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" $\dot{\epsilon} \mu \tilde{\upsilon} \acute{\upsilon} \acute{\upsilon}$ $\tilde{\iota} \acute{\upsilon} \acute{\epsilon}$ " (15, 5)
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 μ μ μ μ μ
 μ μ μ μ μ , μ
." (A. Einstein, 24/01/1936)

μ Ha ,
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 μ μ μ μ μ μ
 μ μ μ μ Gr
 μ μ . μ μ
 μ μ μ μ
 μ (Ha) μ
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 μ μ .
 μ μ μ μ
 μ Burr &

Muller (2002)

μ μ μ .
 μ μ μ
 μ μ Reynolds μ μ μ
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 μ μ μ . μ μ
 μ μ μ Arnoldi, $\mu\mu$ μ
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 $\mu\mu$ μ μ μ μ
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μ μ μ μ Burr & Muller (2002)
 μ Gr μ μ .
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 μ Burr & Muller (2002), μ μ
 μ μ μ μ μ μ
 μ μ μ μ μ μ semi-
implicit μ μ μ μ μ μ
 μ μ μ μ Adams-Bashforth
 μ μ μ μ Crank-Nicolson
 μ μ μ (spectral) modes
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ABSTRACT

Rayleigh-Bénard stability of a liquid metal layer of rectangular cross section is examined in the presence of a strong magnetic field that is aligned with the horizontal direction of the cross section. The latter is much longer than the vertical direction and the cross section assumes a large aspect ratio. The side walls are treated as highly conducting. Linear stability analysis is performed allowing for three dimensional instabilities that develop along the longitudinal direction. The finite element methodology is employed for the discretization of the stability analysis formulation while accounting for the electric conductivity of the box walls. The Arnoldi method provides the dominant eigenvalues and eigenvectors of the problem. In order to facilitate parallel implementation of the numerical solution at large Hartmann numbers, Ha , domain decomposition is employed along the horizontal direction of the cross section. As the Hartmann number increases a real eigenvalue emerges as the dominant unstable eigenmode, signifying the onset of thermal convection, whose major vorticity component in the core of the layer is aligned with the direction of the magnetic field. Its wavelength along the longitudinal direction of the layer is on the order of twice its height and decreases as Ha increases. The critical Grashof was obtained for large Ha and it was seen to scale like Ha^2 signifying the balance between buoyancy and Lorentz forces. For well conducting side walls, the nature of the emerging flow pattern is determined by the combined conductivity of Hartmann walls and Hartmann layers, $c_H + Ha^{-1}$. When poor conducting Hartmann walls are considered, $c_H \ll 1$, the critical eigensolution is characterized by well defined Hartmann and side layers. The side layers are characterized by fast fluid motion in the magnetic field direction as a result of the electromagnetic pumping in the vicinity of the Hartmann walls. Increasing the electric conductivity of the Hartmann walls was seen to delay the onset of thermal convection, while retaining the above scaling at criticality. Furthermore, for both conducting and insulating Hartmann walls and the entire range of Ha numbers that was examined, there was no tendency for a well defined quasi two dimensional structure to develop owing to the convective motion in the core. A connection is made between the above findings and previous experimental investigations indicating the onset of standing waves followed by travelling waves as Gr is further increased beyond its critical value. Asymptotic analysis of the dominant eigenmode performed at criticality confirms the above numerical findings while

designates the onset of highly convective motion $O(Ha)$ as the magnetic field increases. This motion is expected to play central role for the onset of secondary instabilities of hydrodynamic nature.

Three dimensional stability of two dimensional vortical flow of a liquid metal in a cavity of square cross section is examined in order to identify the nature of the emerged time-dependent instability reported by Burr & Muller (2002). Vortices are produced as a result of free convection and internal heating in the cavity in the presence of a magnetic field. The low magnetic Reynolds equations are employed for the base flow and stability formulation. The finite element methodology is used for discretizing the problem. Efficient calculation of the dominant eigenvalues is afforded by the Arnoldi method while neutral stability diagrams are constructed using continuation techniques. The number of vortices exhibited by the base flow switches from one to two as the internal heating crosses a threshold value. The dominant instability mechanism is the Goertler instability for the case of a single vortex and the elliptical instability in the case of two vortices. In the elliptic instability axial vorticity is symmetric, is characterized by two lobed structures aligned with one of the two principal directions of strain and the dominant eigenmode assumes the form of a travelling wave. The magnetic field opposes buoyancy, alters the direction of maximal strain by accentuating wall shear layers in comparison with the vortex pair in the core, and leads to smaller frequencies at criticality. The above flow configuration is assessed to play important role for the onset of travelling wave modes as indicated by experiments of Burr & Muller (2002) with a slight increase of Gr beyond the threshold value.

Finally, nonlinear analysis is carried out of the flow arrangement that was examined by Burr & Muller (2002), seeking a description of the nonlinear evolution of unstable modes beyond criticality, coupling finite element methodology with a spectral approach for the periodic direction of flow. The latter is determined by the configuration that emerges after the onset of the initial thermal instability. A semi-implicit time integration scheme is employed, with the nonlinear convective terms treated explicitly using second order Adams-Bashforth method and linear terms treated in an implicit manner using second order accurate Crank-Nicolson so that decoupling of the different spectral modes is possible while favoring parallel treatment of the solution process. Objective of this study is the investigation for the

onset of saturation that corresponds to the initial thermal instability, identifying the kind of branch that causes as well as the effect on the heat transfer through the liquid metal layer.

1_1H	=	
2_1H	=	
3_2He	=	$3/2$
4_2He	=	$4/2$
	=	μ
b	=	μ
C	=	$\mu \quad \mu \quad \mu$
		$\mu \quad \mu \quad 1/Pr$
		$\mu \quad \mu$
C	=	$\mu \quad \mu \quad \mu \quad \mu$
		$\mu \quad - \quad \mu \quad \mu$
c_H	=	$\mu \quad \mu \quad \text{Hartmann}$
c_p	=	μ
c_s	=	$\mu \quad \mu$
D	=	$\mu \mu$
		$\mu \quad \mu \quad \mu$
		μ
DFT	=	$\mu \quad \mu \quad \mu \quad \text{Fourier}$
d	=	$\mu \quad \mu$
dS	=	$\mu \quad \mu$
e	=	μ

E	=	μ	Ekman,	$= 2 k_0^2 / (\nu / 2)$	
e_m^*	=	μ		μ	1 x m
f	=	residual	m-	μ	Arnoldi
\vec{f}	=	μ	Lorentz		
FDM	=	μ		μ	
FEM	=	μ		μ	
FFT	=	μ	μ	μ	Fourier
FVM	=	μ		μ	
\bar{g}	=	μ			
g_k	=	μ		μ	Fourier
		mode			
Gr	=	μ	Grashof,	$Gr = \frac{g \Delta T h^3}{\nu^2}$	
Gr_{Cr}	=	μ	μ	μ	Grashof
Gr_{Eff}	=	μ	μ	μ	Grashof, $Gr_{Eff} = GrSP_r$
$Gr_{Eff,Cr}$	=	μ	μ	μ	μ Grashof
h	=				
a	=	μ	Hartmann,	$Ha = \sqrt{\frac{h^2 \dagger B_0^2}{\dots \epsilon}}$	
j	=			μ	
J	=				
J_i	=	μ	μ		i
J_x	=		Frechet		μ x

J_x	=	x	μ
J_y	=	y	μ
J_z	=	z	μ
J_{Hx}	=	x Hartmann	μ
J_{Hy}	=	y Hartmann	μ
J_{Sy}	=	μ	μ
k	=	μ μ	
k	=	Fourier mode	
k_{Cr}	=	μ μ μ μ	
L	=		
L	=	μ μ μ	$\mu\mu$ $\mu\mu$
ℓ	=	μ	
\bar{n}	=	μ	
n	=		z
	=	μ μ	
P	=		
p	=		
p_0	=		
p_1	=	μ	

p_0	=	μ
p_{St}	=	μ
Pr	=	μ Prandtl, $Pr = \frac{\mu}{\rho k}$
q	=	μ μ μ
r	=	(residual)
Ra	=	μ Rayleigh, $Ra = \frac{g \beta \Delta T L^3}{\nu \alpha}$
Ra_{Cr}	=	μ μ μ Rayleigh
Re_m	=	μ μ Reynolds, $Re_m = uL/\nu$
S	=	μ μ μ , $S = \frac{qL^2}{\Delta T \dots c_p \epsilon}$
t	=	μ
t	=	μ μ
Av	=	μ μ μ
b	=	μ μ
t_{Co}	=	μ μ
t_{St}	=	μ μ
t	=	μ μ
u	=	x μ
u	=	x μ μ
u	=	
\bar{u}_0	=	
U_0	=	

$$u_0 = x \quad \mu$$

$$u_1 = x \quad \mu \quad \mu$$

$$V =$$

$$v = y \quad \mu$$

$$= y \quad \mu \quad \mu$$

$$v_0 = y \quad \mu$$

$$v_1 = y \quad \mu \quad \mu$$

$$V^{(k)} = \text{Krylov}$$

$$V_m = \mu \text{ Arnoldi}$$

$$w = z \quad \mu$$

$$w = z \quad \mu \quad \mu$$

$$w_0 = z \quad \mu$$

$$w_1 = z \quad \mu \quad \mu$$

$$w_i =$$

$$= \text{stretched} \quad \mu \quad \mu \quad \mu$$

Hartmann

$$x =$$

$$x = \mu$$

$$\bar{x} = \mu$$

$x^{(0)}$ = μ μ GMRES

x_e = μ μ μ μ

x_k = μ μ μ μ μ

x_{0j} =

x_{1j} = μ

y =

y = μ

z =

z = μ

μ

= μ

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+ =

= $\mu\mu$

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du = Dirichlet

nu = Neumann

= μ

= Dirac-
 = μ
 t =
 z = μ
 = μ
 = μ
 y = μ
 = Hessenberg $n \mu$
 h = $\mu \mu$
 = μ
 ' = $\mu \mu$
 0 = μ
 1 = $\mu \mu$
 m = Krylov
 = $\mu \mu \mu$
 Cr = $\mu \mu \mu \mu \mu$
 = μ
 = μ Fourier modes
 N = $\mu \mu \mu - \mu \mu$
 = $\mu \mu \mu$
 j =
 e =

$$\begin{aligned}
&= \mu \quad \mu \quad \mu \quad \mu \\
&= \\
&= \mu \quad \mu \\
&= \mu \\
i &= \\
&= \mu \\
&= \mu \quad \mu \\
&= \mu \\
&= \mu \quad \mu \\
\hat{W} &= \mu \quad \mu \quad \mu \quad \text{Hartmann} \\
s &= \mu \quad \mu \quad \mu \\
0 &= \mu \\
l &= \mu \quad \mu \\
i &= \mu \quad \text{Lagrange} \\
ij &= \text{Lagrange} \\
i &= \mu \quad \mu\mu \quad \text{Lagrange} \\
ij &= \mu\mu \quad \text{Lagrange} \\
&= \\
&= \\
\check{S} &= \mu \\
r &= \mu \quad \mu \quad \mu \quad \mu \quad \mu
\end{aligned}$$

$$\begin{aligned}
i &= \mu & \mu & \mu & \mu & \mu \\
x &= X & & \mu & & \\
yz &= & & \mu & & , h_{yz} \hat{0} \sqrt{h_y^2 < h_z^2} \\
xy &= & & \mu & & , \check{S}_{xy} = \sqrt{\check{S}_x^2 + \check{S}_y^2} \\
z &= Z & & \mu & &
\end{aligned}$$

	μ	
1	Error! Bookmark not defined.
1.1	μ μ μ	Error! Bookmark not defined.
1.2	μ μ μ	
	μ Rayleigh - Benard ,	
	μ μ 10
1.3	μ μ μ	
	μ , μ	
	μ μ μ μ 14
1.4	μ 20
1.5	μ μ (FEM) 21
1.5.1	Lagrange 2 Error! Bookmark not defined.
1.5.2	μμ Lagrange 25
1.6	μ μ (Spectral Method) 26
1.7	μ 26
2	29
2.1	μ μ μ	
	Rayleigh - Benard , μ	
	μ 29
2.2	μμ μ μ μ	
	Rayleigh - Benard , μ	
	μ 36
2.3	μ μ μ	
	, μ μ μ μ	
	μ 38
2.4	μμ μ μ	
	, μ μ μ μ	
	μ Error! Bookmark not defined. 0
3	42

3.1		μ	Galerkin	42
3.1.1		μ	42
3.1.2		μ	(weighted residual)	42
3.1.3			Galerkin	44
3.2	μ	$\mu\mu$	μ	Rayleigh -
	Benard	μ	μ	,
	μ			μ
			45
3.2.1		μ	μ - μ	Arnoldi
3.3	μ	$\mu\mu$	μ	
		μ	μ	,
	μ	μ	μ	μ ..
			52
3.3.1			GMRES	55
3.3.2			GMRES μ preconditioner	57
3.3.3			μ	μ
	μ	-	μ	GMRES
			60
3.4		μ	$\mu\mu$	μ
	Rayleigh - Benard	μ	μ	,
	μ	μ		
			63
3.4.1			μ6Error! Bookmark not defined.
3.4.2			μ 68
3.5			"benchmarks" ..	70
3.5.1	Benchmark		μ	$\mu\mu$
			μ	
			Rayleigh - Benard	μ
			μ	μ
			,	
			μ	μ
			71
3.5.2	Benchmark		μ	$\mu\mu$
			μ	
			μ	μ
			,	
	μ	μ		μ
			μ	μ
			75
3.6		$\mu\mu$	μ	Rayleigh - Benard
		μ	μ	,
				μ
	μ		
				79

3.6.1	μ μ Fourier ...	82
3.6.1.1	μ transpose FFT	84
3.6.1.2	$\mu\mu$ Fourier ...	87
3.6.2	μ $\mu\mu$	88
3.6.3	μ μ $\mu\mu$	89
3.6.4	μ	92
3.6.5	μ	98
3.6.5.1	μ Fourier ...	98
3.6.5.2	μ μ $\mu\mu$	99
3.6.5.3	μ	100
4		104
4.1	μ $\mu\mu$ μ Rayleigh - Benard μ μ ,	
	μ μ	104
4.1.1	μ μ μ Ha	105
4.1.2	μ μ μ Ha	115
4.2	μ $\mu\mu$ μ μ μ μ , μ	
	μ μ μ μ	135
4.2.1	μ μ S Ha=20	146
4.2.2	Ha=20 S=10 ⁵	150
5		154
		162

	μ		
μ 1:		μ4
μ 2:	μ	$\mu\mu$	ITER.....6
μ 3:		μ6
μ 4:	μ	$\mu\mu$	μ
	μ	Helium Cooled Lead Lithium HCLL (Buhler et al. 2010).9
μ 5: (a)	μ	$\mu\mu$	μ
		, (b)	
	μ	(Gr=6000, S=10 ⁵ , Ha=20	
	Pr=0.0321).	19
μ 6:		quadratic Lagrange	μ24
μ 7:		linear Lagrange	μ25
μ 8a:	μ	$\mu\mu$	μ
	μ	29
μ 8b:	μ	μ	μ
	, xz,	, yz,	μ35
μ 9:	μ	μ	μ
	μ	μ	μ
	n		μ
	μ	64
μ 10:	μ	μ	μ
	μ	μ	μ
	n	μ	μ

	μ μ	μ μ		
	μ	μ	μ64
μ 11:	μ	μμ	J	
	μ μ	μ		
	μ μ	μ μ μ	μ	
	ScaLAPACK			69
μ 12:	μμ		μ	
		μ	μμ	μ
	μ	μ70	
μ 13:	μ	μ	Ha=0	μ (a)
	60x30		(b) 80x40	
			72
μ 14:		(a) μ	, (b) x	
		, (c) yz	Ha=100	μ
	110x55	.	μ (d,e)	μ
	μ (b,c)	μ	μμ	μ
	(f,g)	yz	μ (c,e),	
			μ Hartmann74
μ 15:	μ	μμ	μ	Xin &
	Le Quere (2001)			75
μ 16:	μμ			

	μ	benchmark	μ	77
μ 17:	μ	μ		μ	
	μ	μ (mode 4)		μ	
	Ra=1400000, Pr=0.71		μ (a) 20x20	(b) 40x40	
				79
μ 18:	μ	μ		μ	μ
		(a) mode 1, Ra=2400000, Pr=0.71		(b)	μ
	μ, Ra=1400000, Pr=0.71		μ 40x40	79
μ 19:	μ	μ			
	μ	FFT 16	μ	,	m
				84
μ 20:	μ	μ	μ		
	μμ				
	μμ	μ	μ	μ87
μ 21:		μμ		Fourier	
	μ		μ	μ88
μ 22:		-spectral		μ	
	μ	μ	μμ	91
μ 23:		μ (partitioning)		μ, (a)	
	μ		, (b)	μ	Fourier97
μ 24:	μ	(partitioning)	μ		

	μ	Fourier	100
μ 25:	μ	16	μ	
	4 Fourier modes		100
μ 26:	$\mu\mu$		μ	
	μ	μ	$\mu\mu$	μ
	μ	μ	103
μ 27:	$\mu\mu$		(a)	
	μ	μ	Hartmann	(b) μ
	μ	Hartmann	117
μ 28:	μ		y	(a)
	μ	μ	(b) μ Ha.	μ
	y=0.5		120
μ 29:		μ	x	
	μ	Ha.	μ	x=0.95
			121
μ 30:	x	Ha	, (a) Ha=25, (b)
	Ha=100, (c) Ha=200, (d) Ha=400.			x
			
		μ	Hartmann, (e)	
	Ha=25, (f) Ha=100, (g) Ha=200			(h) x
	Ha=400.		124
μ 31:		μ	Ha	
	μ	$c_H=0.00415, c_S=4.5, (a) Ha=25, (b) Ha=100,$		

	(c) Ha=200	(d) Ha=400	126
μ 32:	(J_x, J_y)	xy	$z=0$ (Ha=25)	(a) μ (b) μ (c) (J_y, J_z)
		μ Hartmann	$x=0$ (Ha=800)127
μ 33:		yz	(a)	xy Ha=100,
			μ	(a) Ha=25,
	(b) Ha=50, (c) Ha=100	(d) Ha=200	130
μ 34:		μ		μ
	$c_H=c_S=4.5$	Ha	, (a) Ha=100, (b) Ha=400, (c) Ha=800	
	(d) Ha=2000		133
μ 35:		x		μ $c_H=c_S=4.5$
	Ha	, (a) Ha=100, (b) Ha=400, (c) Ha=800	(d)	
	Ha=2000. (e)	x	Ha=2000134
μ 36:		yz		xy
	μ	$c_H=c_S=4.5$	Ha	, (a) Ha=100, (b) Ha=400,
	(c) Ha=800	(d) Ha=2000	135
μ 37: (a)				(b)
			μ	
	$Gr=Gr_{Cr}=850000, S=0$	Ha=20	137
μ 38:			Ha=0	$S=10^5$ (a)
		, (b)	μ	μ

	(c)	μ	140
μ 39:	-	μ	Ha=0 S=10 ⁵ (a)	
		μ μ	(b)	
		μ	141
μ 40:			Ha=20 S=10 ⁵ (a)	
		, (b)	μ μ	
	(c)	μ	142
μ 41:	-	μ	Ha=20 S=10 ⁵ (a)	
		μ μ	(b)	
		μ	143
μ 42:	$\mu\mu$		(a) Ha=0 () (b) Ha=20	
	()		144
μ 43:			Ha=40 S=10 ⁵ (a, b)	
	(c)	μ	. μ (b)	
			. μ μ	
		μ μ	μ	
	μ ,		145
μ 44:			Ha=20, (a) S=0 (b) S=10 ⁵	147
μ 45:	-	μ	μ μ μ μ	
		Ha=20, (a) S=0	(b) S=25000	148

μ 46:	-	μ	Ha=20, S=50000	μ	(a)	
	μ	μ	(b)	μ	148
μ 47:			Ha=20	S=5000	(a)	
	(b)	μ	μ		148
μ 48:			Ha=20	S=50000	(a)	
	, (b)	μ	μ	(c)		
		μ			149
μ 49:	μμ		=2, Ha=20.0, S=10 ⁵		151
μ 50:	(a)		, (b)	-		
	μ	μ	μ	μ	Ha=20.0	
	S=10 ⁵				152
μ 51:	(a)		, (b)	-		
	μ	μ	μ	μ	μ	
	=7.86, Ha=20.0	S=10 ⁵			153
μ 52:	(a)		, (b)	-		
	μ	μ	μ	μ	μ	
	=23.16, Ha=20.0	S=10 ⁵			153
μ 53:	(a)		, (b)	-		
	μ	μ	μ	μ	μ	
	=20.8, Ha=20.0	S=10 ⁵			153

1:	μ	Error! Bookmark not defined.
2:	μ	CPU	μ .
	μ	cluster	μ
&	μ	68
3:	μ	Hopf	μ (
	Ra)	μ	Pr=0.71.....7 Error! Bookmark not defined.
4:	μ	μ	μ μ Ha
	(a)	μ μ	μ Hartmann (b) μ
	μ	Hartmann.119

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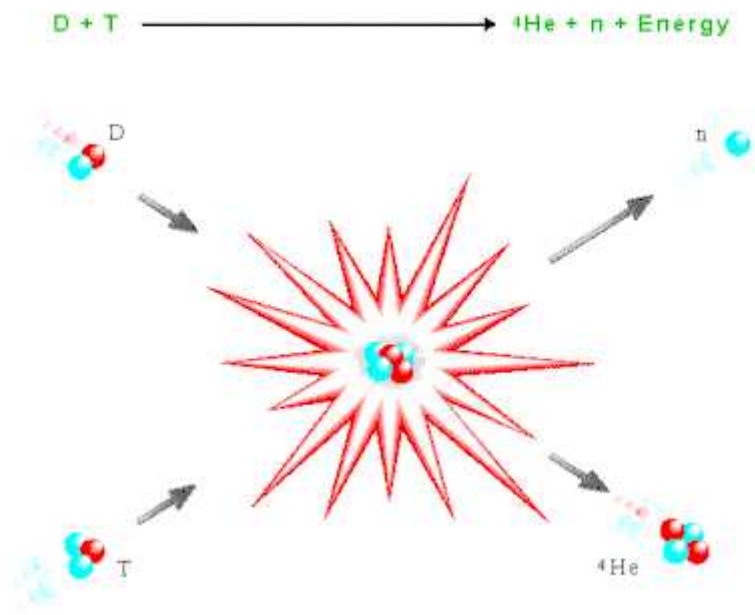
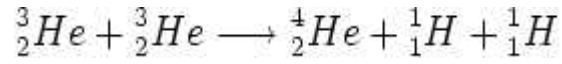
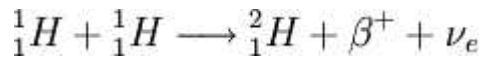
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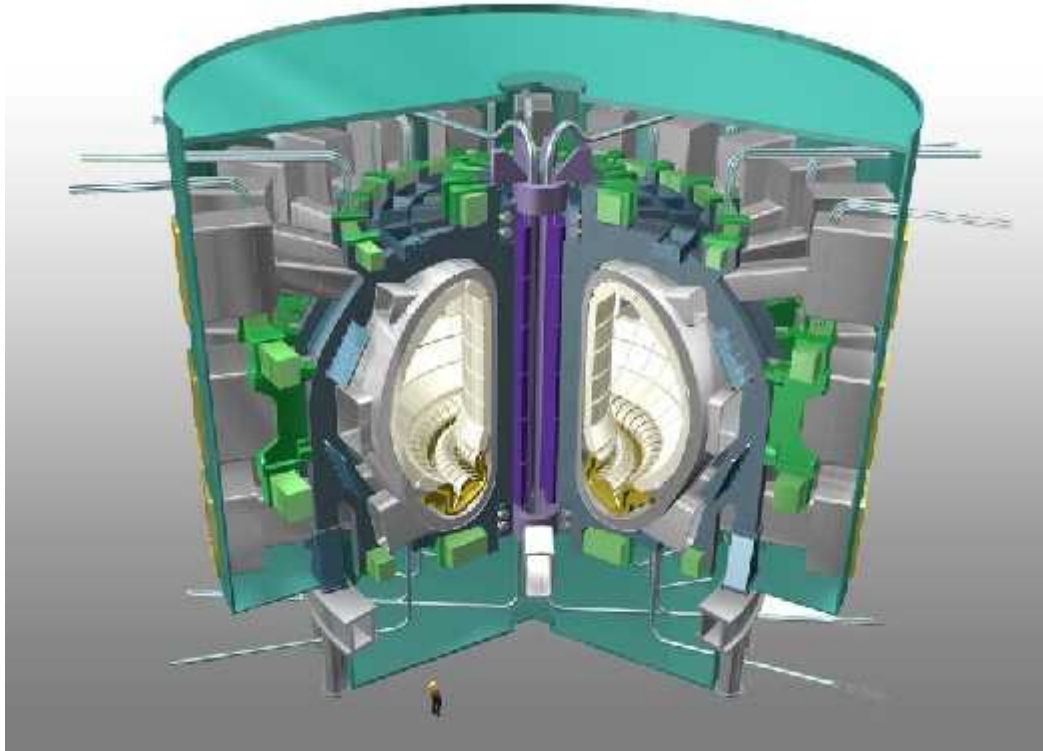
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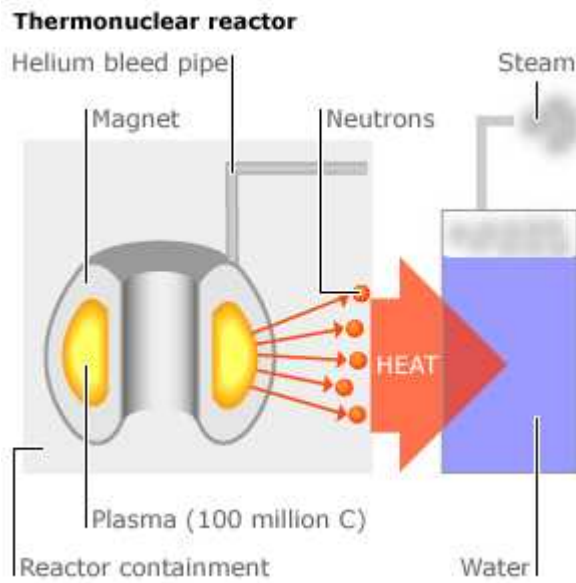
- 1_1H ,
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<p>❖ μ μ ❖ μ μ μ μ μ μ ❖ μ μ ❖ μ μ ❖ ❖</p>	<p>❖ μ μ — — μ — — μ — μ μ ❖ μ μ μ ❖</p>

μ & μ

❖ μ μ	
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1: μ

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Dual

Coolant Lead Lithium, DCLL,

μ μ

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(Buhler & Norajitra 2003,

Fidaros et al. 2008).

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μ SiC μ

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Burr et al.

(2008),

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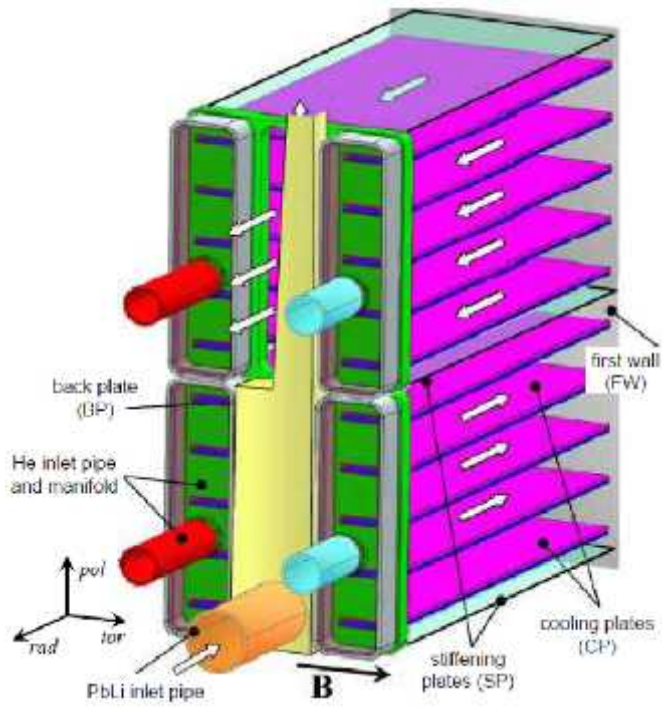
μ

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μ & μ



μ 4 : μ μ μ μ μ
 μ Helium Cooled Lead Lithium HCLL (Buhler & Mistrangelo, 2010)

μ HCLL (. μ 4), μ μ μ
 μ ,

μ μ

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μ μ . μ μ μ μ μ μ

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μ μ μ μ μ μ μ .

