M.Sc. Management of Hydrometeorological Hazards – HYDROHASARDS
Course: Master Thesis
Project: M2R 3rd Semester Master Thesis

PRELEMINARY NON-LINEAR ANALYSIS OF THE RAINFALL STRUCTURE IN A RIVER VALLEY

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January 07, 2013

ABSTRACT

In the present thesis we present a preliminary analysis of rainfall time series of a particular event in the Rhône valley. The analysis concerns both a global statistical analysis of the whole examine area accessed by the radar as well as a detailed local analysis of the rainfall time series at specific cuts inside the Rhône Valley. The local analysis concerns Hurst exponent and Hjorth Mobility and Complexity. These results are examined as a function of the ground elevation and several characteristic regions of behavior are extracted. It seems that the Hurst and Hjorth parameters are quite sensible in the variation of the elevation and related to the system complexity.

RESUME

Dans cette thèse, nous présentons une analyse préliminaire des séries pluviométriques temps d'un événement particulier dans la vallée du Rhône. L'analyse porte à la fois une analyse statistique globale de l'ensemble examiner zone accédée par le radar ainsi que d'une analyse locale détaillée de la série temporelle des précipitations à des réductions spécifiques à l'intérieur de la vallée du Rhône. L'analyse locale concerne l'analyse par l'exposent de Hurst et Hjorth Mobility et Complexity. Ces résultats sont examinés en fonction de l'élévation du sol et plusieurs régions de comportement caractéristiques sont extraites. Il semble que les paramètres de Hurst et Hjorth sont tout à fait sensibles à la variation de l'altitude et qu'elles sont reliées à la complexité du système.

ΠΕΡΙΛΗΨΗ

Η παρούσα Διπλωματική εργασία πραγματεύεται την προκαταρκτική ανάλυση χρονο-σειρών βροχόπτωσης από ένα συγκεκριμένο γεγονός στην κοιλάδας του Rhône. Η ανάλυση αφορά τόσο μια ολική στατιστική ανάλυση ολόκληρης της περιοχής εντός την ακτίνας δράσης του ραντάρ, καθώς και μια λεπτομερή ανάλυση τοπικών χρονο-σειρών βροχόπτωσης σε συγκεκριμένες κατά μήκος τομές μέσα στην κοιλάδα Rhône. Η τοπική ανάλυση αφορά τον συντελεστή Hurst και τις παραμέτρους Hjorth κινητικότητα και πολυπλοκότητα. Αυτά τα αποτελέσματα εξετάζονται ως συνάρτηση του υψομέτρου του εδάφους και παρουσιάζεται η συμπεριφορά των συντελεστών σε χαρακτηριστικές περιοχές. Διαπιστώνουμε ότι οι παράμετροι Hurst και Hjorth είναι αρκετά ευαίσθητοι στις μεταβολές του υψομέτρου και σχετίζονται με την πολυπλοκότητα του συστήματος.

OUTLINE

1. AIM AND SCOPE OF PRESENT RESEARCH

It has been shown in literature that dynamical and adiabatic processes involved in rainfall production could be influenced by mountain ranges. A statistical analysis of rainfall radar data is implemented to analyse the persistent and chaotic character of rainfall over Rhône river valley.

In a first place a global analysis is employed of the whole examine area while in the second place we present a detailed local analysis based on Hurst exponent or Rescaled Range Analysis [see Mandelbort 1982, Papaioanou & Karytinos 1995, Clegg 2006] exponent and Hjorth parameters [Hjorth 1970], tools that are employed in the nonlinear analysis of system dynamics [see for example Vlachos and Kugiumtzis 2010] and which are more sensible to the temporal evolution and dynamics of the system than the usual statistical measures.

We must mention here that the Hurst exponent analysis has been employed in several studies of rainfall temporal analysis on a global scale [see for example Miranda and Andrade 1999, Peters et al. 2001, Bove et al. 2006] and not in a detailed local scale as it is the case in the frame of the present Master Thesis. Our aim was to use notions from the study of other dynamical systems and investigate if they can lead to information about the rainfall structure and the interaction of ground elevation with the cloud dynamics.

