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Eureka Hellas Study case:

"Improving its distribution system on static and probabilistic data, its depot's facility location in Attica based on its clients demands and its forecasting"

by

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The case of Eureka Hellas S.A. in Attica

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Abstract

In this thesis, we try to optimize operations referring to the Eureka Hellas S.A. To begin with, we try to improve the vehicle routing problem of Eureka Hellas S.A. by minimizing its current cost. Moreover, we examine which would be the cost of the implementation of a Stochastic Customers Vehicle Routing Strategy by the company. Furthermore, we propose a forecast of a product of Eureka Hellas S.A. and finally, propose an optimal facility location for the placement of the depot of Eureka Hellas in Attica, by considering the demands of its customers.

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1. Introduction

The term “Operational Research” historically derives from the research to optimize military operations. The use of such operational research methods extended to all areas of human activity by the end of the Second World War. Indicatively, one can mention applications in industry, banking, insurance, agriculture, transport, telecommunications, inventory management, investments, financial management, marketing, technical construction, equipment maintenance, etc. The Operations Research is a wide scientific research field, where are being used optimization, simulation techniques. data probability techniques and statistics.

Combinatorial optimization is a branch of Optimization in Applied Mathematics and Computer Science, which brings together many other scientific fields such as operations research, algorithmic theory, computational complexity, and artificial intelligence. Algorithms of Combinatorial Optimization are used for resolving the problems that are considered quite difficult, due to the large number of possible combinations - solutions. Solving Combinatorial Optimization problems by such algorithms is achieved by reducing the total number of feasible solutions, but with a more efficient, at the same time, search. The scientific area of Combinatorial Optimization is mainly engaged in solving problems that are modeled with network flows. The theory of flows in networks is one of the most important branches of Applied Mathematics and Operations Research. Nowadays. there is a wide range of everyday applications modeled using Combinatorial Optimization problems. This also demonstrates the significant value of this scientific field. Also, it is crucial to highlight that the Combinatorial Optimization deals with deterministic problems and not with problems that involve stochastic processes.

Networks and graphs have the ability to model effectively many practical problems of a very large scale. There have been developed algorithms for graphs which utilize these discrete data structures, such as networks. As a result, these algorithms have a more improved theoretically complexity and computational behavior. Nowadays, the Operational Research problems are so difficult to solve so that it becomes indispensable to solve them with mathematical formulations and modern computer software packages. For example, the problem of the optimal geographic distribution of goods from production centers to consumption centers – customers (Transportation Problem) and the vehicle traffic movement in a road network (Vehicle Routing Problem) are combinatorial flow problems in networks. The Vehicle Routing Problem is one of the most studied combinatorial optimization problem that tries to find an optimal set of routes that needs to be performed by a fleet of vehicles, so as to serve a set of customers.

Furthermore, there are many advantages of using models to solve problems. Some of this is control of alternative scenarios, savings, and generally to minimize risk.

Each modern entity shall seek to maximize the value of the goods / services offered to its customers. More specifically, implementation of logistics tend to coordinate all the efforts made in each link in the supply chain, so that the aimed increase in value in one phase, doesn't cause the reduction of the standard value of any previous or subsequent phase. Operation of logistics is a composition in order to optimize the individual functions: the management of the logistics environment, the administration carrier operation, the storage and control of raw materials, the production management, both planning and forecasting, the product distribution, the customer service and finally the products management.

The increase in value is achieved through five strategic goals that derive from the Logistics operation:

- Design, development, production and delivery of products without defects
- Usage of time as a competitive tool
- Reduction in total cost and increase of the reliability in supply chain
- Establishment and maintenance of technological excellence in a competitive environment
- Minimization of the risks stemming from the external environment logistics

In this thesis, the solution of the problem is based on already developed methods and algorithms of Operational Research and Mathematical Programming.

All mathematical models consist of some basic elements. These are the decision variables, the performance criteria, the objective function, parameters, constraints and the hesitation of the model (feasibleness). It is crucial for a model to be simple, complete, intuitive and adaptive.

The stages that have been followed, in general, in this thesis are:

- Bibliographic research on the subject of the supply chain, the vehicle routing problem and the stock management
- Data collection
- Analyzing the data that have been collected
- Improving the distribution of products and forecasting of Eureka's product 000021.

More specifically:

In Introduction are some general information about Operational Research, Combinational Research and Supply Chain, in the context of including the thesis and raising knowledge and procedures of these.
Chapter 1 is an Introduction to this thesis.
Chapter 2 refers to the issues that are dealt with Logistics.
Chapter 3 refers to the Vehicle Routing Problem and its extensions.
Chapter 4 refers to Inventory Management.
Chapter 5 refers to the case of Eureka Hellas Company. It includes the results of this thesis for improving the distribution of Eureka Hellas based on static and probabilistic customers (therefore and demands) with comparison to the results of the MRP program that is currently being used by the company. Also, it includes an improved forecasting method for product 000021 of Eureka Hellas.

2. Logistics Management

2.1 Introduction to Logistics Management

The term logistics is an ambiguous and complex concept, covering a huge range of procedures for design, implementation and control in the business field. The essential elements that logistics consists of, is the management and strategic planning of the operation, the optimum utilization of the living (human) and inanimate (material) resources, production, storage and distribution of goods from raw materials to the finished product and from production to the shelf. Logistics, theoretically, serve the profitability of a business, ensuring the continuous availability of products and other resources, while enabling, at the same time, a smooth flow for performing the procedures that are mentioned above. Moreover, logistics aims to produce products with the lowest possible cost, to preserve them in the best possible way. In addition, it aims to fully utilize the material resources of the enterprise, to transfer the goods at the lowest cost with the least possible delay and finally, to achieve profitability to the company.

2.1.1 Basic Principles of Logistics Management

Logistics is a complex system that includes sub-processes, which in turn consist of operations and data. The main objectives of this system during its operation are as follows:

- Minimizing the operating cost
- Minimum investing cost
- Optimization of product and service quality
- High level of customer service

The cost of the logistics activities is a crucial part of the total cost of the products, when it reaches the hands of consumers. The cost of the logistics activities includes:

- Transport cost
- Storage and packing costs
- Stock keeping cost
- Physical movement of materials and information cost
- Cost control and stock inventory cost
- Cost of buildings and machinery and their depreciation

It should be noted that it is indispensable in the effort of minimizing the cost, to take into account the total cost and not the individual costs of the various activities. That's because the cost reduction in one area can result in increased costs to another area or even to create problems in the quality of services, mainly due to reduced efficiency.

The optimization of quality products and services include qualitative data. The main ones are:

- Availability: This includes the making of the necessary stocks for the company, so that it can cover at all times the needs of production or service.
- Capacity: This refers to the order execution speed and the reaching of this speed daily.
- Consistency: This refers to the continuous delivery of products that a customer demands, in good condition and quality, error free and properly labeled, so that he can be ensured that he has in his hands the product that he ordered.

The minimizing of costs and the offer of the highest possible quality of products and services, contribute to achieving the best possible level of customer service. The 7s include all areas that a business has to cover in the best way possible to provide the best possible service. Specifically, the 7s are:

- Correct product
- Correct dispatch
- Correct destination place

- Correct timing
- Correct amount
- Correct quality
- Correct price

2.1.2 Logistics management operations

A logistics system can be designed by each company differently, depending on their needs and their form. Nevertheless, the operations that the logistics field should take care of are specific and are as follows:

- *Purchases - Supplies:* The term "markets" refers to the acquisition of products, which can be incorporated in the final product, and can be raw materials, packaging materials, fuel etc, but also services. Market quantities are strongly influenced by the level of customer service that the company has set. At the same time, prices are no longer the main criterion that determines the markets. On the contrary, cooperation of suppliers, industry and buyers is required. The selection of product suppliers and services is a crucial issue for a company, which must organize both their selection and their evaluation. For that reason, are being used criteria of consistency in respect of transport times, of delivery times and of the speed that the supplier can respond to seasonality or possible demand change.

- *Storage:* The warehouse is an important part of logistics, the function of which, is organized with the most modernized methods. More specifically, the operation of a warehouse requires proper organization. This is done by using proper shelving equipment, special ramps, means for moving products, computer systems for better organization and utilization of space and finally, robotic systems for specialized tasks. A warehouse is responsible for processing the following operations: receipt of supplied goods, their placing, their storage and preservation, their quick finding and movement, their monitoring per codes and their preparation for distribution etc.

- *Stocks:* The term "inventory management" refers to the correct level of products that a company must have in order to be able to meet all its obligations. There are two questions that must be answered correctly in order to achieve the desired level. Firstly, the company must answer how many units of each product is needed and then at what time should those orders be done. The objective is to ensure the desired level of customer service with the minimum number of stocks, in order to reduce the direct and indirect cost.

- *Transport:* Transport refers to finding the most efficient way of movement of goods to and from the enterprise. Finding the optimal transport model includes the selection of the appropriate means of transport. If these means of transport are not proprietary, then it includes finding who is the most appropriate carrier. Last but not least, a very important criterion for the right choice of a transport model is the length of time that they will be delivered in comparison to the time that the customers want them. The objective is to be delivered without delays.

- *Distributions:* as transport, distributions refer to the transport from the warehouse or an enterprise to customers. Their difference lies in the fact that transfers refer to the movement of a few products in large quantities in a small number of customers. On the contrary, distributions refer to the movement of many products in small quantities and in a large number of customers.

Finally, it should be noted that an essential part of the logistics system also is information and knowledge. Nowadays, technological developments that have been achieved have made the communication between remote offices easy, and have created databases that allow the recordings and the control of both inventory and distribution.

2.1.3 Benefits of the incorporation of new technologies in logistics

- Better exploitation of materials (inanimate) of the enterprise resources. Physical resources are vehicles, storage facilities, equipment, etc. For example, using a telematics system to the company's vehicles has resulted in more organized movements and fewer routes. On the storage space using a WMS system and a wireless LAN, leads both to exploitation of every inch of the warehouse and to the faster execution of the procedures in it. For instance, the scanner reads the barcode of a carton and automatically sends the information to the central computer equipped with WMS system via wireless LAN.

- Better utilization of the living resources of the enterprise. More precisely, here are included not only the employees but also the customers, suppliers etc. For example, an ERP system or WMS informs about who are the profitable customers, provides valuable working hours for staff and contributes to a better organization of inbound flows from suppliers. A person who is interested for a product, no longer has to to deal with the control of the stock of the product, since this has been taken of by the system itself. The warehouse inventory stock is being done by pressing a button. The same is required to learn the incomes, costs and profits for one day or one month etc. At the same time, a person who is interested for a product, knows which kinds of products are desired and bought more and then configures his orders. In the company’s environment, as it now stands, the objective of the administration is not the detailed check of each part of the company - as this requires a lot of energy and time - but the automation of the processes, with the use of technological tools. That’s because these technologies provide important information to the management and to the executives of the company.

2.2 Role and utility of logistics management in companies

The department of logistics seems to play an important role in business organization nowadays. The existence of a company is inconceivable without having even, with a rudimentary organization, a part of logistics. All companies and organizations have also warehouses where they keep their products and a number of stocks.

The objective for today's enterprises is except from being viable and able to increase their profitability, to be capable of strengthening the areas of logistics and transport. These areas seem to have a great potential, especially in our country, Greece the last few years. Unfortunately, they are affected by the lack of a properly trained staff. As a result, the adaptation of a company's logistics management by their competitors, who don't have any knowledge and therefore use logistics falsely, has become a reality.

In any case and at any time, one will invest in modern technologies in the logistics sector, which may involve the construction of new advanced storage centers.

2.2.1 Position of logistics in the organizational structure of the company

Logistics have a strong presence and interfaces to the entire structure of a company. There aren't few those that connect logistics with the marketing department of the company, without this meaning that this is the only interface that presents the segment logistics.

In fact, logistics have a multidimensional role and develop interfaces with many parts of the enterprise at different stages of the production process. Indicatively we can mention the interfaces developed by the logistics system in the production department and more specifically with the planning and control of the production process. Moreover, we can mention the interfaces developed by the logistics system with the marketing department and the areas of customer service, packaging, order processing etc. Finally, we can also highlight the interfaces developed by the logistics system with the department of the finance of the company, through the maintenance of stock, the inventory control, the control of the distribution cost of products and the equipment costs etc. It's obvious that the objective of every company and the axis around which all activities are developed, is customer satisfaction. That will lead to the creation and expansion of the customer base and thus, increase the company's profitability. To achieve this goal, a very important aspect is the existence of a harmonious cooperation between all the departments of the company and the corresponding departments with suppliers and customers.

For these reasons it is crucial that cooperation, especially on issues relating to the activities of logistics, is achieved between almost all departments of the company. Of course this does not mean, in any way, that the team should only include logisticians. This would lead to inefficiency and wrong actions. Also, we should not forget the relationships that are developed between the company and the clients and we must highlight the importance of such relationships. From them derive a lot of activities in the logistics sector. It is actually a large percentage of logistics activities completed outside the protected and relatively controlled space of a company, which are directly dependent on the existence and maintenance of good relations between the customers and the company. The enterprise is also the one that will then select the clients who will have an important role in its profitability and establish partnerships with them over time.

So it is obvious that everything related to the organization of a company, can determined by logistics. Therefore, issues relating to the operation of logistics in an enterprise, are quite patchy, as they are dealing with more than one area of the factory organization. At the same time, they form together a flow cycle of the product, from the production of the factory, to the delivery in the hands of the consumer, who may also be an entire company.

Figure 1: Traditional organization model (Oxley, 1999).

2.3 Role of human resources in logistics management

Human resources is the most essential asset of the company – throughout the structure of the hierarchy of a company's logistics department – as it consists of the inferior to senior management. Human resources is the one who intervenes and works either spiritually or physically throughout the duration from planning to the total implementation of a system of logistics of a company. So it’s obvious that the human resources department contributes to the existence or not of a successful logistics management. Moreover, there are some parameters that show the importance of the intervention of this department that crews the logistics sector. Initially the department of logistics - like the other departments of a company - is required to adopt specific decisions, that might affect the company’s development. In order to smooth the operation of logistics, these decisions should be taken at the right hierarchical level and at the right time.

Additionally, the department of human resources can increase the productivity and generally the effectiveness and efficiency of the logistics of a company. A measure that shows, if a department of logistics is productive or not, are the indicators like the percentage of errors to the satisfied orders, or even to the satisfaction of the initial goals of the department.

The existence of predetermined objectives for the logistics department, from its creation is a key element for its successful operation.

Actually no one can accurately assess any department of a company, if he doesn’t know what is the purpose of its existence in the business web. Neither the workers can be productive and work as a united team, unless they know what is their work piece and what is the desired result, for which they are working. In logistics the main goal is customer satisfaction by minimizing the costs of the providing services, by the side of the enterprise or organization. Moreover, the empowerment of human resources is crucial, as it aims at a fully and effective utilization of workers’ skills. That leads to increase of the dynamics of the department of logistics, by achieving high efficiency and productivity levels.

2.4 Costs and logistics management

Issues that have to do with cost concern a lot the logistics management of a company. In fact, the cost of the products that arrive to the hands of the consumer is not only the result of the production cost, but also the result of the cost of storage, transport, distribution, administration, packaging and other procedures that take place. The distinctive goal of the company is to minimize all the costs of these logistics procedures that charge the company economically. In any case, the calculation of all the costs of the logistics is required, in view of the fact that the goal of the company is to minimize the sum of these costs.

The most important elements of the logistics costs are:

- Transport cost
- Storage and packaging cost
- Stock conservation cost
- Physical movement cost of materials and information

In general, every enterprise might lose and actually loses money from some customers and possibly from some of the products that it produces. Practically, this happens because according to modern cost control models and more specifically according to ABC analysis, 20% of the best company’s products or customers can bring in 80% of the company’s profits. Empirically, it has been proposed to amend this rule considering that 20% of the best customers bring in 80% of the company's profits, but 30% of these profits are being spent on servicing the less profitable customers.