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μ μ μ , , μ μ

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μ & μ

1.2

μ μ

μ μ μ μ
, μ μ

μ

(Waleffe, 2003, Eckhardt et al., 2008)

μ μ μ μ μ μ μ μ
μ , μ μ μ μ (coherent
structures)

μ μ μ μ μ μ
μ μ μ μ . ,
μ

(Chandrasekhar, 1961, Sommeria & Moreau, 1982). μ μ ,

μ , μ
μ μ .
μ

μ μ μ μ μ μ μ μ
μ μ μ μ μ μ μ μ

μ Hartmann (Hartmann walls)

μ Lorentz μ μ μ μ (side walls)

μ μ μ μ μ μ (Buhler,
1996, 1998). μ μ μ

μ μ μ
(Potherat et al. 2000, 2005).

μ , μ μ μ
μ μ μ μ μ μ μ μ
μ μ μ μ μ μ μ μ
μ

μ & μ
 μ , $\mu\mu$
 μ Hartmann, Ha,
 μ
 μ (Ting et al., 1991). , μ μ
 μ μ μ μ
 μ
 μ μ
 μ μ μ μ
 μ , μ μ μ
 μ μ μ μ
 μ μ μ μ μ μ
 μ , μ μ μ μ μ (first wall)
 μ μ μ μ μ μ
 μ , μ μ μ μ μ jets μ μ μ
 μ μ (Burr et al., 2000).
 μ
 μ μ μ μ , μ
 μ μ μ μ μ μ μ
 μ μ μ μ μ Reynolds
 μ μ μ Ha, μ
Sommeria & Moreau (1982).
, Rayleigh - Benard μ
(Burr & Muller 2002) μ ,
 μ μ μ μ
 μ Ha, μ μ Grashof, Gr,
 μ Ha, $100 < Ha < 1000$,
 μ
 μ μ μ μ

μ & μ

μ

ITER (Buhler & Norajitra 2003, Fidaros et al. 2008, Buhler & Mistrangelo 2010, Kharitsa et al. 2004).

(Kharitsa et al. 2004, Burr & Muller 2002)

Hartmann (Buhler 1998)

Hartmann $Gr \ll Ha^{5/2}$

$Gr/Ha^{5/2} \sim 0.5$

Burr & Muller

Grashof, Gr_{Cr} , Ha

Hartmann

Hartmann

Hartmann

Hartmann

Hartmann

Hartmann

Hartmann

Hartmann

Hartmann

μ & μ

μ

1.3

μ

μ

μ μ
, μ μ μ μ
μ

μ μ
μ μ μ μ μ μ

(Pierrehumbert 1986, Ortega 2003, Bristol et al. 2004)

μ (Pierrehumbert 1986,

Landmann & Saffman 1987). μ , μ

μ μ μ μ ,

μ μ μ

(Landmann & Saffman 1987, Grossmann 2000). μ

μ μ μ μ μ μ

μ μ (Waleffe 1995, Grossmann

2000).

O Crow (1970)

Kelvin μ μ

μ μ μ μ

μ μ μ μ

μ μ μ μ

μ μ , μ

μ μ μ μ Kelvin

μ μ

μ μ μ μ

μ μ μ μ

μ & μ
 $\mu\mu$, μ μ μ
 μ μ , μ μ
(Pierrehumbert 1986) (Gledzer & Ponomarev 1992).
, μ μ μ
 μ μ (Tsai & Widnall 1976). , μ
 μ μ - μ
(Leweke & Williamson 1998, Bristol et al. 2004)
 μ μ μ μ μ
. Particle Image
Velocimetry (PIV) ,
 μ μ μ , μ
 μ
(Leweke & Williamson 1998). ,
 μ μ μ μ μ , μ
 μ μ μ μ . ,
 μ μ
(Gledzer & Ponomarev 1992)
. μ , μ
 μ μ μ μ μ .
 μ μ μ μ
 μ μ μ .
 μ , μ
 μ μ
 μ μ μ . ,
 μ μ μ μ , μ
 $\mu\mu$ (Pierrehumbert
1986) μ ,
 $\mu\mu$ μ
, μ μ μ
 μ μ . , μ
 μ μ (Landman & Saffman 1987) μ μ

μ & μ
 Ekman, μμ
 , μ μ μ μ μ μ .
 μ μ μ μ μ Ekman : =2 k₀²/(/2)
 μ ,
 , μ ℓ = 2f / k₀ μ
 . , μ μμ
 μ (Waleffe 1990), . μ
 (Waleffe 1995) ,
 μ μ μ μ μμ ,
 μ μ μ .
 μ μ μ μ
 μμ . μ
 μ μ μμ -
 μ (Waleffe 1995)
 μ μ μ μ
 (Grossmann 2000).
 , μ μ μ
 μ Reynolds, Re_m = uL/γ , (γ : μ)
 μ
 μ (Sommeria &
 Moreau 1982). , μ , μ
 μ μ μ Navier-Stokes
 μ μ . μ
 μ . , μ μ
 μ μ .
 , μ μ
 μ μ (Burr et al. 2000,
 Burr & Muller 2002) μ

μ & μ

2002)

, ,

μ μ .

μ μ

μ μ . μ ,

μ ITER μ μ

μ μ μ

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μ μ μ

μ μ μ μ

μ μ Burr & Muller (2002). μ

μ μ μ μ

μ μ

μ μ

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. , μ μ μ

μ , μ ,

μμ μ μ . ,

μμ μ μ μ μ

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μ μ (Sommeria

& Moreau 1982). μ μ μ

μ μ μ μ

μ μ μ μ μ .

μ

μ , μ

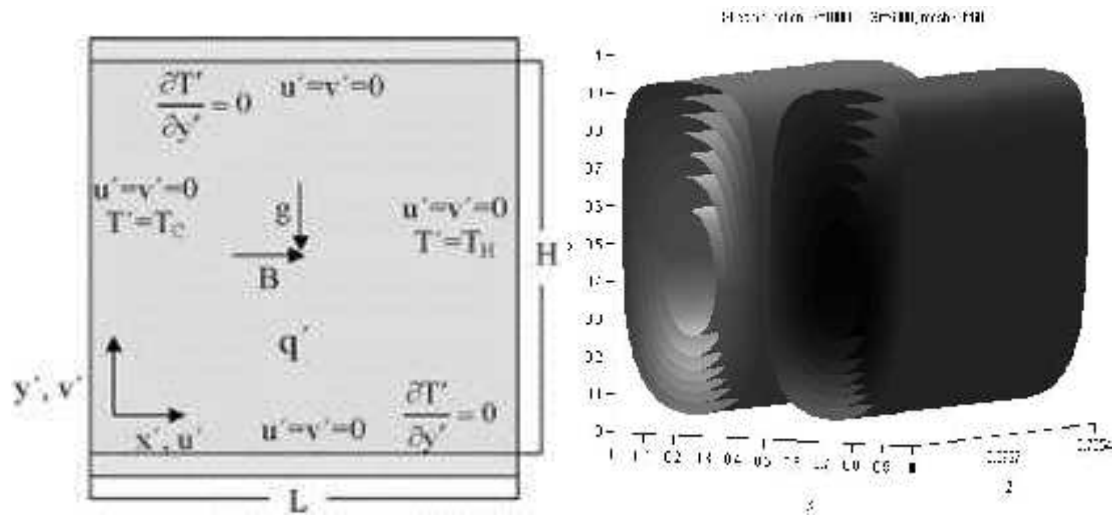
(Buhler & Mistrangelo 2010), . μ 4.

,

μ μ μ

μ & μ
 μ .
 μ μ

Burr & Muller (2002).



μ 5 : (a) μ $\mu\mu$ μ
 , (b) μ

(Gr=6000, S=10⁵, Ha=20 Pr=0.0321).

μ μ (Pelekasis 2006)

μ
 μ μ ,
 , (μ 5a).

μ μ μ
 μ μ , (μ 5b).

μ μ μ Gr
 Hopf
 μ .
 μ , $\mu\mu$
 μ μ , μ
 . μ μ

1.4

μ

μ (Computational Fluid Dynamics, CFD)

μ μ μ .
 μ μ μ .
 μ μ μ

CFD :

μ μ μ μ (hardware),

μ μ μ μ .

μ μ , μ

μ , μ μ μ , μ

μ :

μ (Navier - Stokes)

(). μ μ Navier - Stokes

μ :

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} = -\nabla p + \epsilon (\nabla^2 \vec{u}) \quad (1.1)$$

:

$$\nabla \cdot \vec{u} = 0 \quad (1.2)$$

, t , ε μ , p μ
 (μ μ) ∇ .

, μ μμ
 μ

μ μ μ
 (, . . .). , CFD

μ & μ

μ μ μ

μ .

μ μ μ ,

μ μ μ μ .

μ μ μ -

μ , μ

μ μ

μ

μ μ μ μ

μ μ μ μ

(Finite Difference Method FDM), μ μ

(Finite Volume Method FVM) μ μ (Finite Element Method FEM). μ

(μ , Fletcher et al. 1991, Hirsch 1988, Dick 1993).

μ , μ , μ

μ (spectral method, Gottlieb & Orszag 1977) .

1.5 Η μ μ (FEM)

FEM μ

μ . μ

μ μ μ μ μ

μ μ μ μ μ FVM

μ

μ Gresho & Sani (2000) : " μ

μ μ μ μ μ μ μ

μμ "

μ μ FEM μ

μ μ μ

μ μ μ μ

& μ
 μ , μ , μ
 μ . μ μ μ
 μ , μ (μ) . μ μ μ (unstructured).
 μ μ μ μ μ μ (unstructured).
 μ μ FVM
 FDM.
 μ μ μ μ μ
 (μ μ) μ
 μ . μ
 μ μ . μ
 μ μ μ .
 μ μ μ μ .
 μ μ μ μ (biquadratic)
 Lagrange $i(x,y)$ μ ,
 μ μ μ μ (bilinear) Lagrange $i(x,y)$.
 μ μ μ μ μ μ
 μ μ μ μ μ μ
 μ , μ μ μ
 μ μ μ μ (weighted residual
 formulation).
 μ μ μ μ
 Galerkin μ μ μ μ μ μ
 μ μ μ μ μ μ Petrov -
 Galerkin. μ μ μ μ μ μ
 μ FEM μ μ μ
 Neumann. μ μ μ
 μ FDM FVM. , μ μ μ
 μ : μ μ μ

μ & μ

- i
 - μ
 - μ
 - μ (\dots)
 - μ
 - $u = \sum_{i=1}^n u_i \xi_i$, u
 - e μ μ $[K^{(e)}]\{u^{(e)}\} = \{F^{(e)}\}$.
 - μ μ
 - μ
 - μ
 - μ
 - μ
- μ FEM, μ
- μ μ Gresho & Sani (2000).

1.5.1 Lagrange

Lagrange μ 3 μ μ quadratic

e μ μ μ x_e x_{e+1} quadratic Lagrange

μ μ

μ 6 :

μ & μ

$$\mathbb{E}_1 = -\frac{1}{2}\langle(1-\langle)\rangle$$

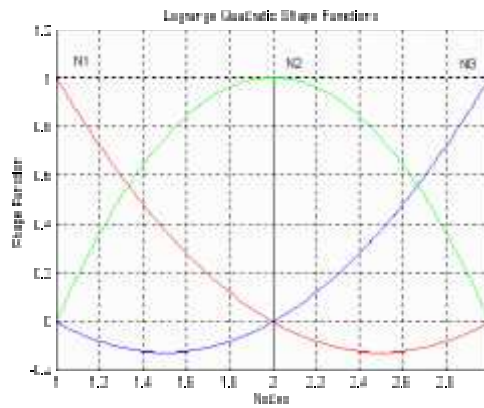
$$\mathbb{E}_2 = (1+\langle)(1-\langle)$$

$$\mathbb{E}_3 = \frac{1}{2}\langle(1+\langle)$$

$$\langle = \frac{2x - (x_e + x_{e+1})}{x_{e+1} - x_e}$$

μ μ -1 1

μ μ μ μ μ



μ 6 :

quadratic Lagrange μ

quadratic Lagrange

μ
Lagrange

e :

$$\mathbb{E}_i^e(\langle_j) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

$$\sum_{i=1}^3 \mathbb{E}_i^e(\langle) = 1$$

μ

μ

μ μ

μ

$$: u = \sum_{i=1}^3 u_i \mathbb{E}_i$$

i

μ

, u_i

μ

μ

i

quadratic Lagrange

μ

μ

μ

μ

:

μ & μ

$$\Psi_{ij}(x, y) = \mathbb{E}_i(x)\mathbb{E}_j(y), \quad i=1,2,3, \quad j=1,2,3$$

1.5.2 μμ Lagrange

μ μ μ linear Lagrange
 μ 3 μ . μ μ
 , μ e μ μ μ x_e
 X_{e+1} linear Lagrange μ

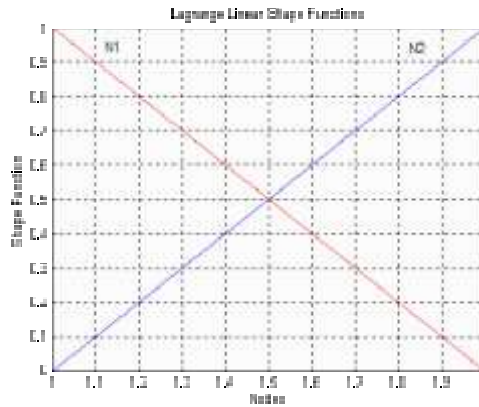
μ
 μ :

$$\mathbb{E}_1 = \frac{1}{2}(1 - \kappa)$$

$$\mathbb{E}_2 = \frac{1}{2}(1 + \kappa)$$

$$\kappa = \frac{2x - (x_e + x_{e+1})}{x_{e+1} - x_e} \quad \mu \quad \mu \quad \mu \quad \mu$$

μ μ μ -1 1 .



μ 7 : linear Lagrange μ .

μ μ μ
 μ μ μ μ μ : $u = \sum_{i=1}^2 u_i \mathbb{E}_i$

μ & μ
 i μ , u_i μ μ
 i .
 , $\mu\mu$ Lagrange
 μ μ μ μ ,
 :
 $\Psi_{ij}(x, y) = \Phi_i(x)\Phi_j(y)$, $i=1,2$, $j=1,2$

1.6 μ (Spectral Method)

μ μ μ
 Fourier μ Chebyshev. μ μ
 μ μ μ
 μ .
 μ μ , μ μ (weighted
 residual approach)
 μ μ μ μ μ (
 collocation).
 μ μ μ . μ , μ μ μ
 μ
 μ .
 μ μ μ μ
 Canuto et al. (1988) Hussaini & Zang (1987).

1.7 μ

μ : 2
 μ μ μ μ μ
 μ (μ μ μ)
 $\vec{\nabla}\Theta // \vec{g}$ (μ μ)
 26

μ & μ
 $\bar{\nabla} \Theta \perp \bar{g}$
 $\mu \mu \mu$
 μ . 3 μ
 μ μ μ FEM. μ
 μ μ μ μ
 μ μ . ,
 μ
 μ
 (scalability) μ $\mu \mu$ $\mu \mu$.
 μ (benchmark cases) μ μ
 μ μ μ 3.5
 μ μ μ . ,
 3.6, μ μ μ $\mu \mu$ μ
 μ μ μ μ μ
 μ μ $\mu \mu$ μ $\mu \mu$.
 , μ μ μ μ μ
 μ domain Fourier decomposition μ
 μ μ
 $\mu \mu$ $\mu \mu$.
 4 μ μ μ
 μ
 μ $\mu \mu$. μ μ
 μ μ μ μ
 μ μ μ μ
 μ . , μ μ
 μ μ μ μ μ μ
 Gr_{Cr} μ Ha μ
 μ μ . μ
 μ (c_H) μ (c_S)
 μ .
 μ μ μ $\mu \mu$

&

Burr & Muller (2002).

=1 =2

5

Ha, $100 < Ha < 2000$.

Rayleigh-Benard

3

Burr & Muller (2002).

(adaptive) FEM

(Ainsworth & Oden 1992, 1993, Demkowicz 2007).

μ & μ

II.

2.1

μ **Rayleigh - Benard** μ ,

μ μ

μ μ μ μ μ μ μ , μ

8a.

μ μ

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: $= (T_b - T_t) / 2$.

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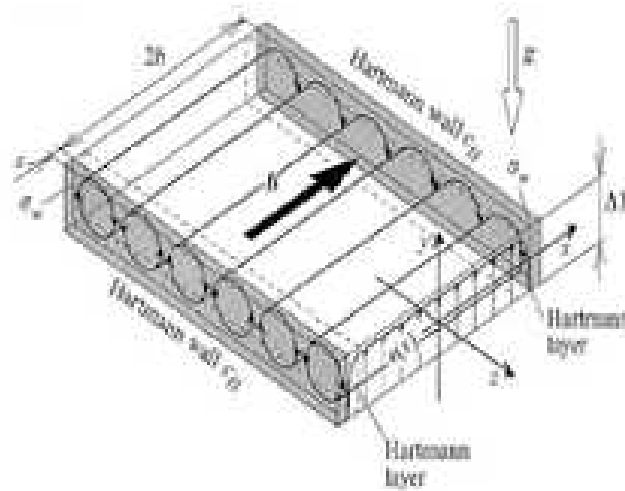
, (

μ

8a),

Burr & Muller

(2002).



μ 8a :

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$\mu\mu$

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$\mu\mu$

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:

μ & μ

$$\bar{u}_0 = 0, \quad T_0 = (T_t - T_b)y'/h + T_b = -2\Delta T y'/h + T_b \quad (2.1)$$

y μ μ .

μ Boussinesq μ ,

$$\rho = \rho_0 [1 - \beta(T - T_{Av})], \quad T_{Av} = (T_b + T_t)/2, \quad (2.2)$$

μ μ μ

<<1, μ μ μ

. μ μ

:

$$\rho'_0 = \rho'_{st} + \rho'_0 = -\rho_0 g y' + \rho'_0 \quad \rho'_0 = (\rho_0 g h) \beta \Delta T \left[\left(\frac{y'}{h} \right) - \left(\frac{y'}{h} \right)^2 \right] \quad (2.3)$$

μ μ

Rayleigh - Benard,

μ μ Gr μ

$$\mu, \quad Gr = \frac{g S \Delta T h^3}{\epsilon^2} \quad \mu$$

μ μ . , μ μ μ Gr,

Gr_{Cr} 865, μ

μ . μ

μ (Drazin &

Reid 1981).

μ μ Lorentz

μ μ μ μ

μ μ

μ . , μ μ μ

μ , x, μ

$$\vec{B} = B \vec{e}_x . \quad \mu \quad \mu$$

μ μ μ μ

Burr & Muller (2002)

μ μ . μ Hartmann

μ & μ

$$5.7 \times 10^7 \text{ m}^{-1}, \quad 1.37 \times 10^6 \text{ m}^{-1},$$

$$2.47 \times 10^{-6} \text{ m}^{-1}, \quad \text{Na}^{22} \text{K}^{78}$$

Reynolds, $\text{Re}_m N \sigma \mu_0 \mathbf{u} \mathbf{h}$

$$\rho \sigma \mu_0 : >^1, \quad \text{Re}_m \text{ (Low Re}_m \text{ model).}$$

$$\text{Ohm, } \vec{j}' = \sigma \left[-\vec{\nabla} \phi' + \vec{u}' \times \vec{B} \right]$$

, μ Lorentz
 μ : $\vec{f} = \vec{j}' \times \vec{B}$.

$$\vec{\nabla} \cdot \vec{j}' = 0 \rightarrow \nabla^2 \phi' = \vec{\omega}' \cdot \vec{B}, \quad \vec{\omega}' \equiv \vec{\nabla} \times \vec{u}' \quad (2.4)$$

$$(x, y, z) = \left(\frac{x'}{h}, \frac{y'}{h}, \frac{z'}{h} \right), \quad t = \frac{t'}{\sqrt{h/(g\beta\Delta T)}}, \quad p = \frac{p'}{\rho g \beta \Delta T h}, \quad \vec{u} = \frac{\vec{u}'}{\sqrt{g\beta\Delta T h}} \quad (2.5)$$

$$\Theta = \frac{T - T_{Av}}{T_b - T_{Av}}, \quad \phi = \frac{\phi'}{B h \sqrt{g\beta\Delta T h}} \quad (2.5)$$

μ & μ

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{2.6}$$

μ x, y, z

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{\partial P}{\partial x} + Gr^{-1/2} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \tag{2.7}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial P}{\partial y} + Gr^{-1/2} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \Theta - \frac{Ha^2}{Gr^{1/2}} \left(v + \frac{dw}{dz} \right) \tag{2.8}$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial P}{\partial z} + Gr^{-1/2} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{Ha^2}{Gr^{1/2}} \left(w - \frac{\partial w}{\partial y} \right) \tag{2.9}$$

$$\frac{\partial \Theta}{\partial t} + u \frac{\partial \Theta}{\partial x} + v \frac{\partial \Theta}{\partial y} + w \frac{\partial \Theta}{\partial z} = \frac{1}{Gr^{1/2} Pr} \left(\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} + \frac{\partial^2 \Theta}{\partial z^2} \right) \tag{2.10}$$

μ

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \tag{2.11}$$

(2.7-2.9)

x, y, z

μ Lorentz

(2.10)

(2.6, 2.11)

μ

μ

μ

μ

(2.6-2.11)

μ

μ

μ

μ

μ

μ & μ

Dirichlet.

μ ,

(Neumann), μ μ

μ .

μ μ

μ , μ μ μ μ :
μ μ μ μ μ μ

$$\bar{u}(x=0) = \bar{u}(x=A) = \bar{u}(y=0) = \bar{u}(y=1) = 0, \quad (2.12)$$

$$\Theta(y=0) = 1, \quad \Theta(y=1) = -1, \quad \frac{\partial \Theta}{\partial x}(x=0) = \frac{\partial \Theta}{\partial x}(x=A) = 0 \quad (2.12)$$

μ μ μ

μ (Walker 1981),

$$-\vec{j} \cdot \vec{n} = c_H \nabla_s^2 w \rightarrow \frac{\partial w}{\partial x}(x=0, y, z) = -c_H \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right), \quad \frac{\partial w}{\partial x}(x=A, y, z) = c_H \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (2.13)$$

$$-\vec{j} \cdot \vec{n} = c_S \nabla_s^2 w \rightarrow \frac{\partial w}{\partial y}(x, y=0, z) = -c_S \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right), \quad \frac{\partial w}{\partial y}(x, y=1, z) = c_S \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (2.13)$$

n μ μ

∇_s^2 Laplace

μ . μ c_S c_H μ

μ μ μ μ μ μ

μ μ . S H μ

Hartmann μ

μ , . ,

$$\mu : c_H = \frac{c_{St} t_{St}}{ch}, \quad c_S = \frac{c_{Co} t_{Co}}{ch}$$

t_{St}, t_{Co}

μ , , c, c_{St}, c_{Co}

μ μ ,

, .

μ μ 2.13.

μ

μ . μ

μ μ

Laplace.

μ & μ
 μ ,
 μ μ μ μ , μ
 μ μ μ μ μ μ ,
 μ μ μ μ μ μ ,
 μ μ μ μ μ μ . , μ μ
 μ μ

$$\ddot{w}_{St} N \neq 0 \Leftrightarrow \int_{xN > t_{St}} \frac{\partial^2 w_{St}}{\partial x^2} < \frac{\partial^2 w_{St}}{\partial y^2} < \frac{\partial^2 w_{St}}{\partial z^2} dx N \neq 0$$

$$\frac{\partial w_{St}}{\partial x} \Big|_{xN > t_{St}} < \frac{\partial^2 w_{St}}{\partial y^2} \int_{xN > t_{St}} w_{St} dx < \frac{\partial^2 w_{St}}{\partial z^2} \int_{xN > t_{St}} w_{St} dx N \neq 0 \Leftrightarrow \frac{\partial w_{St}}{\partial x} \Big|_{xN > t_{St}} N \neq 0$$

$$t_{St} \frac{\partial^2 \bar{w}_{St}}{\partial y^2} < \frac{\partial^2 \bar{w}_{St}}{\partial z^2} N > \frac{\partial w_{St}}{\partial x} \Big|_{xN0} N > \frac{c}{c_{St}} \frac{\partial w_l}{\partial x} \neq 0 \quad :$$

$$\frac{\partial^2 \bar{w}_{St}}{\partial y^2} < \frac{\partial^2 \bar{w}_{St}}{\partial z^2} N > \frac{ch}{t_{St} c_{St}} \frac{\partial w_l}{\partial x} \Big|_{xN0} \neq 0 \quad \rho_{w_{St}} \rho_{xN0}, y, z : \bar{w}_{St}(y, z) N w_l \rho_{xN0}, y, z : :$$

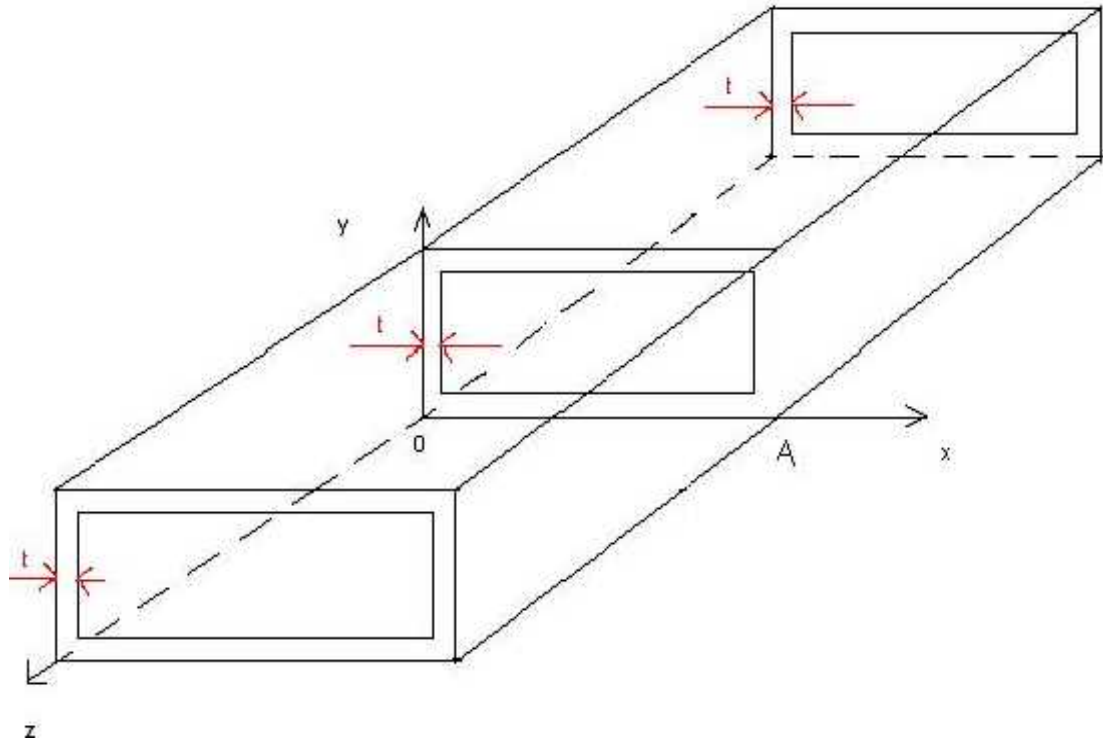
$$\frac{\partial w_l}{\partial x} \Big|_{xN0} N > c_H \frac{\partial^2 w_l}{\partial y^2} < \frac{\partial^2 w_l}{\partial z^2} \Big|_{xN0} \quad (r)$$

l, St μ μ
 μ , (.
 μ 8b). μ μ
 μ μ μ .
 μ - μ $l = St =$,
() μ μ :

$$\frac{\partial w}{\partial x}(x=0, y, z) = -c_H \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

M μ μ
 μ (. 2.13),
 μ μ
 μ μ c_S c_H ,

μ & μ



μ $8b$: μ μ μ
 $, xz,$ $, yz,$ μ .

μ : μ μ μ

$$\bar{u}(z) = \bar{u}(z + \ell), \quad \Theta(z) = \Theta(z + \ell), \quad p(z) = p(z + \ell), \quad \phi(z) = \phi(z + \ell) \quad (2.14)$$

$z-$ μ

μ $\mu\mu$.
 $, \mu$ μ ,

μ μ μ μ μ μ .

μ μ μ μ μ μ .

μ μ μ μ μ z .

