2.0712	2.0264	1.9795	1.9302	1.8780

20	4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928	2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3660	2.3210	2.2504	2.1757	2.0960	2.0540	2.0102	1.9645	1.9165	1.8657	1.8117
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.9380	1.8894	1.8380	1.7831
23	4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201	2.2747	2.2036	2.1282	2.0476	2.0050	1.9605	1.9139	1.8648	1.8128	1.7570
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002	2.2547	2.1834	2.1077	2.0267	1.9838	1.9390	1.8920	1.8424	1.7896	1.7330
25	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821	2.2365	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.7110
26	4.2252	3.3690	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655	2.2197	2.1479	2.0716	1.9898	1.9464	1.9010	1.8533	1.8027	1.7488	1.6906
27	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501	2.2043	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7306	1.6717
28	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360	2.1900	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541
29	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229	2.1768	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6376
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107	2.1646	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223
40	4.0847	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240	2.0772	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.0970	2.0401	1.9926	1.9174	1.8364	1.7480	1.7001	1.6491	1.5943	1.5343	1.4673	1.3893
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.1750	2.0868	2.0164	1.9588	1.9105	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.4290	1.3519	1.2539
∞	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799	1.8307	1.7522	1.6664	1.5705	1.5173	1.4591	1.3940	1.3180	1.2214	1.0000

Bonferroni adjustment

For Bonferroni adjustment you divide the p value to be achieved for significance by the number of paired comparisons to be made. So if you have three groups, and the overall test of significance between them comes out significant (using .05 as the threshold value), you might want to compare all the pairs to see which are significantly different. How many possible pairs are there to compare? The formula $k(k-1)/2$ tells you – where k is the number of groups or conditions. So here $k(k-1)/2 = 3 * 2 / 2 = 3$. So any pair has to achieve a sig value on a paired test smaller than $.05/3 = .017$ to be sig at the .05 level. With four groups, and again wanting to compare all possible pairs ($k(k-1)/2 = 6$), then p for any pair has to be smaller than $.05/6 = .0083$ to be sig. Put simply, the adjusted p value (or alpha level as it is sometimes called) for n paired comparisons is:

$$\frac{\text{Target p value}}{n}$$

Bonferroni adjustment is often seen as a bit too conservative, however, i.e. it protects against the danger of overclaiming the number of significant differences between pairs of values/ conditions/ groups when doing multiple followup comparisons, but it does so at the cost of possibly underclaiming.

Post Hoc Tests

If the ANOVA show that there are significant differences between the groups then we can make individual comparisons between groups, e.g. To compare a group with the group b, using the t-test. However, this t-test differs from the previous in SE [here calculated using the random variation, the error].

A complete example in SPSS

	group	baseline	thirdmonth	diff
1	1.00	150.00	120.00	30.00
2	1.00	157.50	126.67	30.83
3	1.00	153.33	120.00	33.33
4	1.00	165.17	113.33	51.84
5	1.00	165.00	120.00	45.00
6	1.00	180.00	120.00	60.00
7	1.00	165.00	136.67	28.33
8	1.00	150.00	118.33	31.67
9	1.00	147.50	133.33	14.17
10	2.00	150.33	130.00	20.33
11	2.00	160.00	133.33	26.67
12	2.00	180.00	12.33	147.67
13	2.00	151.67	120.00	31.67
14	2.00	154.17	123.33	30.84
15	2.00	155.00	126.67	28.33
16	2.00	155.83	116.67	39.16
17	2.00	151.67	116.67	35.00
18	3.00	163.33	165.77	-2.44
19	3.00	157.21	150.34	-6.13
20	3.00	178.81	176.56	-2.27
21	3.00	167.98	157.37	-10.61
22	3.00	157.90	161.22	-3.32
23	3.00	169.34	171.56	-2.22
24	3.00	196.45	170.23	26.22
25	3.00	158.68	162.34	-3.66

Oneway

ANOVA

diff

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	7729.135	2	3864.568	6.192	.007
Within Groups	13730.900	22	624.132		
Total	21460.035	24			

We observe that the Pvalue = 0.007 < 0.05 so our assumption is wrong since there is a difference in the effectiveness between drugs and placebo. In the post-hoc tests we will find detailed comparisons between drugs and placebo.