In each case, the medium-sized customers are considered the most profitable ones, as they require good service but they pay the price of the product, without having any discounts on the prices. On the contrary, the biggest customers reduce the amount of the company's profits with discounts on the prices of the products that are being bought. In this logic, the company should be aware that any attempt to reduce the cost of the logistics will entail a fall in the quality or to the level of customer service. In reverse, any increase in the quality of customer service will be accompanied by a corresponding increase in costs for the company. On the other hand, the cost of logistics services doesn’t increase linearly with the increasing of the level of customer service, but for every increase in the level of the customer service, the expenditure is higher.

Another equally important factor in the formation of the cost of the logistics system of a company is the organization of the warehouses and the stock conservation. Nowadays, there are many ways to organize the warehouse in order to achieve the best management of products at the lowest possible cost. A recent trend in order to maintain the stocks of a company is the system of Just in Time (JIT). This system essentially proposes the minimization of the inventory that is being maintained by the company in the warehouse, without of course, provoking any lack in the coverage of the customer demands. In reality, the misplacement of products in the warehouse, for example, may increase the duration of the cycle of the products. For this reason it is very expensive to maintain stocks at every stage of the production process, such as packaging materials stocks, stocks of raw materials or finished products. Generally, the procedures performed in the warehouse must be accurate, quickly and economically performed, as they significantly affect the final configuration of the logistics cost.

Finally, it is worth to highlight the great contribution that the transport costs have in the total cost of the logistics system. It’s a factor that must be calculated precisely and accurately at the design of the logistics system. The transportation cost includes both the calculation of the costs of the transport fleet of the company and the time that should be spent to transport the products to the customer in the desired time. Indicative of the importance of transport costs in the total cost of the logistics system is the choice of the place where the raw materials needed to manufacture the product are produced, as the company’s location.

Furthermore, quality is a key element in achieving low cost in logistics management. As already pointed out before, mistakes in the logistics process are very expensive because they cost double the company. In such case the company is forced, for example, when a wrong product has been sent to a customer, to take back the product, ship it back, put it again in the warehouse, and send to the customer the right product. As a result, apart from the inconvenience and the time delay that the costumer endures in order to get the desired product, the company pays increased transport cost. To be precise, it pays costs of double tracks and disruption of the delivery schedule that might be drawn up.

To conclude with, many people believe that the best method to minimize the cost of logistics of an enterprise is by having the right system design from the start and by defining the objectives through this logistics system. Cost reduction occurs when the management of the logistics company minimizes the deviations from the predetermined objectives of the system.

2.5 Role of logistics in the economy

Productivity growth has a positive effect on prices of the sales of goods and services, the value of money, the possibility of more effective competition in foreign markets. Moreover, it has a positive effect in the economic growth, leading to increased employment levels and thus lower unemployment. By comparing the costs of logistics with other social activities of the seven most developed countries, its role in the economy becomes better understood. It turns out that the cost of logistics is ten times more than advertising, double the amount given for defense expenditure and equal to the amount given annually for drugs. This conclusion is reasonable when you consider that logistics is the third largest cost faced by a typical company when operated as shown in the following figure.

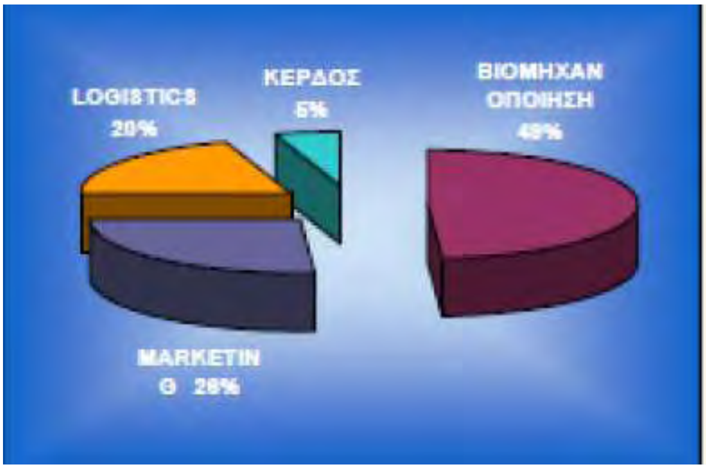

Figure 2: Participation rates of all costs of a typical enterprise.

2.6 The seven principles of supply chain management

In general, the strategy of logistics has three objectives:

- Reduce costs
- Reduce the employed capital
- Improve the service

Cost reduction is a strategy that aims at minimizing the variable costs that are related with transportation and storage. It is achieved by choosing alternative transportation and storage models. In this case, the primary objective is maximizing the profit while maintaining service levels stable.

The reduction of the employed capital is a strategy that aims to minimize the level of invested capital in the logistics system with incentive to maximize efficiency. Improved services are strategies that recognize that the benefits depend on the level of the supply of logistics service. It is possible - despite the rapid increase in costs brought from the increased service levels - to obtain profits from sales, which overcome the increase in overall costs. For such a service strategy to become successful, it should be developed in opposition to the service offered by their competitors.

The design of logistics handles four major categories of problems which are:

- The customer service levels
- The location of facilities
- Measures for storage
- The decisions on the transport of products

Each of these categories has a significant impact on system design as explained below.

The customer service objectives determine the provided service levels, which affect more than any other factor the system design. High levels of service enable centralized inventories in a few warehouses and also enable the use of less costly means of transport, in contrast to low levels. However, the maximum service levels eject the logistics costs, thus making the problem of defining service levels very difficult.

The strategy of positioning the premises, namely the geographical positioning of them is a draft on which logistics will follow. The route, by which the products are led to the market, is determined by the number, location and size of the installation and the market demand. In this problem, must also be considered, the movements of each product and the cost of the source or the intermediate storage points in the consumption area, whose selection affects the overall cost of distribution. The whole essence of the problem lies in finding the assignments minimum or maximum cost installation.

The inventory decisions relate to the way stocks are being managed. There are many strategies such as the promotion of stocks in depots or filling these points with feedbacks. These strategies follow specific rules or the placement of stocks in local or regional warehouses, based on specific products or with various continuous methods of controlling stocks. This policy of the company affects placement decisions of the installations and therefore should be counted in the strategy of logistics. Transport strategies include options of the transportation means, load sizes, routing, planning and decisions that affect the proximity of the warehouses to the customers. Even stock levels thought their load sizes affect transport decisions.

In summary, the above problem, as allocated to four major categories before, are important design areas because of the effect they have on profitability, capital flows and on the investment performance of the company. It should also be noted that any decision area shouldn’t be studied separately but in relation to the other, so as to take into account the outcome of their balancing.

2.7 Profits of logistics and future challenges

Apart from the positive effects that logistics have in a country's economy, their proper management has also positive effects at the company itself. The fact that logistics are essentially a fixed asset makes substantially difficult to be copied by other competing companies. If logistics are managed properly then it is also possible to reduce the costs in the requirements or to reduce the investment in stocks.

Above all, logistics can be a cause of a competitive advantage that a company can have a good pricing, a good promotion and a good product. Indeed, there is the advantage that the company can create a distribution network, designed in such a way that it cannot be copied by competitors, which will contribute to the increase of the company's customers. Furthermore, nowadays, a good management of logistics is a key factor for the competitiveness of the company but to become the logistics "weapon" of aggressive marketing, logistics department should be treated as a crucial integral part of the company’s strategy. Finally, it must be noted that the extent to which logistics are an integral part to the company's strategic planning process, might determine the future success of the company.

3. The Vehicle Routing Problem and its extensions

3.1 Introduction to the Travelling Salesman Problem

One of the most interesting and tried to solve difficult problems of operational research is the Travelling Salesman problem (TSP). More particularly, this problem belongs to the NP- hard problems, which basically means that the exact solution of the problem can be measured only for a small number of clients (e.g. 30-40). The complexity of the TSP problem increases exponentially with the increase of nodes. For this reason there have been studied and proposed until now many heuristics (heuristic) and meta-heuristics (metaheuristic) approaches to solve the TSP, which actually give very good results. The formulation of the problem of the traveling salesman involves finding the shortest route that connects a number of nodes from a given starting point, passing through each node exactly once and finally returning to the starting point. Namely, the street vendor is called to serve a particular set of clients (nodes), by visiting each customer exactly once and returning to the point from which he started (depot), while covering the shortest distance possible.

3.2 Introduction to the Vehicle Routing Problem

The problem of vehicle routing (VRP) is a generalized form of the traveling salesman problem (TSP) and plays nowadays an important role in the fields of physical distribution and logistics. The only difference is that unlike the TSP, the VRP refers to the routing of a fleet of vehicles for the distribution of the products. More specifically, the objective of VRP is for given distances and vehicle capacities, finding paths for a homogeneous fleet of vehicles such that:

- Cost minimization is being succeeded
- The demand doesn't exceed the capacity of the vehicle
- Each customer who has a predefined demand, must be served by only one vehicle visit
- Each vehicle must leave the warehouse (depot) and after having served all the customers, it must return back to the depot

Apart from the basic vehicle routing model, there are multitude variations of it that cover limitations arising in cases of real applications such as vehicle capacity, the heterogeneity of the fleet, the stochastic customer demand etc.

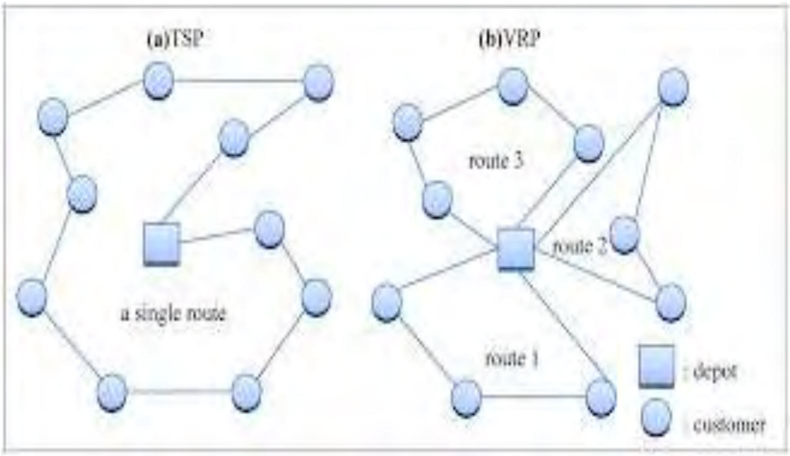

Figure 3: TSP and VRP example.

3.3 Extensions of the VRP problem

3.3.1 Vehicle Routing Problem with limited capacity of the fleet (CVRP)

The vehicle routing problem with limited capacity is one of the most common extensions of the VRP in cases of real applications. More specifically, according to the CVRP a given number of homogeneous fleet-with specific capacity vehicles- that depart from a common warehouse (depot) must serve the known requirements of customers, for the same type of products. Essentially a CVRP problem is the same as a classic VRP problem with the only difference that we have an additional constraint to take into consideration. This additional constraint is that each vehicle must have an upper capacity limit in the number of products that carries for the customers. The objective of the CVRP problem is to find such routes that minimize the total cost and simultaneously satisfy all the constraints for each vehicle. With other words, CVRP is an attempt to minimize the cost of the customer service so that all the quantities that are demanded by the customers and included in a route, with not exceeding the maximum capacity of the vehicle. The problem of the optimal routing of vehicles with limited capacity can be represented by a graph G = (V, E), which represents a network of routes. The set of vertices V = {0, 1, ... n} includes the base of the vehicle, namely the depot (this peak is usually the top 0) and any other peak represents a customer of the n, who must be served. Each arc (i, j) of all E represents a non-negative cost cij (travel cost) and tij time (travel time). Also each client i∈ {1,..., n} has a non-negative demand, which is denoted by di (we consider do = 0) and which has to be served. Also, the vehicle fleet consists of K vehicles, each of which has a predetermined capacity.

3.3.2 Vehicle Routing Problem with heterogeneous fleet

The problem of vehicle routing with heterogeneous fleet is one of the most common variants of the VRP in cases of real applications. More specifically, according to HVRP, a given number of known heterogeneous fleet capacity vehicles should serve the known requirements of the customers by a common warehouse (depot), while simultaneously minimizing the cost. It should be noted that heterogeneity of the fleet of vehicles may have either to do with the different capacity of the vehicles or due to the different types of vehicles of the fleet. Finally, a HVRP problem is essentially identical to a classic problem VRP with the only difference that we have two additional constraints to take into account. The first additional constraint is that the number of routes carried by all types of vehicles must not exceed the available total number of vehicles we had in our warehouse initially. The second additional constraint is that a customer must be served only once by one type of vehicle. As far as the formulation of the HVRP is concerned, a directed graph G = (V, A) is given, where V = {0, 1… n} is the set of n + 1 nodes and A is the set of arcs. Node 0 represents the depot, while the remaining node set V’= V \ {0} corresponds to the n customers. Each customer i ∈ V’ requires a supply of q_i units from the depot (we assume q_0 = 0). A heterogeneous fleet of vehicles is stationed at the depot and is used to supply the customers. The vehicle fleet is composed by m different vehicle types, with M = {1,..., m}. For each type k ∈ M, m_k vehicles are available at the depot, each having a capacity equal to Q_k. Each vehicle type is also associated with a fixed cost, equal to F_k, e.g., rental or capital amortization costs. In addition, each arc (i, j) ∈ A and each vehicle type k ∈ M has a non-negative routing cost c_{kij}. A route is defined as the pair (R, k), where R is a simple circuit in G containing the depot, and k is the type of vehicle associated with the route.

3.3.3 Capacitated Vehicle Routing Problem with heterogeneous fleet

The Vehicle Routing Problem with limited capacity and with heterogeneous fleet is a mix of CVRP problems with HVRP, and responds to real cases of vehicle routing problems applications. More specifically, according to HCVRP a given number of heterogeneous fleet with known capacity of each vehicle must serve the already known customer requirements from a common depot. This must be done without all customers' requirements exceeding the maximum capacity of the vehicle, while simultaneously ensuring the minimization of the cost. The capacitated vehicle routing problem with heterogeneous fleet (HCVRP) can be represented by a graph G = (V, E), which represents a network of routes. The set of vertices V = {0, 1..., n} includes the base of the vehicles, namely the depot (this peak is the peak 0). Every other peak represents each customer we serve and belong to all customers V '= V \ {0}. Each arc (i, j) of the total E corresponds to a non-negative cost cij (travel cost) and tij time (travel time). We also know that each customer i ∈ {1, ..., n} has a non-negative demand, which is denoted by di (we think $d_o = 0$), which is analyzed in Ukg_i and Ult_i depending on the weight and volume required by customer i order. The sum of the quantities of customers belongs to the set D (d_i ∈ D), while as a whole C define C = DU {0}. The heterogeneous fleet of vehicles is primarily in the warehouse (depot) and is used for the supply of customers. Composed of k different types, and for each type of vehicle k ∈ K, n_k vehicles are initially available in depot, each having a maximum capacity Qw_k in kg and Qv_k in lt. Each type of vehicle also has a fixed cost Fk, e.g. due to rental etc. The total number of vehicles available at the depot initially is n. In the following modelization we consider that heterogeneity exists only in the capacity of the vehicles.

The HCVRP mathematically is stated as follows:

$$\min\left(\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=0}^{K} D_{ij} * X_{ijk} * VC_k + \sum_{j=0}^{n}\sum_{k=0}^{K} X_{0jk} * FC_k\right)$$

St.