$\mu\mu$ y μ
 0 $/2,$

$\mu\mu$ $x = /2 :$

μ & μ

$$x \text{ N A} / 2: \frac{\partial v}{\partial x} \text{ N } \frac{\partial w}{\partial x} \text{ N } \frac{\partial b}{\partial x} \text{ N } \frac{\partial w}{\partial x} \text{ N } 0, \quad u \text{ N } 0 \quad (2.15)$$

μ

μ μ :

$$Gr = \frac{g \Delta T h^3}{\epsilon^2}, \quad Ha = \sqrt{\frac{h^2 \dagger B_0^2}{\dots \epsilon}}, \quad Pr = \frac{\epsilon}{r}, \quad A = \frac{L}{h}, \quad c_H = \frac{c_{st} t_{st}}{ch}, \quad c_S = \frac{c_{co} t_{co}}{ch} \quad (2.16)$$

=k/(c_p)

μ

μ

μ

.

μ

μ

μ

μ

μ

μ

μ

μ

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μ

μ

μ

μ

Burr & Muller (2002).

2.2

μμ

μ

μ

μ

Rayleigh - Benard

,

μ

μ

μ μ

μ

μ

μ

μ

(2.1-2.3)

μ

:

$$\bar{u}_0 = 0, \quad \Theta_0 = -2y + 1, \quad p_0 = y - y^2, \quad \phi = 0 \quad (2.17)$$

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(2.6-2.14)

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(2.17).

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Hartmann

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$$\begin{Bmatrix} \bar{u} \\ p \\ \Theta \\ \phi \end{Bmatrix} = \begin{Bmatrix} \bar{u}_0 \\ p_0 \\ \Theta_0 \\ \phi_0 \end{Bmatrix} + \begin{Bmatrix} \bar{u}_1(x, y) \\ p_1(x, y) \\ \Theta_1(x, y) \\ \phi_1(x, y) \end{Bmatrix} e^{\omega t} e^{ikz}, \quad \bar{u} \equiv (u, v, w) \quad (2.18)$$

μ & μ
 $=_{r+i}$ μ μ
 t $k=2 /$ μ μ μ
 $0, 1$ μ ,
 μ μ μ μ ,
 μ μ μ , μ μ
 μ μ μ μ r .
 μ μ μ μ μ
 μ μ μ μ μ .
 μ , μ μ μ r μ μ .
 μ i μ μ , μ μ ,
 Hopf, .
 (2.6-

2.11) μ $(^2)$ μ
 , μ μ μ .

$$\check{S}u_1 = -\frac{\partial P_1}{\partial x} + Gr^{-1/2} \left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} - u_1 k^2 \right) \quad (2.19)$$

$$\check{S}v_1 = -\frac{\partial P_1}{\partial y} + Gr^{-1/2} \left(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} - v_1 k^2 \right) + \Theta_1 - \frac{Ha^2}{Gr^{1/2}} (v_1 + ikw_1) \quad (2.20)$$

$$\check{S}w_1 = -ikP_1 + Gr^{-1/2} \left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} - w_1 k^2 \right) - \frac{Ha^2}{Gr^{1/2}} \left(w_1 - \frac{\partial w_1}{\partial y} \right) \quad (2.21)$$

$$\check{S}\Theta_1 + v_1 \frac{\partial \Theta_0}{\partial y} = \check{S}\Theta_1 - 2v_1 = \frac{Gr^{-1/2}}{Pr} \left(\frac{\partial^2 \Theta_1}{\partial x^2} + \frac{\partial^2 \Theta_1}{\partial y^2} - \Theta_1 k^2 \right) \quad (2.22)$$

$$\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} - w_1 k^2 \right) = \frac{\partial w_1}{\partial y} - ikv_1 \quad (2.23)$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + ikw_1 = 0 \quad (2.24)$$

μ μ μ μ μ μ
 μ μ k
 μ (2.16), μ $\bar{u}_1, \Theta_1, p_1, \phi_1,$

μ & μ

μ μ
μ .

2.3

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Boussinesq

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μ Reynolds

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(Low Re_m model).

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μ

(2.6-2.11) μ

(2.10)

$S/Gr^{1/2}$,

S

μ

μ

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,

μ

$=(-A_v)/(T_h - T_{Av})$,

$A_v = (T_h + T_c)/2$.

μ

μ

μ

$T_h - T_c$

μ

μ

μ

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(2.25).

(Landau & Lifshitz (1987)) : $v|_{\Gamma} = v^{(w)}|_{\Gamma}$,

$w|_{\Gamma} = w^{(w)}|_{\Gamma}$.

$$\Theta(x=0)=-1 \quad \Theta(x=1)=1, \quad \frac{\partial \Theta}{\partial y}(y=0)=\frac{\partial \Theta}{\partial y}(y=1)=0, \quad \Theta(z)=\Theta(z+\ell) \quad (2.25)$$

$$\bar{u}(x=0)=\bar{u}(x=1)=\bar{u}(y=0)=\bar{u}(y=1)=0, \quad \bar{u}(z)=\bar{u}(z+\ell) \quad (2.26)$$

$$\frac{\partial w}{\partial x}(x=0)=\frac{\partial w}{\partial x}(x=1)=0, \quad \frac{\partial w}{\partial y}(y=0)=\frac{\partial w}{\partial y}(y=1)=0, \quad w(z)=w(z+\ell) \quad (2.27)$$

$$w(x=0,5, y=0.5, z)=0, \quad (2.27)$$

$$Gr = \frac{gS\Delta TL^3}{\epsilon^2}, \quad Ha = \sqrt{\frac{L^2 \dagger B_0^2}{\dots \epsilon}}, \quad S = \frac{qL^2}{\Delta T \dots c_p \epsilon}, \quad Pr = \frac{\epsilon}{r}, \quad A = \frac{H}{L} = 1 \quad (2.28)$$

Gr, Ha, Lorentz, SPr, Pr, GrSPr=Gr_{Eff}, Gr

μ & μ
 μ μ μ μ
 μ μ Gr.
 μ
 μ μ μ μ μ
 μ μ μ
 μ μ (Pb-17Li) μ
 μ Pr=0.0321. μ μ
 μ μ μ
 μ S=10⁵
 μ μ μ μ ,
 μ μ μ μ ,
 μ Ha μ μ μ .
 μ
 μ μ μ μ μ

2.4 $\mu\mu$ μ μ
 μ , μ
 μ μ μ

H μ μ μ
 μ μ a=0 μ μ
 μ Gr, Ha, Pr. μ
, μ μ
, (Pelekasis 2006).
 μ μ z
 μ μ Hopf
 μ Gr μ
 μ Ha.

3.1 Galerkin

μ μ Galerkin,
μ μ .

3.1.1 μ

μ μ μ μ μ μ
:

$$L(u(x))=f(x), x \in \Omega \tag{3.1}$$

μ

$$\left. \begin{aligned} u &= \bar{u} \\ \frac{\partial u}{\partial x} &= q \end{aligned} \right\} \tag{3.2}$$

(3.1)

μ μ : du

Dirichlet nu Neumann.

3.1.2 μ (weighted residual)

To μ , μ μ μ FEM Galerkin,
μ μ (3.1) μ .
μ μ μ (3.1, 3.2)

μ , μ μ μ .
μ μ , "

μ . Gresho & Sani (2000) μ : "

μ ... μ () μ
, , ()."

$$u^h(x) = \sum_{j=1}^n u_j N_j(x) \quad (3.1-3.2)$$

$$u^h(x) = \sum_{j=1}^n u_j N_j(x) \quad (3.3)$$

$$u_j = \int_{\Omega} u_j N_j(x) dx \quad (3.1)$$

$$r_{\Omega} = L(u^h) - f \quad (3.3)$$

$$r_{\Omega} = L(u^h) - f \quad (3.4)$$

$$\int_{\Omega} (r_{\Omega})^2 d\Omega \quad (3.5)$$

$$\int_{\Omega} w_i r_{\Omega} d\Omega \quad (3.6)$$

$$W = \{w_i : i=1,2,\dots,n\}$$

$$\int_{\Omega} w_i r_{\Omega} d\Omega \quad (3.6)$$

$$W = \{w_i : i=1,2,\dots,n\}$$

3.1.3 Galerkin

μ & μ
 μ μ
 μ μ , μ μ μ
 μ FEM
 μ FDM FVM.
 μ collocation Dirac- n μ
 :
 $w_i = u(x - x_i), i=1,2,\dots,n$ (3.7)

μ μ , μ μ
 μ μ μ FDM.
 collocation, μ μ - (step-
 discontinuous) μ :
 $w_i = \begin{cases} 1, & x_i \leq x \leq x_{i+1} \\ 0, & \end{cases}$ (3.8)

μ n . FVM, μ
 μ μ ,
 μ .
 μ μ μ , μ
 μ ,
 :
 $w_i = N_i$ (3.10)

H μ , FEM, μ
 Galerkin. μ μ
 μ μ μ
 μ . , μ μ
 μ μ , μ μ μ
 $\mu\mu$. μ μ μ

3.2

Rayleigh - Benard

(2.19-2.24)

FEM.

Lagrange $\psi_i(x,y)$

Lagrange $\psi_i(x,y)$

(Pelekasis 2006, Dimopoulos & Pelekasis 2012) :

$$\begin{bmatrix} u_i \\ v_i \\ w_i \\ \Theta_i \\ \phi_i \end{bmatrix} (x, y) = \sum_{i=1}^N \begin{bmatrix} u_i \\ v_i \\ w_i \\ \Theta_i \\ \phi_i \end{bmatrix} \Phi_i(x, y), \quad P_i(x, y) = \sum_{i=1}^M p_i \Psi_i(x, y); \quad (3.11)$$

O

FEM

Galerkin

$$\left[\iint \Psi_i \frac{\partial \Phi_j}{\partial x} dx dy \right] u_j + \left[\iint \Psi_i \frac{\partial \Phi_j}{\partial y} dx dy \right] v_j + \left[\iint ik \Psi_i \Phi_j dx dy \right] w_j = 0, i=1, M \quad (3.12)$$

x, y, z

$$\begin{aligned} \sum_{j=1}^N \left[\check{S} \iint \Phi_i \Phi_j dx dy \right] u_j &= - \sum_{j=1}^M \left[\iint \frac{\partial \Phi_i}{\partial x} \Psi_j dx dy \right] P_j \\ -Gr^{-1/2} \left[\sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} dx dy \right] u_j + \sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} dx dy \right] u_j + \sum_{j=1}^N \left[k^2 \iint \Phi_i \Phi_j dx dy \right] u_j \right] \end{aligned} \quad (3.13)$$

$$\begin{aligned} \sum_{j=1}^N \left[\check{S} \iint \Phi_i \Phi_j dx dy \right] v_j &= \sum_{j=1}^M \left[- \iint \Psi_j \frac{\partial \Phi_i}{\partial y} dx dy \right] P_j \\ -Gr^{-1/2} \left[\sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} dx dy \right] v_j + \sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} dx dy \right] v_j + \sum_{j=1}^N \left[k^2 \iint \Phi_i \Phi_j dx dy \right] v_j \right] \\ + \sum_{j=1}^N \left[\iint \Phi_i \Phi_j dx dy \right] \Theta_j - \frac{Ha^2}{Gr^{1/2}} \left[\sum_{j=1}^N \left[\iint \Phi_i \Phi_j dx dy \right] v_j + \sum_{j=1}^N \left[ik \iint \Phi_i \Phi_j dx dy \right] w_j \right] \end{aligned} \quad (3.14)$$

$$\begin{aligned} \sum_{j=1}^N \left[\check{S} \iint \Phi_i \Phi_j dx dy \right] w_j &= \sum_{j=1}^M \left[-ik \iint \Psi_j \Phi_i dx dy \right] P_j \\ -Gr^{-1/2} \left[\sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} dx dy \right] w_j + \sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} dx dy \right] w_j + \sum_{j=1}^N \left[k^2 \iint \Phi_i \Phi_j dx dy \right] w_j \right] \\ - \frac{Ha^2}{Gr^{1/2}} \left[\sum_{j=1}^N \left[\iint \Phi_i \Phi_j dx dy \right] w_j - \sum_{j=1}^N \left[\iint \Phi_i \frac{\partial \Phi_j}{\partial y} dx dy \right] w_j \right] \end{aligned} \quad (3.15)$$

$$\begin{aligned} \sum_{j=1}^N \left[\check{S} \iint \Phi_i \Phi_j dx dy \right] \Theta_j &= \\ - (Gr^{-1/2} / Pr) \left[\sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} dx dy \right] \Theta_j + \sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} dx dy \right] \Theta_j + \sum_{j=1}^N \left[k^2 \iint \Phi_i \Phi_j dx dy \right] \Theta_j \right] \end{aligned} \quad (3.16)$$

$$\begin{aligned} & - \int_0^1 c_H \frac{\partial \Phi_i}{\partial y} \frac{\partial w}{\partial y} dy \Big|_{x=0} - \int_0^1 c_H \frac{\partial \Phi_i}{\partial y} \frac{\partial w}{\partial y} dy \Big|_{x=A} - \int_0^A c_S \frac{\partial \Phi_i}{\partial x} \frac{\partial w}{\partial x} dx \Big|_{y=1} - \int_0^A c_S \frac{\partial \Phi_i}{\partial x} \frac{\partial w}{\partial x} dx \Big|_{y=0} + \\ & \Phi_i \left(c_H \frac{\partial w}{\partial y} (x=0, y=1) - c_S \frac{\partial w}{\partial x} (x=0, y=1) \right) - \Phi_i \left(c_H \frac{\partial w}{\partial y} (x=0, y=0) + c_S \frac{\partial w}{\partial x} (x=0, y=0) \right) + \\ & \Phi_i \left(-c_H \frac{\partial w}{\partial y} (x=A, y=0) + c_S \frac{\partial w}{\partial x} (x=A, y=0) \right) + \Phi_i \left(c_H \frac{\partial w}{\partial y} (x=A, y=1) + c_S \frac{\partial w}{\partial x} (x=A, y=1) \right) = \end{aligned}$$

μ & μ

μ μ μ

(Pelekasis 2006)

3.2.1 Arnoldi

μ μ Arnoldi (Arnoldi 1951)

μ μ μ μ Hessenberg,

μ :

$$\begin{pmatrix}
 a_{1,1} & a_{1,2} & a_{1,3} & \dots & a_{1,n-3} & a_{1,n-2} & a_{1,n-1} & a_{1,n} \\
 a_{2,1} & a_{2,2} & a_{2,3} & \dots & a_{2,n-3} & a_{2,n-2} & a_{2,n-1} & a_{2,n} \\
 0 & a_{3,2} & a_{3,3} & \dots & a_{3,n-3} & a_{3,n-2} & a_{3,n-1} & a_{3,n} \\
 0 & 0 & a_{4,3} & \dots & a_{4,n-3} & a_{4,n-2} & a_{4,n-1} & a_{4,n} \\
 0 & 0 & 0 & \dots & \vdots & \vdots & \vdots & \vdots \\
 0 & 0 & 0 & 0 & a_{n-2,n-3} & a_{n-2,n-2} & a_{n-2,n-1} & a_{n-2,n} \\
 0 & 0 & 0 & 0 & 0 & a_{n-1,n-2} & a_{n-1,n-1} & a_{n-1,n} \\
 0 & 0 & 0 & 0 & 0 & 0 & a_{n,n-1} & a_{n,n}
 \end{pmatrix}$$

, Arnoldi μ μ μ μ

μ μ μ μ

μ μ

Saad (Saad 1980)

μ μ non-Hermitian,

μ (Bai et al. 2000).

μ Arnoldi μ

n, μ n μ

μ μ μ μ μ Hessenberg

: AV = VH, Hessenberg n μ

V

V μ v₁=Ve₁

μ μ .

μ μ

μ V_m n x m μ

Hessenberg m m : AV_m-V_mH_m

= fe_m^{*}, μ f 1 x n residual m-

μ & μ
 μ Arnoldi e_m^* μ
 μ $1 \times m$
 μ μ Arnoldi $\mu\mu$
 :
 : μ μ m μ μ v_1 .
 : (V_m, H_m, f) $AV_m - V_m H_m = f e_m^*$
 $j=1, 2, \dots, m-1$
 $w = AV_j$
 w V_j ($h_{1:j,j}$)
 $h_{j+1,j} = \|w\|_2$
 A $h_{j+1,j} = 0$, stop
 $v_{j+1} = w / h_{j+1,j}$

 $f = Av_m$
 f V_m ($h_{1:m,m}$)
 $s = \|f\|_2$

 μ μ μ μ
 Krylov $m(A, v_1)$. T μ μ
 V_m μ μ Arnoldi.
 $w=0$ μ , μ
 m μ residual
 Arnoldi μ $K_j(A, v_1)$, $j < m$
 . μ
 μ μ μ .
 , μ μ μ ,
 μ , μ μ μ μ μ μ
 μ . μ μ μ , μ
 μ μ μ Cayley , 0

Hopf.
 Krylov
 Arnoldi.
 (Pelekasis 2006,
 Dimopoulos & Pelekasis 2012)
 .

3.3

H 3.2
 :

$$\left[\iint \Psi_i \frac{\partial \Phi_j}{\partial x} dx dy \right] u_j + \left[\iint \Psi_i \frac{\partial \Phi_j}{\partial y} dx dy \right] v_j + \left[\iint ik \Psi_i \Phi_j dx dy \right] w_j = 0, i=1, M \quad (3.22)$$

$$\sum_{j=1}^N \left[\iint \Phi_i \Phi_j dx dy \right] u_j + \sum_{j=1}^N \left[\iint \Phi_i \frac{\partial u_0}{\partial x} \Phi_j dx dy \right] u_j + \sum_{j=1}^N \left[\iint \Phi_i \frac{\partial \Phi_j}{\partial x} u_0 dx dy \right] u_j + \sum_{j=1}^N \left[\iint \Phi_i \frac{\partial \Phi_j}{\partial y} v_0 dx dy \right] u_j + \sum_{j=1}^N \left[\iint \Phi_i \frac{\partial u_0}{\partial y} \Phi_j dx dy \right] v_j = - \sum_{j=1}^M \left[\iint \frac{\partial \Phi_i}{\partial x} \Psi_j dx dy \right] P_j - Gr^{-1/2} \left[\sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} dx dy \right] u_j + \sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} dx dy \right] u_j + \sum_{j=1}^N \left[k^2 \iint \Phi_i \Phi_j dx dy \right] u_j \right] \quad (3.23)$$

$$\begin{aligned}
 & \sum_{j=1}^N \left[\dagger \iint \Phi_i \Phi_j dxdy \right] v_j + \sum_{j=1}^N \left[\iint \Phi_i \Phi_j \frac{\partial v_0}{\partial y} dxdy \right] v_j + \sum_{j=1}^N \left[\iint \Phi_i \frac{\partial \Phi_j}{\partial y} v_0 dxdy \right] v_j + \\
 & \sum_{j=1}^N \left[\iint \Phi_i \frac{\partial \Phi_j}{\partial x} u_0 dxdy \right] v_j + \sum_{j=1}^N \left[\iint \Phi_i \Phi_j \frac{\partial v_0}{\partial x} dxdy \right] u_j = \sum_{j=1}^M \left[-\iint \Psi_j \frac{\partial \Phi_i}{\partial y} dxdy \right] P_j \\
 & -Gr^{-1/2} \left[\sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} dxdy \right] v_j + \sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} dxdy \right] v_j + \sum_{j=1}^N \left[k^2 \iint \Phi_i \Phi_j dxdy \right] v_j \right] \\
 & + \sum_{j=1}^N \left[\iint \Phi_i \Phi_j dxdy \right] \Theta_j - \frac{Ha^2}{Gr^{1/2}} \left[\sum_{j=1}^N \left[\iint \Phi_i \Phi_j dxdy \right] v_j + \sum_{j=1}^N \left[ik \iint \Phi_i \Phi_j dxdy \right] w_j \right]
 \end{aligned} \tag{3.24}$$

$$\begin{aligned}
 & \sum_{j=1}^N \left[\dagger \iint \Phi_i \Phi_j dxdy \right] w_j + \sum_{j=1}^N \left[\iint \Phi_i \frac{\partial \Phi_j}{\partial x} u_0 dxdy \right] w_j + \\
 & \sum_{j=1}^N \left[\iint \Phi_i \frac{\partial \Phi_j}{\partial y} v_0 dxdy \right] w_j = \sum_{j=1}^M \left[-ik \iint \Psi_j \Phi_i dxdy \right] P_j \\
 & -Gr^{-1/2} \left[\sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} dxdy \right] w_j + \sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} dxdy \right] w_j + \sum_{j=1}^N \left[k^2 \iint \Phi_i \Phi_j dxdy \right] w_j \right] \\
 & - \frac{Ha^2}{Gr^{1/2}} \left[\sum_{j=1}^N \left[\iint \Phi_i \Phi_j dxdy \right] w_j - \sum_{j=1}^N \left[\iint \Phi_i \frac{\partial \Phi_j}{\partial y} dxdy \right] w_{1j} \right]
 \end{aligned} \tag{3.25}$$

$$\begin{aligned}
 & \sum_{j=1}^N \left[\dagger \iint \Phi_i \Phi_j dxdy \right] \Theta_j + \sum_{j=1}^N \left[\iint \Phi_i \Phi_j \frac{\partial \Theta_0}{\partial x} dxdy \right] u_j + \sum_{j=1}^N \left[\iint \Phi_i \frac{\partial \Phi_j}{\partial x} u_0 dxdy \right] \Theta_j + \\
 & \sum_{j=1}^N \left[\iint \Phi_i \frac{\partial \Phi_j}{\partial y} v_0 dxdy \right] \Theta_j + \sum_{j=1}^N \left[\iint \Phi_i \Phi_j \frac{\partial \Theta_0}{\partial y} dxdy \right] v_j = \\
 & - \left(\frac{Gr^{-1/2}}{Pr} \right) \left[\sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} dxdy \right] \Theta_j + \sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} dxdy \right] \Theta_j + \sum_{j=1}^N \left[k^2 \iint \Phi_i \Phi_j dxdy \right] \Theta_j \right]
 \end{aligned} \tag{3.26}$$

$$\begin{aligned}
 & \int \Phi_i \frac{\partial w}{\partial n} dS = 0 = -(c_s - c_H) \int \Phi_i k^2 w dS + \sum_{j=1}^N \left[\iint \Phi_i \frac{\partial \Phi_j}{\partial y} dxdy \right] w_j - \sum_{j=1}^N \left[ik \iint \Phi_i \Phi_j dxdy \right] v_j + \\
 & \sum_{j=1}^N \left[k^2 \iint \Phi_i \Phi_j dxdy \right] w_j + \sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} dxdy \right] w_j + \sum_{j=1}^N \left[\iint \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} dxdy \right] w_j, \quad i=1, M
 \end{aligned} \tag{3.27}$$

μ & μ

(3.22-3.27)

μ μ μ

$$\int_{\Gamma} \Phi_i \frac{\partial q}{\partial n} d\Gamma \quad (q = \{u, v, w, \dots, p\},$$

μ

μ $\partial q / \partial n$

)

Dirichlet

μ

μ

μ

.

μ

μ

μ

μ

μ

μ

μ

Cayley (Cliffe et al. 1993)

μ

Arnoldi

μ

μ

μ

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μ

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μ

μ

:

$$Jx - sBx = \dagger Bx - sBx \rightarrow (J - sB)x = (\dagger - s)Bx \rightarrow$$

$$\dagger x = (J - sB)^{-1} Bx, \quad \dagger \equiv \frac{1}{\dagger - s} \quad (3.28)$$

(3.28)

μ

(3.21).

s

μ

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μ

μ

μ

Arnoldi.

μ

μ

J-sB,

μ

μ

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μ

μ

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$\mu\mu$

Pelekasis (2006)

μ

μ

μ

(S=10⁵)

.

μ

$\mu\mu$

μ

μ

(2.1)

μ

μ

$\mu\mu$.

,

μ

GMRES (General Minimal Residual),

μ

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Stokes

preconditioner

μ

μ

μ

μ

μ

Ha, Gr

μ

μ k

μ & μ

μμ μ μ
CPU.

3.3.1 GMRES

μ μ x=b μ

. , μ

μ .

μ μ μ

μ

μ i+1 x_{i+1} μ :

$$Kx_{i+1} = Kx_i + b - \Lambda x_i - Kx_i \xrightarrow{r_i = b - Ax_i} Kx_{i+1} = Kx_i + r_i$$

μ μ μ ,

μ (Elman et al. 2005). μ

μ μ .

μ . μ

μ μ

μ GMRES (General Minimal Residual). μ

1986 Saad Schultz μ , μ

μμ μ (Saad & Schultz 1986).

μ 50

Lanczos Arnoldi

μ μ .

, μ μ

. Lanczos

x_i μ ,

$[A, r^{(0)}]$, μ $r^{(0)}, Ar^{(0)}, A^2r^{(0)}, \dots, A^{i-1}r^{(0)}$ $r^{(0)}=b-$

$Ax^{(0)}$ μ .

μ & μ
 μ Krylov i , Lanczos
 μ
 μ GMRES Krylov k
 $x^{(k)} \in x^{(0)} + K_k(A, r^{(0)})$ μ μ
 $V^{(k)} = \{v^{(1)}, v^{(2)}, \dots, v^{(k)}\}$ μ $v^{(1)} = r^{(0)} / \|r^{(0)}\|$.
 μ V_k Krylov
 $K_k \equiv \text{span}\{v^{(1)}, Av^{(1)}, \dots, A^{k-1}v^{(1)}\} = K_k[A, r^{(0)}]$ μ μ Arnoldi.
 upper-

Hessenberg $H_k = [h_{ij}] \quad 1 \leq i, j \leq k, \quad h_{ij} = 0 \quad j < i - 1$

:

$$H_k = V_k^T A V_k$$

$$A V_k = V_k H_k + h_{k+1,k} [0, \dots, v^{(k+1)}] \rightarrow A V_k = V_{k+1} \tilde{H}_k, \quad \tilde{H}_k = [h_{i,j}]_{1 \leq i \leq k+1, 1 \leq j \leq k}$$

$$O \quad \mu \quad \cdot \quad v^{(1)} = r^{(0)} / \|r^{(0)}\| \quad \mu$$

$$\mu \quad , \quad v^{(k+1)},$$

$$\mu \quad \text{Krylov} \quad k.$$

$$v^{(k)} \quad \mu \quad \mu \quad \text{Gram-Schmidt (Elman et$$

al. 2005).

:

$$x^{(k)} = x^{(0)} + V_k y^{(k)} \rightarrow Ax^{(k)} - b = Ax^{(0)} - b + A V_k y^{(k)}$$

$$\mu \quad \mu \quad y^{(k)}. \quad \mu \quad H_k$$

μ :

$$r^{(k)} = r^{(0)} - V_{k+1} \tilde{H}_k y^{(k)} = V_{k+1} \left(\|r^{(0)}\| e^{(1)} - \tilde{H}_k y^{(k)} \right), \quad e^{(1)} = (1, 0, \dots, 0)^T$$

$$v^{(k)} \quad :$$

$$\|r^{(k)}\| \equiv S_k = \|S_0 e^{(1)} - \tilde{H}_k y^{(k)}\|, \quad \|r^{(0)}\| \equiv S_0$$

$$y^{(k)} \quad \mu$$

$$\|r^{(k)}\| = \|b - Ax^{(k)}\|^2.$$

$$\mu \quad , \quad \mu$$

$$\mu \quad \mu \quad H_k$$

$$, \quad \mu \quad \mu \quad \mu$$

$$, \quad \mu \quad \mu \quad H_k,$$

μ & μ
 μ m μ , m μ
 μ μ . μ GMRES
 (Saad & Schultz 1986):

- μ $x^{(0)}$
 $r^{(0)} = f - Ax^{(0)}$ $v^{(1)} = \frac{r^{(0)}}{\|r^{(0)}\|}$.
- μ $v^{(k)}$
 μ $j=1,2,\dots,m$
 $h_{i,j} = (Av^{(j)}, v^{(i)})$, $i=1,2,\dots,j$
 $\hat{v}^{(j+1)} = Av^{(j)} - \sum_{i=1}^j h_{i,j} v^{(i)}$
 $h_{j+1,j} = \|\hat{v}^{(j+1)}\|$, $v^{(j+1)} = \frac{\hat{v}^{(j+1)}}{h_{j+1,j}}$
- $x^{(m)} = x^{(0)} + V_m y^{(m)}$
 $y^{(m)} = \arg \min_{y \in R^m} \|Se_1 - \tilde{H}_m y\|$, $y \in R^m$.
- H $r^{(m)} = f - Ax^{(m)}$.
 $s_k < \dagger s_0$, $t \square 1$, μ μ .
 μ $x^{(0)} = x^{(m)}$, $v^{(1)} = \frac{r^{(m)}}{\|r^{(m)}\|}$ μ
 μ .

3.3.2 GMRES μ preconditioner

GMRES μ μ μ μ μ
 μ μ μ , μ
 μ μ μ μ μ
 preconditioner.