2. DESCRIPTION OF THE EXAMINE AREA AND THE EVENT

The examine area is located in southern France, see Fig.1. It is an area that particularly in the autumn experiences prolonged rain events that are capable of producing catastrophic floods. It includes two mountain regions; Massif Central and Alps foothill. Rhone valley is between them and below is Nimes plain area; around Nimes city till the Mediterranean Sea line see Fig.2.

Figure 1. The light blue square indicated the examine area.

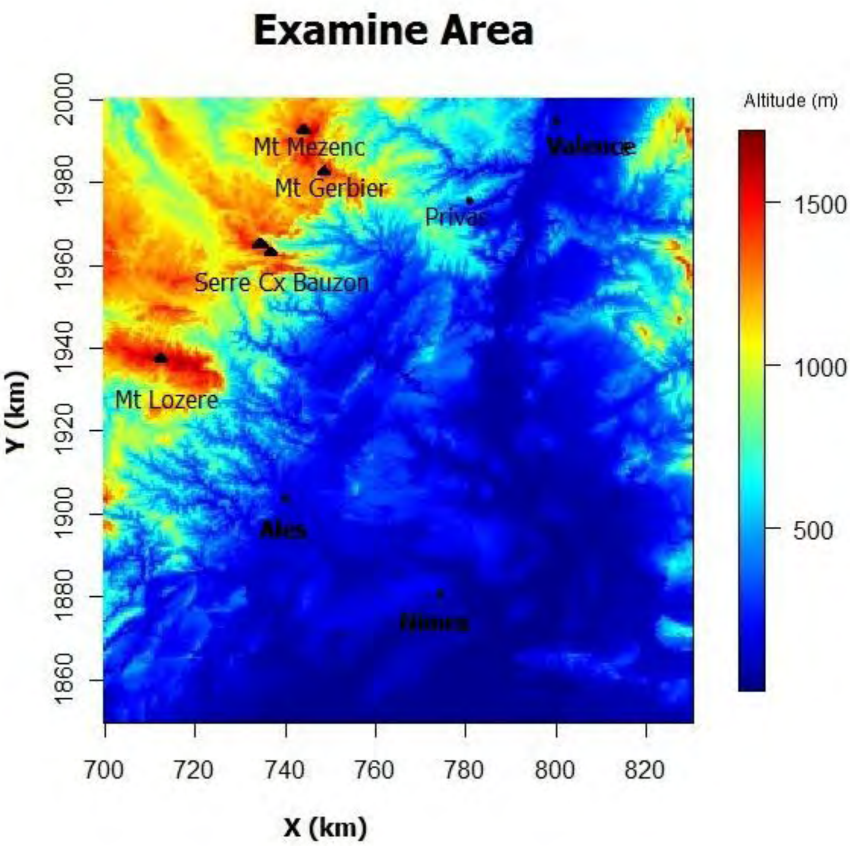

Figure 2. The examine area with its relief.

We choose a rainfall event scanned by Nimes Radar during Fall 2011. The data concern a 24 hour event on 06 November 2011 from 00:00 to 23:45 (hours). The sampling time is 15 min. The direction of clouds is from North to South. In Fig.3 we present some snapshots of the radar images. The event above all the examine area has strong rainfall from the begging at 00:00 hours till 04:00 hours, it continuous with less stronger than before but still intense rainfall from 04:15 hours till 19:00 hours and from 19:15 hours till 23:45 hours we have almost no rainfall.