1. $\sum_{j=1}^{n}\sum_{k=0}^{K} X_{0jk} \le n$

2. $\sum_{j=1}^{n}\sum_{k=0}^{K} X_{i0k} \le n$

3. $\sum_{i=0}^{n}\sum_{k=0}^{K} X_{ijk} = 1 \qquad \forall j \in D$

4. $\sum_{j=0}^{n} X_{0jk} \le 1 \qquad \forall k \in K$

5. $\sum_{i=0}^{n} X_{ipk} - \sum_{j=0}^{n} X_{pjk} = 0 \qquad \forall p \in C, \ \forall k \in K$

6. $\sum_{i=0}^{n}\sum_{j=0}^{n}\left[Time_{ij} * X_{ijk} + g * Ukg_{j} \right] \le T \qquad \forall k \in K$

7. $\sum_{i=0}^{n}\sum_{j=0}^{n}\left[Time_{ij} * X_{ijk} + g * Ult_{j} \right] \le T \qquad \forall k \in K$

8a. $yw_{i0k} = 0 \qquad \forall i,k$

8b. $yv_{i0k} = 0 \qquad \forall i,k$

9a. $\sum_{i=0}^{n}\sum_{k=0}^{K} yw_{ijk} - \sum_{i=0}^{n}\sum_{k}^{K} yw_{jik} = Ukg_{j} \qquad \forall j \in D$

9b. $\sum_{i=0}^{n}\sum_{k=0}^{K} yv_{ijk} - \sum_{i=0}^{n}\sum_{k}^{K} yv_{jik} = Ult_j \qquad \forall j \in D$

10a. $\sum_{j=0}^{n}\sum_{k=0}^{K} yw_{0jk} = \sum_{j=1}^{n} Ukg_j$

10b. $\sum_{j=0}^{n}\sum_{k=0}^{K} yv_{0jk} = \sum_{j=1}^{n} Ult_j$

11a. $yw_{ijk} \leq Qw_k * X_{ijk} \qquad \forall i, j, k$

11b. $yv_{ijk} \leq Qv_k * X_{ijk} \qquad \forall i, j, k$

The first constraint states that the total number of the vehicles that leave the depot to go and serve customers cannot exceed the total number of the vehicles that are available initially at the depot. The second constraint assures us that the total number of all vehicles, after having served all their customers and having returned to the depot, cannot exceed the initially available total number of vehicles in the depot. Moreover, constraint 3 states that every customer i must be visited at most once and constraint 4 that every vehicle k leaves from the depot at most once. In addition, constraint 5 states that for every customer p and for every vehicle k that has serviced him, every vehicle k must leave from that client i (flow in-flow out). Constraint 6 states that every vehicle k can do its route and serve its customers the kg that are being demanded, in time less than the maximum possible T. Constraint 7 also ensures that every vehicle k can do its route and serve its customers the lt that are being demanded, in time less than the maximum possible T. Constraints 8a and 8b state that there must be no return in kg or lt back to the depot from the vehicles. Constraints 9a and 9b state that the demand of every customer i in kg and in lt must be satisfied exactly. Constraint 10a and 10b state that the total quantity in kg and in lt that leaves from depot by all the vehicles is equal to the demand in kg and in lt of all the customers that are being served by all the vehicles. Last but not least, constraints 11a and 11b are constraints of insurance that while there is available capacity in kg and in lt in vehicle k, the flow of quantities is being permitted.

3.3.4 Vehicle Routing Problem with time windows

The VRPTW is an important generalization of the VRP and a basic distribution management problem that can model many real-world problems. The VRPTW problem consist of designing a set of minimum cost routes, originating and terminating at a central depot, for a fleet of vehicles which serve a set of customers with known demands. The customers must be assigned exactly once to vehicles such that the vehicle capacities are not exceeded. The service at a customer must begin within the time window defined by earliest time and the latest time when the customer permits the start of service. Some of the more useful applications of the VRPTW include bank deliveries, postal deliveries, industrial refuse collection, national franchise restaurant deliveries, and school bus routing and security patrol services.

The VRPTW is given by a fleet of homogeneous vehicles denoted by V, a set of customers C and a directed graph G = (V, C). The graph consists of $|C| + 2$ vertices, where the customers are denoted 1, 2, …, n and the depot is represented by the vertex 0 (the driving-out depot) and n + 1 (the returning depot).

The VRPTW has multiple objectives. Such goals are minimizing not only the number of vehicles required, but also the total travel time, waiting time, and the total travel distance incurred by the fleet of vehicles. The set of arcs denoted by A represents connections between the depot and the customers and among the customers. No arc terminates in vertex 0, and no arc originates from vertex $n + 1$. With each arc (i, j), where $i \neq j$, we associate a cost c_{ij} and a time t_{ij}, which may include service time at customer i.

Each vehicle has a capacity q and each customer i a demand d_i. Each customer i has a time window $[a_i, b_i]$. A vehicle must arrive at the customer before b_i. It can arrive before a_i but the customer will not be serviced before. The depot also has a time window $[a_0, b_0]$. Vehicles may not leave the depot before a_0 and must be back before or at time b_{n+1}.

It is assumed that q, a_i, b_i, d_i, c_{ij} are non-negative integers, while the t_{ij} are assumed to be positive integers. It is assumed that the triangular inequality is satisfied for both the c_{ij} and the t_{ij}. The model contains two sets of decision variables x_{ijk} and s_{ik}. For each arc (i, j), where $i \neq j$, $i \neq n+1$, $j \neq 0$, and each vehicle k we define $x_{ijk} = 1$, if and only if the optimal solution, arc (i, j) is traversed by vehicle k and equal 0, otherwise.

The decision variable s_{ik} is defined for each vertex i and each vehicle k and denotes the time vehicle k starts to service customer i. In case the given vehicle k does not service customer i, s_{ik} does not mean anything. We assume $a_0 = 0$ and therefore $s_{0k} = 0$, for all k. In the VRPTW we want to design a set of minimal cost routes, one for each vehicle, such that: each customer is visited exactly once, every route originates at vertex 0 and ends at vertex $n + 1$, and the time windows and capacity constraints observed.

The VRPTW mathematically is stated as follows:

Min

St.

1.

2.

3.

4. ,

5.

6. ,

7. ,

8. ,

The first constraint states that each customer is visited exactly once, and the second that no vehicle is loaded with more than its capacity allows it to. Constraints 3,4,5 ensure that each vehicle leaves the depot (node 0) and that after arriving at a customer. the vehicle leaves again, and finally arrives at the depot. Constraint 6 states that a vehicle k cannot arrive at j before $s_{ik} + t_{ij}$ if it is travelling from *i* to *j*. Here K is a large scalar. Finally, constraint 7 ensures that time windows are observed and constraint 8 assures that the integrality constraints are not being violated. Note that an unused vehicle is modeled by driving the empty route (0, *n* + 1).

3.3.5 Vehicle Routing Problem with pick-up and delivery

The vehicle routing problem that includes pick-ups during the time of service constitutes a VRP problem in which, the probability that some customers may return some products is being predicted. Hence, in such type of problems the forecast of the necessary space in the vehicle for the placement of the returnable products is essential. This constraint makes the problem harder to solve and might result in poor utility of the capacity of vehicles in cases of long covered distances.

As far as the definition of the VRPPD is concerned, the cost of the route is the same with the cost of a simple VRP problem but with considering the new constraints that are being set and concern the number of deliveries and the volume of the products that has to be distributed. The VRPPD can be represented by a graph G = (V, E), which represents a network of routes. The set of vertices V = {1..., n} includes the clients, while the set V_o includes all the customers and the depot (client 0), $V_o=\{0,.,n\}$, thus $V_o=V \cup 0$. The set V_p (V_d) is the set that includes all the pick-up and delivery customers, whose number is shown by N_p (N_d), when N is the total number of all customers. The distance between customers i and j is denoted by c_{ij} and the vehicle capacity is denoted by Q. The pick- up demand of client i is denoted by p_i, whereas the delivery demand of client i is denoted by d_i. Moreover, the total number of vehicles is NV, y_{ij} is demand picked-up from clients and transported in arc (i, j). Also z_{ij} is the demand to be delivered to customers routed after node and transported in arc (i, j). Finally, x_{ij} is a binary decision variable which shows us if its equal to 1 that arc (i, j) belongs to the optimal set of routes.

The modelization of the VRPPD is:

Min

St.

1.

2.

3.

4.

5.

6.

7.

8. $z_{ij} \geq 0$

9.

The objective function seeks to minimize the total distance covered, meaning to minimize the total cost. The first two constraints ensure us that each client is visited by exactly one vehicle. Constraint 3 sets a limit on the vehicles that are being used. Furthermore, constraints 4 and 5 are flow equations for pick-up and delivery demands, respectively and guarantee that both demands are satisfied for each customer. Restriction 6 establishes that pick-up and delivery demands will only be transported using arcs included in the solution and impose an upper limit on the total volume transported by a vehicle in any given section of the route. To conclude with, constraints 7-9 define the nature of the decision variables.

3.3.6 Vehicle Routing Problem with multiple depots

The Multi-Depot Vehicle Routing Problem (MDVRP) is a generalization of the single depot VRP. In the MDVRP, multiple vehicles leave from multiple depots and must return to that depots, from where they have left, after completing their assigned tours. The objective of the MDVRP is to form a route with lowest cost to serve all customers from multiple depots. In general, the objective of the MDVRP is to minimize the total delivery distance or time spent in serving all customers.

The MDVRP can be represented by a graph, which represents a network of routes. In the modelization of the MDVRP we use some sets, some parameters, indexes and of course some decision variables. Precisely the set I includes all depots, while the set J includes all customers. A set K includes all vehicles, whose total number is denoted by N. The distance travelled between point i and j is denoted by c_{ij} and the capacity of each vehicle k is denoted by Q_k. Furthermore, the demand of each customer j is stated by d_j and Vi is the maximum throughput of depot i. As far as the decision variables used in the modelization of the MDVRP are concerned, two binary decision variables are being used. Firstly, x_{ijk} is equal to 1 if i immediately proceeds j on route k and equal to 0 if it’s otherwise. Also, z_{ij} decision variable is equal to 1 if customer j is allotted to depot i and 0 otherwise.

The modelization of the MDVRP is:

$$\text{Min} \sum_{i \in I \cup J} \sum_{j \in I \cup J} \sum_{k \in K} c_{ij} * x_{ijk}$$

1.

2. $k \in K$

3.

4.

5. $k \in K$

6.

7.

8.

9.

10.

The objective function seeks to minimize the total distance covered, meaning to minimize the total cost. The first constraint states that each customer has to be assigned to a single route. The second restriction assures us that there will be no exceeding in the capacity of the vehicles. Also, constraint 3 gives us the sub-tour elimination restriction. Constraint 4 is the flow in-flow out constraint. Moreover, constraint 5 ensures us that each route can be served at most once, while constraint 6 includes the capacity restrictions for the depots. Constraint 8 specifies that a customer can be assigned to a depot only if there is a route from the depot going through the customer. Finally, constraints 9 and 10, respectively, include the binary and positive values restrictions.

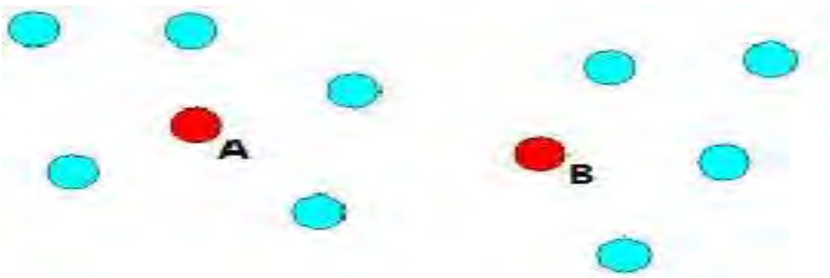

Figure 4: A multi depot VRP example (dots A and B are depots and the other ones customers).

Chapter 4: Inventory Models

4.1. Introduction to Inventory Models

For a company it is indispensable to manage efficiently its stock. As stock is defined the quantity of the products imported into the enterprise and exceeds the quantity of what is extracted from the enterprise. The creation of stocks can either be designed in order to resolve the presented differences between the supply and the demand of the products, or the result of various factors such as poor planning or extraordinary phenomena. The necessity of having stocks is mainly in light of the fact that there is uncertainty regarding the supply and the demand of goods. This uncertainty needs to be dealt for the satisfaction of the customers' needs.

The control of stocks (inventory control) is a technique with scientific bases designed to monitor the stock of the goods and takes decisions. Such decisions are when and in what quantity should the materials that are needed for the production to be ordered.

Furthermore, an inventory management system is a system that includes all the regulations and checks. These determine the level of stocks, when the stocks should be updated and how large should the order be. In a productive system, stocks are divided into raw materials, finished products that are ready for sale, intermediate products and supplies. Stocks are created also in services in the sense of material goods and supplies that support this service.

The main objective of an inventory management system is to determine firstly when the products should be ordered and secondly how big the order should be. Some companies prefer maintaining long-term relationships with their suppliers for the satisfaction of their needs for almost a whole year. In this case, an inventory management system will determine when and what amount will be distributed. It must be highlighted that an effective inventory management system, saves resources for the enterprise, while simultaneously it minimizes the cost.

Moreover, the concept of stocks is general and covers a wide range of economic phenomena. At the same time, the problem of inventory management is very important for all companies, as their stocks bind usually a large proportion of their capital and as a result have considerable maintenance costs. There are categories of enterprises such as super markets where their stocks cover about 50% of their assets.

As a result, the inventory control problem has become a major subject the recent years, where significant efforts have been made to overcome it. More precisely, in the theoretical approach to the problem, numerous published scientific studies have been made. These studies have formulated many theories for the inventory management models. But from a practical term, only a small part of such theories has been applied to real operational environment.

The inventory management problem is generally defined as a balancing problem between the cost of having lack of a product with the cost of having inventory surplus of a product. A proper inventory management planning disconnects the productive system from deviations in demand and maintains a smooth flow of production. This way it increases the production rate and reduces the cost.

4.2 The purpose of stock conservation

Stock management is one of the most important functions in a production system for various reasons. If the demand of a product was known, then the company could produce this product in such a quantity that corresponds exactly to the demand. Since in reality the demand is rarely known, by maintaining stocks, the enterprise has the opportunity to disconnect the productive system from the demand of customers. As a result, the company is capable of coping with any change done to the demand of customers.

Therefore, making stocks helps in the acceleration and improvement of the delivery time of the products by reducing the chances of non-fulfillment of an order or late delivery. The presence of raw material inventory and intermediate products ensures the continuous supply of the production system and the smooth flow of production, without being affected by any delays of the suppliers.

It also ensures the independent functioning of the productive stages, the increasing of the rate of production and the reduction of the production costs. For example, maintaining stocks reduces the costs for changing machinery from the production of a product to the production of another.

Each new order involves costs for the enterprise, which do not depend on the order quantity. Therefore, the higher the order, the smaller will be the total number of orders, and thus the smaller will be the cost of them. Finally, a company with stock conservation has the opportunity to reduce its payments to suppliers by making greater orders in periods when the prices of the suppliers, are low.

4.3 Inventory Costs

For the decision-making concerning the amount of stock, the enterprise should take into account the following costs:

a. *The holding cost of the stock:* This category includes the storage cost, the cost of the committed capital, the inventory insurance costs, obsolescence inventory costs and finally, the handling cost during storage and transport. Specifically, the committed capital cost arises because the company is forced to invest its capital for the maintenance of stocks over the other alternative uses of its capital. The cost of the committed capital is always equal or greater than the value that the enterprise would have if it had invested the capital in financial products of very low risk.

b. *The inventory procurement costs:* It includes both the fixed cost for placing an order to the suppliers of the company and the cost of buying inventory from them. In the case where the company does not purchase any goods but produces everything on its own facilities, the fixed production costs relate to the costs of preparation of the production process (setup), while variable costs relate to the production cost.

c. *Shortage cost:* Typically, is the cost of unsatisfied demand. If the stocks of the enterprise run out, the company is obliged to delay or cancel the order and therefore loses profits reputation.

4.4 Basic types of stocks

Stocks, according to their conservation causes, can be divided into four categories:

- cycle inventory
- security stocks
- anticipation inventory
- pipeline inventory

These different types of stock are not distinguished by their physical form.

Cycle inventory is part of the total stock and is directly determined by the size of the order. The amount of the cycle inventory depends on the time between two orders. For example, if an order is made each month, the amount of the cycle inventory will be equal to the monthly demand. The longer the period of time between two orders, the greater will be the cycle stock. In order to avoid customer service problems and unavailability, companies often keep a safety stock. Safety stocks are useful when suppliers do not deliver the required amount at the predetermined date in acceptable quality, or when prepared objects have been damaged, or require further corrections and finally they are useful when demand is stochastic.

The maintenance of *security stocks* ensures the smooth functioning of production process in case of such problems. For the preservation of safety stocks, an enterprise makes an order earlier than it really needs or in a larger amount.