μ & μ
 , preconditioning μ
 $\mu\mu$ μ
 . μ μ μ
 μ μ μ μ
 μ μ .
 μ , μ
 μ μ μ
 preconditioning preconditioner
 μ . μ preconditioning
 μ μ $x=b$ μ
 μ

μ μ :
 $[M^{-1}A]x = M^{-1}b, [AM^{-1}][Mx] = b$

μ μ preconditioning.
 μ preconditioning
 μ . ,
 μ μ^{-1}
 (Elman et al. 2005). $V^{(k)} = \{v^{(1)}, v^{(2)}, \dots, v^{(k)}\}$,

Krylov $K_k \equiv span\{r^{(0)}, AM^{-1}r^{(0)}, (AM^{-1})^2 r^{(0)}, \dots, (AM^{-1})^{k-1} r^{(0)}\}$
 k μ :
 $x^{(k)} = x^{(0)} + M^{-1}V_k y^{(k)}$.

μ^{-1} μ
 μ^{-1} .
 μ
 preconditioner μ Incomplete LU
 . , μ^{-1} μ L U.
 μ μ a_{ij} μ $i, j=1, \dots, n$
 μ ILU L
 U R=LU-A ,
 μ .

μ & μ

O μ ILU Gauss

μ μ μ (Saad 1996) :

1. For $i=2,\dots,n$ Do:
2. For $k=1,\dots,i-1$ and if $(i,k) \notin P$ Do:
3.
$$a_{ik} = \frac{a_{ik}}{a_{kk}}$$
4. For $j=k+1,\dots,n$ and for if $(i,k) \notin P$ Do:
5.
$$a_{ij} = a_{ij} - a_{ik}a_{kj}$$
6. End Do
7. End Do
8. End Do

μ μ μ μ :

$$P \subset \{(i, j) \mid i \neq j, 1 \leq i, j \leq n\}$$

μ i μ μ

μμ L U i μμ μ

μ μμ . μ μμ L U 1, ...,

i-1 μ .

μ

Incomplete LU GMRES

μ μ

Youcef Saad.

μ

μ μ Krylov

GMRES μ Incomplete LU .

, μ

μ μ , CSR (Compressed

Sparse Row Format). μ μ μ μ μ -

μ .

μ μ

μ -μ , μ -

μ & μ

μ μ

$\mu - \mu$ $\mu - \mu$ $\mu .$

3.3.3

μ μ

$\mu - \mu$

GMRES

μ $\mu\mu$ μ μ

μ μ

μ

$\mu .$ μ μ

,

.

,

$\mu\mu$

μ

μ μ ,

μ μk μ

Gr μ μ Ha.

μ

μ μ μ μ μ μ Gr

μ μ Ha. μ μ

μ Gr μ . μ Gr

μ

.

μ μ μ μ

μ μ (μ μ μ)

μ μ μ (Pelekasis 2006)).

μ μ μ . μ μ

μ μ μ ,

μ μ GMRES.

μ μ $\mu\mu$ μ μ

μ x (in & Le Quere 2001) :

$$(J - \dagger B)x = 0 \tag{3.29}$$

μ & μ
 μ μ μ^k μ μ , ... Gr,
 μ μ μ μ k_0 Gr₀.
 μ μ parametric continuation μ
 μ μ μ μ μ \mathbf{x}
 μ μ
 μ μ (3.29).

$x_{ij} + ix_{ij} = 1 + 0i$ (3.30)

μ μ μ (3.29) (3.30)
 μ μ Newton-Raphson (NR).
 k NR $\mu\mu$ μ .

$$\begin{bmatrix} J_x - \dagger^k B & Bx^k \\ 0 \dots 1 \dots 0 & 0 \end{bmatrix} \begin{Bmatrix} ux^k \\ u\dagger^k \end{Bmatrix} = \begin{Bmatrix} (J_x - \dagger^k B)x^k \\ x_{ij}^k + ix_{ij}^k - 1 \end{Bmatrix} \quad (3.31)$$

μ μ
 μ J_x Frechet J
 μ \mathbf{x} μ $u\dagger^k = \dagger^{k+1} - \dagger^k$. $J_x - \dagger^k B$
 $n \times n$ Bx^k μ n ,
 (eigenpair) $\{ux^k, u\dagger^k\}$ μ μ μ
 μ μ μ
 B μ .
 μ μ μ GMRES
 preconditioner, μ
 μ μ , μ Stokes o o
 μ μ μ μ $\mu\mu$ μ
 μ μ
 x, y, z (2.7-2.10):

μ & μ

$$B\dot{x} = Jx = \left(\frac{C}{Gr^{0.5}}L + N \right)x \quad (3.32)$$

μ L N μ μ
 μ μ μ - μ μ μ μ
 μ , , μ C : i) μ μ
 μ , ii) μ μ
 1/Pr μ μ .
 semi-implicit μ μ μ :

$$\begin{aligned} \frac{Bx^{n+1} - Bx^n}{\Delta t} &= \frac{C}{Gr^{0.5}}Lx^{n+1} + Nx^n \Rightarrow \\ \left(B - \frac{\Delta t C}{Gr^{0.5}}L \right) x^{n+1} &= (\Delta t N + B)x^n \Rightarrow \\ x^{n+1} &= \left(B - \frac{C}{Gr^{0.5}}\Delta t L \right)^{-1} (\Delta t N + B)x^n \Rightarrow \\ x^{n+1} &= \left(B - \frac{C}{Gr^{0.5}}\Delta t L \right)^{-1} \left(\Delta t N + B - \frac{C}{Gr^{0.5}}\Delta t L + \frac{C}{Gr^{0.5}}\Delta t L \right) x^n \Rightarrow \\ x^{n+1} &= x^n + \left(B - \frac{C}{Gr^{0.5}}\Delta t L \right)^{-1} \left(\Delta t N + \frac{C}{Gr^{0.5}}\Delta t L \right) x^n \quad (3.33) \\ \frac{x^{n+1} - x^n}{\Delta t} &= \left(B - \frac{C}{Gr^{0.5}}\Delta t L \right)^{-1} \left(N + \frac{C}{Gr^{0.5}}L \right) x^n \end{aligned}$$

μ μ μ
 μ (3.33) Stokes
 μ μ μ
 . μ μ (3.33)
 μ μ μ μ . , (3.31), μ
 Stokes preconditioner
 μ μ GMRES (Saad 2003), μ μ :

$$\begin{bmatrix} \left(B - \frac{C}{Gr^{0.5}}\Delta t L \right)^{-1} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ \left[J_x - \dagger^k B \quad Bx^k \right] \\ 1 \end{bmatrix} \begin{bmatrix} u x^k \\ u \dagger^k \end{bmatrix} = \begin{bmatrix} \left(J_x - \dagger^k B \right) x^k \\ x_{rj}^k + ix_{ij}^k - 1 \end{bmatrix} \quad (3.34)$$

t μ μ μ μ
 μ (Mamun & Tuckerman 1995).

μ & μ
 Stokes μ μ preconditioner
 μ (3.31) μ
 μ GMRES. μ
 μ Arnoldi μ μ μ
 μ (3.31) μ μ μ .
 μ μ , $\mu\mu$ μ
 μ .

3.4

Rayleigh - Benard μ $\mu\mu$ μ

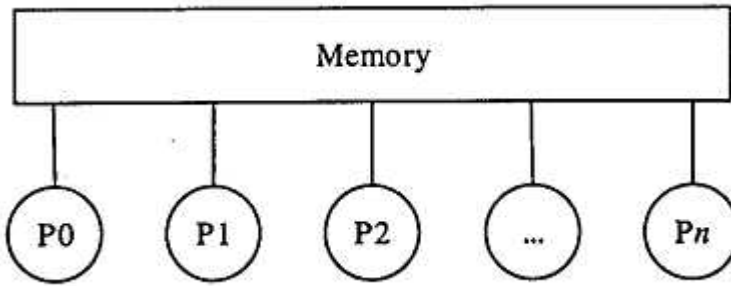
μ μ

μ μ
 MPI (Message Passing Interface)
 ScaLAPACK (Scalable Linear Algebra Package), (Blackford et al.
 1997). , μ μ μ
 μ μ μ μ (shared-memory computer) μ
 μ μ μ μ (distributed-memory system)

μ μ . μ
 μ μ μ μ μ , OpenMP
 (Multi-Processing)

μ μ μ μ $\mu\mu$, μ
 μ μ μ μ
 μ μ .
 μ μ μ .

μ & μ



μ 9: μ μ μ μ μ ,
 n μ μ

μ .

μ μ μ μ

μ .

μ

μ μ μ μ

μ ,

μ

μ .

μ μ

$\mu\mu$

μ

μ

$\mu\mu$

μ .

μ

μ

μ μ μ

, $\mu\mu$

μ

μ μ μ

μ

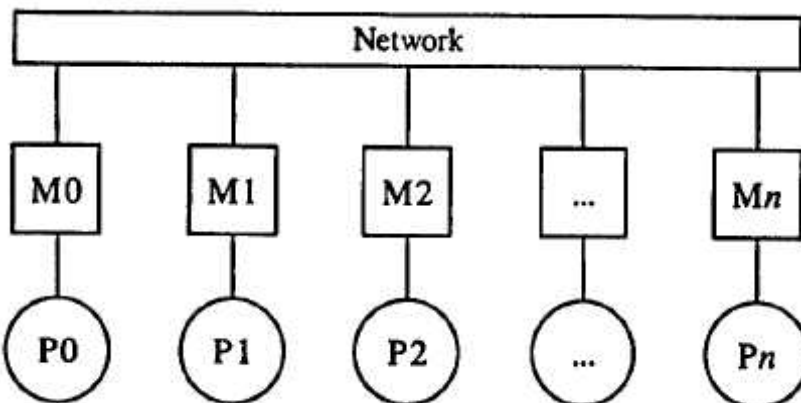
μ .

μ

μ

MPI

(Pacheco 1996) PVM (Parallel Virtual Machine), (Geist et al. 1994).



μ 10: μ μ μ μ μ ,
 n μ μ μ μ

μ & μ

Compiler : μ compiler
μ .

: μ μ μ μ μ
μ ,
μ μ , μ
μ μ μ μ .

μ

scalability μ μ μ μ μ
μ μ μ μ (cluster)
μ & μ μ
cluster , HELIOS.

3.4.1 μ

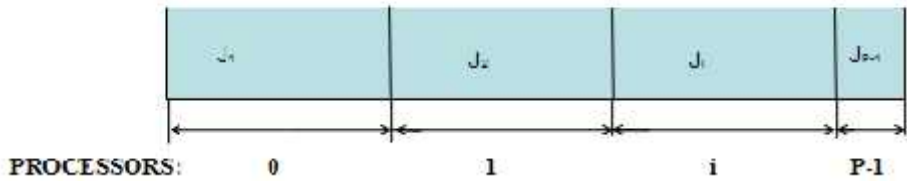
μ μ
μ μ μ μ
μ 10⁴ μ μ μ 6
μ Intel Pentium Quad – Cores μ μ μ 8 Gb.

Ha Gr_{Cr} μ μ
Rayleigh-Benard μ μ μ μ
μ (scalability)
μ μ , μ CPU

	μ	&	μ		
Mesh	Ha		Gr _{Cr}	processors	CPU in minutes
40x20	0		41400	1	25
40x20	0		41400	2	16
40x20	0		41400	4	9
60x30	0		42900	1	68
60x30	0		42900	2	36
60x30	0		42900	4	20
60x30	100		53000	1	68
60x30	100		53000	4	20
80x40	100		53000	1	109
80x40	100		53000	4	30
140x70	100		53000	1	1854
140x70	100		53000	16	450
110x55	800		460000	1	732
110x55	800		460000	16	187
140x70	800		460000	1	1854
140x70	800		460000	16	450

μ & μ
 μ divide-and-conquer ScaLAPACK. μ
 μ
 J , μ μ
 μ , μ
 μ μ μ , (3.21). μ μ
 μ μ μ μ
 0 -1 μ μ
 0 -2 μ μ
 -1 μ
 μ , (μ 11). μ
 μ μ
 μ (Planas et al. 2011).

J_i : parts of the Jacobian matrix correspond to the i -th processor



μ 11 : μ $\mu\mu$ J
 μ μ μ
 μ μ μ μ μ μ ScaLAPACK.
 μ μ J μ
 μ μ μ μ μ
 ScaLAPACK. μ , μ $-$ μ
 μ , μ μ
 μ x μ , μ
 μ J , μ , μ μ
 Arnoldi μ , μ μ
 μ 150 μ μ μ
 μ μ μ .



μ 12 : $\mu\mu$ μ μ μ μ μ

3.5 "benchmarks"

μ μ μ μ μ μ μ μ

(benchmark test)

μ μ μ μ μ μ

μ & μ

3.5.1 Benchmark

μ μμ

μ Rayleigh - Benard μ

μ , μ

μ

μ μ

μ μ

μ μ μ μ μ μ

μ μ μ .

μ μ Rayleigh-Benard

μ μ

Ha=0,

μ

μ benchmark μ ,

μ μ Gr_{Cr} μ μ μ k. μ Pr μ 0.02

μ μ (Na²²K⁷⁸),

μ μ μ Burr & Muller (2002). O

μ μ μ μ z

Ha=0 μ μ

Gr_{Cr}≈42900, μ μ μ k=0

10 μ x

μ μ μ μ c_{Cr}≈ /10.

μ μ =20 μ

, μ μ

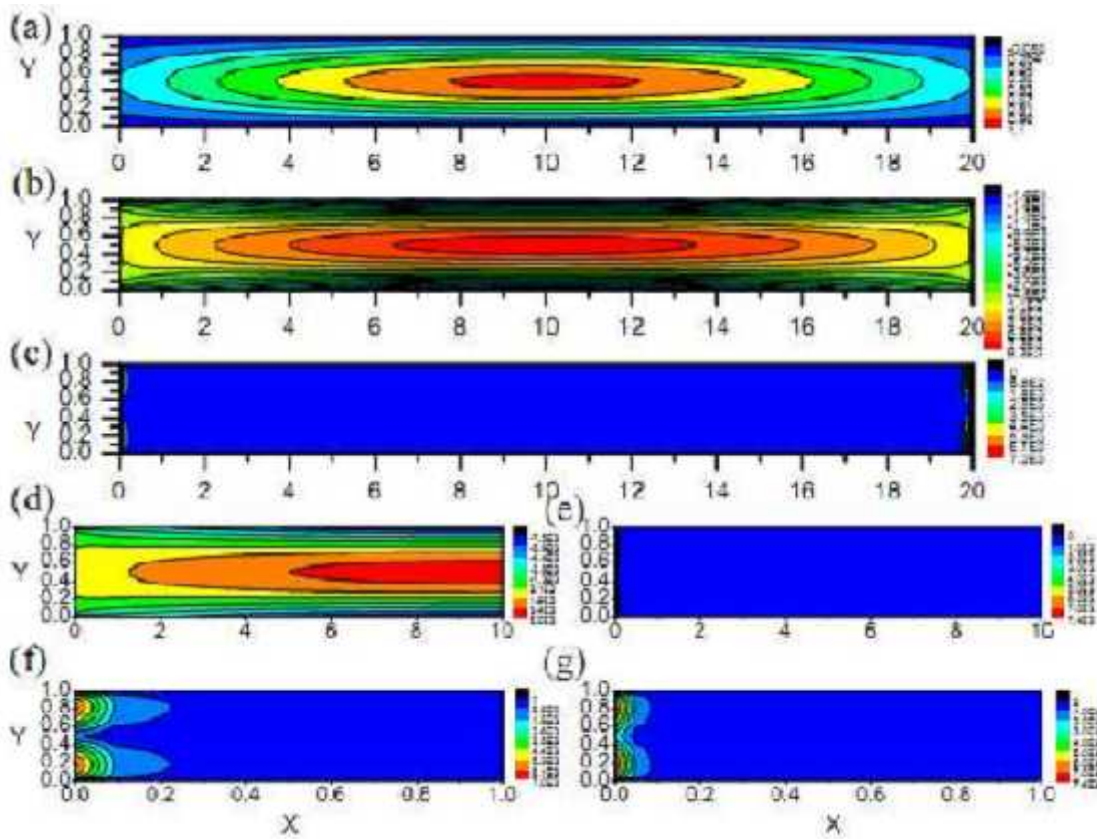
μ c_{Cr}≈2h. μ 13a,b

μ μ μ μ

μ 60x30 80x40 , .

μ μ μ Gr_{Cr} μ 100.

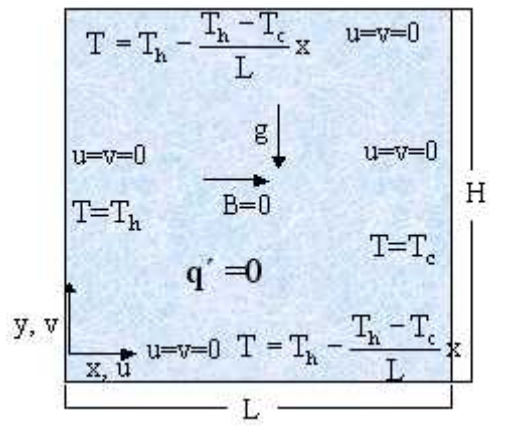
μ & μ
 yz (μ 14e,f). μ
 μ Hartmann
 μ . μ
 μ μ μ $\mu\mu$
 (2.15). μ μ μ
 μ μ μ
 μ , μ μ ,
 $\mu\mu$ $x=A/2$.



μ 14 : (a) μ , (b) x
 , (c) yz $Ha=100$ μ 110x55
 . μ (d,e) μ μ (b,c)
 μ $\mu\mu$. μ (f,g) yz
 μ (c,e), ,
 μ Hartmann.

3.5.2 Benchmark

(Henkes & Le Quere 1996, Gelfgat & Tanasawa 1994).
 Xin & Le Quere (2001)



15: Xin & Le Quere (2001).

Hopf
 $Ra = 1.64 \times 10^6$,
 Navier - Stokes
 $Ra = 1.6 \times 10^6$
 $k \in [0, 24]$
 Rayleigh.
 (1000)

μ & μ

μ Arnoldi μ μ ,

μ μ (mode 4). μ μ

μ μ Hopf

(k=0) μ

Ra=1.64x10⁶ $\mu\mu$ μ

μ .

μ μ k μ μ^2 μ μ ,

=2 /k, μ μ

· μ μ μ μ μ μ

μ μ μ μ μ

μ . μ

μ μ benchmark μ μ

μ μ .

μ μ μ

μ μ μ

μ , μ $\mu\mu$

(μ 16) μ Hopf

· μ μ

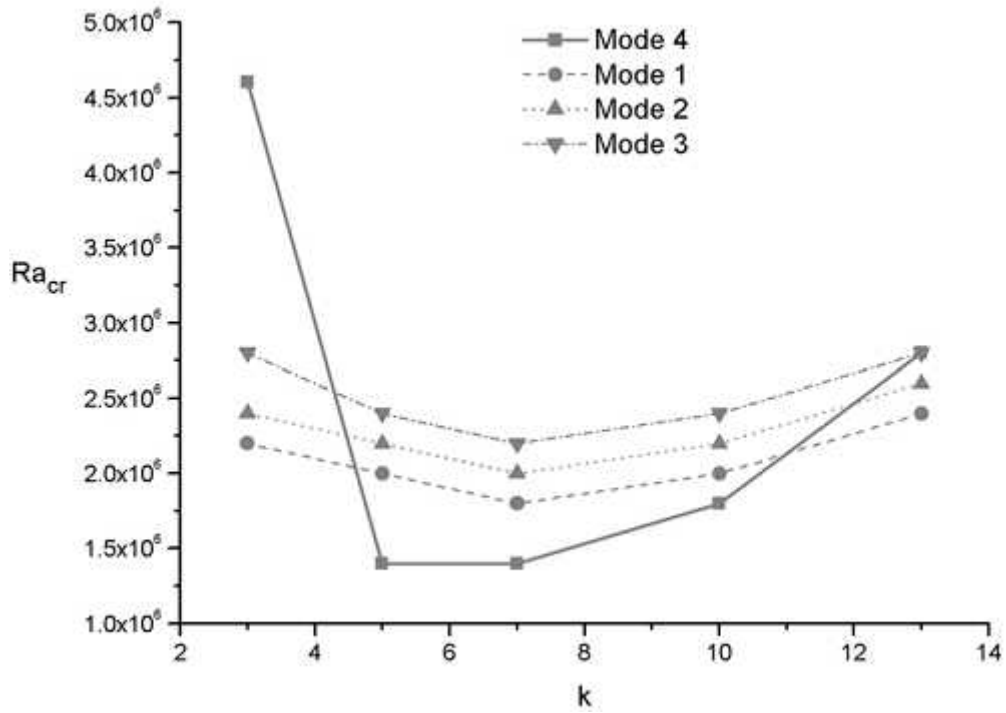
μ μ μ μ , (modes 1,

2, 3). μ μ μ μ , (mode

4), μ μ k 4. To $\mu\mu$

μ μ μ .

μ & μ



μ 16 : $\mu\mu$

μ benchmark μ .

3

μ

μ

Hopf

μ

μ

Xin & Le Quere (2001)

μ

μ

μ

μ

$\mu\mu$

.

μ

μ

μ

μ

μ

μ

(mode 4)

vortical tubes

μ

μ

μ

20x20

40x40

, (μ 17a,b).

μ

μ

μ

μ

.

μ

(iso-surfaces)

μ

μ

μ

μ

=1.39 (mode

1, μ 18a)

μ

(mode 4,

μ 18b).

-

μ 18a

μ

μ

μ

μ

μ

μ

.

,

μ

.

,

μ 18b

μ & μ
 μ 17 μ μ
 μ . M μ μ
 μ μ μ μ 18a
 μ
 μ , μ $\mu\mu$
 μ μ μ k_{Cr} 7,
 Goertler μ μ $\mu\mu$
 μ μ .

	mode 1	mode 2	mode 3
(20x20) ,	1.38	1.16	1.61
(40x40) ,	1.39	1.16	1.61
Xin & Le Quere, 41x41 collocation μ	1.414 84	1.192 73	1.621 84

3 : μ Hopf μ (
 Ra) μ Pr=0.71.

μ & μ

(2002)

μ

μ , μ μ . ,
μ μ μ

μ μ μμ

μ μ

μ .

μ , μ μ , μ ,

. μ

μ μ μ

μ

μ μ

.

μ μ μ μ

μμ μ μ 8 μ

μ μμ

μ μ μ

μ . μ

μ μ μμ

μ . ,

μ (coherent structures)

μ μ μμ .

μ μ μ

(Burr & Muller 2002).

, μ

μ μ

μ . μ μ μ

μ μ

μ μ

μ μ μ .

FEM

μ μ μ

μ & μ

spectral modes z (Gottlieb & Orszag 1977, Patera 1984)

μ μ μ μ μ

$\mu\mu$ (Orszag & Kells 1980). μ

μ μ μ (Orszag & Kells 1980) μ

(Dimas & Triantafyllou 1994) μ

(Snyder & Degrez 2002 & Vanden - Abeele et al. 2004). μ

μ μ μ μ μ

μ μ

μ μ $\mu\mu$

$\mu\mu$, . μ μ

μ μ μ μ μ

μ μ .

μ μ μ μ μ μ

μ μ μ semi-implicit

. μ $\mu\mu$ μ μ

μ Adams-Bashforth , $\mu\mu$

μ μ μ μ Crank-Nicolson

, μ

μ μ

. μ μ

(Streamwise Upwinding Petrov Galerkin).

μ μ μ

μ μ μ Fourier modes

μ mode,

: () domain decomposition

μ μ μ

, μ μ (Dimopoulos & Pelekasis

2012) z () Fourier decomposition

μ μ $x=b$ Fourier modes

μ . μ μ

.

μ μ μ

μ μ μ

CPU, ScaLAPACK, FFTE (Fast Fourier Transform East), MPI, BLACS

3.6.1 Discrete Fourier Transform

The Discrete Fourier Transform (DFT) is a mathematical operation that converts a discrete-time signal into its frequency domain representation. The DFT is defined as:

$$Y[k] = \sum_{n=0}^{N-1} X[n] e^{-j2\pi kn/N}$$

where $X[n]$ is the input signal, $Y[k]$ is the output signal, and N is the number of samples. The DFT is a linear transformation, and its inverse is given by:

$$X[n] = \frac{1}{N} \sum_{k=0}^{N-1} Y[k] e^{j2\pi kn/N}$$

The computational complexity of the DFT is $O(N^2)$.

Cooley & Tukey (1965) introduced the Fast Fourier Transform (FFT), which is an efficient algorithm for computing the DFT. The FFT reduces the computational complexity to $O(N \log_2 N)$.

The FFT is based on the divide-and-conquer principle. It decomposes the DFT into smaller DFTs, which are then combined to form the final result. The FFT is a linear transformation, and its inverse is given by:

$$X[n] = \frac{1}{N} \sum_{k=0}^{N-1} Y[k] e^{j2\pi kn/N}$$

$$Y[i] = \sum_{k=0}^{n-1} X[k] \check{S}^{ki}, 0 \leq i < n \tag{3.35}$$

$\check{S} = e^{2\pi j \sqrt{-1}/n}$ are the twiddle factors.

μ & μ

(3.35), μ Y[i] n

μ μ μ . μ . DFT

μ μ μ (n²). O FFT μ μ

(n log₂ n). μ n μ 2.

FFT n-point DFT μ (n/2-

point) DFTs, :

$$\begin{aligned}
 Y[i] &= \sum_{k=0}^{(n/2)-1} X[2k] \tilde{S}^{2ki} + \sum_{k=0}^{(n/2)-1} X[2k+1] \tilde{S}^{(2k+1)i} \\
 &= \sum_{k=0}^{(n/2)-1} X[2k] e^{2(2f\sqrt{-1}/n)|i} + \sum_{k=0}^{(n/2)-1} X[2k+1] \tilde{S}^i e^{2(2f\sqrt{-1}/n)|i} \\
 &= \sum_{k=0}^{(n/2)-1} X[2k] e^{2f\sqrt{-1}|i/(n/2)} + \tilde{S}^i \sum_{k=0}^{(n/2)-1} X[2k+1] e^{2f\sqrt{-1}|i/(n/2)}
 \end{aligned} \tag{3.36}$$

$$W_0 = e^{2f\sqrt{-1}/(n/2)} = \tilde{S}^2 \quad \text{n-point DFT}$$

(n/2-point) DFTs :

$$Y[i] = \sum_{k=0}^{(n/2)-1} X[2k] W_0^{ki} + \tilde{S}^i \sum_{k=0}^{(n/2)-1} X[2k+1] W_0^{ki} \tag{3.37}$$

n μ , DFT μ

μ μ , μ μ (μ DFT

).

μ μ , (μ 19),

μ μ

FFT 16 μ . μ μ μ .

μ μ μ . μ μ ,

μ . μ μ μ μ

, μ μ w^p,

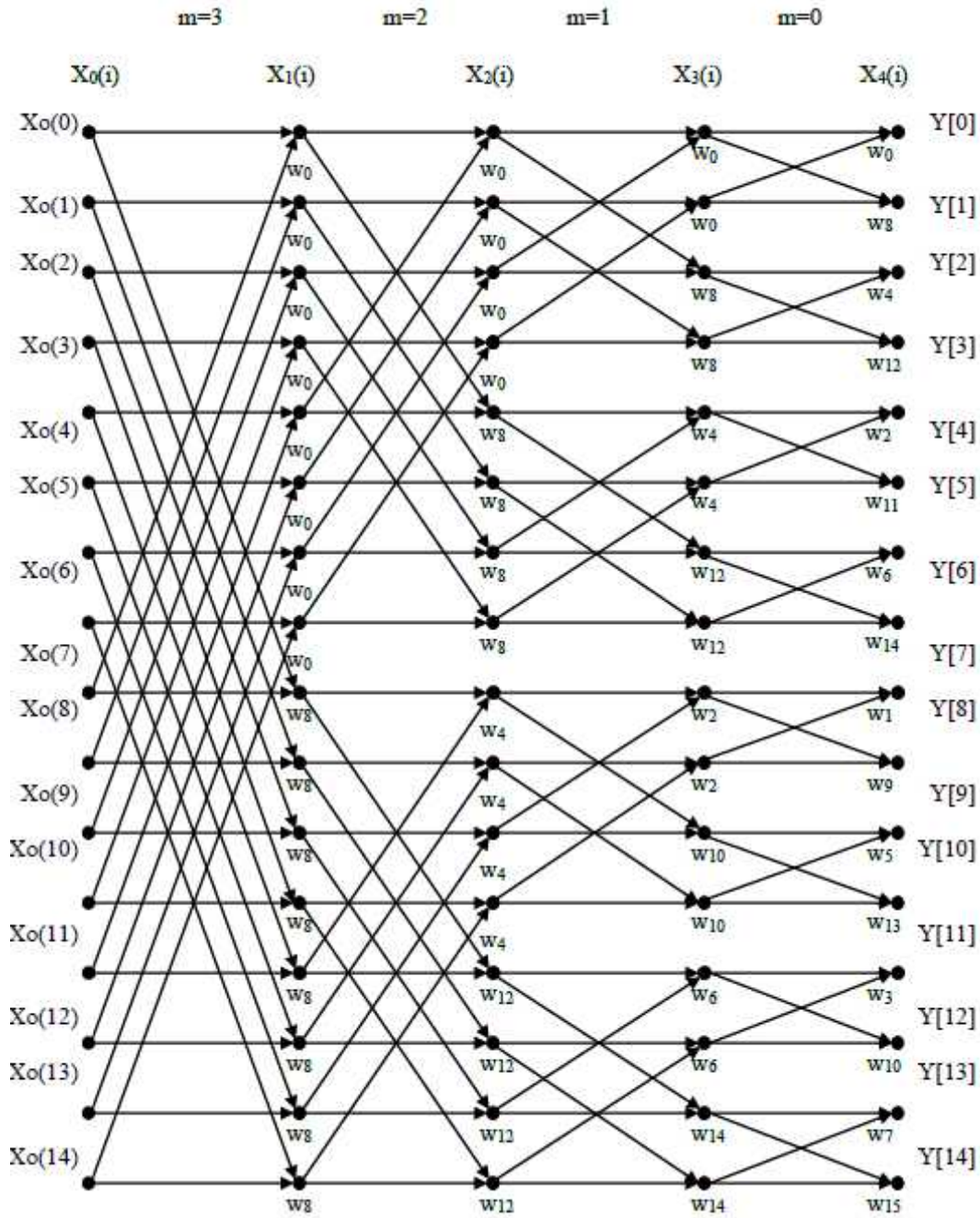
, μ p μ μ

0 15. w^p w^p = 1.