Chapter 3 – Results in Python

Software

Our application works as follows:

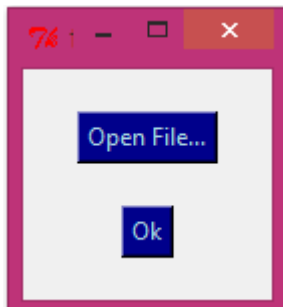
Window 1:

The user can press one of the 2 Buttons Independent T-test or One-Way Anova:

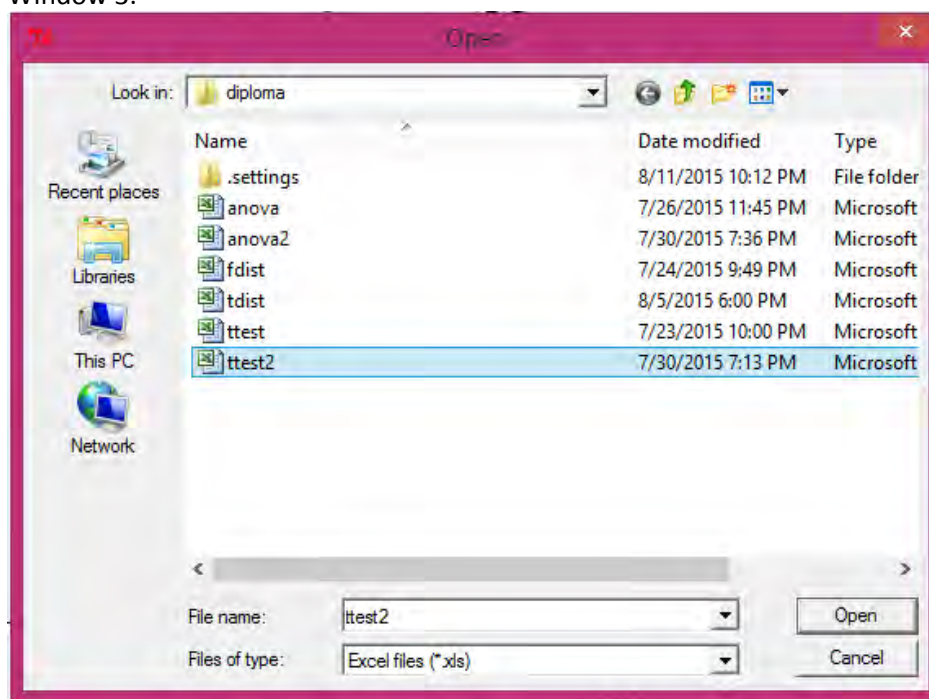


We saved our measures in a excel file. The second window, which automatically opens when the user press one of the 2 buttons above, helps us to browse in our hard disk(window 3) and open it then click Ok. With this click, we close the window 1 and appears the following window 3, the application keeps the file's path and then we can select between the two buttons for a t-test or an one-way anova.

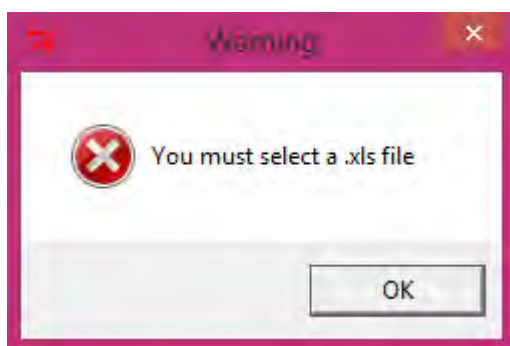
Window 2:



Window 3:



If we press the OK button (window 2) without selecting a file at first, a warning message appears that reminds us to select one file if we want to continue.



If we press the Button: Independent T-test

We will run the same example of page 8

The results with our application are the same and shown below



We run and another one example with SPSS and with our application.

	group	fev
1	1.00	2.12
2	1.00	1.63
3	1.00	2.00
4	1.00	2.30
5	1.00	2.02
6	1.00	1.74
7	1.00	2.49
8	1.00	2.30
9	1.00	2.19
10	2.00	2.22
11	2.00	2.20
12	2.00	3.71
13	2.00	2.49
14	2.00	2.64
15	2.00	1.38
16	2.00	2.95
17	2.00	2.49
18	2.00	2.68
19	2.00	2.42
20	2.00	1.96

```

T-TEST GROUPS=group(1 2)
/MISSING=ANALYSIS
/VARIABLES=fev1
/CRITERIA=CI(.95).