As *anticipation inventory* is determined the stock that is used to absorb dissimilar demand at different times. For example, 90% of the annual demand for heating oil appears in four months' time. By maintaining anticipation inventory companies do not have to make considerable changes in production that cost money. Anticipation inventory are also used in cases of uncertainty, concerning the offering of a product.

Stocks that are moving from one point of the material flow system to another are called stocks in motion (*pipeline inventory*).

4.5 Independent versus Dependent demand

Inventory control models assume that demand for an item is either independent of or dependent on the demand for other items. For example, the demand for refrigerators is usually independent of the demand for toaster ovens. Many inventory problems, however, are interrelated; the demand for one item is dependent on the demand for another item. For example, a manufacturer of small power lawn mowers. The demand for lawn mower wheels and spark plug are needed for each finished lawn mower. Usually when the demand for different items is dependent, the relationship between the items is known and consistent. Thus, management schedules production based on the demand for the final products and computes the requirements for components.

4.6 Types of Inventory Models

There are four independent demand models:

- The economic order quantity (EOQ) model
- The production order quantity model
- The back order inventory model
- The quantity discount model

The economic order quantity (EOQ) model is one of the oldest and most commonly known inventory control techniques. It was first seen in 1915 in a publication of Ford W. Harris and is still being used by a large number of enterprises nowadays. The assumptions that this model has are the above. Firstly, this model considers that the demand is known and constant, and that the receipt of the inventory is simultaneous. Moreover, in this model, as lead-time is considered the time between the placement of the order and the receipt of the order, which is known and constant. Furthermore, quantity discounts are not possible and the only variable costs are the cost of setting up or placing an order (setup cost) and the cost of holding or storing the inventory over time (holding cost). Finally, this model also assumes that stockouts can be completely avoided, if orders are placed at the right time.

With the EOQ model, the optimal order quantity will occur at the point where the total setup cost is equal to the total holding cost. By using these facts there have been developed some equations that solve directly for Q*. The necessary steps that are needed is to develop an expression for setup or order cost, to develop another one for holding cost, to set setup cost equal to holding cost and finally to solve the equation in order to find the best order quantity.

In order to solve Q* we determine the following variables:

Q: is the number of pieces per order
Q*: is he optimum number of pieces per order (EOQ)
D: is the annual demand in units for the inventory item
S: is the setup cost or the ordering cost for each order
H: is the holding or carrying cost per unit per year

Annual setup cost = annual demand / number of units in each order = $D * S / Q$
Annual holding cost = (Average inventory level) (holding cost/unit/year) = $Q * H / 2$
The optimal order quantity (Q*) is found simply by solving this equation:

$$Q* = \sqrt{2DS / H}$$

Moreover, the *total annual* inventory cost (TC) is the sum of the setup and the holding costs:

$$TC = D * S / Q + Q * H / 2$$

Now, that we have found the optimum number of pieces per order, we shall decide when we should order. *The reorder point (ROP)* is given as:

$$ROP = d * L$$

d: the demand per day, where $d = D / \mathrm{w}$ and w = number of working days in a year
L: lead time for a new order in days

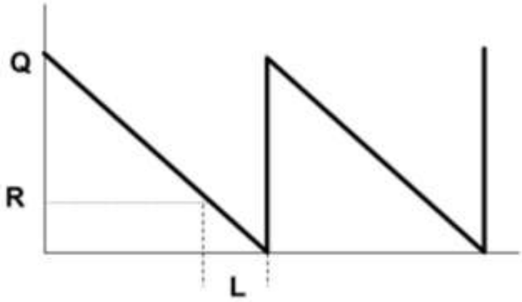

Figure 5: The reorder point (ROP) curve.

The Production Order Quantity model

In the EOQ model, it is assumed that the entire inventory order was received at one time. However, there are times when the company may receive its inventory over a period of time. Such cases require a different model, the Production Order Quantity model in light of the fact that this does not require the instantaneous receipt assumptions. This model is applicable when inventory continuously flows or builds up over a period of time, after an order has been placed or when units are produced and sold simultaneously. This model is also called, Production Order Quantity model, in view of the fact that it is suitable for the production environment. It is useful when the inventory continuously flows or builds up over time and the traditional economic order quantity assumptions are valid. This model is derived from setting ordering or setup costs equal to holding costs and solving for Q*.

In order to solve Q* we determine the following variables:

Q = Number of pieces per order
H = Holding cost per unit per year
p = Daily production rate
d= Daily demand rate, or usage rate
t = Length of the production run in days

Annual Inventory Holding Cost = *(*Average inventory level)*(Holding cost per unit per year) = Average inventory level*H

Average Inventory Level = (Maximum Inventory Level)/2

Maximum Inventory Level = (Total produced during the production run) – (Total used during the production run) = pt-dt

But pt=Q and t=Q/p so

Maximum Inventory level = p (Q/p)-d (Q/p) =Q (1-d/p)

Annual Inventory Holding cost (or simply holding cost) = (Maximum Inventory level)*H/2 = $Q/2 * [1 - d/p] * H$

Using the expression for holding cost above and the expression for setup cost developed in the EOQ model, we solve for the optimal number of pieces per order by equating setup cost and holding cost.

Setup cost = D*S/Q

Holding cost = $Q/2 * [1 - d/p] * H$

By setting ordering cost equal to holding cost we obtain Q*,the optimum order:

D*S/Q = $Q/2 * [1 - d/p] * H \Rightarrow Q*_p = \sqrt{2 * D * S / H * [1 - (d/p)]}$

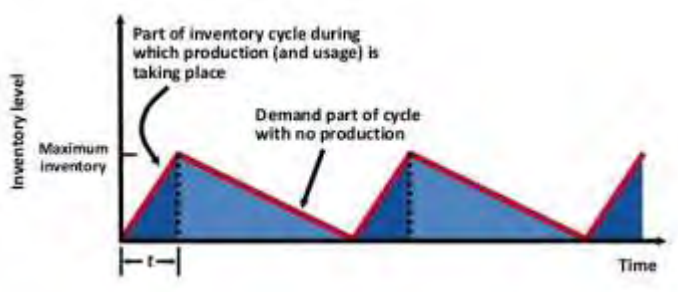

Figure 6: Change in Inventory Levels Over Time for the Production Model.

Back-Order Inventory Model

In the Back-Order Inventory model we assume that stockouts and back ordering is allowed. This model is also called Planned Shortages Inventory model. In this model the basic assumptions are the same as the ones we had in the previous models. In addition, however, in this model we assume that sales will not be lost due to a stockout.

In order to solve this model, we will use some variables that are described below:

Q = Number of pieces per order
D = Annual demand in units
H = Holding cost per unit per year
S = Setup cost for each order
B = Back-ordering cost per unit per year
b = Remaining units after the back order is satisfied
Q – b = Amount back-ordered

The total cost must include the cost of being out of stock (back-ordering cost) so:

Total cost (TC) = Setup cost + Holding cost + Back-ordering cost = D*S/Q + (Average Inventory Level)*H + (Average Back-Ordering cost)*B

By solving the equation above, we can find the *optimum order size in units (Q*):*

$$Q* = \sqrt{(2SD/H)} * \sqrt{(\mathrm{H+B/\,B})}$$

The optimum remaining units after back ordering (b) is found by the equation above:

$$b* = Q*(B/B+H)$$

The optimum amount back-ordered in units (Q*-b*) is found by the equation above:

$$Q* - b* = Q*(1 - B/B+H)$$

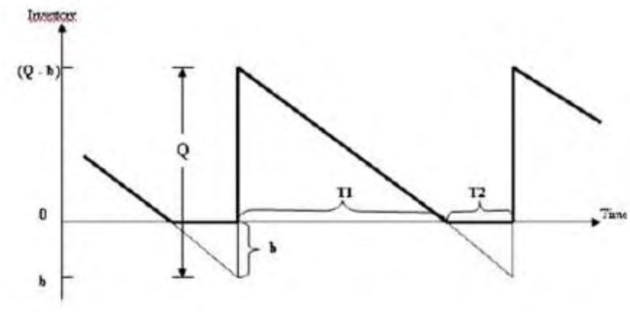

Figure 7: Change in Inventory Over Time with Back-Orders.

Quantity Discount Models

To increase sales, many enterprises offer quantity discounts to their customers. A quantity discount is simply a reduced price (P) for the item when it is purchased in larger quantities. It is not uncommon to have a discount schedule for larger orders. A typical quantity discount schedule is shown below.

Discount Number	Discount Quantity	Discount (%)	Discount Cost ($)
1	0-999	0	**5.00 (normal cost)**
2	1,000 – 1,999	4	4.80
3	2,000 – over	5	4.75

Table 1: A typical quantity discount schedule.

As it can be seen from the table in figure 8, the normal price of the item is 5 $. When 1000 to 1999 items are being ordered at one time, then the price per unit drops to 4.8 $ and when 2000 and more items are being ordered at one time the price drops to 4.75 $ per unit. Management has to decide when and how much items to order. With quantity discounts model the objective is the same as the other models, thus to minimize the total cost.

Total cost (TC) = Setup cost + Holding cost + Product = $D*S/Q + Q*H/2 + P*D$

Where:

D = Annual Demand in units

S = Ordering or setup cost per order or per unit

P = Price per unit

H = Holding cost per unit per year

In order to find the optimum quantity of the order ($Q*$) and minimize the total cost, in the Quantity Discounts model we have to follow the steps above:

- Calculate a value for $Q*$, using the equation : $Q* = \sqrt{2DS / I * P}$

  We use $I*P$ instead of the holding cost (H) in light of the fact that we cannot assume that H is a constant, as the price of the time is a factor in annual holding cost.
- For any discount, if the order quantity is too low to qualify for the discount, adjust the order quantity upward to the lowest quantity that will qualify for the discount.
- Using the total cost equation, that is already been stated, compute a total cost for every $Q*$ determined in steps 1 and 2. If you had to adjust $Q*$ upward in view of the fact that it was below the allowable quantity range, ensure to use the adjusted value for $Q*$.
- Select the $Q*$ that has the lowest total cost, as computed in step 3.This will be the quantity that minimizes the total cost.

5. Eureka Company

5.1 Presentation of the company

The Eureka Company was founded in Famagusta from Xanthos Sarris in 1959 and introduced the "brightener Eureka" in sachet.

In 1960 is built the first chemical plant of Eureka in Famagusta and at the same time Xanthos Sarris establishes a second company in Greece, named Eureka Hellas, and starts from a small rented factory in Athens.

In the decade 1960-1970 are produced and placed on the market and many other products such as Eureka liquid washer, insecticide Aroxol, shampoo Dor, jellies and candy Mon ami, the Famozo for windows and Tik Tak for the toilet.

In the decade 1970-1980, and specifically in 1971 starts the production of high foaming Bing washing powder. At the same time, the factory food Safeway Foods is built and starts its operation. First product was Cremolina, the first soft margarine produced in Cyprus and that takes the 30% of the market share in one year. The products Sky vegetable shortening and cooking oil Favoro are following.

Tragic milestone in the Group's history was the complete destruction of the company in 1974, due to the Turkish invasion. The chemical and food factories were abandoned because of the occupation of Famagusta by the Turkish invaders. Nevertheless, Xanthos Sarris operates in Limassol and one month after supporting the refugee workers, starts the operation of a small craft, under very difficult conditions, while simultaneously proceeding in the construction of a new owned factory. With this immediate start of operation after the Turkish invasion, Eureka managed to remain close to its customers and maintained the value of its brands. In 1976, it launched the Roklin cleaner for large surfaces and in 1977 starts the operation of the proprietary factory Eureka Hellas SA Volos, in modern production facilities for all chemicals. The company headquarters is now moved to the Industrial Area of Volos. In 1978 Eureka starts strong export activity with first stop the Arab countries, followed by expansion in central and Eastern Europe.

In 1981 passes away the great visionary Xanthos Sarris and the eldest son Nikos Sarris becomes CEO of Greek Company and cooperates with the important partner Sotos Sotiriou, who takes over the duties of General Manager. In Cyprus Company remains CEO Christodoulos Neophytou. Christodoulos Neophytou was the proximate cause of Eureka Ltd when as a representative of bleach product discovered the great potential that existed, which Xanthos Sarris decisively and dynamically evaluated and exploited. In subsequent development, follow large investments in technological equipment and building facilities that helped develop the competitive strength of the company.

In 1990-2000 decade, the production of new products dynamically continues in the Cyprus market with the introduction of cleaner and aromatic Fioro toilet fluid in 1992. In 1994 follows the production of Aroxol Mec in trigger form, one of the most innovative products in the category of insecticides / insect repellents. In 1996 is launched the detergent powder laundry Eureka Extra Power and in 1997 starts the collaboration with the French company Compo and begins the distribution of Algoflash, brand household fertilizers for plants, which does not just take the No. 1 spot in sales, but actually created the category in supermarkets. In 2000 the Eureka Hellas SA makes the acquisition of international brand CHIRTON in air fresheners, enhancing the growth of Volos plant productivity and exports to Russia and many European countries.

In 2001 the range of products Eureka continues to develop with the production of liquid detergent for washing machines under the brand name Eureka Marseille, which holds the No. 1 position in the segment of liquid detergent with Marseille soap. In 2003, it launched the liquid laundry detergent Eureka liquid and a specialized series of liquid detergents for black and dark clothes Eureka Care Black, while in 2004 it launched the Eureka Marseille detergent powder.

In 2005, the Eureka Group acquires a new dynamic emerging company in Romania, Interstar Chim S.A., which owns a small company in Ukraine. This acquisition transforms the Group's profile, multinational and gives strong impetus to the development of the EUREKA Group.

In 2007, are launched two new detergents Eureka Care Color for colored clothes and Eureka Care Wool. In 2008 Cyprus Eureka, evolves as a service provider for third parties penetrating into new categories, with brands such as Elite stationery products, Septona cotton products, Castania Lebanese nuts, the Artemis vinegar, oil and others.

In 2009, 50 years after the foundation of the first company, Eureka Group has successful business activities in four countries Cyprus, Greece, Romania and Ukraine, and three modern factories in Volos, Limassol and Bucharest and exports to more than twenty international markets.

Nowadays, the Eureka Group, investing in the triptych of man, quality and trust aims to continue and expand the dynamic presence in the market.

5.2 Philosophy and objectives of the company

With main axis "the beliefs" of the company, which was written in 1967 by the creator Xanthos Sarris and with a humanistic philosophy that extends to all partners within and outside the Group, is built the present and the future of the company. Focusing on the human condition of the consumer and not in a 'dry' trade relationship, Eureka's primary goal is the continuous satisfaction of care with quality products to facilitate everyday life. Through research, technical knowledge and constant vigilance ensure that any product with the signing Eureka offers unique advantages. Consumer confidence is the result of the company's relationship towards honesty and the confidence that selects quality products, friendly income.

The Eureka Group is being expressed dynamically in the social field with social responsibility activities, ensuring that any development objective is linked with responsibility towards people and the development of modern life. The dedication of the consumer will always be the strongest incentive Eureka Group's evolution, proving that even in the era of speed, the respect of the human remains the largest value.

The objectives of the company are the construction and marketing of first policy products for residential use and with the conduction of operations with sincerity, honesty and straightness. The beliefs of the company are:

-Ensure the cooperation and assessment of the staff of the company.

-Cultivate the confidence of the consumers.

-Be successful in the honest cooperation with the resellers.

-Protect the estate of the company and hope that the profit will be enough for the maintenance, renewal, improvement and extension of installations and for the increase of the cycle of operations of the company.

-Offer the best possible working conditions for the staff, in a spirit of mutual respect and respect of the personality of each other.

-The staff must perceive that the management wishes not only their continuous employment at the operations, but also the promotion of each other in hierarchy, without any other obstacle than their personal worth and the existence of opportunities, which management wants and promotes.

-Improve the organization and conduction of the operations, thus that the company continues the operations and after the death of its founders.

-Any company must satisfy the reasonable requirements of the staff, before the owners reap the profits.

-Any suggestion is welcomed and negotiable.