μ μ . μ

μ 2(2) : 2(2) = 1(2) + 1(6) * w₀.

μ & μ



μ 19 : μ μ
 μ FFT 16 μ , m
 , (μ 2004).

3.6.1.1 μ transpose FFT

μ & μ

μ FFT : μ

binary exchange transpose. μ

μ $\mu\mu$ μ ,

μ .

μ FFT μ μ

FFTE (Takahashi 2000)

μ μ μ μ

μ μ μ μ

ALLTOALL MPI (Vanden-Abeelee

et al.), μ μ μ transpose

μ .

μ μ μ transpose

μ μ \sqrt{n} μ

μ , μ n , μ $\sqrt{nx\sqrt{n}}$.

μ μ n $\sqrt{nx\sqrt{n}}$ FFT n

μ μ μ FFTs \sqrt{n} μ $\mu\mu$

μ μ μ FFTs \sqrt{n} μ .

μ μ μ $\sqrt{nx\sqrt{n}}$ μ μ

\sqrt{n} μ $\mu\mu$ FFTs, μ μ

μ μ \sqrt{n} μ $\mu\mu$ FFTs

μ μ μ . μ transpose μ

μ FFT, μ $\mu\mu$

$\sqrt{nx\sqrt{n}}$.

transpose FFT μ μ . μ μ

FFT \sqrt{n} μ $\mu\mu$. μ

μ μ μ , $\mu\mu$

$\mu\mu$. μ μ μ

FFT \sqrt{n} μ $\mu\mu$ μ μ μ

μ 1 3 μ μ

μ & μ
 μ μ^2 , μ
 μ ALLTOALL .
 μ p ($1 < p < n$)
 μ $\mu \sqrt{n}/p$ $\mu\mu$, μ
 μ . μ 1 3 \sqrt{n}/p FFTs
 \sqrt{n} μ . , μ transpose FFT
 μ μ μ μ μ μ
 μ . μ 20 μ μ μ μ
 $\mu\mu$ μ μ μ . μ
 μ X[0], X[4], X[8], X[12] μ μ ,
 μ μ μ .
 μ μ X[4], X[5], X[6], X[7] μ μ μ .
 μ $\mu\mu$.

spectral decomposition. $\hat{g}(-f) = [\hat{g}(f)]^*$ (3.38)

Fourier modes $k = 0, 1, \dots, N/2 - 1$ are real, and modes $k = N/2, N/2 + 1, \dots, N - 1$ are complex conjugates of modes $k = 0, 1, \dots, N/2 - 1$.

$$\begin{array}{l}
 k = 0 \\
 k = 1 \\
 \vdots \\
 k = N/2 - 1 \\
 k = N/2 \\
 k = N/2 + 1 \\
 \vdots \\
 k = N - 1
 \end{array}
 \left. \begin{array}{l}
 \rightarrow real \\
 \\
 \\
 \\
 \rightarrow real \\
 \\
 \\
 \rightarrow \hat{g}_k = [\hat{g}_{N-k}]^*
 \end{array} \right\}$$

μ 21 : μ μ Fourier μ

3.6.2 μ $\mu\mu$

μ & μ

μ

μ Crank-Nicolson. μ μ

:

$$\frac{df}{dt} = \gamma f \quad (3.39)$$

μ μ μ

:

$$\frac{f^{n+1} - f^n}{\Delta t} = \gamma \frac{f^{n+1} + f^n}{2} \quad (3.40)$$

n+1 μ n

μ . μ (3.40) μ Crank-Nicolson

μ n+1/2

μ μ μ μ

μ , μ μ μ

μ . , μ μ .

3.6.3 μ μ μμ

μ μ

Adams-Bashforth. μ μ (3.39)

μ μ :

$$\frac{f^{n+1} - f^n}{\Delta t} = \gamma \left(\frac{3}{2} f^n - \frac{1}{2} f^{n-1} \right) \quad (3.41)$$

μ μ μ μ μ

μ μ .

μ μ μ ,

μ μ

. μ μμ . μ μ

μ μμ μ

μ & μ

μ μ

Fourier modes

μ , μ μ μ μ ,

μ μ μ μ (Vanden-Abeelee et al.).

, μ μ

μ μ

μ μ (2.7-2.10) :

$$\left(\hat{h}_{xk}\right)_i^e = \sum_{n=0}^{N-1} \int_{\Omega_e} \Phi_i \left(u_n \frac{\partial u_n}{\partial x} + v_n \frac{\partial u_n}{\partial y} + w_n \frac{\partial u_n}{\partial z} \right) \text{Exp} \left[-\frac{2f kn}{N} I \right] d\Omega_e \quad (3.42)$$

$$\left(\hat{h}_{yk}\right)_i^e = \sum_{n=0}^{N-1} \int_{\Omega_e} \Phi_i \left(u_n \frac{\partial v_n}{\partial x} + v_n \frac{\partial v_n}{\partial y} + w_n \frac{\partial v_n}{\partial z} \right) \text{Exp} \left[-\frac{2f kn}{N} I \right] d\Omega_e \quad (3.43)$$

$$\left(\hat{h}_{zk}\right)_i^e = \sum_{n=0}^{N-1} \int_{\Omega_e} \Phi_i \left(u_n \frac{\partial w_n}{\partial x} + v_n \frac{\partial w_n}{\partial y} + w_n \frac{\partial w_n}{\partial z} \right) \text{Exp} \left[-\frac{2f kn}{N} I \right] d\Omega_e \quad (3.44)$$

$$\left(\hat{h}_{\Theta k}\right)_i^e = \sum_{n=0}^{N-1} \int_{\Omega_e} \Phi_i \left(u_n \frac{\partial \Theta_n}{\partial x} + v_n \frac{\partial \Theta_n}{\partial y} + w_n \frac{\partial \Theta_n}{\partial z} \right) \text{Exp} \left[-\frac{2f kn}{N} I \right] d\Omega_e \quad (3.45)$$

μ μ μ ^ μ μ μ μ
 Fourier . k, e, n μ k- Fourier mode, μ
 z, i(x,y)
 Lagrange (3.42-3.45) k- mode
 spectral .

μ
 () μ
 Fourier μ Fourier modes.

μ μ (2) μ μ .
 , μ -spectral μ
 Orszag (1969, 1971) Orszag & Kells (1980).

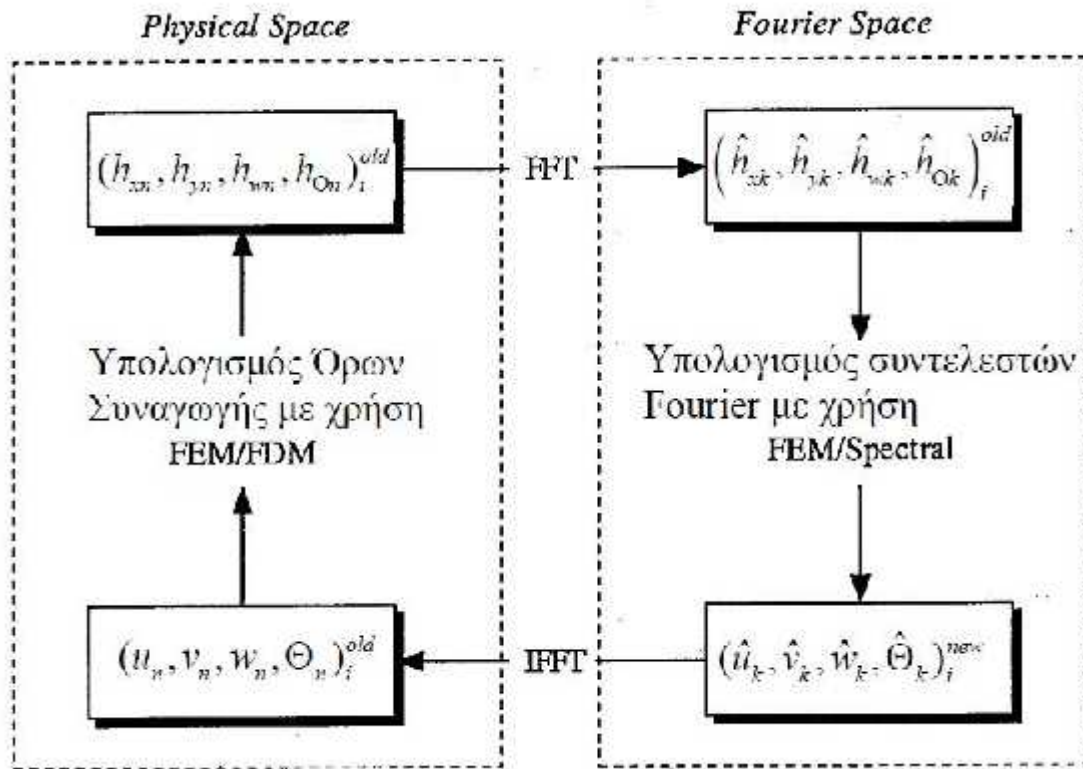
μ μ Fourier μ
 μ . μ μ , μ
 μ μ μμ

μ & μ

$(u_n, v_n, w_n, \Theta_n)_i^{old}$ μ

μ . , μ

$(h_{xn}, h_{yn}, h_{wn}, h_{\Theta n})_i^{old}$ μ μ FEM FDM.



μ 22 : -spectral μ

μ μ μ .

μ

μ

$(h_{xn}, h_{yn}, h_{wn}, h_{\Theta n})_i$ μ

u, v, w μ . ,

:

$$(h_{xn})_i^e = \int_{\Omega_e} \Phi_i \left(u_n \frac{\partial u_n}{\partial x} + v_n \frac{\partial u_n}{\partial y} + w_n \frac{\partial u_n}{\partial z} \right) d\Omega_e \quad (3.46)$$

$$(h_{yn})_i^e = \int_{\Omega_e} \Phi_i \left(u_n \frac{\partial v_n}{\partial x} + v_n \frac{\partial v_n}{\partial y} + w_n \frac{\partial v_n}{\partial z} \right) d\Omega_e \quad (3.47)$$

$$(h_{zn})_i^e = \int_{\Omega_e} \Phi_i \left(u_n \frac{\partial w_n}{\partial x} + v_n \frac{\partial w_n}{\partial y} + w_n \frac{\partial w_n}{\partial z} \right) d\Omega_e \quad (3.48)$$

μ & μ

$$(h_{\Theta_n})_i^e = \int_{\Omega_e} \Phi_i \left(u_n \frac{\partial \Theta_n}{\partial x} + v_n \frac{\partial \Theta_n}{\partial y} + w_n \frac{\partial \Theta_n}{\partial z} \right) d\Omega_e \quad (3.49)$$

(∂/∂z)_n μ

:

$$\left(\frac{\partial f_n}{\partial z} \right) = \frac{f_{right} - f_{left}}{2\Delta z} \quad (3.50)$$

z μ μ :

$$\Delta z = \frac{L}{N} \quad (3.51)$$

L μ Fourier

modes. f_{right} f_{left} :

$$f_{left} = f_{n-1}, f_{right} = f_{n+1} \quad (3.52)$$

n=0 n=N-1.

μ μ μ :

$$n=0 \rightarrow f_{left} = f_{N-1}, f_{right} = f_1 \quad (3.53)$$

$$n=N-1 \rightarrow f_{left} = f_{N-2}, f_{right} = f_0 \quad (3.54)$$

μ

Fourier $(\hat{h}_{xk}, \hat{h}_{yk}, \hat{h}_{wk}, \hat{h}_{\Theta k})_i^{old}$ μ FFT

μ . 2^n

μ FFTs (n) μ

μ (log₂N) μ μ .

3.6.4 μ

μ

(2.6-2.11) μ

μ . u, v, w, , p μ μ

μ μ μ μ 12

μ μ μ μ

μ . μ μ ,

$$u \hat{q}_k = \hat{q}_k^{n+1} - \hat{q}_k^n \quad (3.55)$$

$$\hat{q}_k = \{ \hat{u}_k, \hat{v}_k, \hat{w}_k, \hat{\Theta}_k, \hat{W}_k, \hat{p}_k \}.$$

, μ μ μ μ Fourier μ
 μ :

$Au = b \Rightarrow$

$$A(\hat{q}_k^{n+1} - \hat{q}_k^n) = \left(D \hat{q}_k^n + \frac{3}{2} C^n - \frac{1}{2} C^{n-1} \right) \Rightarrow \quad (3.56)$$

$$(\hat{q}_k^{n+1} - \hat{q}_k^n) = A^{-1} \left(D \hat{q}_k^n + \frac{3}{2} C^n - \frac{1}{2} C^{n-1} \right)$$

D $\mu\mu$
 μ μ μ μ
 μ , C μ μ $\mu\mu$
 μ - μ μ .

$$\sum_{j=1}^N \int_{\Omega_e} \left(\Psi_i \frac{\partial \Phi_j}{\partial x} u \hat{u}_{k,j} + \Psi_i \frac{\partial \Phi_j}{\partial y} u \hat{v}_{k,j} + \left(\frac{2fk}{L} I \right) \Psi_i \Phi_j u \hat{w}_{k,j} \right) d\Omega_e = \quad (3.57)$$

$$- \sum_{j=1}^N \int_{\Omega_e} \left(\Psi_i \frac{\partial \Phi_j}{\partial x} \hat{u}_{k,j}^n + \Psi_i \frac{\partial \Phi_j}{\partial y} \hat{v}_{k,j}^n + \left(\frac{2fk}{L} I \right) \Psi_i \Phi_j \hat{w}_{k,j}^n \right) d\Omega_e$$

μ **x, y, z**

$$\begin{aligned}
 & \left[\frac{1}{\Delta t} \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e + \frac{Gr^{-1/2}}{2} \left(\sum_{j=1}^N \int_{\Omega_e} \left(\frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right) d\Omega_e + \left(\frac{2fk}{L} \right)^2 \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right) \right] u \hat{u}_{k,j} \quad (3.58) \\
 & - \left[\frac{1}{2} \sum_{j=1}^N \int_{\Omega_e} \frac{\partial \Phi_i}{\partial x} \Psi_j d\Omega_e \right] u \hat{p}_{k,j} = \frac{3}{2} (\hat{h}_{yk})_i^n - \frac{1}{2} (\hat{h}_{yk})_i^{n-1} \\
 & - \left[Gr^{-1/2} \left(\sum_{j=1}^N \int_{\Omega_e} \left(\frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right) d\Omega_e + \left(\frac{2fk}{L} \right)^2 \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right) \right] \hat{u}_{k,j}^n \\
 & + \left[\sum_{j=1}^N \int_{\Omega_e} \frac{\partial \Phi_i}{\partial x} \Psi_j d\Omega_e \right] \hat{p}_{k,j}^n
 \end{aligned}$$

$$\begin{aligned}
 & \left[\frac{1}{\Delta t} \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e + \frac{Gr^{-1/2}}{2} \left(\sum_{j=1}^N \int_{\Omega_e} \left(\frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right) d\Omega_e + \left(\frac{2fk}{L} \right)^2 \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right) \right] u \hat{v}_{k,j} \\
 & - \left[\frac{1}{2} \sum_{j=1}^N \int_{\Omega_e} \frac{\partial \Phi_i}{\partial y} \Psi_j d\Omega_e \right] u \hat{p}_{k,j} - \left[\frac{1}{2} \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right] u \hat{\theta}_{k,j} + \left[\frac{1}{2} \frac{Ha^2}{Gr^{1/2}} \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right] u \hat{v}_{k,j} \\
 & + \left[\frac{1}{2} \frac{Ha^2}{Gr^{1/2}} \left(\frac{2fk}{L} I \right) \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right] u \hat{w}_{k,j} = \frac{3}{2} (\hat{h}_{yk})_i^n - \frac{1}{2} (\hat{h}_{yk})_i^{n-1} \\
 & - \left[Gr^{-1/2} \left(\sum_{j=1}^N \int_{\Omega_e} \left(\frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right) d\Omega_e + \left(\frac{2fk}{L} \right)^2 \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right) \right] \hat{v}_{k,j}^n \\
 & + \left[\sum_{j=1}^N \int_{\Omega_e} \frac{\partial \Phi_i}{\partial y} \Psi_j d\Omega_e \right] \hat{p}_{k,j}^n + \left[\sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right] \hat{\theta}_{k,j}^n \quad (3.59) \\
 & - \left[\frac{Ha^2}{Gr^{1/2}} \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right] \hat{v}_{k,j}^n - \left[\frac{Ha^2}{Gr^{1/2}} \left(\frac{2fk}{L} I \right) \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right] \hat{w}_{k,j}^n
 \end{aligned}$$

$$\begin{aligned}
 & \left[\frac{1}{\Delta t} \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e + \frac{Gr^{-1/2}}{2} \left(\sum_{j=1}^N \int_{\Omega_e} \left(\frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right) d\Omega_e + \left(\frac{2fk}{L} \right)^2 \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right) \right] u \hat{w}_{k,j} \\
 & + \left[\frac{1}{2} \left(\frac{2fk}{L} I \right) \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Psi_j d\Omega_e \right] u \hat{p}_{k,j} + \left[\frac{1}{2} \frac{Ha^2}{Gr^{1/2}} \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right] u \hat{w}_{k,j} + \left[\frac{1}{2} \frac{Ha^2}{Gr^{1/2}} \sum_{j=1}^N \int_{\Omega_e} \Phi_i \frac{\partial \Phi_j}{\partial y} d\Omega_e \right] u \hat{w}_{k,j} \\
 & = \frac{3}{2} (\hat{h}_{yk})_i^n - \frac{1}{2} (\hat{h}_{yk})_i^{n-1} \\
 & - \left[Gr^{-1/2} \left(\sum_{j=1}^N \int_{\Omega_e} \left(\frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right) d\Omega_e + \left(\frac{2fk}{L} \right)^2 \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right) \right] \hat{w}_{k,j}^n \\
 & - \left[\left(\frac{2fk}{L} I \right) \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Psi_j d\Omega_e \right] \hat{p}_{k,j}^n \quad (3.60) \\
 & - \left[\frac{Ha^2}{Gr^{1/2}} \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right] \hat{w}_{k,j}^n - \left[\frac{Ha^2}{Gr^{1/2}} \sum_{j=1}^N \int_{\Omega_e} \Phi_i \frac{\partial \Phi_j}{\partial y} d\Omega_e \right] \hat{w}_{k,j}^n
 \end{aligned}$$

$$\left[\frac{1}{\Delta t} \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e + \frac{Gr^{-1/2}}{2Pr} \left(\sum_{j=1}^N \int_{\Omega_e} \left(\frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right) d\Omega_e + \left(\frac{2fk}{L} \right)^2 \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right) \right] u \hat{\Theta}_{k,j} \quad (3.61)$$

$$= \frac{3}{2} (\hat{h}_{\Theta k})^n - \frac{1}{2} (\hat{h}_{\Theta k})^{n-1}$$

$$- \left[\frac{Gr^{-1/2}}{Pr} \left(\sum_{j=1}^N \int_{\Omega_e} \left(\frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right) d\Omega_e + \left(\frac{2fk}{L} \right)^2 \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right) \right] \hat{\Theta}_{k,j}^n$$

μ ^ μ μ μ μ Fourier .
 k, e, n μ k- Fourier mode, μ
 μ μ , . i(x,y), i(x,y)

μμ Lagrange, .

$$\left(\hat{h}_{xk}, \hat{h}_{yk}, \hat{h}_{zk}, \hat{h}_{\Theta k} \right) \quad \mu \quad \mu\mu$$

(3.42-3.45) L μ .

μ μ x, y, z ,

. μ μ

μ (2.13) μ μ μ . ,

μ :

$$\sum_e \left[\int_{\Omega_e} \Phi_i \nabla_{xy}^2 \hat{w}_k d\Omega_e \right] = \sum_e \left[\left(\frac{2fk}{L} \right)^2 \int_{\Omega_e} \Phi_i \hat{w}_k d\Omega_e + \int_{\Omega_e} \frac{\partial \Phi_i}{\partial y} \hat{w}_k d\Omega_e - \int_{\Omega_e} \left(\frac{2fk}{L} I \right) \Phi_i \hat{v}_k d\Omega_e \right] \Rightarrow$$

$$\sum_e \left[\int_{\Omega_e} \nabla_{xy} \left(\Phi_i \nabla_{xy} \hat{w}_k \right) d\Omega_e \right] - \sum_e \left[\int_{\Omega_e} \nabla_{xy} \Phi_i \nabla_{xy} \hat{w}_k d\Omega_e \right] =$$

$$\sum_e \left[\left(\frac{2fk}{L} \right)^2 \int_{\Omega_e} \Phi_i \hat{w}_k d\Omega_e + \int_{\Omega_e} \frac{\partial \Phi_i}{\partial y} \hat{w}_k d\Omega_e - \int_{\Omega_e} \left(\frac{2fk}{L} I \right) \Phi_i \hat{v}_k d\Omega_e \right] \Rightarrow$$

$$\begin{aligned}
 \sum_e \int_{dS} \Phi_i \frac{\partial \hat{W}_k}{\partial n} dS &= \sum_e \left[\int_{\Omega_e} \left(\frac{\partial \Phi_i}{\partial x} \frac{\partial \hat{W}_k}{\partial x} + \frac{\partial \Phi_i}{\partial y} \frac{\partial \hat{W}_k}{\partial y} \right) d\Omega_e \right] + \\
 \sum_e \left[\left(\frac{2fk}{L} \right)^2 \int_{\Omega_e} \Phi_i \hat{W}_k d\Omega_e + \int_{\Omega_e} \frac{\partial \Phi_i}{\partial y} \hat{W}_k d\Omega_e - \int_{\Omega_e} \left(\frac{2fk}{L} I \right) \Phi_i \hat{W}_k d\Omega_e \right] &\Rightarrow \\
 \int_0^1 \Phi_i \frac{\partial \hat{W}_k}{\partial x} dy \Big|_{x=A} - \int_0^1 \Phi_i \frac{\partial \hat{W}_k}{\partial x} dy \Big|_{x=0} + \int_0^A \Phi_i \frac{\partial \hat{W}_k}{\partial y} dx \Big|_{y=1} - \int_0^A \Phi_i \frac{\partial \hat{W}_k}{\partial y} dx \Big|_{y=0} &= I \Rightarrow \\
 \left(\Phi_i c_H \frac{\partial \hat{W}_k}{\partial y} \Big|_{0,x=A}^{1,x=A} - \int_0^1 c_H \frac{\partial \Phi_i}{\partial y} \frac{\partial \hat{W}_k}{\partial y} dy \Big|_{x=A} \right) + \left(\Phi_i c_H \frac{\partial \hat{W}_k}{\partial y} \Big|_{0,x=0}^{1,x=0} - \int_0^1 c_H \frac{\partial \Phi_i}{\partial y} \frac{\partial \hat{W}_k}{\partial y} dy \Big|_{x=0} \right) & \\
 + \left(\Phi_i c_S \frac{\partial \hat{W}_k}{\partial x} \Big|_{0,y=1}^{A,y=1} - \int_0^A c_S \frac{\partial \Phi_i}{\partial x} \frac{\partial \hat{W}_k}{\partial x} dx \Big|_{y=1} \right) + \left(\Phi_i c_S \frac{\partial \hat{W}_k}{\partial x} \Big|_{0,y=0}^{A,y=0} - \int_0^A c_S \frac{\partial \Phi_i}{\partial x} \frac{\partial \hat{W}_k}{\partial x} dx \Big|_{y=0} \right) &= \\
 I + \int_0^1 \Phi_i c_H \left(\frac{2fk}{L} \right)^2 \hat{W}_k dy \Big|_{x=A} + \int_0^1 \Phi_i c_H \left(\frac{2fk}{L} \right)^2 \hat{W}_k dy \Big|_{x=0} & \\
 + \int_0^A \Phi_i c_S \left(\frac{2fk}{L} \right)^2 \hat{W}_k dx \Big|_{y=1} + \int_0^A \Phi_i c_S \left(\frac{2fk}{L} \right)^2 \hat{W}_k dx \Big|_{y=0} &\Rightarrow \\
 \Phi_i c_H \frac{\partial \hat{W}_k}{\partial y} \Big|_{0,x=A}^{1,x=A} + \Phi_i c_H \frac{\partial \hat{W}_k}{\partial y} \Big|_{0,x=0}^{1,x=0} + \Phi_i c_S \frac{\partial \hat{W}_k}{\partial x} \Big|_{0,y=1}^{A,y=1} + \Phi_i c_S \frac{\partial \hat{W}_k}{\partial x} \Big|_{0,y=0}^{A,y=0} &= \tag{3.62} \\
 I + \int_0^1 c_H \left[\Phi_i \left(\frac{2fk}{L} \right)^2 \hat{W}_k + \frac{\partial \Phi_i}{\partial y} \frac{\partial \hat{W}_k}{\partial y} \right] dy \Big|_{x=A} + \int_0^1 c_H \left[\Phi_i \left(\frac{2fk}{L} \right)^2 \hat{W}_k + \frac{\partial \Phi_i}{\partial y} \frac{\partial \hat{W}_k}{\partial y} \right] dy \Big|_{x=0} & \\
 + \int_0^A c_S \left[\Phi_i \left(\frac{2fk}{L} \right)^2 \hat{W}_k + \frac{\partial \Phi_i}{\partial x} \frac{\partial \hat{W}_k}{\partial x} \right] dx \Big|_{y=1} + \int_0^A c_S \left[\Phi_i \left(\frac{2fk}{L} \right)^2 \hat{W}_k + \frac{\partial \Phi_i}{\partial x} \frac{\partial \hat{W}_k}{\partial x} \right] dx \Big|_{y=0} &
 \end{aligned}$$