```

T-Test

[DataSet1] C:\Users\Ayvika\Desktop\DM\all\ergasia1\w0m1a8.sav

Group Statistics

group	N	Mean	Std. Deviation	Std. Error Mean
fev1 1.00	9	2.1878	.27526	.09175
2.00	11	2.4873	.58548	.17653

Independent Samples Test

		Levene's Test for Equality of Variances		T-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
fev1	Equal variances assumed	1.489	.241	-1.784	18	.091	-.37949	.21278	-.80953	.06754
	Equal variances not assumed			-1.907	14.784	.076	-.37949	.19895	-.80406	.04509

T-Test

Group Statistics

group	N	Mean	Std. Deviation	Std. Error
1	9	2.09	0.28	0.09
2	11	2.47	0.71	0.21

Independent Samples Test

t	df	Mean Diff.	Std. Error Diff.	95% CI Lower	95% CI Upper
-1.784	18	-0.38	0.213	-0.868	0.109

Sig=0.091

Significance of the difference

Comparing the value t (the sign is ignored) with the value 5% point of the t-distribution (df shown in the table above):

$1.78 < 2.10$

The value t is smaller than 5% point of the t-distribution. Then, we can claim that the t is a random value and not a significant one (ie not different from zero), with a probability error (P-value) $P < 0.05$ (ie a small error probability). Thus, we may argue that the difference between the two means is not significant with a probability error $P < 0.05$

Zero is included in the 95% CI and thus, there is not significance difference between the two treatments.

Close

If we press the Button: One-way Anova

We will execute the same example of page 14

The results with our application are the same and shown below



Results					
Group I - Group J	Mean Difference	Error	Sig	95% CI Upper Bound	95% CI Lower Bound
0 - 1	-12	2	0	-18	-5
0 - 2	30	2	0	24	36
1 - 0	12	2	0	5	18
1 - 2	42	3	0	36	48
2 - 0	-30	2	0	-36	-24
2 - 1	-42	3	0	-48	-36
Close					

We run and another one example with SPSS and with our application.

	treatment	data
1	1.00	62.00
2	1.00	74.00
3	1.00	86.00
4	1.00	74.00
5	1.00	91.00
6	1.00	37.00
7	2.00	69.00
8	2.00	43.00
9	2.00	100.00
10	2.00	94.00
11	2.00	100.00
12	2.00	98.00
13	3.00	50.00
14	3.00	120.00
15	3.00	100.00
16	3.00	288.00
17	3.00	4.00
18	3.00	76.00

Oneway

ANOVA

data

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	3897.333	2	1948.667	.558	.584
Within Groups	52388.667	15	3492.578		
Total	56286.000	17			

The P-value between groups (Sig.) $P = 0.584$, so there is no significant difference between groups ($P < 0.05$). From the 95% CI the difference between groups are compared in pairs is containing zero.

ANOVA

Groups	SumOfSquares	df	MeanSquare	F	sig.
BetweenGroups	3897.190	2	1948.595	0.558	0.584
WithinGroups	52388.667	15	3492.578		
Total	56285.857	17			

Conclusion:

$0.558 < 3.68$

The value f is smaller than 5% point of the F -distribution. Then, we can claim that the f is a random value and not a significant one (ie not different from zero), with a probability error (P-value) $P < 0.05$ (ie a small error probability). Thus, we may argue that the non difference between the groups is not significant with a probability error $P < 0.05$

The p -value is larger than the significance level of 0.05, so we can't reject the null hypothesis. The null hypothesis is that the groups were the same.

Post-hoc Tests

Chapter 4 – Conclusion

The independent samples t-test is used to test the hypothesis that the difference between the means of two samples is equal to 0 (this hypothesis is therefore called the null hypothesis). The program displays the difference between the two means, and the 95% Confidence Interval (CI) of this difference. Next follow the test statistic t , the Degrees of Freedom (DF) and the two-tailed probability P . When the P -value is less than the conventional 0.05, the null hypothesis is rejected and the conclusion is that the two means do indeed differ significantly.

The purpose of ANOVA is almost the same as the t tests. The goal is to determine whether the mean differences that are obtained for the sample data are sufficiently large to justify a conclusion that there are mean differences between the populations from which the samples were obtained

ANOVA allows researcher to evaluate all of the mean differences in a single hypothesis test using a single alpha-level (in our examples $\alpha=0.05$) and, thereby, keeps the risk of a Type I error under control no matter how many means are being compared. However, what if we just compared each of the groups in a pairwise manner like the Bonferroni's correction, using a 'testwise' α -level = α -level / (number of tests)

Chapter 5 - Acknowledgements

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