-The group of the department heads in close cooperation between their internal issues will improve the outlook progress of the company and consequently of the staff.

-Anyone is allowed to be involved in any political party, and his actions concerning his political beliefs in or out of the company do not cause any damage in the company.

-Anybody is free to discuss, without fear for retaliation, with the Section Head or with the management if any decision or action contradicts with the beliefs of the company.

5.3 Management and human resources of the company

Nowadays the Eureka Group of Companies, having experience 52 whole years, is consisted by three companies: Eureka Ltd - which is established in Cyprus and has there in Limassol the headquarters of the Group - , Eureka Hellas A.E – which is established in Greece and has in Volos its headquarters – and finally Interstar Chim S.A. – which is established in Romania and Ukraine.

The Eureka Group is managed by members of the founder and owner family Sarri with the collaboration of experienced managers. Together as a group, give dynamic strategic directions for further development and activities of the Group.
Key management personnel are Nikolaos Sarris, Elena Sarri Varnava, Alkis Sarris, Christodoulos Neophytou, Dimitrios Dimitriou, Paris Heracleous, Jason Neophytou, Natia Kizi and more.

As far as the human resources is concerned, the most substantial reward for those who work in Eureka is to be involved in an enterprise that highlights the human values. The Group Eureka considers the employees the most important factor in its development, the driving force of its business and the key factor of its success. With this philosophy, the Group implements policies and programs aimed at climate of continuous rewards of employees' efforts and in the configuration of a single working culture across the entire range of the Group. Specifically the company offers each member the opportunity to grow and evolve within the company, cares for the systematic upgrading of the knowledge and skills of staff, by applying progressive education and training programs. Moreover, the enterprise of Eureka encourages open communication, teamwork and good cooperation, while simultaneously supports initiatives and enhances creativity and innovation. It aims for a pleasant, safe and quality environment, which respects and supports the employee.

Nowadays, the Group has 520 employees of whom 186 work in Greece, 71 in Cyprus, 218 in Romania and 45 in Ukraine. The average age of our employees is around 42 years. This shows the youthful and dynamic character of the Group and actuality trust company for young people. Finally, the employees feel part of a large business "family", which supports their work by all possible means, reward their efforts and recognize their contribution to development every success. Therefore, it is no coincidence that the average service term of human resources is around 14 years.

5.4 Operations of the company

The *export activity* of the Eureka Group started in 1978 and now located in more than 20 international markets such as Russia, Ukraine, Eastern Europe, USA, Asia, the Arab countries and the Middle East. In these, the company operates through local distributors. The Eureka Group dynamically claims success outside the Greek borders, always with the same principle: to provide quality products at competitive prices that makes the lives of the consumers easier and contributes to a better quality of life.

Thanks to advanced mechanical equipment and expertise of human resources, the production units owned by the Group Eureka, are among the most modern and efficient in domestic and international market. In the industrial areas of Volos in Greece, in total surface facilities 34.000m², has an annual *production* capacity of 24,000 tons of chemical products in one shift and 700 tons of plastic bottles and caps in three shifts. With equally high standards operates the factory Interstar Chim, based in Bucharest, Romania. It covers a total surface of 13.000m² facilities with an annual production capacity of 8,000 tons of chemical products in one shift and 300 tons of plastic bottles and caps in three shifts.

Besides the products manufactured in factories in Greece (Volos), Cyprus (Limassol), Romania (Bucharest) represents in Greece, Cyprus and Romania the disposal of other products resulting from collaborations with major companies abroad. The first *strategic collaboration* of the Group Eureka began in 1994 with the American industrial Dow Chemicals for the production of AROXOL Mec. In 1997 started the collaboration with the French company Compo, with successful product household care plant ALGOFLASH, which first placed this category in supermarkets. Up to now, it still holds the first place in the Greek and Cypriot market.

Another area where the Group operates dynamically from 2000 is to provide *comprehensive services to other production companies (3PL).* The services provided to their colleagues covering the fields of physical distribution, storage, product placement in stores (merchandising), debt collection as well as the development of strategic marketing. With the above services, the Eureka Group actively supports its partners by contributing effectively to build their own brands. Warranty of the Eureka Group is a well-organized system and service for storing and distributing it. In Greece there are being serviced by the Group Eureka about 570 customers with a total of 3100 stores as well as 4500 ‘smaller’ customers through special partners. In Cyprus there are being serviced by the Group Eureka 1000 customers and 770 ‘smaller’ customers through special partners.

Group Eureka, besides the many successful products it produces for its own brands, it proposes, creates and develops integrated products, depending on the requirements and needs of each client (*private labeling*).

Furthermore, the Eureka Group by utilizing its successful and long experience and reliability in the production factor offers *the service Business to Business*, which is addressed in other companies engaged in retail products and aims at the manufacturing of their products or tailor products to their needs. This is ensured through direct service, personalization of services provided per case, as well as effective and continuous communication.

The *Golden Club* is a new, innovative service that appeals to the consumer of Eureka in Cyprus. When a member is registered in the Golden Club, gets the right of receipt and installation of Eureka products in his personal space, with the main objective to facilitate and convenience his everyday life.

5.5 Social responsibility of the company

The respect for people, the environment and the contribution to society are the basic values that govern the Group's action Eureka since its inception. Group Eureka implements a CSR model whose aim is sustainable development in conjunction with the well-being of all those affected by its activities. Closely connected with the local communities in which it operates, Eureka Group supports their activities, offers sponsorships and product donations to nonprofit organizations, supports scientific research and assist - with feeling ecological consciousness - in projects which care for the environment and for the sustainable development.

5.6 The products of the company

In Greece the products that the company offers are fabric detergents such as Eureka Massalias in 1L, 2L and 4L, Eureka Massalias with Chios Mastic in 2L, Eureka Massalias with Lavender and Ylang-Ylang in 1L and 2L, Eureka Care Liquid for Black & Dark in 0,75L and 1,5L, Eureka Care for Wool and Delicate ones in 0,75L, Eureka Sport in 1,5L and finally Eureka Express liquid in 1L. Also the company offers baby fabric detergents such as Eureka Baby in 1L and 2L and wash additives such as Eureka Classic in sachet of 60gr, of 120 gr, 240 gr and in box of 500gr and 1 kg. More wash additives are Eureka Classic Blossom of Orange in sachet of 60gr and box of 500gr, Eureka Classic Liquid in 1L, Eureka tablets in box of 12 tablets, Eureka Bright in sachet of 60gr and box of 500gr, Eureka Bright Energy gel in 1L, Eureka Bright Pre-Wash Spray in 450ml and finally Eureka Senso in 1L and 2L. Moreover, the company produces and offers in Greece color absorbers, such as Eureka Color Block and Fabric Softener (10 sheets) and Eureka Color Block (10 and 20 sheets).As far as water softeners are concerned the company produces and sells in Greece Eureka Anti-Kalk dust in a sachet of 54gr and of 950gr, Eureka Anti-Kalk gel in 750ml and Eureka Anti-Kalk tablets in a box of 15 and 30 tablets. Dishwashing liquids from the enterprise offered in Greece are Eureka Power Bubbles in 500ml, Eureka Regular in 500ml and in 750ml, Eureka Crystal Care in 500ml and Eureka Hygiene in 500ml. As far as the insecticides and the insect repellents are concerned, the company offers in Greece Aroxol for Flies and Mosquitoes in 300ml and 400ml, Aroxol for Cockroaches and Ants in 300ml, Aroxol for Ants in 250ml, Aroxol Mec Instant in 400ml and 750ml, Aroxol Total Protection, Aroxol Mat (box of 30 tablets and 60 tablets), Aroxol Liquid 60 nights, Aroxol Spiral, Aroxol Stick, Aroxol Full Season Miticide of box with 2 items and box with 6 items, Aroxol Full

Season sachets, Aroxol Full Season Gel of 3,6 and 12 items, Aroxol Moth Paper, Aroxol Food Moth Paper, Aroxol Earth Choice for Cockroaches and Ants, Aroxol Earth Choice Mat, Aroxol Earth Choice Liquid 60 Nights and finally Aroxol Earth Choice for Spirals. As carpet cleaners, the company offers in Greece Eureka Carpet Care Shampoo in 500ml and in 1L and Eureka Carpet Care Spray. Eureka offers all-purpose cleaners in Greece such as Topine Gel ultra in 750ml and in 1250ml, Topine Mutli Chlor Spray, Eureka Cream for General Cleaning, Eureka for Furniture and finally Eureka for All Surfaces. In addition, the toilet cleaners that the enterprise offers in Greece are Tik Tak Fresh and Tit Tak Dust, while the glass cleaner that it offers is Eureka for the Glasses. Disinfectants that the company also offers are Topine Plus, Topine Total Power, and finally Torpine Germicide Disinfectant. The drain openers that the firm offers in Greece are Flup Occlusive Gel and Flup Occlusive Dust, while the air fresheners that it offers are Fresh Air Moments and Fresh Air Euphoria. Finally the iron aid that the company offers in Greece is Eureka Easy Iron, while the professional products that it offers are Eureka Professional Brightener, Eureka Professional Dust Active Care, Eureka Professional Dust Super Plus, Eureka Professional Softener, Eureka Professional Liquid Dish Washer Lemon Fresh, Eureka Professional Liquid Dish Washer Regular, Eureka Professional Germicide – Disinfectant Topine Plus, Eureka Professional Viscous and Aromatic Chlorine, Eureka Professional Liquid for General Cleaning, Eureka Professional Liquid Windows Cleaner Antistatic and finally Eureka Professional Soap.

5.7 The Eureka case in Attica

In this section, we will consider the case of a real application of HCVRPTW in Eureka Hellas that distributes its products to its customers with its own vehicle fleet consisting of seven vehicles. The depot of the company is located in Loutrou 97 in Athens. As far as the vehicles of the company are concerned, they differ in terms of capacity, which creates the heterogeneity of the fleet to our problem. More specific, information about our fleet of vehicles are presented in the following page.

Vehicles	Maximum Capacity	Type	Fuel Consumption lt/100 km	Daily kms
MERCEDES 1827	18.000	Closed	31,78	74
MERCEDES 412 D	4.600	Closed	10,67	40
MERCEDES 1317	13.500	Closed	24,49	36
MERCEDES 1828L	19.000	Closed	20,47	60
MERCEDES 1828L	19.000	Closed	26,90	65
MERCEDES	3.500	Closed	11,81	82
MERCEDES	4.600	Closed	14,62	50

Table 2: Details for the fleet of vehicles of Eureka Hellas.

Moreover, as far as the time windows of the problem are concerned, the fleet of vehicles must service the clients between 7:00 am -13:30 pm. Also, some specific clients that are described in the table below, must be serviced the specific day that they want, or else the company can make an arrangement with them so that they would be serviced another day of the week. All the other customers of the company can be serviced any day of the week.

Clients	Date of desired service
1	Monday
123	Wednesday
124	Thursday
125	Monday
126	Tuesday
142	Wednesday

Table 3: Date Preferences of clients matrix.

This thesis in order to be completed, has utilized data from May and June of Eureka Hellas in 2016, concerning its clients demands and placement. In addition, data from the sales of a product for monthly basis for the last three years has been used for the fulfillment of the forecasting of the product.

5.8 Improving the distribution system on static demands

In this section, we tried to improve the distribution system of Eureka Hellas company on static demands of June 2016. To be more precise, in June 2016 the company had to fulfil the demands of 151 customers all over the region of Attica. In order to succeed in the improvement of its distribution system, we ran for each day of June a heuristic approach of HCVRPTW. That was done in order for us to compare the total sum of all costs that we found, with the total sum of the costs that the company had this month for distributing its products. The procedure that has been followed is being described below with a presentation of the heuristic approach used and an example of the implementation of the heuristic of the HCVRPTW for the 20th of June 2016.

The Heuristic Approach

Nomenclature

$V = \{1,2,3,\ldots|V|\}$ = set of vehicles

$R_{V\{v=1,2,\ldots|V|\}}$ = is the route schedule for every v belongs to V

$RC_{V\{v=1,2,\ldots|V|\}}$ = is the route cost for every v belongs V

N = is the number of trip requests. Each trip request is consisted by one pickup and one deliver node.

$TR_{i\{i=1,2,\ldots N\}}$ = is the sequence of trip demands.

$EPTi_{\{i=1,2,\ldots N\}}$ = Earlier pickup time of trip request i

BIP_{iv} = the best insertion position of i-th trip request TR_i to route schedule R_v

$MinCostBIP_{iv}$ = is the cost of the best insertion position of the trip request TR_i to route schedule R_v

A description of the insertionH algorithm in pseudo code is as follows:

Step0: for every **v** *in* **V** *build an empty* R_v
Step1: Sort R_v *in descending order according to* RC_v
Step2: Sort TR_i *in ascending order according to demands* EPT_i
Step3: for every $TR_{i\,\{i=1,2,\ldots N\}}$

Step3.1: Sort R_v *in ascending order according to* RC_v
Step3.2: for every $R_{v\,\{v=1,2,\ldots|V|\}}$ *do*
find the BIP_{iv}
Calculate $MinCostBIP_{iv}$
Step3.3: If no $BIP_{iv\,\{v=1,2,\ldots|V|\}}$ *found goto step 3.6*
Step3.4: From all $MinCostBIP_{iv}$ *select the minimum one*
Step3.5: Assign that TR_i *to the appropriate* R_v *and goto step3*
Step3.6: Reject that trip request and repeat step3
end for

Figure 8: A presentation of the Heuristic Approach used in this thesis. (Lois, A., 2016, On the Online Dial-a-Ride Problem, LAP LAMBERT Academic Publishing, 60.)

Example of the implementation of the Heuristic

Data

	Depot	Clients					
		29	99	109	127	128	131
Depot	-	10,6	7,9	12,2	23,4	15,8	36,7
29	11,1	-	11,2	15,6	26,7	5,6	40
99	7,7	11	-	7,2	18,3	16,1	31,6
109	18,8	22,1	15,2	-	11,1	27,2	24,4
127	16,2	19,5	12,6	5,1	-	24,6	29,5
128	16,4	6,3	16,6	21	32,1	-	45,4
131	36,6	39,9	33	25,5	20,4	45	-

Table 4: Distances matrix of 20th of June 2016.

	Depot	Clients					
		29	99	109	127	128	131
Depot	-	13	17	17	22	17	32
29	15	-	16	16	21	8	31
99	17	14	-	14	19	18	29
109	20	17	16	-	9	21	19
127	17	14	13	5	-	18	24
128	21	11	21	21	26	-	36
131	34	32	30	23	18	36	-

Table 5: Time distances matrix of 20th of June 2016.

Clients	Demands in kg
29	3.329,00
99	565,74
109	28,00
127	605,42
128	1.462,66

Table 6: Demands matrix of clients for the 20th of June 2016.

Results

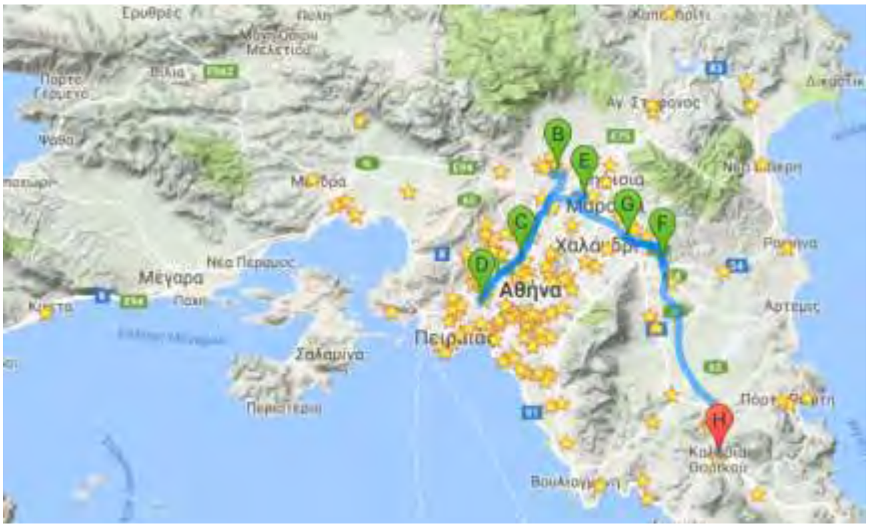

Figure 9: The optimal route for the 20th of June 2016 for the distribution of Eureka's products to its clients in Attica: Depot, 29, 99, 109, 128, 127, 131, Depot.