(3.62) μ μ μ

:

$$\begin{aligned}
 & \left[-\sum_{j=1}^N \int_{\Omega_e} \left(\frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right) d\Omega_e - \sum_{j=1}^N \left(\frac{2fk}{L} \right)^2 \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right] u \hat{w}_{k,j} + \\
 & \left[\sum_{j=1}^N \int_{\Omega_e} \Phi_i \frac{\partial \Phi_j}{\partial y} d\Omega_e \right] u \hat{w}_{k,j} + \left[\left(\frac{2fk}{L} I \right) \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right] u \hat{v}_{k,j} + \\
 & \sum_{j=1}^N \int_0^1 c_H \left[\left(\frac{2fk}{L} \right)^2 \Phi_i \Phi_j + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right] dy \Big|_{x=A} u \hat{w}_{k,j} + \\
 & \sum_{j=1}^N \int_0^1 c_H \left[\left(\frac{2fk}{L} \right)^2 \Phi_i \Phi_j + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right] dy \Big|_{x=0} u \hat{w}_{k,j} + \\
 & \sum_{j=1}^N \int_0^A c_S \left[\left(\frac{2fk}{L} \right)^2 \Phi_i \Phi_j + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right] dx \Big|_{y=1} u \hat{w}_{k,j} + \\
 & \sum_{j=1}^N \int_0^A c_S \left[\left(\frac{2fk}{L} \right)^2 \Phi_i \Phi_j + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right] dx \Big|_{y=0} u \hat{w}_{k,j} - c_H \Phi_i \frac{\partial \Phi_j}{\partial y} \Big|_{0,x=A}^{1,x=A} u \hat{w}_{k,j} \\
 & - c_H \Phi_i \frac{\partial \Phi_j}{\partial y} \Big|_{0,x=0}^{1,x=0} u \hat{w}_{k,j} - c_S \Phi_i \frac{\partial \Phi_j}{\partial x} \Big|_{0,y=1}^{A,y=1} u \hat{w}_{k,j} - c_S \Phi_i \frac{\partial \Phi_j}{\partial x} \Big|_{0,y=0}^{A,y=0} u \hat{w}_{k,j} = \\
 & \left[+\sum_{j=1}^N \int_{\Omega_e} \left(\frac{\partial \Phi_i}{\partial x} \frac{\partial \Phi_j}{\partial x} + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right) d\Omega_e + \sum_{j=1}^N \left(\frac{2fk}{L} \right)^2 \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right] \hat{w}_{k,j} - \\
 & \left[\sum_{j=1}^N \int_{\Omega_e} \Phi_i \frac{\partial \Phi_j}{\partial y} d\Omega_e \right] \hat{w}_{k,j} - \left[\left(\frac{2fk}{L} I \right) \sum_{j=1}^N \int_{\Omega_e} \Phi_i \Phi_j d\Omega_e \right] \hat{v}_{k,j} \\
 & - \sum_{j=1}^N \int_0^1 c_H \left[\left(\frac{2fk}{L} \right)^2 \Phi_i \Phi_j + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right] dy \Big|_{x=A} \hat{w}_{k,j} \\
 & - \sum_{j=1}^N \int_0^1 c_H \left[\left(\frac{2fk}{L} \right)^2 \Phi_i \Phi_j + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right] dy \Big|_{x=0} \hat{w}_{k,j} \\
 & - \sum_{j=1}^N \int_0^A c_S \left[\left(\frac{2fk}{L} \right)^2 \Phi_i \Phi_j + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right] dx \Big|_{y=1} \hat{w}_{k,j} \\
 & - \sum_{j=1}^N \int_0^A c_S \left[\left(\frac{2fk}{L} \right)^2 \Phi_i \Phi_j + \frac{\partial \Phi_i}{\partial y} \frac{\partial \Phi_j}{\partial y} \right] dx \Big|_{y=0} \hat{w}_{k,j} \\
 & + c_H \Phi_i \frac{\partial \Phi_j}{\partial y} \Big|_{0,x=A}^{1,x=A} \hat{w}_{k,j} + c_H \Phi_i \frac{\partial \Phi_j}{\partial y} \Big|_{0,x=0}^{1,x=0} \hat{w}_{k,j} \\
 & + c_S \Phi_i \frac{\partial \Phi_j}{\partial x} \Big|_{0,y=1}^{A,y=1} \hat{w}_{k,j} + c_S \Phi_i \frac{\partial \Phi_j}{\partial x} \Big|_{0,y=0}^{A,y=0} \hat{w}_{k,j}
 \end{aligned} \tag{3.63}$$

μ & μ

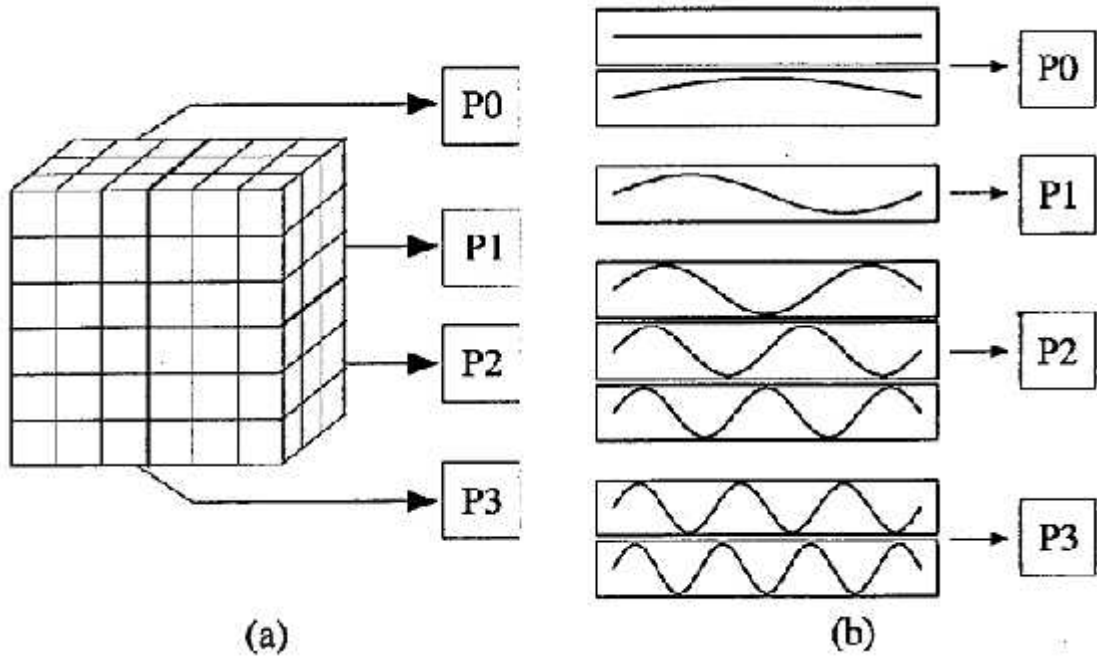
μ_{CH}, μ_{CS} Hartmann
(3.57-
3.61, 3.63) μ μ μ
 μ .

3.6.5 μ

3.6.5.1 μ Fourier

H
 $\mu\mu$ μ Fourier mode
 μ . , μ μ μ
 μ μ μ . μ
 μ μ μ ()
Fourier mode node () μ
(partitioning) μ μ
 μ , μ 23a. μ ,
 μ , μ μ
 , μ (,
) threads.
 μ μ μ
 , . ,
 μ μ μ
 μ . , μ μ
 μ μ .

μ & μ



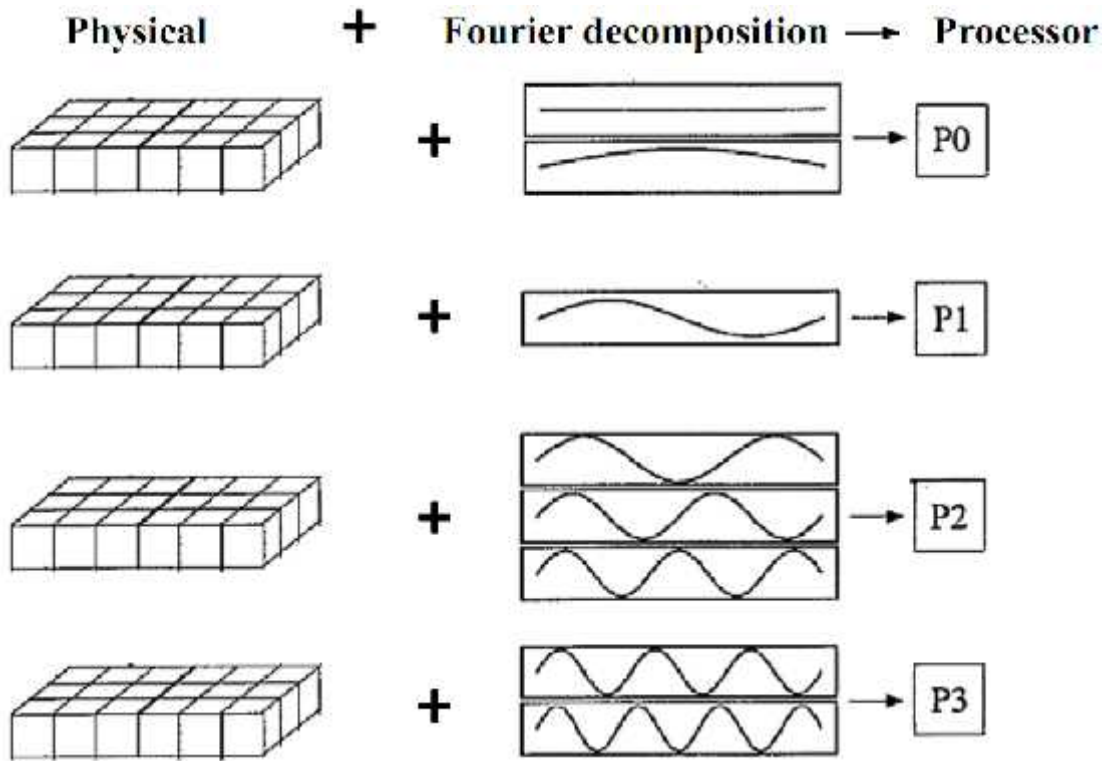
μ 23 : μ (partitioning) μ , (a)
 μ , (b) μ Fourier.
 μ , μ Fourier, μ 23b, μ
 μ . , μ
 μ . , Fourier modes
 μ μ μ
 modes μ μ
 μ mode μ μ
 μ Fourier mode μ μ
 μ μ μ
 μ Fourier, μ 24. ,
 Fourier mode μ thread ,
 μ
 μ

μ & μ

$\mu\mu$ μ

Fourier

mode.



μ 24 : μ (partitioning) μ
 μ Fourier.

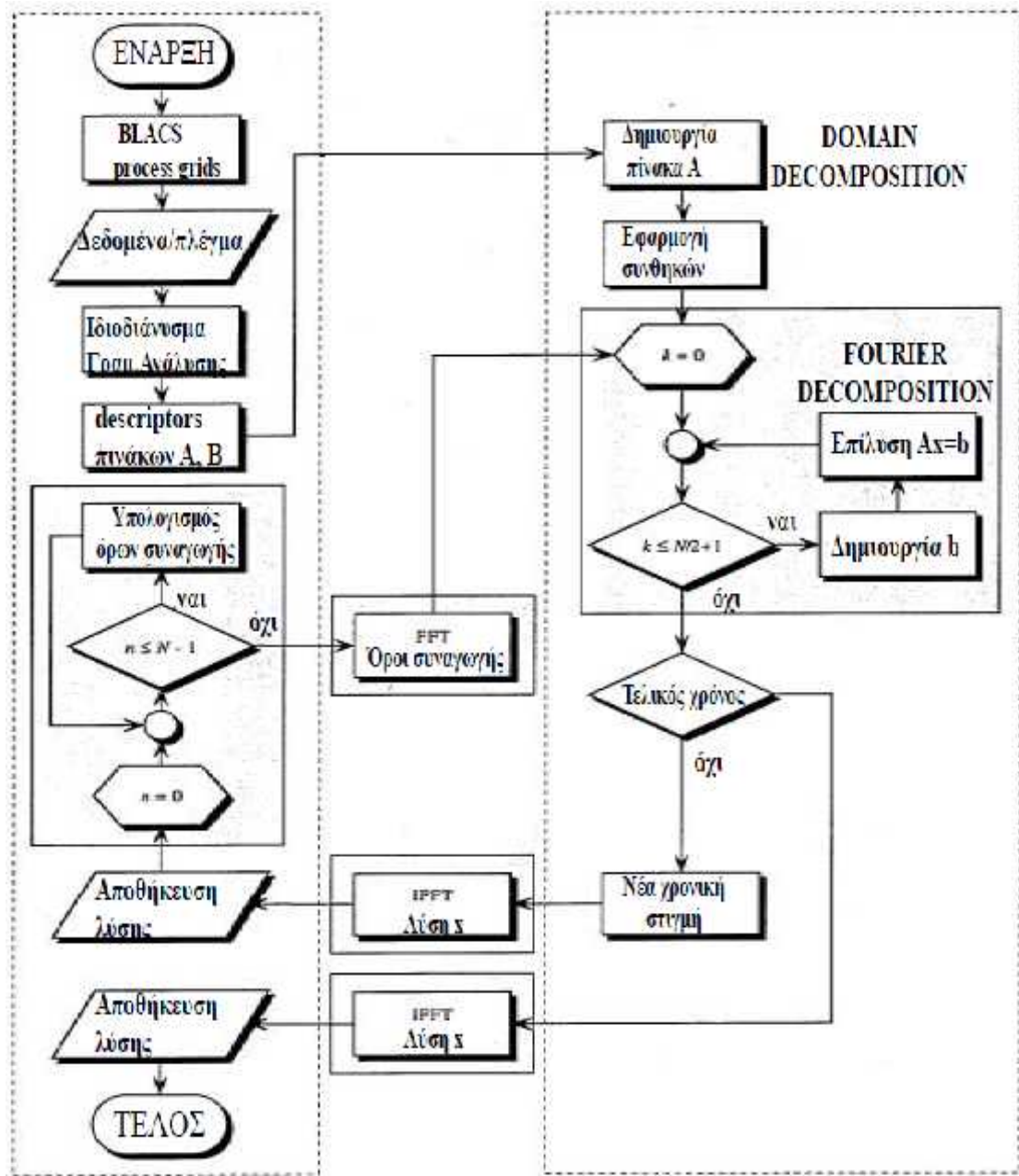
	#1	#2	#3	#4
Mode 0	0	1	2	3
Mode 1	4	5	6	7
Mode 2	8	9	10	11
Mode 3	12	13	14	15

μ 25 : μ 16 μ
 4 Fourier modes.

μ μ 16 μ μ μ
 μ 4 Fourier modes, mode 4 ,
 100

μ & μ
 (μ 25), node. H μ
 4 μ
 μ $Ax=b$, node, Fourier
 decomposition μ domain decomposition,
 μ 24. μ , μ
 $\mu\mu$
 μ . μ , 10
 $\mu\mu$,
 μ Fourier mode 2
 mode. O μ
 μ μ μ b
 (Dimopoulos & Pelekasis 2012) μ μ
 8, 9 11 mode, μ
 μ $Ax=b$
 Fourier mode 2.
 μ $\mu\mu$
 μ mode μ
 (process grid) , μ μ μ
 μ μ μ modes. μ $\mu\mu$
 μ communicator, μ μ μ
 μ BLACS
 μ Fourier mode.
 μ μ μ
 . scalability
 μ μ CPU.
 , μ μ μ .
 μ , $\mu\mu$ μ
 Fourier μ $\mu\mu$ μ μ -spectral
 μ .
 , μ μ μ μ FFT.
 μ
 Fourier modes μ μ μ
 $\mu\mu$ μ μ . μ

FOURIER



μ 26 : μμ μ μ μ μ μ

IV.

4.1 μ μμ μ Rayleigh - Benard μ μ

, μ μ

μ μ μ μ μ

Rayleigh -

Benard

μ

μ μ μ Ha.

μ

μ . μ ,

Gr_{Cr} μ

Ha

μ μ

μ

μ

Hartmann

. μ

μ

μ

μ

μ

μ

Burr & Muller (2002).

,

μ

μ =20 μ h=20mm

μ

Na²²K⁷⁸ μ

μ Gr, Ha

Pr. Pr

μ 0.02,

Gr Ha

μ

μ

μ

μ

μ

μ

.

μ

μ

μ

Ha μ

100

1000

μ

μ Gr_{Cr}

μ

μμ

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μ

μ

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μ

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μ

μ

μ

μ

Hartmann

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μ

μ

μ

μ

,

c_H=0.00415

c_S=4.5.

μ

μ

μ

μ

μ

μ

μ

Hartmann

.

$$Gr_{Cr} Pr = Ra_{Cr} \sim Ha^2$$

$$U_0 \hat{O} \dots gUT / \theta \dagger B_0^2; N \theta Gr / Ha^2; \theta \epsilon / h:$$

$$Ra_{Cr} \sim Ha^2$$

(Davoust et al. 1999).

Ha

μ

Lorentz

Buhler (1998),

(4.3-4.5) :

$$\bar{e} \hat{I} \bar{e} P N \frac{1}{Ha^2} \bar{e}^2 \bar{U} < T \bar{e}_y < \bar{J} \hat{I} \bar{e}_x \bar{E} \frac{1}{Ha^2} \bar{e}^2 \bar{h} < \frac{\partial}{\partial x} \bar{J} > ik T \bar{e}_x < \frac{\partial T}{\partial x} \bar{e}_z N 0 \quad (4.9)$$

$$\bar{e} f \hat{O} \bar{e}_x \frac{\partial f}{\partial x} < \bar{e}_y \frac{\partial f}{\partial y} < ik f \bar{e}_z$$

Ha $\rightarrow \infty$

Ohm

$$\frac{\partial^2 W}{\partial x^2} N > ik T \bar{E} \int_{A/2}^x \frac{\partial^2 W}{\partial r^2} dr N ik T dr \frac{\partial W}{\partial x} \Big|_x N ik T dr \bar{E} \quad (4.10)$$

$$W|_x N W|_0 < ik \int_0^x dp T dr$$

(4.10)

(4.7)

μ & μ

$$h_x N \hat{e}_0^2 \hat{W} < ik \frac{dp}{0 p} \hat{e}^2 T dr N \hat{e}_0^2 \hat{W} < ik \frac{dp}{0 p} \hat{e}^2 > 2V : dr, \tag{4.11}$$

$$\hat{e}_0 f \hat{0} \hat{e}_y \frac{\partial f}{\partial y} < \hat{e}_z ik f, \hat{e}^2 f \hat{0} \frac{\partial^2 f}{\partial x^2} < \frac{\partial^2 f}{\partial y^2} > k^2 f$$

μ μ μ Hartmann.

, x

μ μ Hartmann

μ . μ

x μ ,

Ha → ∞ x

μ . μ

y, z μ , y, z

μ μ μ μ ,

μ . μ

μ (4.11) μ

x

μ Hartmann μ

μ μ y μ , μ

μ Ha⁻¹ , μ

V, W μ (Ha) (1) U (1) (Ha⁻¹),

μ μ Hartmann, μ

μ Hartmann. ,

(4.7) μ x

μ Hartmann, J_{Hx} = (1) O(Ha⁻¹)

μ μ μ μ , .

y , J_{Hy}, μ (1)

Hartmann. μ , μ Hartmann μ

μ μ μ μ

μ , (. (2.13 ,)). , μ

μ Hartmann

μ . , μ Hartmann

μ μ

μ & μ

μ μ μ

Hartmann.

μ

μ

Hartmann

Hartmann

μ

μ

x

:

$$\frac{1}{Ha^{>2}} \frac{\partial^2 W_H}{\partial X^2} < \frac{\partial^2 W}{\partial x^2} N h_{Hx} < h_x \frac{\partial W_H}{\partial x} >$$

$$\frac{1}{Ha^{>2}} \frac{\partial^2 W_H}{\partial X^2} N h_{Hx} N O(1), \quad X \hat{O} \frac{x}{Ha^{>1}}$$

(4.12)

$$\tilde{E} J_{Hx} N > \frac{\partial W_H}{\partial x} N > Ha \frac{\partial W_H}{\partial X} N O(Ha^{>1})$$

$$\frac{\partial J_{Hx}}{\partial x} N Ha \frac{\partial J_{Hx}}{\partial X} N > Ha^2 \frac{\partial^2 W_H}{\partial X^2} N > h_{Hx} N O(1)$$

x J_{Hx}

x

x

μ

stretched

μ

μ

μ

Hartmann,

μ μ

μ

μ

Hartmann.

,

(4.9)

:

$$\frac{\partial^2 h_{Hx}}{\partial X^2} < \frac{\partial J_{Hx}}{\partial x} < \frac{\partial J_x}{\partial x} > ikT N O \tilde{E} \frac{\partial^2 h_{Hx}}{\partial X^2} < \frac{\partial J_{Hx}}{\partial x} N \frac{\partial^2 h_{Hx}}{\partial X^2} > h_{Hx} N O$$

(4.13)

x

, x⁺ Hx,

μ Hartmann,

=0,

Hx

X → ∞

x

μ

μ

:

$$h_{Hx} N > h_x \theta_x N O: e^{>x}$$

$$Ha \frac{\partial J_{Hx}}{\partial X} N > h_{Hx} \tilde{E} Ha J_{Hx} |_{xN0} N > \frac{\partial h_{Hx}}{\partial X} |_{xN0}$$

(4.14)

$$\tilde{E} J_{Hx} |_{xN0} N > \frac{\partial W_H}{\partial x} |_{xN0} N > Ha^{>1} h_x |_{xN0}$$

μ μ

μ

Hartmann

μ

μ

(2.13 ,) :

μ & μ

$$\langle c_H \ddot{e}_0^2 \hat{W} | N \frac{\partial W}{\partial x} \Big|_{x=0} \rangle < \frac{\partial W_H}{\partial x} \Big|_{x=0} \rangle \ddot{E} \frac{\partial W}{\partial x} \Big|_{x=0} \rangle N \langle c_H \ddot{e}_0^2 \hat{W} \rangle \text{Ha}^{-1} h_x \Big|_{x=0} \quad (4.15)$$

$$\ddot{E} \langle \frac{\partial W}{\partial x} \Big|_{x=0} \rangle N \langle c_H \langle \text{Ha}^{-1} \rangle \ddot{e}_0^2 \hat{W} \rangle \quad (4.10),$$

$$\langle ik \frac{A/2}{0} Tdr N \langle c_H \langle \text{Ha}^{-1} \rangle \ddot{e}_0^2 \hat{W} \rangle \ddot{E} \ddot{e}_0^2 \hat{W} N \frac{\langle 1 \rangle}{c_H \langle \text{Ha}^{-1} \rangle} ik \frac{A/2}{0} Tdr \quad (4.16)$$

$$h_x N \frac{\langle 1 \rangle}{c_H \langle \text{Ha}^{-1} \rangle} ik \frac{A/2}{0} Tdr \langle ik \frac{x}{0} \frac{A/2}{p} \rangle \ddot{e}_0^2 \hat{W} : \quad (4.17)$$

(4.16) μ (4.10)

μ μ μ Hartmann
 μ μ μ , ~ a, μ
 μ μ , ~ (1). μ
 μ μ Hartmann μ
 μ μ . (4.17)
 μ μ μ
 μ μ Hartmann braking effect, μ μ
 μ μ μ μ μ ,
 μ .

$$V N \langle ikW \rangle \langle T \rangle \frac{\partial P}{\partial y}, \quad w N \frac{\partial W}{\partial y} \rangle ikP, \quad (4.18a,b)$$

, μ y z μ , .
 (4.4,4.5), μ μ

$$\ddot{e}_0^2 P N \frac{\partial T}{\partial y} \quad (4.19)$$

μ
 μ μ Ha μ
 μ , . (4.3), μ

μ & μ
 U
 μ (1) Ha
 V, W μ μ μ $1/(c_H < Ha^{>1})$.
 μ , μ Hartmann μ
 μ Hartmann μ μ , μ μ
 μ Hartmann μ μ , μ μ V
 μ (Ha) μ μ μ μ
 Ha^{-1} . μ , μ Hartmann μ
 $1/ c_H$, Hartmann μ V
 μ μ μ μ
 μ Ha μ . μ μ
 μ μ μ μ
 μ x μ μ .
 $\mu\mu$, μ (4.17), μ
 μ μ Hartmann μ
 μ μ (4.17) (1) μ
 μ μ . , μ μ μ
 μ μ μ μ μ μ
 μ μ μ μ Hartmann x
 μ μ μ $\mu\mu$ μ .
 μ Hartmann, μ c_H
 μ μ μ μ
 μ μ . μ μ μ μ Hartmann, $c_H \approx 0$,
 μ μ μ Hartmann
 μ μ μ μ Ha^{-1} . , x
 (Ha) μ Hartmann
 V (Ha) , μ

&

Hartmann

(1)

(4.9)

(4.7)

Ha -> infinity

(4.18)

= a^-1/2

U=O(Ha)

Hartmann, x

(4.3)

(Ha^1/2)

(Buhler 1998) jets

Hartmann

(Walker 1981).

μ & μ

μ Hartmann, μ
 μ c_H .

4.1.2 μ μ μ **Ha**

μ μ μ μ μ
 , μ μ μ Grashof μ μ μ
Hartmann μ μ μ Hartmann
 μ μ

μ μ Pr=0.02. μ μ 27a,b
 μ μ ,

$c_H=0.00415$, μ , $c_H=4.5$, μ Hartmann, ,
 μ μ Gr μ μ Ha

Gr~Ha² μ μ Ha,
 μ Lorentz μ μ

μ μ . μ μ μ μ
 μ μ μ μ μ

$x=A/2$. μ , $\mu\mu$ μ
 μ $\mu\mu$.

μ μ 27a, μ
 μ μ μ Burr & Muller (2002),

μ μ μ μ .

μ μ Gr Ha.

μ , μ μ (Gr~Ha²) μ
 μ Hartmann μ μ μ

μ , $c_H=c_S=4.5$, μ 27b. ,
 μ μ μ Hartmann

μ μ μ μ

μ μ μ

μ & μ

μ . μ μ

μ μ μ

μ , μ μ μ

Hartmann, c_{H+Ha}^{-1} ,
 Hartmann. ,

μ , μ μ x

μ μ

29a b, μ μ 27a \widehat{Gr}_{Cr} N $2Gr_{Cr}$

μ μ Gr Burr & Muller (2002),

Gr_{Cr} μ μ μ Hartmann

μ .

μ μ .

μ μ μ ,

μ μ μ

Ha, μ μ

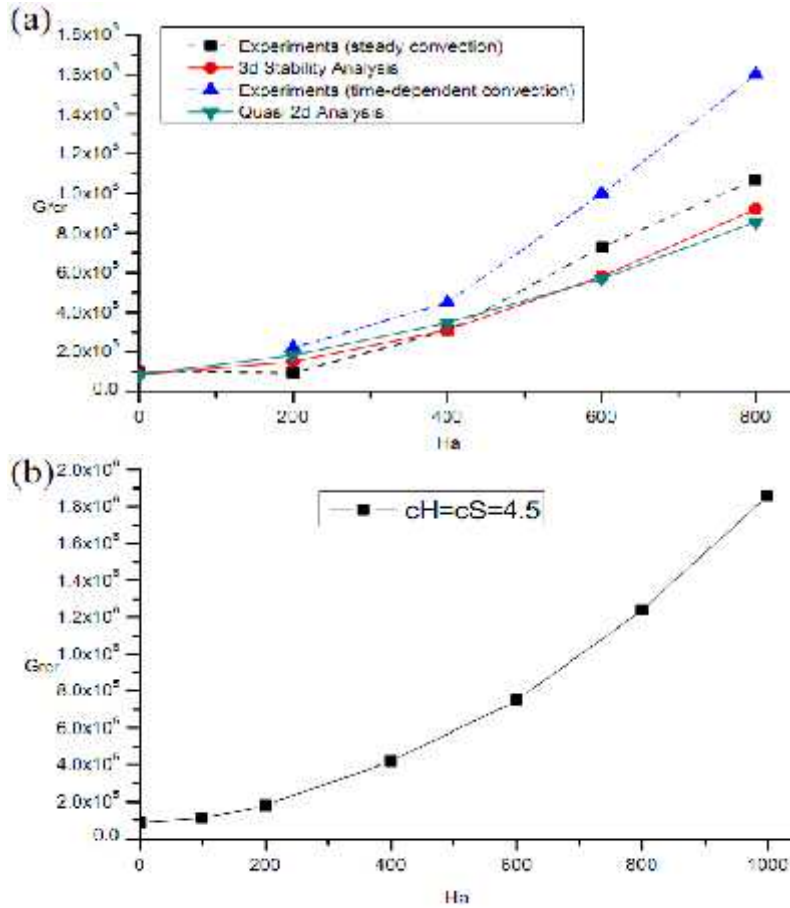
μ U_0 N ...s gUT / θ † B_0^2 : . ,

μ μ \bar{u}, w, \bar{j}

(4.2) :

\bar{U} N Ha \bar{u} , W N Ha w , \bar{J} N Ha \bar{j} (4.19)

μ & μ



Hartmann & Muller, 2002).

$Gr_{Cr} \sim Ha^2$.

$T(x=A/2, y=0.5)=1$.

Hartmann, $0 < Ha < 1000$,

$k_{Cr} \approx 3$,

$c_r = 2 / k_{Cr} \approx 2 \rightarrow c_r \approx 2h$.

Lorentz

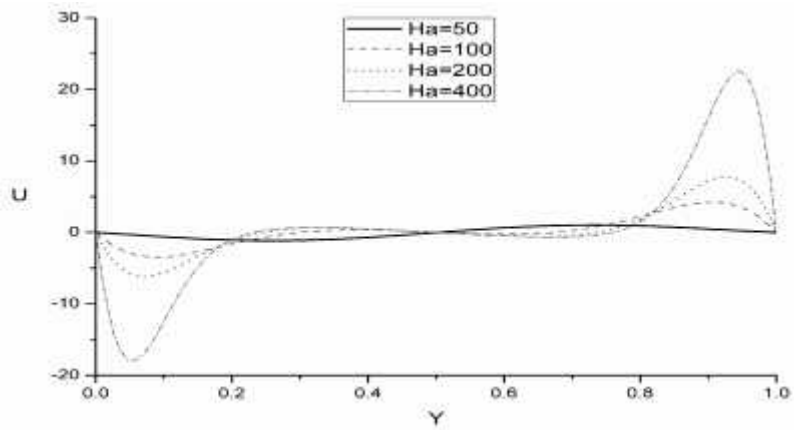
μ & μ
 $\mu\mu$ μ μ μ μ x μ μ
 μ μ Hartmann.
 μ
 μ μ μ μ μ Ha
 μ μ μ μ μ μ μ μ
 . μ μ Burr & Muller
 (2002) μ μ
 μ μ Ha , μ
 . μ
 μ μ μ
 $\mu\mu$ μ μ $\mu\mu$ μ
 μ Ha. μ μ μ μ
 (Davidson 2001),
 μ μ Gr_{Cr} μ μ μ μ Ha,
 Rayleigh-Bénard μ ,
 μ Hartmann .