With the same method in this section, we find the optimal route for servicing the clients of Eureka Hellas for all days of June 2016 with a heuristic approach of HCVRPTW. The total results found are presented below:

Date	Total Heuristic kms	Vehicles used	Heuristic Cost
1/6/2016	230,2	1	73,15756
2/6/2016	224,9	2	67,82009
3/6/2016	116,1	1	36,89658
6/6/2016	109	1	34,6402
7/6/2016	160,2	1	50,91156
8/6/2016	109,2	1	34,70376
9/6/2016	244,6	4	44,60353
10/6/2016	178,2	1	56,63196
13/6/2016	136	1	43,2208
14/6/2016	167,7	1	53,29506
15/6/2016	191,4	2	52,00294
16/6/2016	238,3	2	72,07861
17/6/2016	201,1	1	63,90958
20/6/2016	101	1	32,0978
21/6/2016	116,1	1	36,89658
22/6/2016	100,9	1	32,06602
23/6/2016	245,7	3	60,25516
24/6/2016	109,7	1	34,86266
27/6/2016	163,3	1	51,89674
28/6/2016	175,1	1	55,64678

29/6/2016	76,4	1	24,27992
30/6/2016	270	2	80,69571
Total:			1092,569 €

Table 7: Results of the implementation of the heuristic approach of HCVRPTW on daily basis for June 2016 for Eureka Hellas.

Vehicles	Eureka kms of June 2016	Eureka Cost of June 2016
2297	1.268	402,97
5116	804	85,79
7389	589	144,25
8165	740	151,48
8166	1.193	320,92
7776	1.512	178,57
9421	287	41,96
Total:		1325,92 €

Table 8: A table presenting the kms and the costs that Eureka had for its distribution system in Attica without the 3PL products transportation for June 2016.

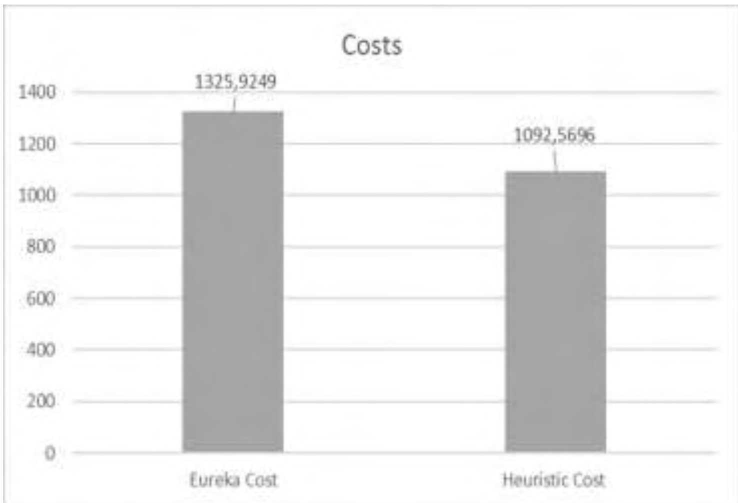

Figure 10: A figure comparing the cost of Eureka for its distribution system for Attica (without the 3PL products transportation) for June 2016 with the cost found by this thesis with the heuristic approach of HCVRPTW.

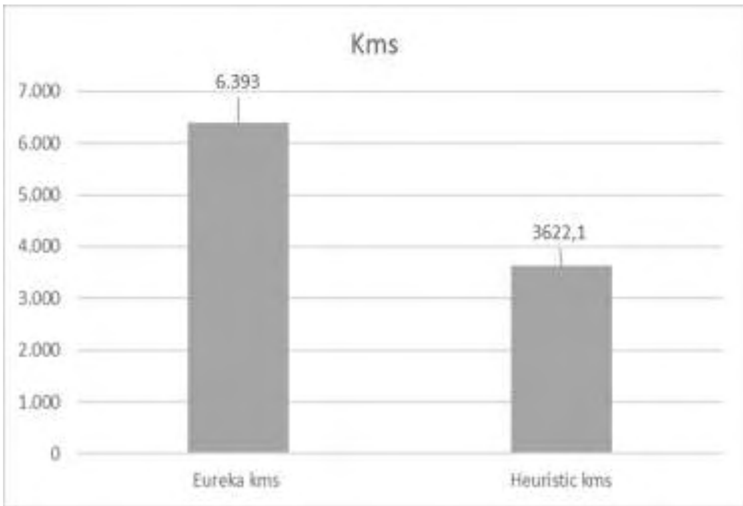

Figure 11: A figure comparing the kms of Eureka for its distribution system for Attica (without the 3PL products transportation) for June 2016 with the cost found by this thesis with the heuristic approach of HCVRPTW.

As we can see, the results presented in this section show that with the implementation of this heuristic approach, which is based on static demands, we could minimize by 18% the costs of the distribution of the products of the company to its customers based on the kms. That is a significant amount of money as it refers to savings of 233 €/month and 2800 €/year.

Furthermore, we then considered the costs of the drivers of the vehicles that have been used by both scenarios. The standard cost of each driver for each month is 1000 €.

Heuristic kms Cost	**Eureka kms Cost**	**Max vehicles used in Heuristic**	**Max vehicles used by Eureka**	**Heuristic Staff Cost**	**Eureka Staff Cost**	**Total Cost Heuristic**	**Total Cost Eureka**
1092,569	1325,9249	4	7	4000	7000	5092,569	8325,9249

Table 9: A table presenting the costs of the Heuristic scenario and of the Eureka scenario (based on the kms), the maximum number of vehicles used in each scenario and the formulation of the total cost of each scenario.

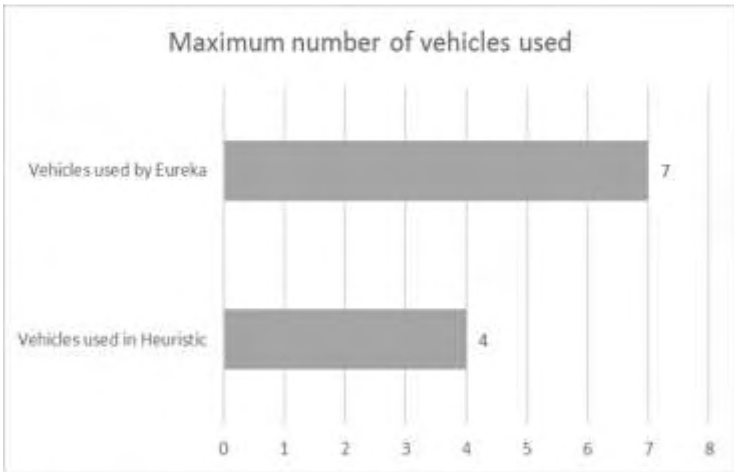

Figure 12: A figure presenting the maximum number of vehicles used in each scenario for the distribution of Eureka's Hellas products to its customers in Attica.

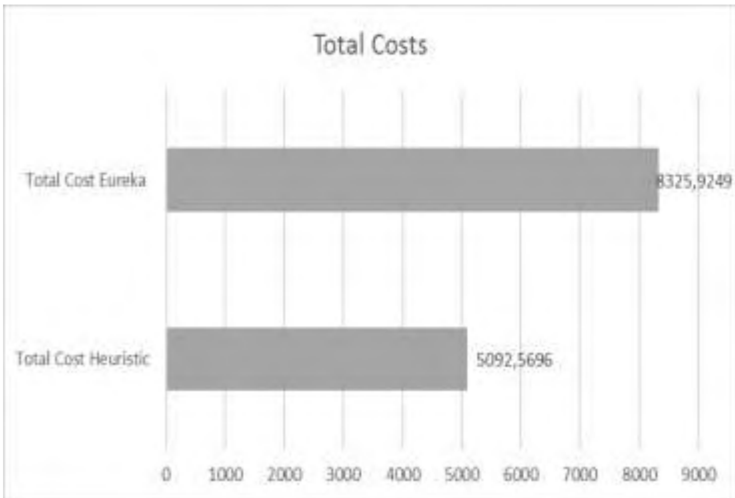

Figure 13: A figure presenting the total costs of each scenario for the distribution of Eureka's Hellas products to its customers in Attica based on kms and staff.

5.9 Improving the distribution system on probabilistic customers

In this section, we tried to improve the distribution system of Eureka Hellas Company on probabilistic customers for June 2016. To be more precise, we initially did a HCVRPTW for each day of the week of June by using a heuristic approach of HCVRPTW with a random selected probability that refers to the probability of the customers that need to be satisfied (P=0,4). In this case, with the selection of probability P=0,4 we define as clients all the clients that have ordered twice in the same month and in the same day of the week (Monday etc). As demand for each customer, we used the static amount of his demands of June. The result of this method concluded in finding 775,3691 € total cost, which is 29% smaller than the total cost we had found by using the HCVRPTW heuristic approach for static customers (1092,5696 €). In addition it is smaller from the total cost of Eureka Hellas for June 2016 (1619,7534 €).

However, the first scenario we did with probability P=0,4 for customers wasn't enough. We found the worst-case scenario of each day of the week. As worst-case scenario of each day of the week, we define the day etc of all Mondays which has the most clients to be serviced. Then, we did a HCVRPTW for these worst-case scenarios of each day of the week, by adding also as clients all the customers that had to be serviced this day of the week for all June. For the clients added in the HCVRPTW approach, we used as demands the average of their demands (while taking into account the probability of demanding of its client). The process and results of this scenario are being described below:

- Monday

Initially we made scenarios for Mondays. We found that we have the worst-case scenario for Mondays in Monday 13/6/2016 when we have to service 23 clients. Moreover, this worst-case scenario probability, meaning the probability where we have to service all 48 customers is found P=0,2. In the worst case scenario, the cost of the heuristic approach of HCVRPTW is found 95,71638 €, while its cost for the simple scenario with probability P=0,4 is being found 43,2208 €.

Method	Cost
Stochastic Customers WC Scenario HCVRPTW Heuristic	95,71638
Scenario HCVRPTW Heuristic	43,2208

Table 10: A matrix presenting the cost of Eureka, the cost of the Stochastic Customers Worst-Case Scenario HCVRPTW and the cost of the simple HCVRPTW scenario for Monday 13/6/2016.

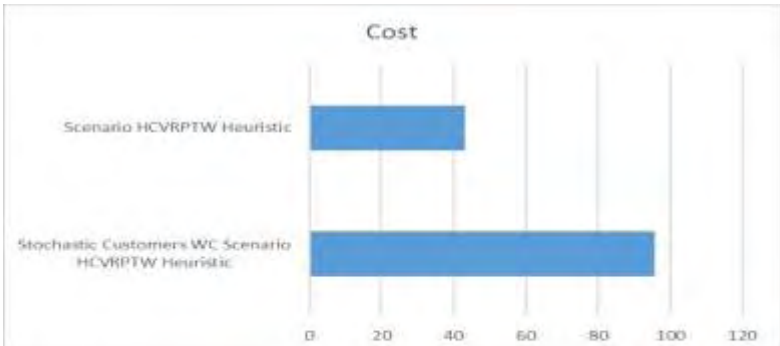

Figure 14: A figure comparing the cost of Eureka with the cost of the Stochastic Customers Worst-Case Scenario HCVRPTW and with the cost of the simple HCVRPTW scenario for Monday 13/6/2016.

- Tuesday

Moreover, we made scenarios for Tuesdays. We found that we have the worst-case scenario for Tuesdays in Tuesday 14/6/2016 when we have to service 20 clients. Moreover, this worst-case scenario probability, meaning the probability where we have to service all 55 customers is found P=0,2. In the worst case scenario, the cost of the heuristic approach of HCVRPTW is found 152,52682 €, while its cost for the simple scenario with probability P=0,4 is being found 53,29506 €.

Method	Cost
Stochastic Customers WC Scenario HCVRPTW Heuristic	152,5268
Scenario HCVRPTW Heuristic	53,29506

Table 11: A matrix presenting the cost of Eureka, the cost of the Stochastic Customers Worst-Case Scenario HCVRPTW and the cost of the simple HCVRPTW scenario for Tuesday 14/6/2016.

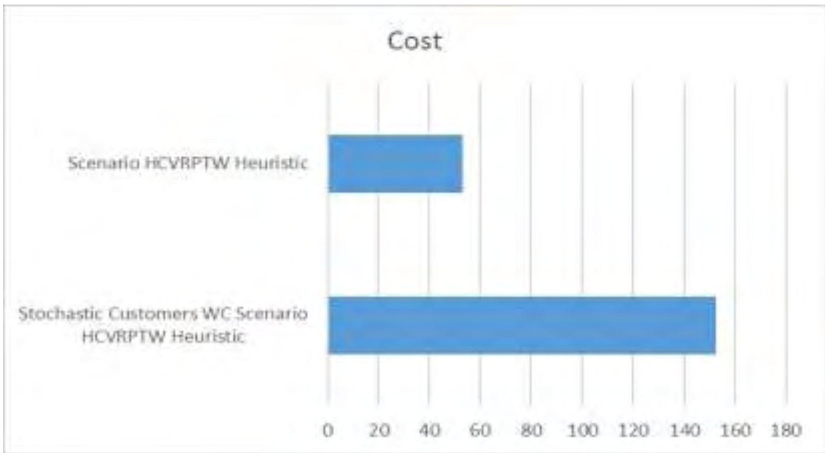

Figure 15: A figure comparing the cost of Eureka with the cost of the Stochastic Customers Worst-Case Scenario HCVRPTW and with the cost of the simple HCVRPTW scenario for Tuesday 14/6/2016.

- Wednesday

Furthermore, we made scenarios for Wednesdays. We found that we have the worst-case scenario for Wednesdays in Wednesday 29/6/2016 when we have to service 20 clients. Moreover, this worst-case scenario probability, meaning the probability where we have to service all 57 customers is found P=0,2. In the worst case scenario, the cost of the heuristic approach of HCVRPTW is found 102,51554 €, while its cost for the simple scenario with probability P=0,4 is being found 24,27992€.

Method	Cost
Stochastic Customers WC Scenario HCVRPTW Heuristic	102,5155
Scenario HCVRPTW Heuristic	24,27992

Table 12: A matrix presenting the cost of Eureka, the cost of the Stochastic Customers Worst-Case Scenario HCVRPTW and the cost of the simple HCVRPTW scenario for Wednesday 29/6/2016.

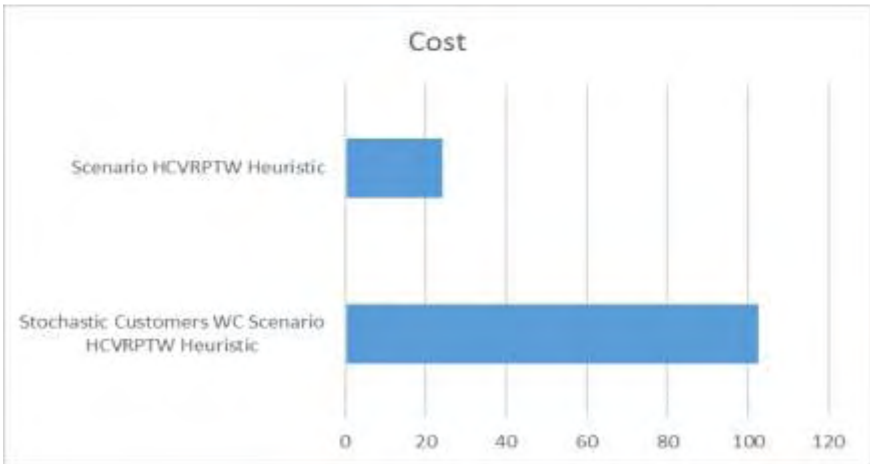

Figure 16: A figure comparing the cost of Eureka with the cost of the Stochastic Customers Worst-Case Scenario HCVRPTW and with the cost of the simple HCVRPTW scenario for Wednesday 29/6/2016.