HARTMANN

HARTMANN

Ha	Gr_{Cr}	k_{Cr}	Ha	Gr_{Cr}	k_{Cr}
0	42 900	0	0	42 900	0
100	53 000	2.7	100	56 000	3.0
200	75 000	3.1	200	90 000	3.4
400	155 000	3.3	400	210 000	4.0
600	290 000	3.4	600	375 000	5.0

μ & μ



μ 29 : μ x
 μ Ha. μ x = 0.95.

μ μ μ μ μ μ 30
 μ x μ Ha

μ 30a,b,c,d μ
 μ (Ha) x
 μ Ha=400. μ

μ μ , x=10, μ ,
 μ , y z μ

Hartmann x μ μ μ
 μ μ , μ .

μ μ Hartmann μ 30e,f,g
 μ μ

μ Hartmann. μ μ
Hartmann

μ , μ .
 μ 30h μ x

Ha=400,

μ Gr μ μ .
 μ μ μ μ

μ & μ

$\ell N 2f / k_{cr} \bar{O} 1.6$

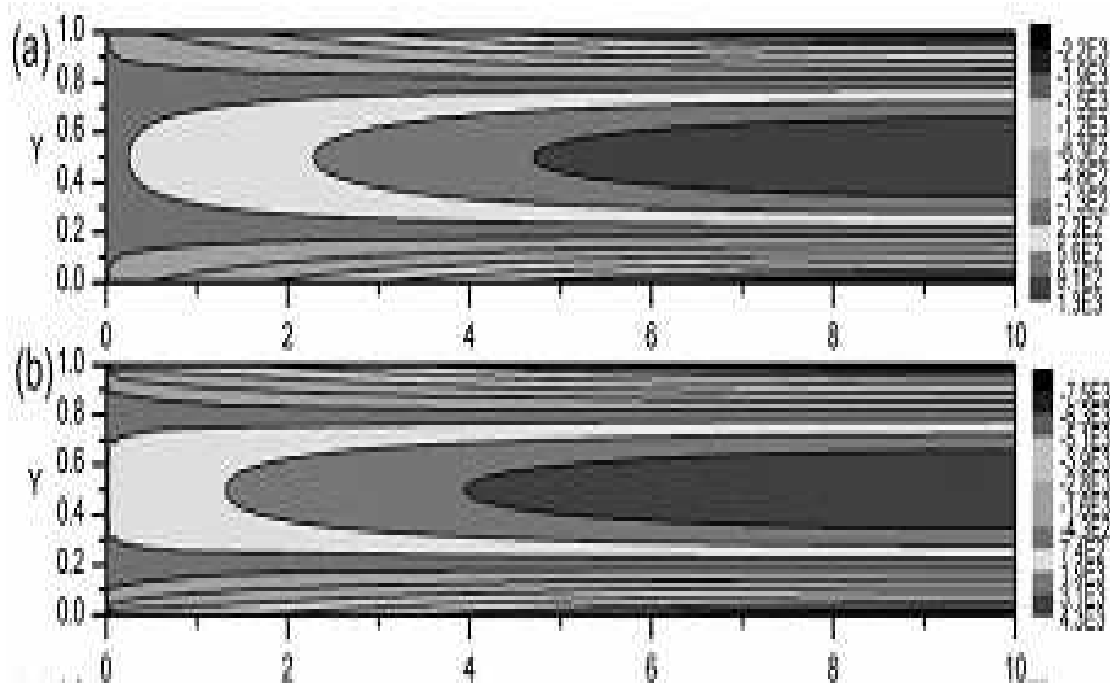
μ - μ μ μ μ
 μ . μ μ μ

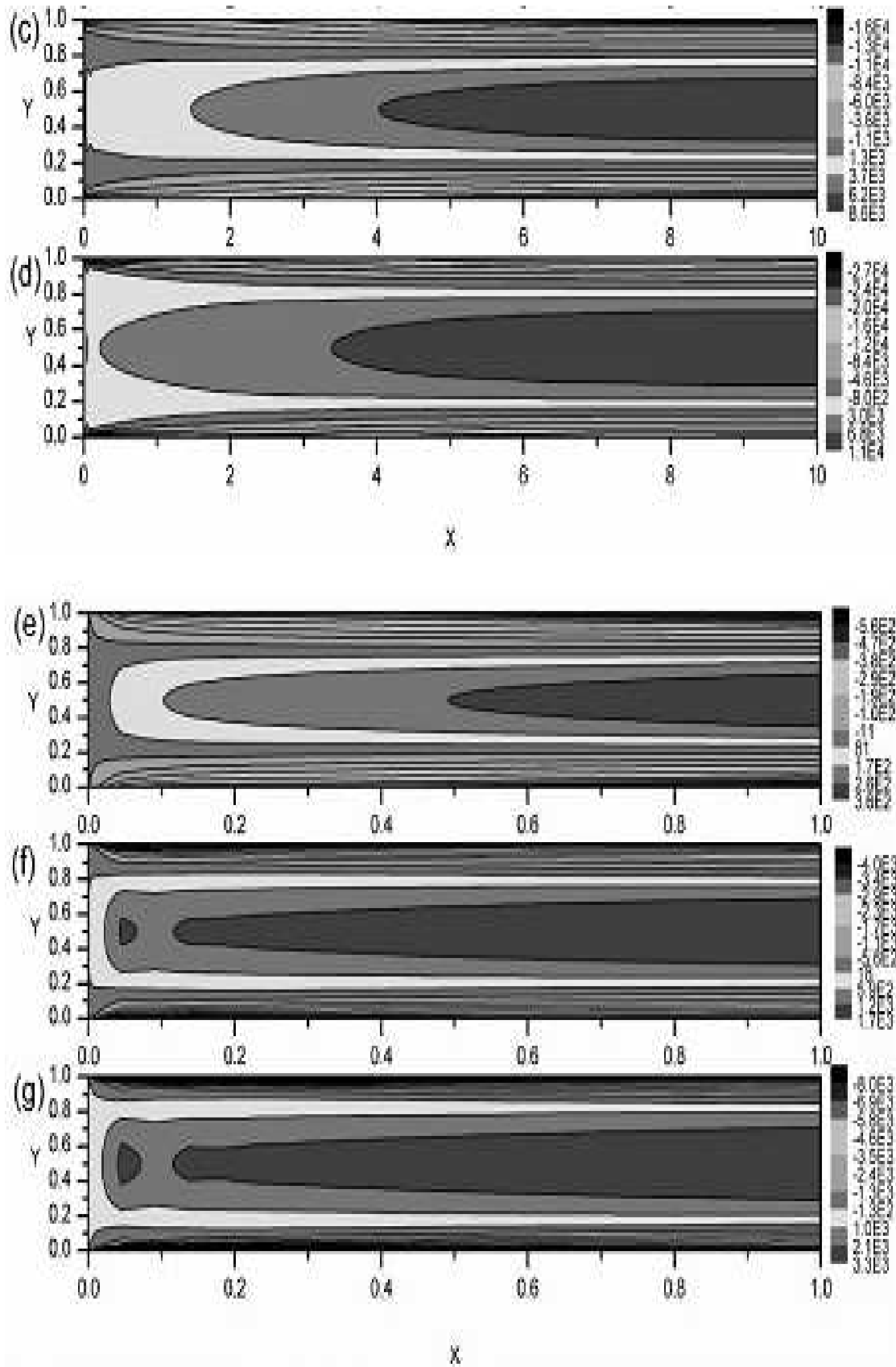
μ 30h

μ μ

μ .

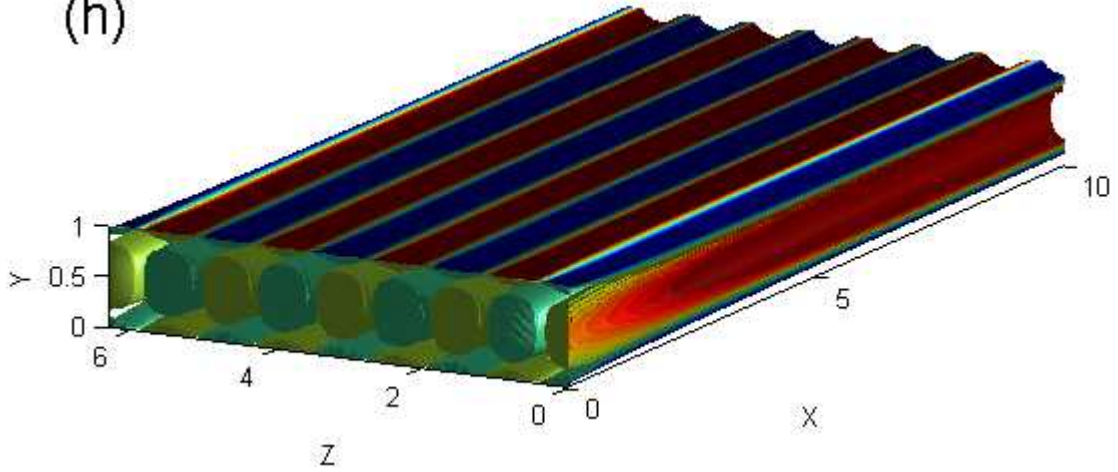
μ μ





μ & μ

(h)



$\mu = 30$: x Ha , (a) $Ha=25$, (b) $Ha=100$, (c) $Ha=200$, (d) $Ha=400$. x
 μ Hartmann, (e) $Ha=25$, (f) $Ha=100$, (g) $Ha=200$ (h) x $Ha=400$.

μ μ μ , μ
 μ ,
 μ Hartmann. ,
 μ Ha . μ 31a,b,c,d
 μ μ μ ,
 μ μ μ ,
 μ Hartmann
 μ .
 μ μ
Hartmann, μ μ Ha
 μ μ (4.16).
 (Ha) μ
Hartmann μ μ μ μ
 μ μ . ,
 (Ha) μ , J_y J_z ,
 μ J_x μ (1), (. (4.10)).
 μ Hartmann μ μ Lorentz,

μ & μ

$$\bar{J} \hat{I} \bar{B} N J_z \bar{e}_z \hat{I} \bar{e}_y,$$

V, μ Hartmann μ

μ (Davidson 2001).

μ 32a,b

(xy z=0)

μ Hartmann (yz x=0).

μ 32a

μ μ

μ

μ μ

Hartmann

Ha=25,

μ μ Hartmann,

(. μ 28b).

μ

μ

, μ

μ μ

Hartmann

μ ,

= (Ha⁻¹),

μ

, $J_{Hx} N \partial W_H / \partial x N Ha \partial W_H / \partial X N O(1)$,

μ

μ μ Hartmann,

Hartmann,

$\partial J_x / \partial x N O(Ha)$,

μ

μ

y , $\partial J_y / \partial y N O^9 Ha$: .

,

μ

μ Hartmann ,

,

μ 32a,

μ

μ xy .

μ 32b

μ μ

μ

μ Hartmann

μ

μ

μ

μ

μ ,

Ha=800

μ

μ μ $k_{cr}=4$

μ

μ

$\ell N 2f / k \bar{O} 1.6$.

μ

μ

,

μ J_y

J_z

μ

μ

Hartmann

μ

μ

μ ,

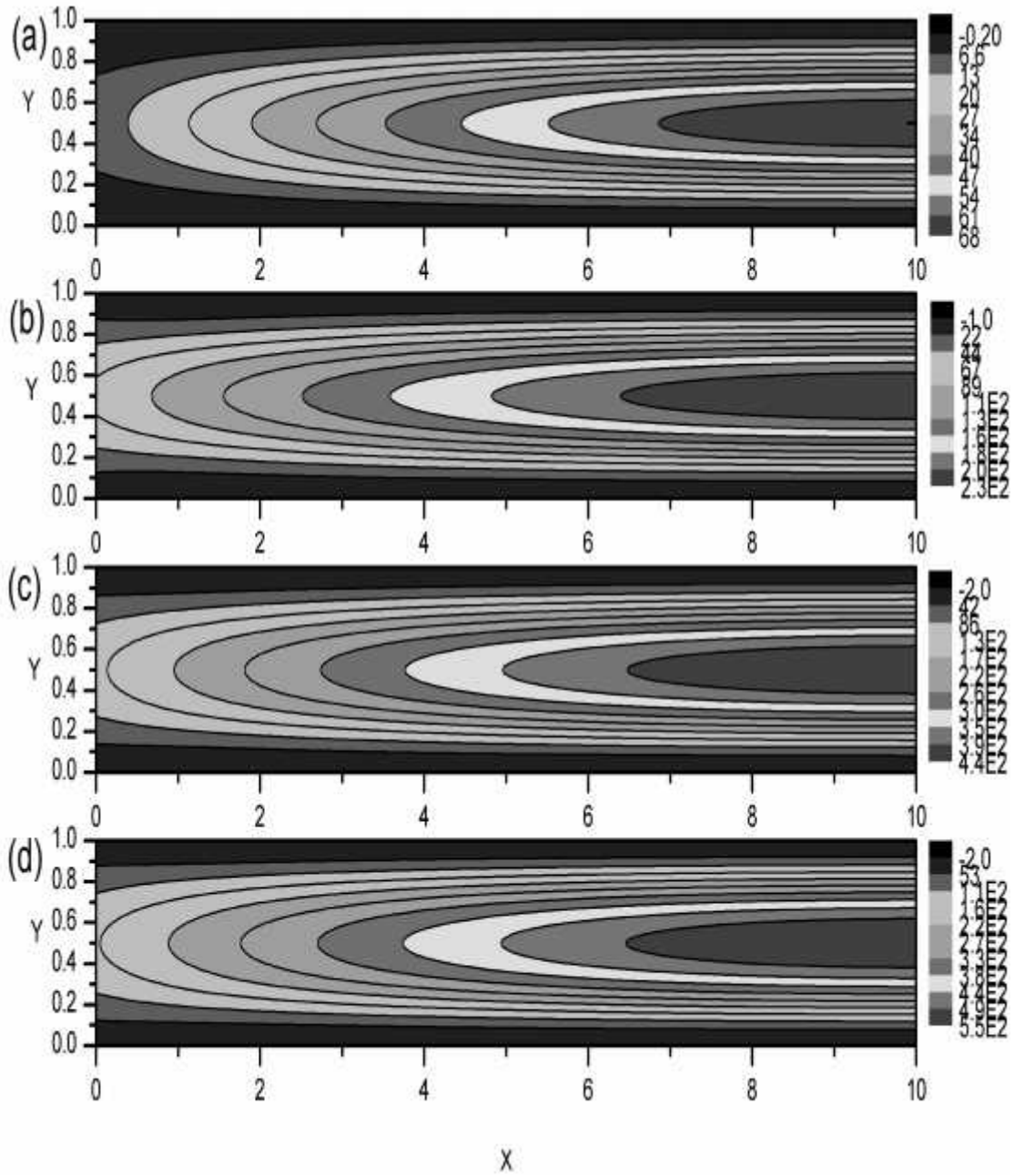
μ

μ

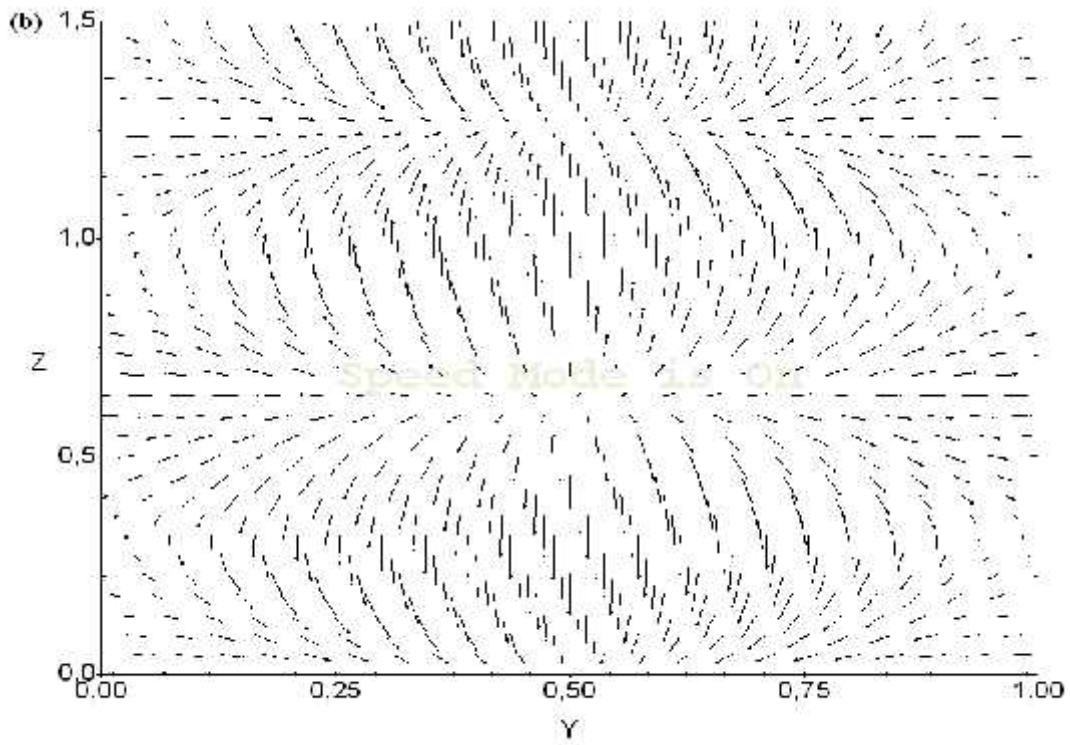
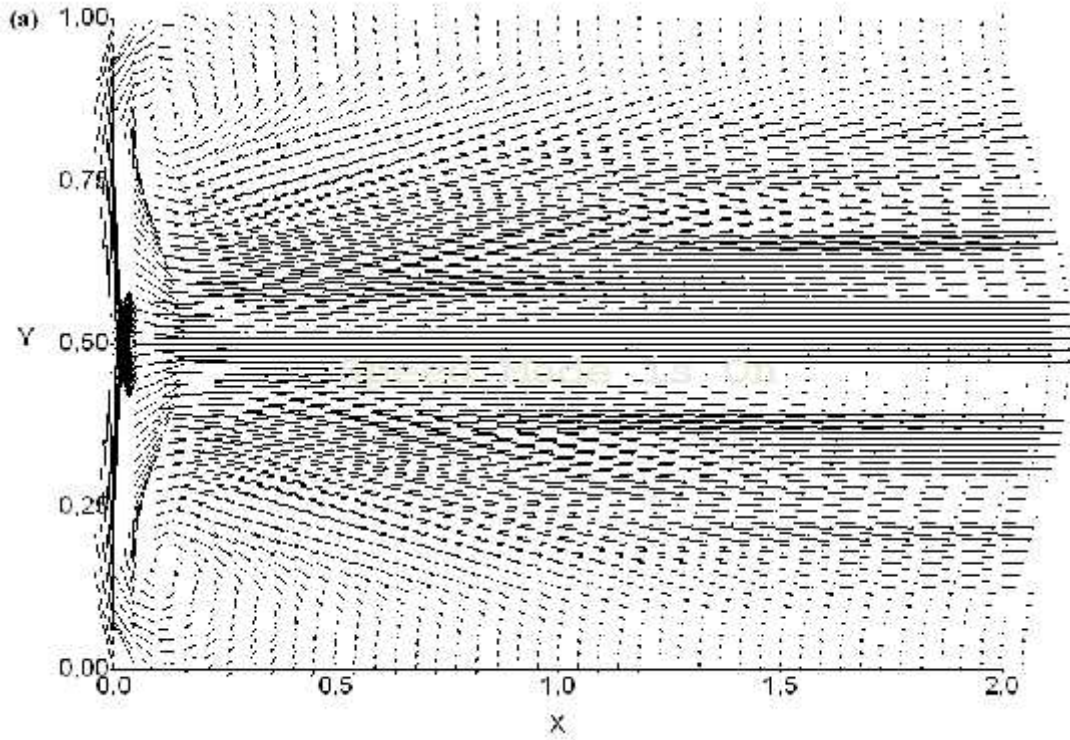
μ

Hartmann

μ & μ



μ 31 : μ Ha
 μ $c_H=0.00415, c_S=4.5, (a) Ha=25, (b) Ha=100, (c)$
 $Ha=200 (d) Ha=400.$



μ 32 : μ μ (a) μ
 (J_x, J_y) xy $z=0$ (Ha=25) (b) μ (J_y, J_z) μ
 Hartmann $x=0$ (Ha=800).

μ & μ

μ μ

Hartmann

μ
 $\mu\mu$ μ

$$, h_{yz} \approx \sqrt{h_y^2 + h_z^2},$$

μ μ , μ 33a

μ μ $Ha=100.$

Hartmann

μ μ

μ , $\partial V_H / \partial x = O(Ha^2),$

μ Hartmann. μ 33b,c,d,e

μ

Hartmann

μ

μ Hartmann

μ μ

Hartmann

μ μ

μ

$$yz \sim O(Ha).$$

Hartmann

$$O(Ha^2)$$

yz

μ μ μ ,

μ 25 Ha 200. μ 33b,c,d,e

μ , Ha ,

μ μ μ

μ

μ

μ

μ

μ μ . ,

$$\frac{\partial V_H}{\partial y}$$

Hartmann

$$\frac{\partial U}{\partial x}$$

z

$\partial P / \partial x \approx O(1):$

μ

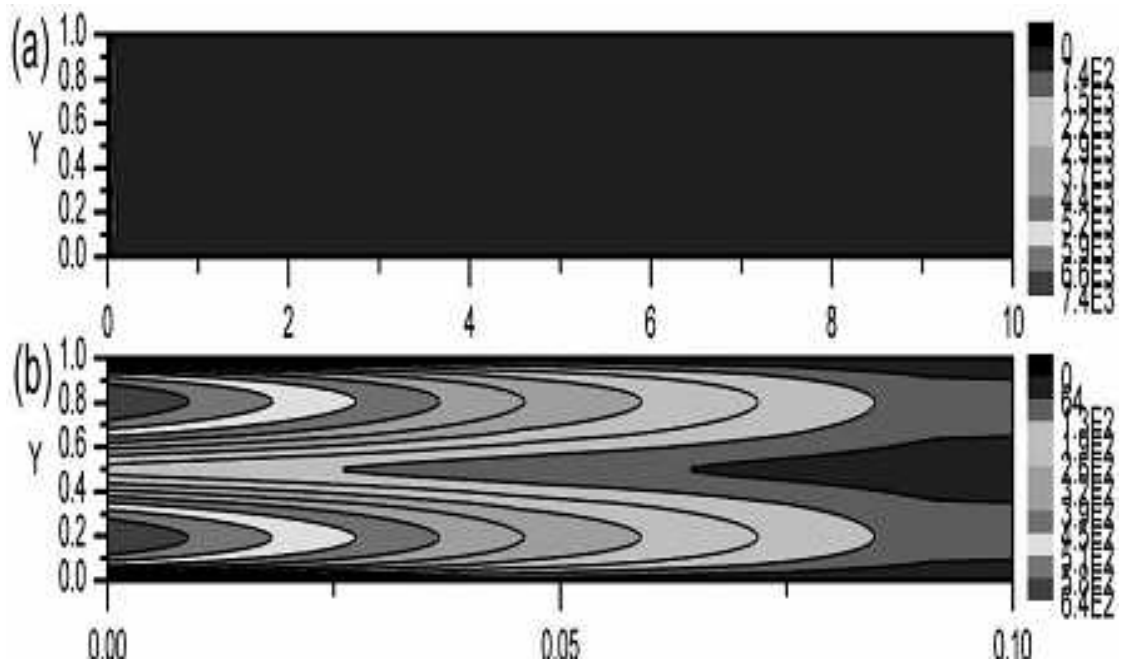
μ

x

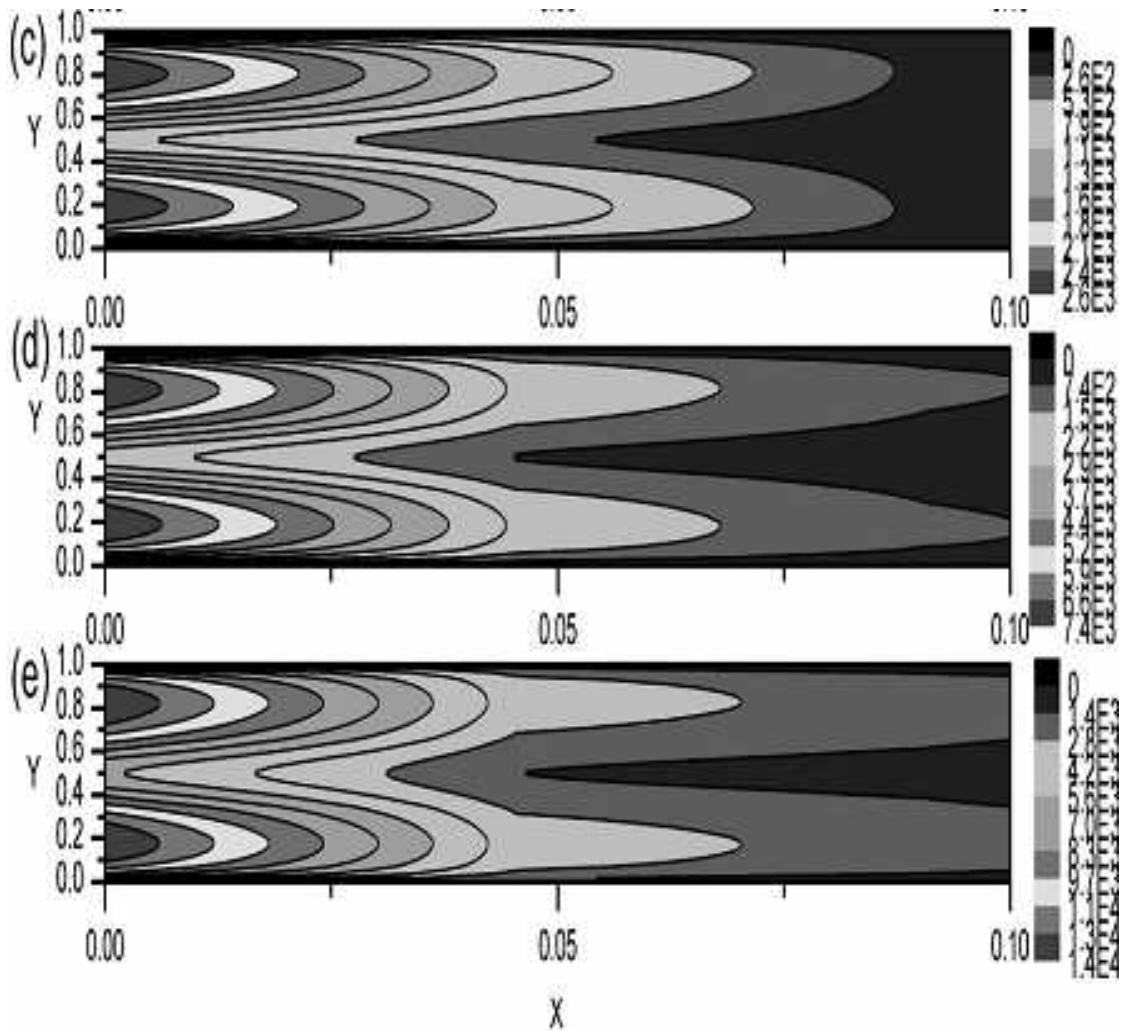
μ $\partial P / \partial x \ll 1$,

μ y ,

μ & μ
 , yz, μ μ
 μ Hartmann μ
 , μ
 μ μ μ μ μ ,
 μ
 Hartmann,
 , μ , μ μ
 ,
 μ μ μ μ Ha (μ 33b,c,d,e)
 μ .
 , μ μ
 μ Buhler (1998)
 μ μ μ μ μ
 $\mu\mu$. μ μ
 μ μ μ
 Davoust et al. (1999)
 μ μ μ .