- Thursday

In addition, we made scenarios for Thursdays. We found that we have the worst-case scenario for Thursdays in Thursday 16/6/2016 when we have to service 26 clients. Moreover, this worst-case scenario probability, meaning the probability where we have to service all 64 customers is found P=0,2. In the worst case scenario, the cost of the heuristic approach of HCVRPTW is found 128,74055 €, while its cost for the simple scenario with probability P=0,4 is being found 72,07861 €.

Method	Cost
Stochastic Customers WC Scenario HCVRPTW Heuristic	128,7406
Scenario HCVRPTW Heuristic	72,07861

Table 13: A matrix presenting the cost of Eureka, the cost of the Stochastic Customers Worst-Case Scenario HCVRPTW and the cost of the simple HCVRPTW scenario for Thursday 16/6/2016.

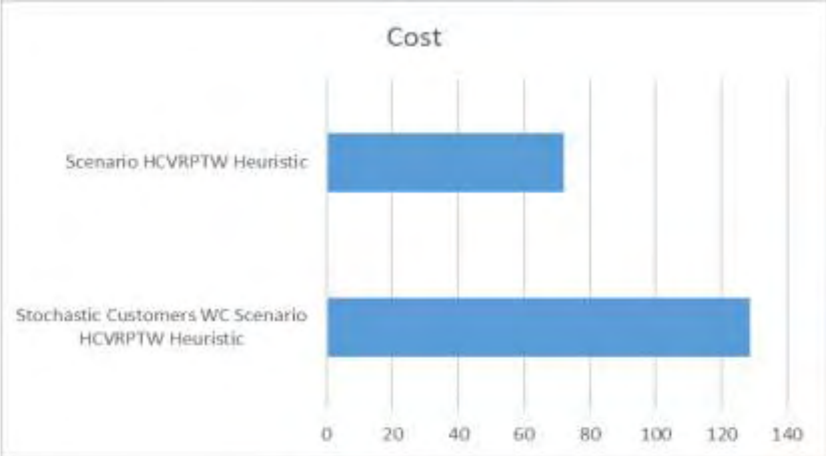

Figure 17: A figure comparing the cost of Eureka with the cost of the Stochastic Customers Worst-Case Scenario HCVRPTW and with the cost of the simple HCVRPTW scenario for Thursday 16/6/2016.

- Friday

Last but not least, we made scenarios for Fridays. We found that we have the worst-case scenario for Fridays in Friday 10/6/2016 when we have to service 23 clients. Moreover, this worst-case scenario probability, meaning the probability where we have to service all 52 customers is found P=0,2. In the worst case scenario, the cost of the heuristic approach of HCVRPTW is found 107,44248 €, while its cost for the simple scenario with probability P=0,4 is being found 56,63196 €.

Method	Cost
Stochastic Customers WC Scenario HCVRPTW Heuristic	107,4425
Scenario HCVRPTW Heuristic	56,63196

Table 14: A matrix presenting the cost of Eureka, the cost of the Stochastic Customers Worst-Case Scenario HCVRPTW and the cost of the simple HCVRPTW scenario for Friday 10/6/2016.

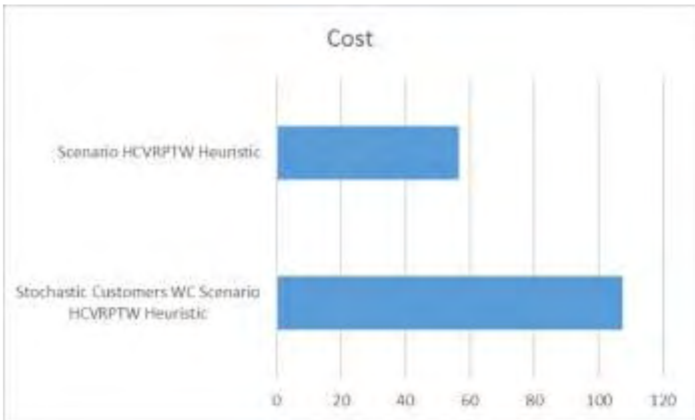

Figure 18: A figure comparing the cost of Eureka with the cost of the Stochastic Customers Worst-Case Scenario HCVRPTW and with the cost of the simple HCVRPTW scenario for Friday 10/6/2016.

Date	**Stochastic Customers WC Heuristic Cost**
Monday	95,71638
Tuesday	152,52682
Wednesday	102,51554
Thursday	128,74055
Friday	107,44248
Stochastic Customers WC Heuristic Cost /week :	586,94177 €
Stochastic Customers Cost for June :	2347,76708 €
Eureka Cost for June :	1619,7534 €

Table 15: A table presenting the costs of the Stochastic Customers WC Scenario case for June and of Eureka for June 2016.

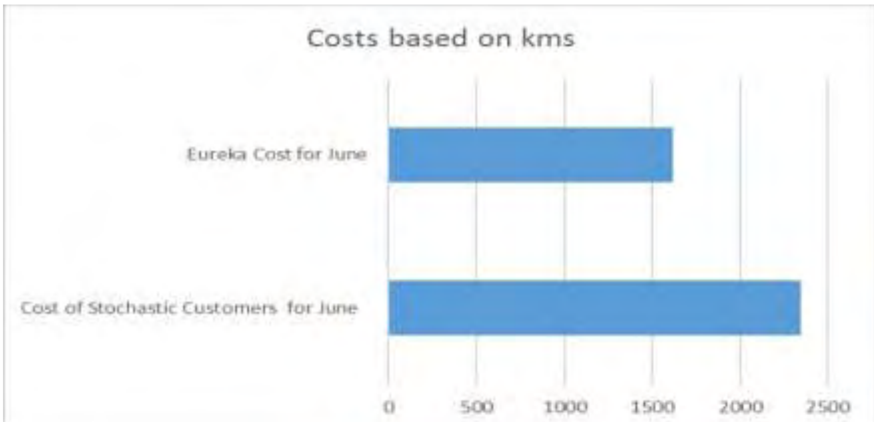

Figure 19: A histogram comparing the costs based on kms of the Stochastic Customers WC Scenario case for June and of Eureka for June 2016.

Furthermore, we then considered the costs of the drivers of the vehicles that have been used by both scenarios. The standard cost of each driver for each month is 1000 €.

Max number of vehicles used by Stochastic Customers Scenario for June	Cost of Drivers of Stochastic Customers Scenario	Max number of vehicles used by Eureka	Cost of Drivers of Eureka
4	4000 €	7	7000 €

Table 16: A table presenting the maximum number of vehicles used by both Stochastic Customers and Eureka's scenarios and its participation in the formulation of the total cost of each scenario.

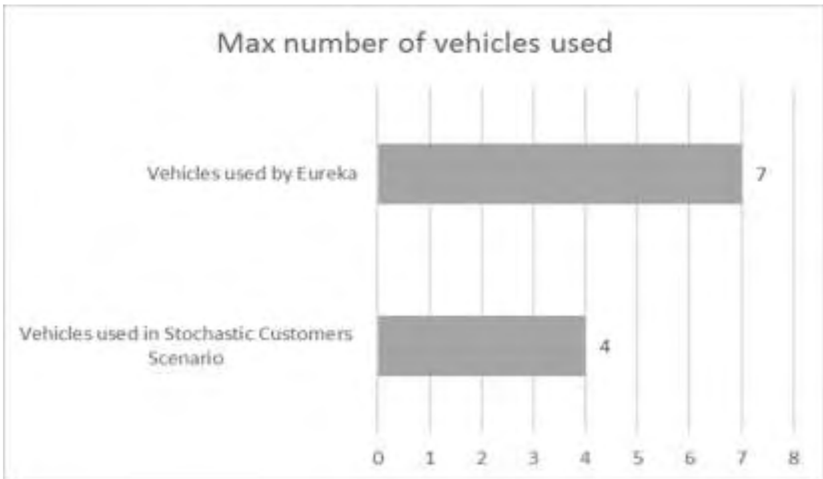

Figure 20: A figure presenting the maximum number of vehicles used by both Stochastic Customers and Eureka's scenarios.

Cost of Stochastic Customers for June	Eureka Cost for June	Total Cost of Stochastic Customers for June	Total Cost of Eureka for June
2347,76708	1325,9249	6347,76708	8325,9249

Table 17: A table presenting the total costs of the Stochastic Customers scenario and the Eureka's scenario for June 2016.

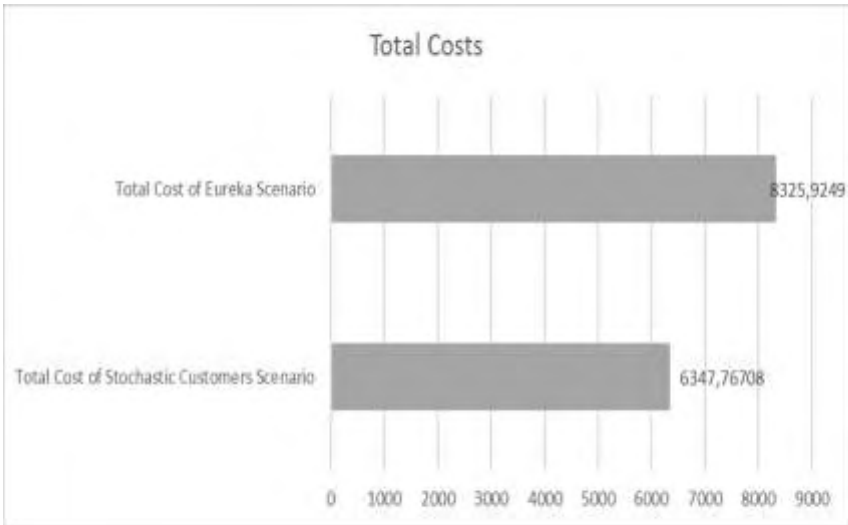

Figure 21: A figure presenting the total costs of the Stochastic Customers scenario in Attica and the Eureka's scenario for June 2016.

To conclude with, in this chapter, we found out that if Eureka Hellas S.A. changes its strategy and adopts the Stochastic Customers scenario strategy, it would have a reduction in its expenses in the worst case scenarios by 24%, or 1.978 €/month or 23.727 €/year .

5.10 Proposing the forecasting of product 000021 of Eureka

In this section of this thesis, we tried to improve the forecasting of a product of Eureka Hellas Company. More precisely, we tried to propose a forecasting of product 000021, based on the demands of the clients for this product for the last three years (2014-2016). The curve of the demands of product 000021 for the last three years is presented in the figure below.

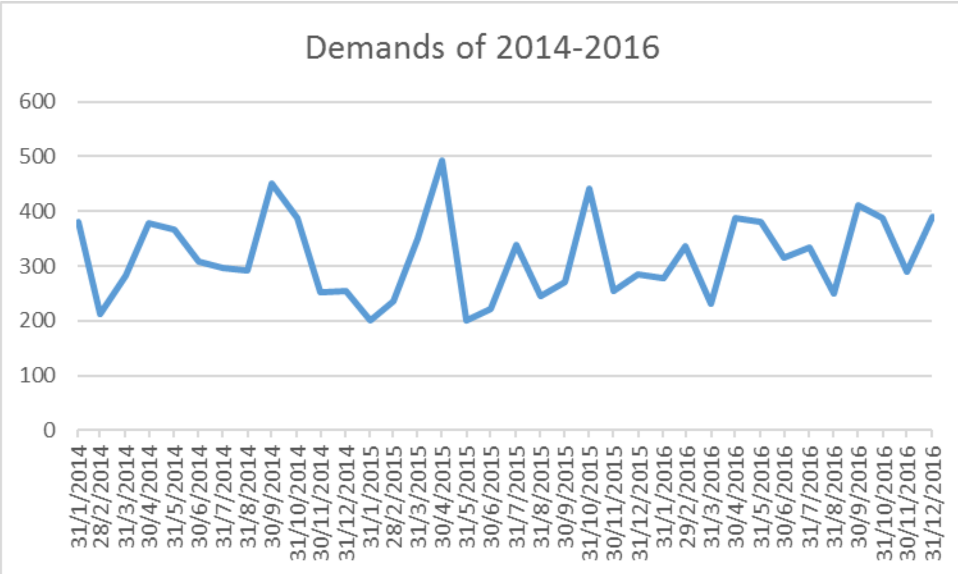

Figure 22: The curve of the demands for product 000021 for the last three years.

In order to succeed, we ran a forecast on a three-year basis (2014, 2015 & 2016), whose outcome was to have a great deviation (223 boxes), so it became a necessity to find the reason why there is such a great deviation.

Date	Demand	Forecast	Lower Trust Limit	Higher Trust Limit
31/1/2014	380	-	-	-
28/2/2014	213	-	-	-
31/3/2014	283	-	-	-
30/4/2014	379	-	-	-
31/5/2014	367	-	-	-
30/6/2014	309	-	-	-
31/7/2014	296	-	-	-
31/8/2014	293	-	-	-
30/9/2014	451	-	-	-
31/10/2014	387	-	-	-

30/11/2014	252	-	-	-
31/12/2014	254	-	-	-
31/1/2015	201	-	-	-
28/2/2015	236	-	-	-
31/3/2015	351	-	-	-
30/4/2015	492	-	-	-
31/5/2015	200	-	-	-
30/6/2015	221	-	-	-
31/7/2015	338	-	-	-
31/8/2015	245	-	-	-
30/9/2015	270	-	-	-
31/10/2015	441	-	-	-
30/11/2015	254	-	-	-
31/12/2015	285	-	-	-
31/1/2016	278	-	-	-
29/2/2016	336	-	-	-
31/3/2016	232	-	-	-
30/4/2016	388	-	-	-
31/5/2016	380	-	-	-
30/6/2016	316	-	-	-
31/7/2016	333	-	-	-
31/8/2016	249	-	-	-
30/9/2016	411	-	-	-
31/10/2016	387	-	-	-
30/11/2016	289	-	-	-
31/12/2016	389	389	389,00	389,00
31/1/2017	-	348,4980661	206,62	490,38
3/3/2017	-	328,1890823	169,43	486,95
31/3/2017	-	428,4567084	254,40	602,52
1/5/2017	-	488,599038	300,42	676,78
31/5/2017	-	340,910498	139,55	542,27
1/7/2017	-	336,242771	122,47	550,01
31/7/2017	-	351,9206225	126,32	577,52
31/8/2017	-	331,6116388	94,78	568,44
30/9/2017	-	428,6448253	181,39	675,90
31/10/2017	-	492,0215944	234,07	749,98
30/11/2017	-	349,0972009	81,47	616,73
31/12/2017	-	339,6653274	62,04	617,29

Table 18: Forecasting results for product 000021 on a three-year scale (2014-2016).

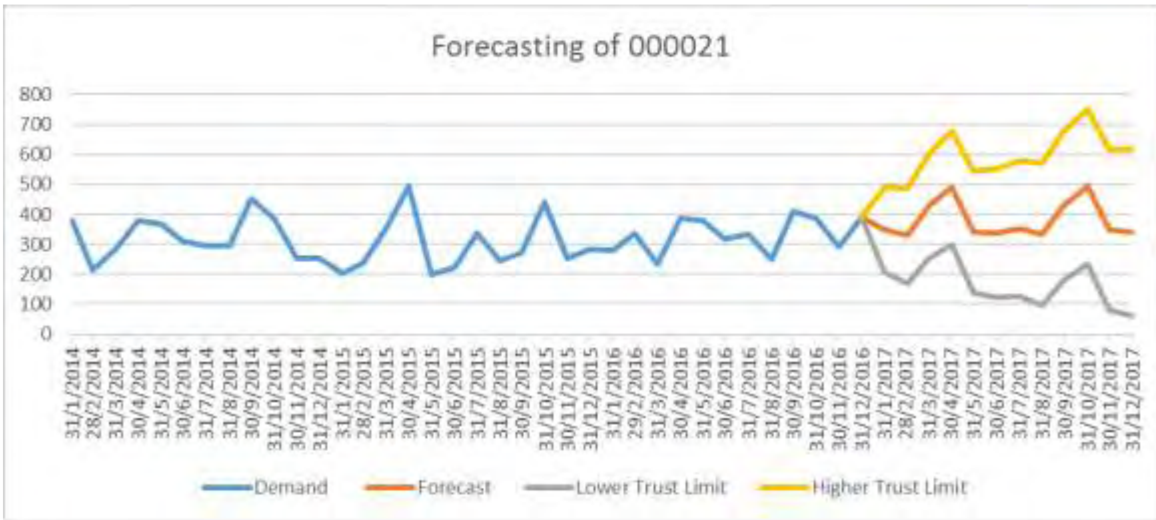

Figure 23: Forecasting curve for product 00021 on a three-year basis (2014-2016).