μ & μ

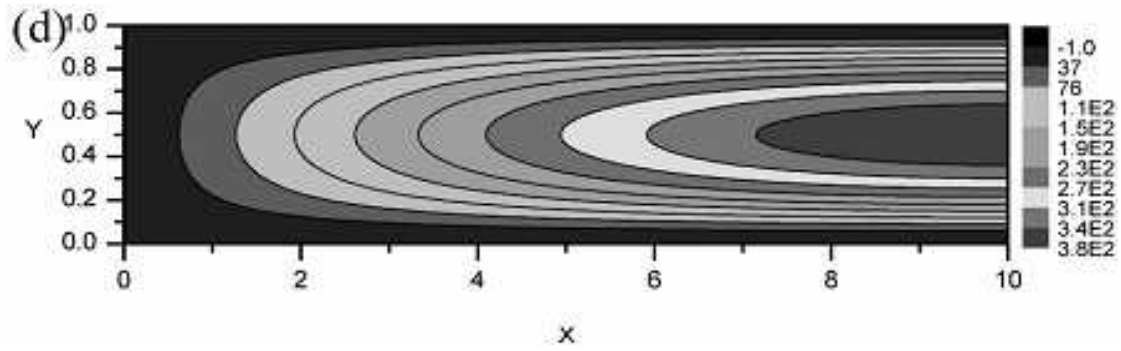


μ 33 : yz (a) xy $Ha=100,$
 μ (b) $Ha=25,$ (c) $Ha=50,$ (d)
 $Ha=100$ (e) $Ha=200.$

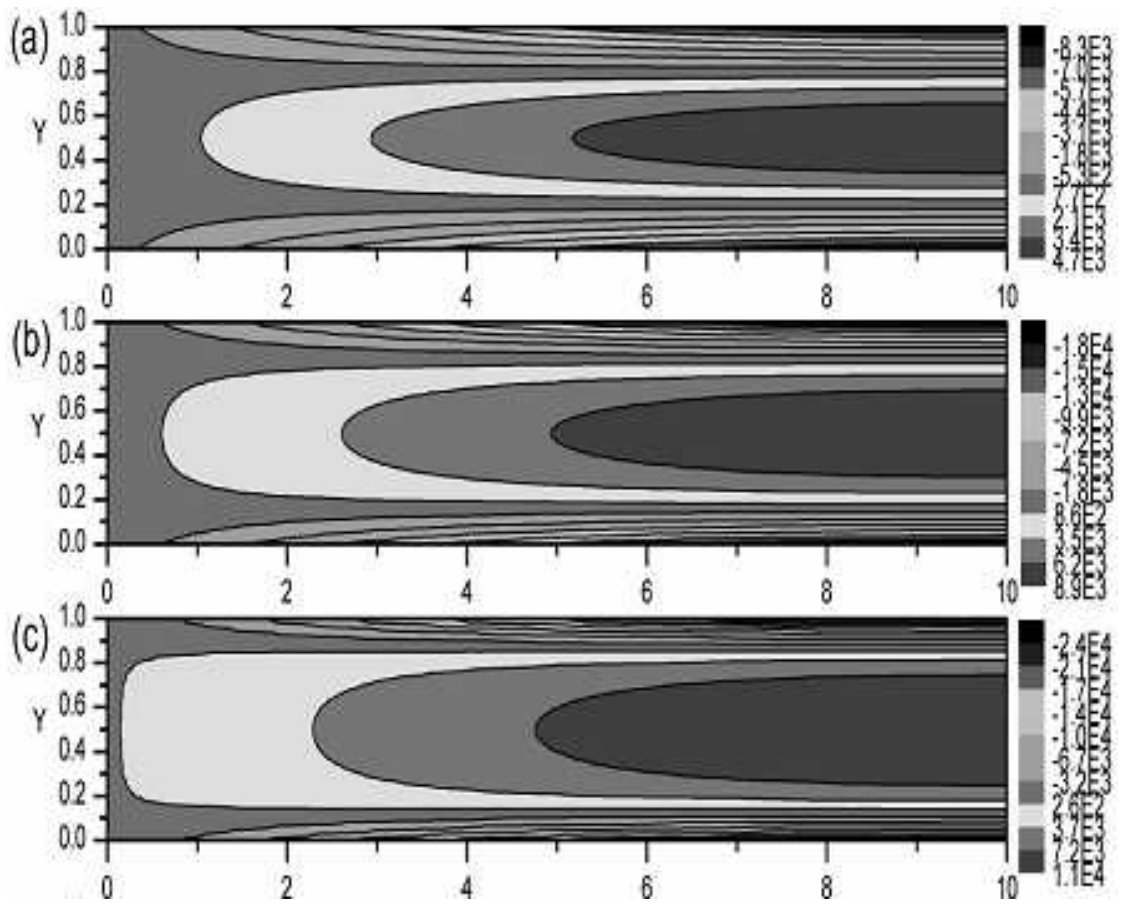
μ μ μ Hartmann μ
 $c_S=c_H,$ μ μ μ
 μ μ $Gr_{Cr} \sim Ha^2$ μ
 μ , μ μ
Hartmann μ μ Ha
 μ μ μ
 μ μ $Ha=2000.$ μ
 μ μ Hartmann, x
 V μ μ (1)
Hartmann, (. (4.17) (4.18a)). μ

μ & μ
 μ μ μ ,
 μ μ . , μ
 μ μ μ Hartmann
 μ μ .
 μ μ μ μ ,
 μ μ μ y-
 μ μ Hartmann.
 μ μ
 μ μ μ μ
Hartmann. μ μ
 μ μ μ
. μ 34a,b,c,d μ
 μ μ Ha μ μ
 μ μ Hartmann
. x μ
35a,b,c,d x μ (1) μ μ μ
Hartmann μ μ Ha
 μ . μ 35a,b,c,d
 μ μ μ
 μ , μ μ μ μ
 μ μ . μ x
 μ 30h, μ μ 35e μ μ
 μ μ μ .
 μ 33, 34 x
 μ μ μ μ
 μ Hartmann. yz x
, μ . μ ,
 μ yz
 μ Ha
 μ 36a,b,c,d μ Hartmann μ
Hartmann $O(Ha^{-1})$ μ .

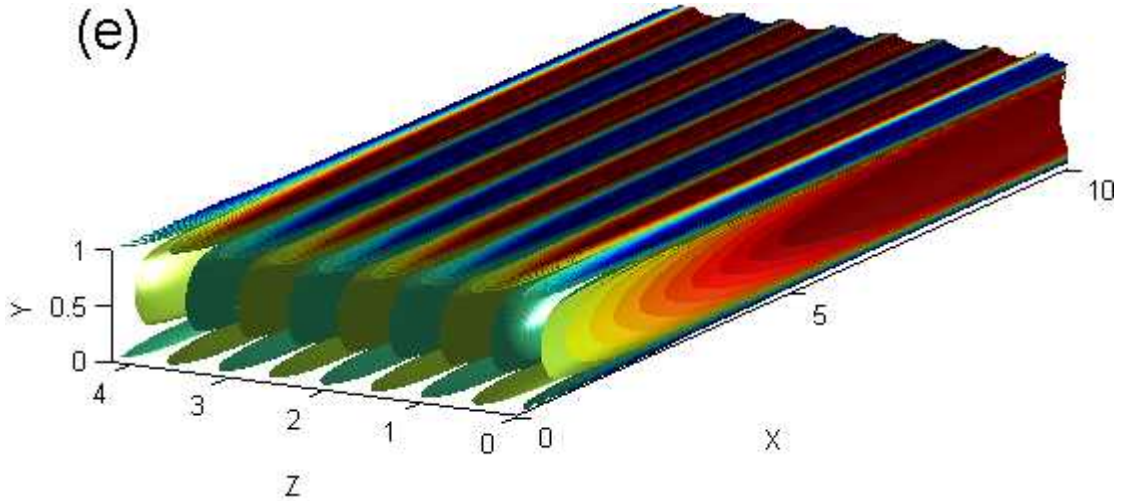
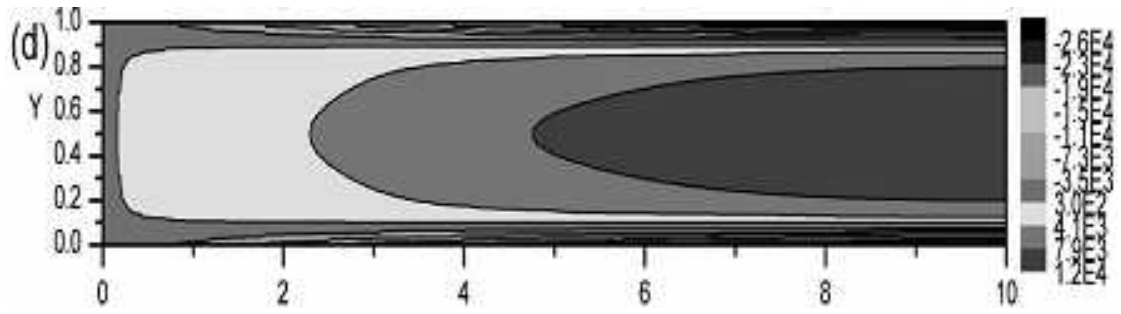
μ & μ



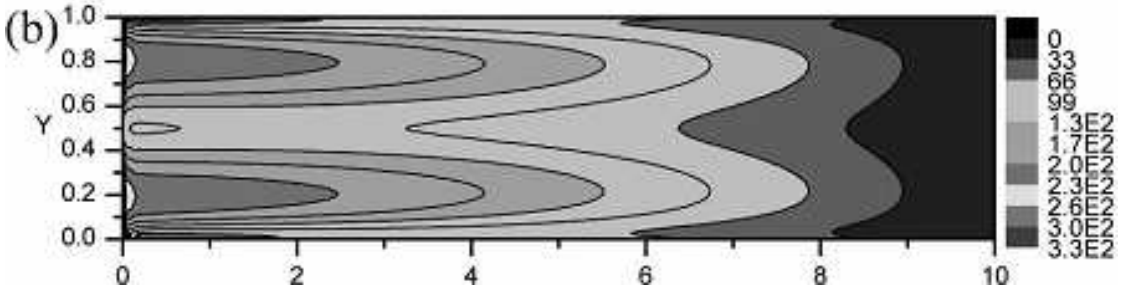
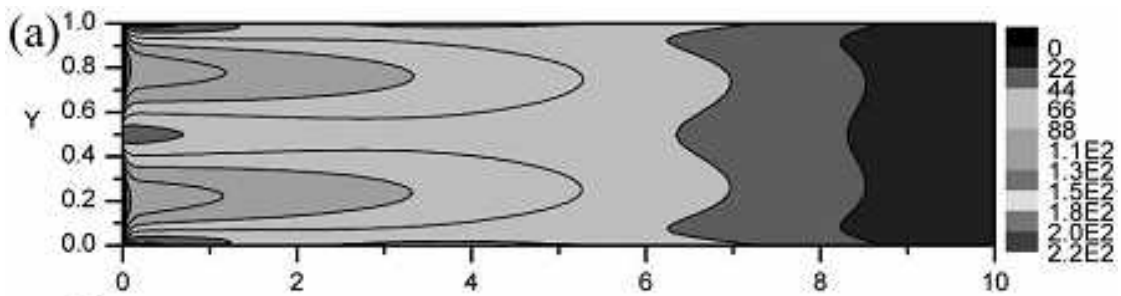
μ 34 : μ μ μ
 $c_H=c_S=4.5$ Ha , (a) Ha=100, (b) Ha=400, (c) Ha=800 (d)
 Ha=2000.



μ & μ



μ 35 : μ $c_H=c_S=4.5$
 Ha , (a) Ha=100, (b) Ha=400, (c) Ha=800 (d) Ha=2000. (e)
 x Ha=2000.



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xy , (μ 37b),

$$\tilde{S}_z = \partial v / \partial x - \partial u / \partial y,$$

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, (Leweke & Williamson 1998).

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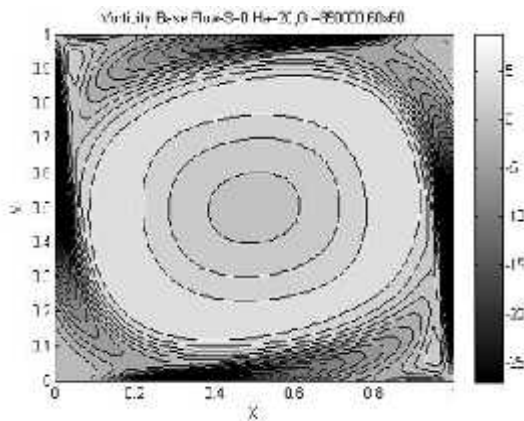
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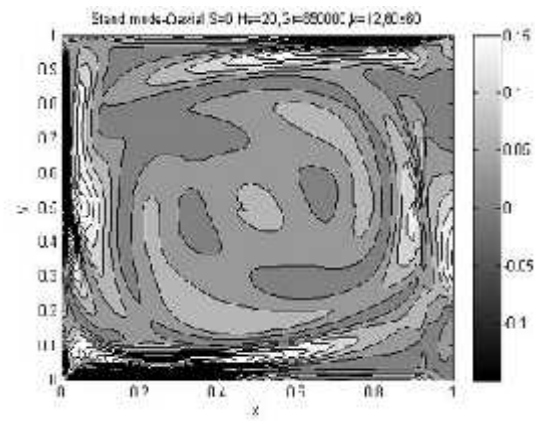
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μ 37 : (a)



(b)

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$Gr=Gr_{Cr}=850000, S=0 \quad Ha=20.$

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μ μ Goertler.

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(μ 38a) μ μ μ Gr μ

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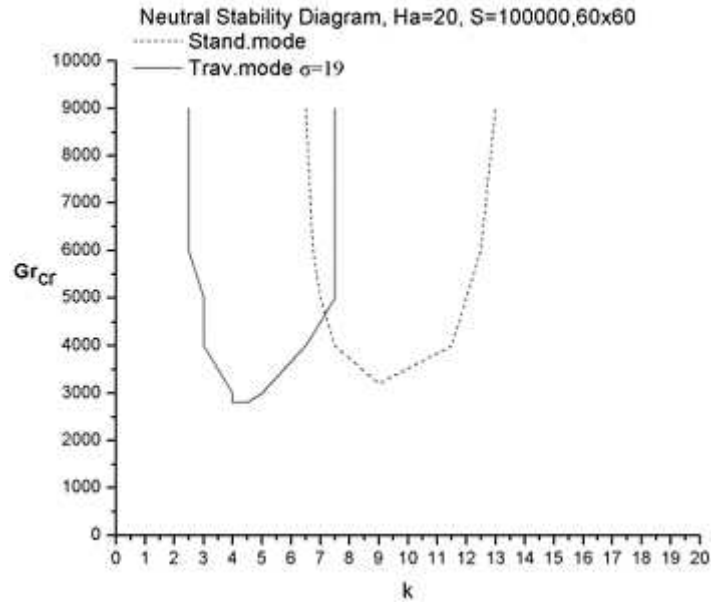
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Landman & Saffman (1987)

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$$E = 64f^3 / \check{S} / Gr^{1/2} . \mu (,$$

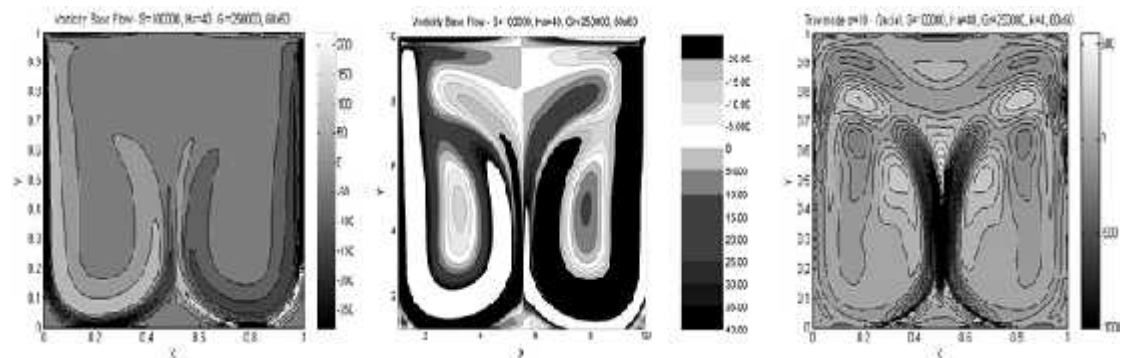
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Landman & Saffman (1987)

μ $5000 < S \leq 10^5$ μ Ha=0

$0 \leq Ha < 40$ μ $S=10^5$.



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4.2.1 μ μ S Ha=20

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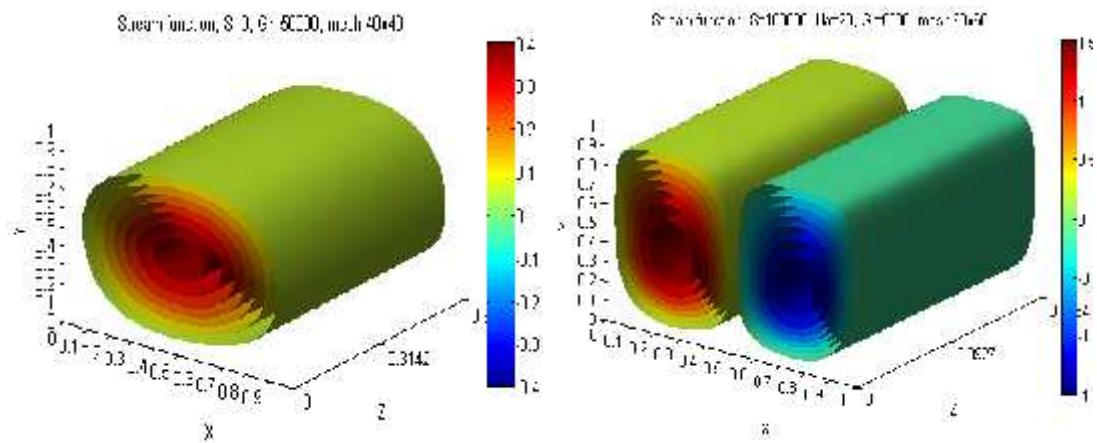
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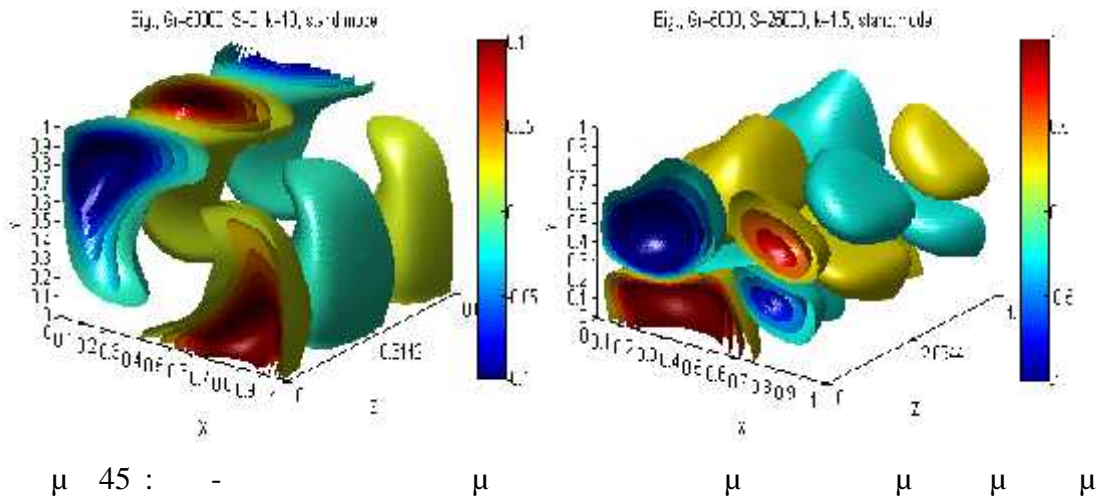
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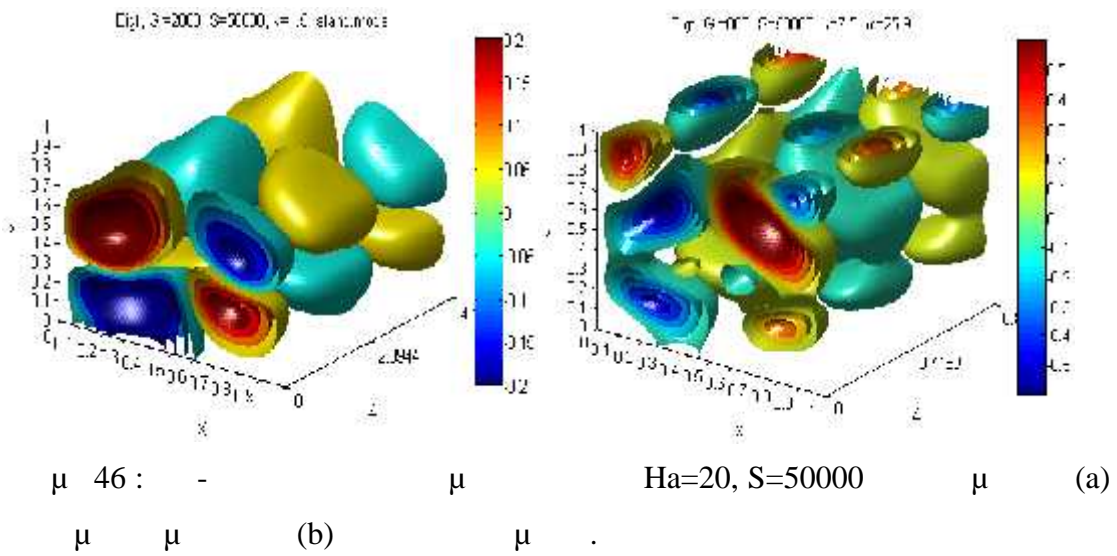
μ 44 :

Ha=20, (a) S=0 (b) S=10⁵.

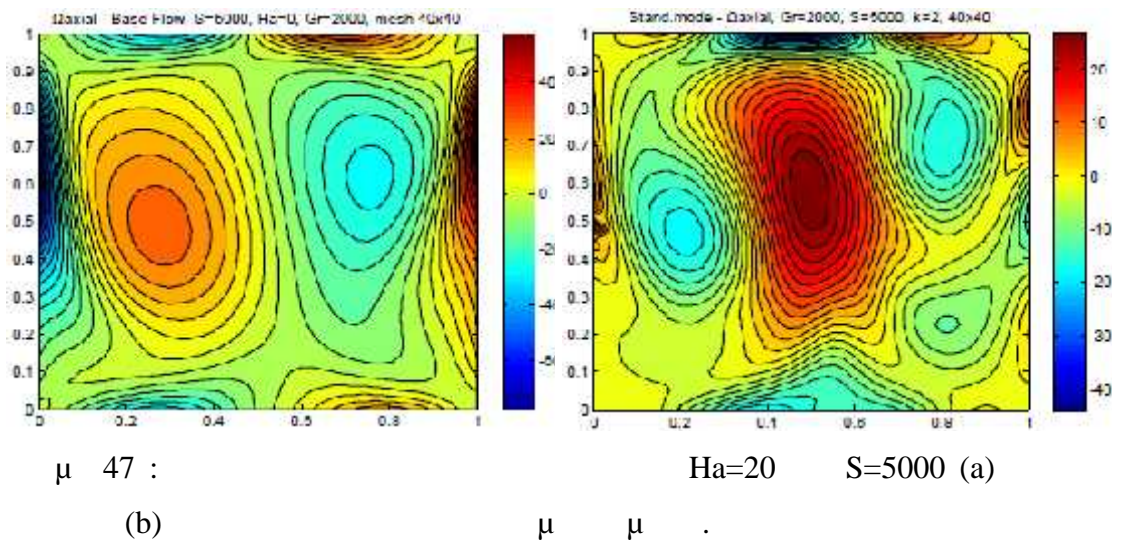
μ & μ



Ha=20, (a) S=0 (b) S=25000.

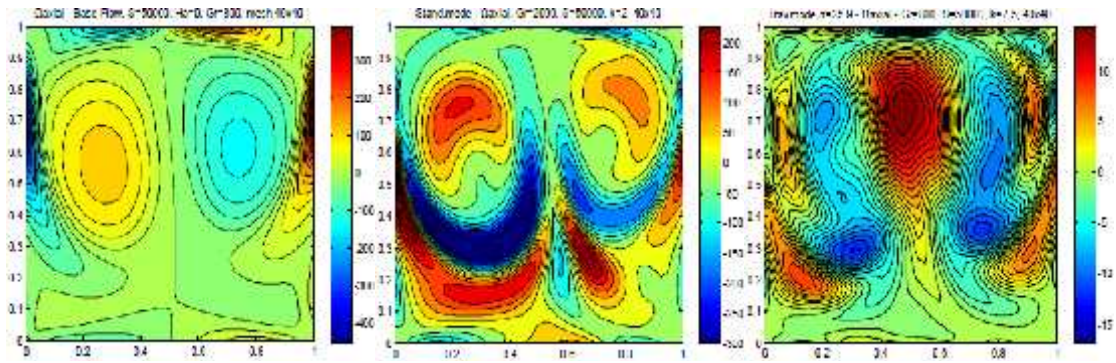


Ha=20, S=50000 (a) (b)



Ha=20 S=5000 (a) (b)

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μ 48 : Ha=20 S=50000 (a)
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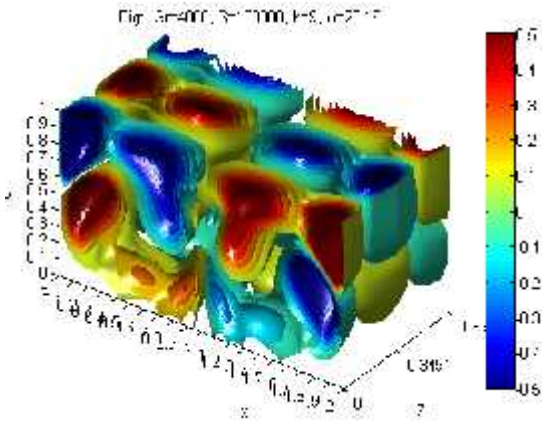
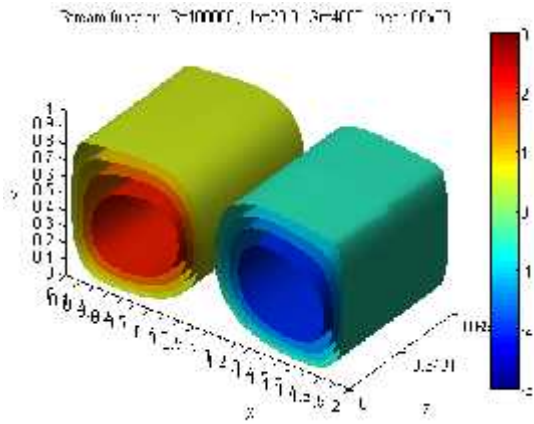
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S=0, μ

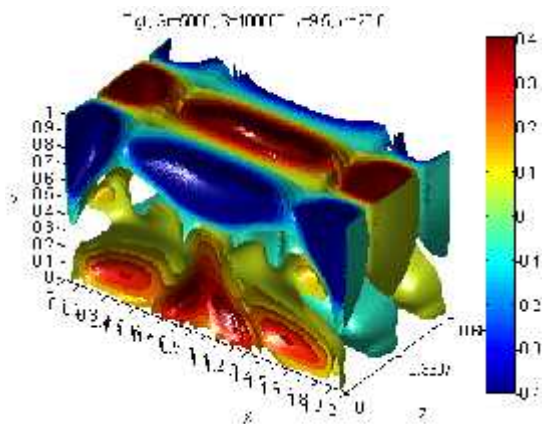
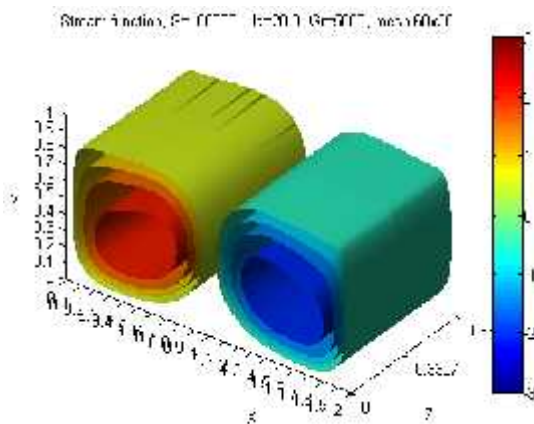
Goertler.

μ & μ

$\mu = 51$: (a) , (b) -
 $\mu = \mu = \mu = 7.86$,
 $Ha=20.0 \quad S=10^5$.



$\mu = 52$: (a) , (b) -
 $\mu = \mu = \mu = 23.16$,
 $Ha=20.0 \quad S=10^5$.



$\mu = 53$: (a) , (b) -
 $\mu = \mu = \mu = 20.8$,
 $Ha=20.0 \quad S=10^5$.

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