In order to justify this great deviation found after the forecasting, we communicated with Eureka Hellas and asked for some information. The company stated that this phenomenon is happening due to the existence of the "cleaning seasons", when customers order more quantities of the product. Therefore, we asked for a scale analysis of the demands from 1-10, where as 10 is stated a "cleaning season" with great demands of the product. We did that in order to find a positive correlation between this scale analysis of the demands (answers) and the demands, which would justify the deviation.

Period	Answer	Demands
31/1/2016	4	278
28/2/2016	3	336
31/3/2016	4	232
30/4/2016	10	388
31/5/2016	6	380
30/6/2016	4	316
31/7/2016	7	333
31/8/2016	3	249
30/9/2016	8	411
31/10/2016	9	387
30/11/2016	3	289
31/12/2016	5	389

Table 19: A table presenting the scale analysis of demands given by the company.

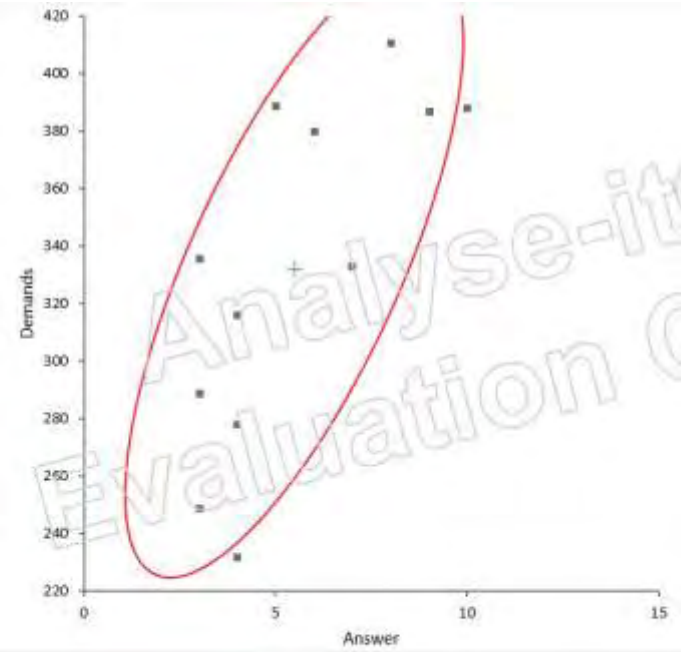

Figure 24: A figure presenting the Demands- Answers correlation.

Correlation

Pearson's r	Answer	Demands
Answer	-	0,734
Demands	0,734	-

Table 20: A table presenting the positive correlation found between Demands and Answers.

5.11 Improving the facility location of Eureka's depot in Attica

In this section of this thesis, we tried to improve the facility location of Eureka's Hellas depot in Attica, based on the demands of its 151 clients for May & June 2016. More precisely, we calculated the total demands of June & May and defined a weight coefficient for each customer based on the amount of his demands in the total demands of these months. Furthermore, we created four clusters, in which clients of Eureka are being divided, and then calculated the weight coefficient of demands of each cluster. The procedure followed is being described below.

Figure 25: A map of Attica, where as stars are the clients of Eureka Hellas S.A. that are being divided in four clusters.

Cluster 1

Clients	Demands of May	Demands of June	Total Demands	Weight coefficient of demands
1	22152,95	28395,91	50548,86	7,8595
21	449,71	269,98	719,69	0,1119
24	114,66	121,24	235,9	0,036678
30	301,76	298,8	600,56	0,09338
58	70	28,28	98,28	0,01528
97	579,93	541,46	1121,39	0,17436
123	25.901,03	24.576,85	50477,88	7,84852
147	0	842,41	842,41	0,13098
149	6.779,14	21.561,11	28340,25	4,40646
Total				20,677058

Table 21: A matrix presenting the clients that are included in Cluster 1 and their weight coefficient of demands.

Cluster 2

Clients	Demands of May	Demands of June	Total Demands	Weight coefficient of demands
20	677,31	401,85	1079,16	0,1678
22	566,15	188,45	754,6	0,1173
33	645,61	300,83	946,44	0,14715
41	326	167,86	493,86	0,07678
43	207,14	155,07	362,21	0,056318
81	799,13	376,01	1175,14	0,182715
115	977,29	1066,84	2044,13	0,31783
121	178,47	220,54	399,01	0,06204
Total				1,127933

Table 22: A matrix presenting the clients that are included in Cluster 2 and their weight coefficient of demands.

Cluster 4

Clients	Demands of May	Demands of June	Total Demands	Weight coefficient of demands
38	507,13	185,76	692,89	0,10773
40	672,98	100,9	773,88	0,12032
48	367,57	144,93	512,5	0,07968
86	330,86	370,74	701,6	0,10908
104	2.772,44	1253,62	4026,06	0,62598
116	537,94	87,62	625,56	0,09726
119	294,73	430,63	725,36	0,11278
124	17.511,57	21.634,92	39146,49	0,06086
131	2.438,40	13578,34	16016,74	0,0249
135	582,28	488,49	1070,77	0,16648
137	102,06	106,24	208,3	0,03238
138	380,8	714,09	1094,89	0,17024
139	362,34	276,77	639,11	0,09937
140	54,47	155,86	210,33	0,0327
151	12.555,07	7.589,50	20144,57	3,13216
Total				4,97192

Table 23: A matrix presenting the clients that are included in Cluster 4 and their weight coefficient of demands.

Cluster 3

Clients	Demands of May	Demands of June	Total Demands	Weight coefficient of demands
2	974,68	617,61	1592,29	0,24757
3	659,58	276,78	936,36	0,145589
4	672,2	459,93	1132,13	0,176028
5	626,58	348,23	974,81	0,151567
6	592,38	263,12	855,5	0,13301
7	55,4	29,68	85,08	0,013228
8	119,28	175,66	294,94	0,04585
9	182,81	173,96	356,77	0,05547
10	325,85	240,37	566,22	0,088038
11	489,94	347,8	837,74	0,130255
12	355,45	141,95	497,4	0,07733
13	217,38	58,81	276,19	0,04294
14	370,23	304,38	674,61	0,10489
15	799,22	535,89	1335,11	0,20758
16	246,45	70,35	316,8	0,04925
17	545,32	213,26	758,58	0,117947
18	965,66	305,08	1270,74	0,19758
19	502,21	191,75	693,96	0,10789
23	483,36	391,64	875	0,136048
25	305,28	244,34	549,62	0,08545
26	103,54	225,91	329,45	0,05122
27	241,06	217,91	458,97	0,07136
28	212,09	381,02	593,11	0,09221
29	1060,52	6.672,41	7732,93	1,20235
31	353,47	243,83	597,3	0,09287
32	1296,14	608,12	1904,26	0,29608
34	285,23	185,11	470,34	0,07313
35	472,64	448,64	921,28	0,14324
36	654,53	279,38	933,91	0,1452
37	676,83	410,18	1087,01	0,169
39	469,7	350,6	820,3	0,12754
42	579,84	350,19	930,03	0,1446
44	392,63	418,1	810,73	0,126
45	134,05	108,74	242,79	0,03775
46	542,24	471,69	1013,93	0,1576
47	776,4	275,68	1052,08	0,16358
49	216,05	154	370,05	0,05753
50	723,39	229,06	952,45	0,14809
51	470,33	354,18	824,51	0,1282
52	271,89	256	527,89	0,082
53	318,74	249,18	567,92	0,0883
54	51,2	231,93	283,13	0,044
55	298,21	15,42	313,63	0,04876

56	10,9	114,93	125,83	0,01956
57	105,45	429,38	534,83	0,08315
58	70	28,28	98,28	0,0152
59	1732,72	348,2	2080,92	0,32355
60	345,51	617,57	963,08	0,14974
61	70,09	70,09	140,18	0,02179
62	1110,36	1147,13	2257,49	0,351
63	32,8	12,2	45	0,69967
64	1692,27	901,01	2593,28	0,4032
65	0	77,72	77,72	0,01208
66	203,3	67,39	270,69	0,042
67	1814,33	1633,3	3447,63	0,5360
68	326,59	63,5	390,09	0,0606
69	308,29	258,44	566,73	0,0881
70	0	47,76	47,76	0,7425
71	0	42,78	42,78	0,6651
72	105,18	187,88	293,06	0,04556
73	0	42,06	42,06	0,65396
74	140,21	52,56	192,77	0,02997
75	227,89	144,66	372,55	0,05792
76	357,59	422,03	779,62	0,12121
77	0	34,34	34,34	0,5339
78	650,08	575,68	1225,76	0,19058
79	122,08	52,56	174,64	0,02715
80	274,67	254,24	528,91	0,08223
82	759,6	609,13	1368,73	0,2128
83	3.386,36	1672,76	5059,12	0,7866
84	2066,71	1521,51	3588,22	0,5579
85	2.249,45	2037,08	4286,53	0,6664
87	135,75	130,39	266,14	0,04138
88	347,3	14,97	362,27	0,0563
89	455,21	33,34	488,55	0,07596
90	0	140,18	140,18	0,02179
91	1147,03	1136,1	2283,13	0,35499
92	1551,78	2130,98	3682,76	0,57261
93	392,38	777,27	1169,65	0,18186
94	151,44	59,19	210,63	0,0327
95	28	51,5	79,5	0,01236
96	917,37	825,57	1742,94	0,27099
98	593,73	329,59	923,32	0,14496
99	5.532,57	5592,97	11125,54	1,72984
100	1008	816,71	1824,71	0,28371
101	1103,23	1328,25	2431,48	0,37805
102	5.017,86	5002,98	10020,84	1,558
103	323,31	2212,52	2535,83	0,3942
105	1511,29	1268,61	2779,9	0,4322
106	1281,32	1477,8	2759,12	0,4289
107	392,5	360,72	753,22	0,1171
108	328,15	196,72	524,87	0,0816
109	1255,06	477,9	1732,96	0,2694

110	0	43,74	43,74	0,0068
111	822,67	499,86	1322,53	0,2056
112	262,8	349,7	612,5	0,0952
113	823,29	798,27	1621,56	0,2521
114	1422,71	1114,69	2537,4	0,3945
117	4675,16	2920,47	7595,63	1,181
118	920,08	1414,89	2334,97	0,36305
120	561,34	766,41	1327,75	0,2064
122	97,94	148,92	246,86	0,03838
125	1311,98	2190,54	3502,52	0,54458
126	107.745,25	74.082,44	181827,7	28,27135
127	4578	3019,42	7597,42	1,1812
128	6.118,06	4725,66	10843,72	1,68602
129	131,03	165,69	296,72	0,0461
130	1.223,11	955,67	2178,78	0,33876
132	1072,93	1597,78	2670,71	0,41525
133	744,89	819,79	1564,68	0,24328
134	1162,86	1384,21	2547,07	0,396
136	360,15	543,78	903,93	0,14054
142	5.551,74	16.953,90	22505,64	3,49927
143	10.977,67	16.049,22	27026,89	4,20225
144	1.749,96	2329,92	4079,88	0,63435
145	95,08	174,28	269,36	0,04188
146	0	5.354,50	5354,5	0,83254
148	519,62	424,53	944,15	0,1468
150	0	612,98	612,98	0,0953
152	16.874,45	4240,96	21115,41	3,28311
Total				70,36405

Table 24: A matrix presenting the clients that are included in Cluster 3 and their weight coefficient of demands.

In the following table are being presented the weight coefficient of demands of each cluster.

Clusters	**Weight coefficient of demands (hi)**
Cluster 1	20,677058
Cluster 2	1,127933
Cluster 3	70,36405
Cluster 4	4,97192

Table 25: A matrix presenting presented the weight coefficient of demands of each cluster.

Moreover, we selected a client in each cluster, and then filled the distances matrix of all these four clients of each cluster. After that, we just solve the median facility location problem that has been created.

	Client cluster 1	Client cluster 2	Client cluster 3	Client cluster 4
Client cluster 1	-	50,2	36,6	69,5
Client cluster 2	50,3	-	10	30,1
Client cluster 3	37,2	9,3	-	40,4
Client cluster 4	71,6	32,6	42,1	-

Table 26: A D_{ij} Matrix presenting the distances between the clients of the clusters that we have selected to represent them.

	Client cluster 1	Client cluster 2	Client cluster 3	Client cluster 4
Client cluster 1	-	50,2*h2	36,6*h3	69,5*h4
Client cluster 2	50,3*h1	-	10*h3	30,1*h4
Client cluster 3	37,2*h1	9,3*h2	-	40,4*h4
Client cluster 4	71,6*h1	32,6*h2	42,1*h3	-

Table 27: A modified Dij Matrix with the weight coefficient of demands of each cluster in order to solve the Median Facility Location problem.

	Client cluster 1	Client cluster 2	Client cluster 3	Client cluster 4	Sum of row
Client cluster 1	-	56,622	2575,324	445,548	3077,494
Client cluster 2	1040,056	-	703,6405	149,654	1893,3505
Client cluster 3	769,186	10,489	-	200,865	980,54 **min**
Client cluster 4	1480,477	36,770	209,317	-	1726,564

Table 28: A matrix presenting the results of the Median Facility Location problem for Eureka Hellas SA.

Figure 26: A map of the area of Attica, presenting with the dots the selected clients of each cluster, and thus presenting the best location for the depot of Eureka in Attica in Cluster 3.

5.12 SWOT analysis of Eureka

Strenghts

- Its facilities in Volos and Bucharest have a total surface of 34.000m² and 13.000m², with well-organized storage and distribution system.
- Eureka already cooperates with a 3pl company for the transportation of its products to customers who are located outside of the Attica region.
- Thanks to the advanced mechanical equipment used and to the expertise of human resources, the production units owned by the Group EUREKA, are among the most modern and efficient in domestic and international market.
- In 1997, the company started the collaboration with the French company Compo, with successful products of household care planting, named "ALGOFLASH" that introduced a new category in the supermarket where until today ranks in the first place in Greek and Cypriot market.

Weaknesses

- Still high current distribution cost of the products in the region of Attica.
- Great seasonality in most products EUREKA Group, therefore considerable variation after the forecasting of the demands of the products.
- Dependence of the whole Vehicle Routing system upon the known base of demands. (Inapplicability of the Stochastic Customers Strategy).

Opportunities

- A better utilization of the installations of Interstar CHIM S.A. in Bucharest, with a capacity of production to one shift 8,000 tons of chemical products and 300 tons of plastic bottles and caps in three shifts.
- The application of optimization strategies at every level (Routing, Forecasting, Facility Location).
- The production of private label products by the Group Eureka has won the last years the trust of many large supermarket chains within and outside Greece and this goal is to continue.

Threats

- Country's economic crisis complicates the implementation of development projects.
- The introduction of competitive companies with highly optimized processes.

5.13 Conclusions

- We improved the Vehicle Routing of Eureka Hellas S.A. cost for June of 2016 (by considering the kms) by 18%, or by 233 €/month and 2.800 €/year.

- We improved the Vehicle Routing of Eureka Hellas S.A. distribution system (considering kms and staff) expenses by 38,7%, or by 3233 €/month and 38.800 €/year.

- We justified that a change in the current strategy of Vehicle Routing of the company, to a strategy with Stochastic Customers can reduce its distribution system (considering kms and staff) expenses by 24%, or 1.978 €/month and 23.727 €/year.

- We optimized the facility location of the depot of Eureka Hellas S.A. in Attica, based on its clients demands.

- We introduced a basic forecasting for the demands of the products.

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