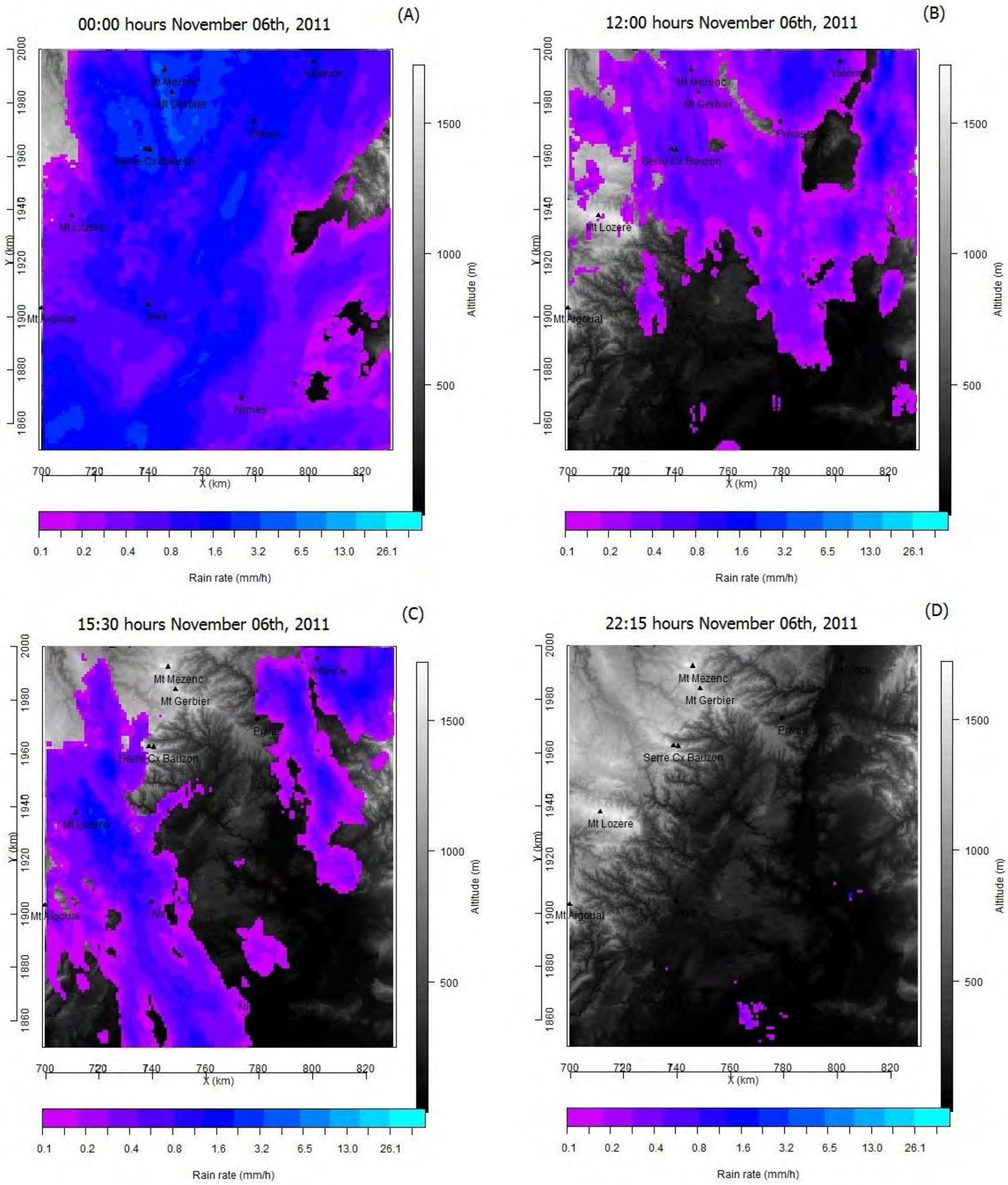

Figure.3 Snapshots from the examine event at November 06th, 2011 at (A) 00:00 hours, (B) 12:00 hours, (C) 15:30 hours and (D) 23:45 hours.

In a first stage we performed a statistical analysis over the whole examine area for all the recordings available. The obtained the statistical data concerning standard deviation, mean, skewness, kurtosis, standard deviation divided by mean, max values as well as the intermittency are presented in Fig.4. We draw the attention to the fact that the intermittency presented here is number of the number zeros (no rain) during the event and is defined as the number of pixels of the radar image where the value is zero divided by the total number of pixels of the image. All statistical data were calculated from rainfall data scanned by Nimes Radar, they are temporal and concern all the examine area. It's a result of working on row radar data (reflectivity) with 'mm' units for rainfall intensity. All parameters computed on positive rainfall intensities not including zeros except of the intermittency calculation where we counted zeros.

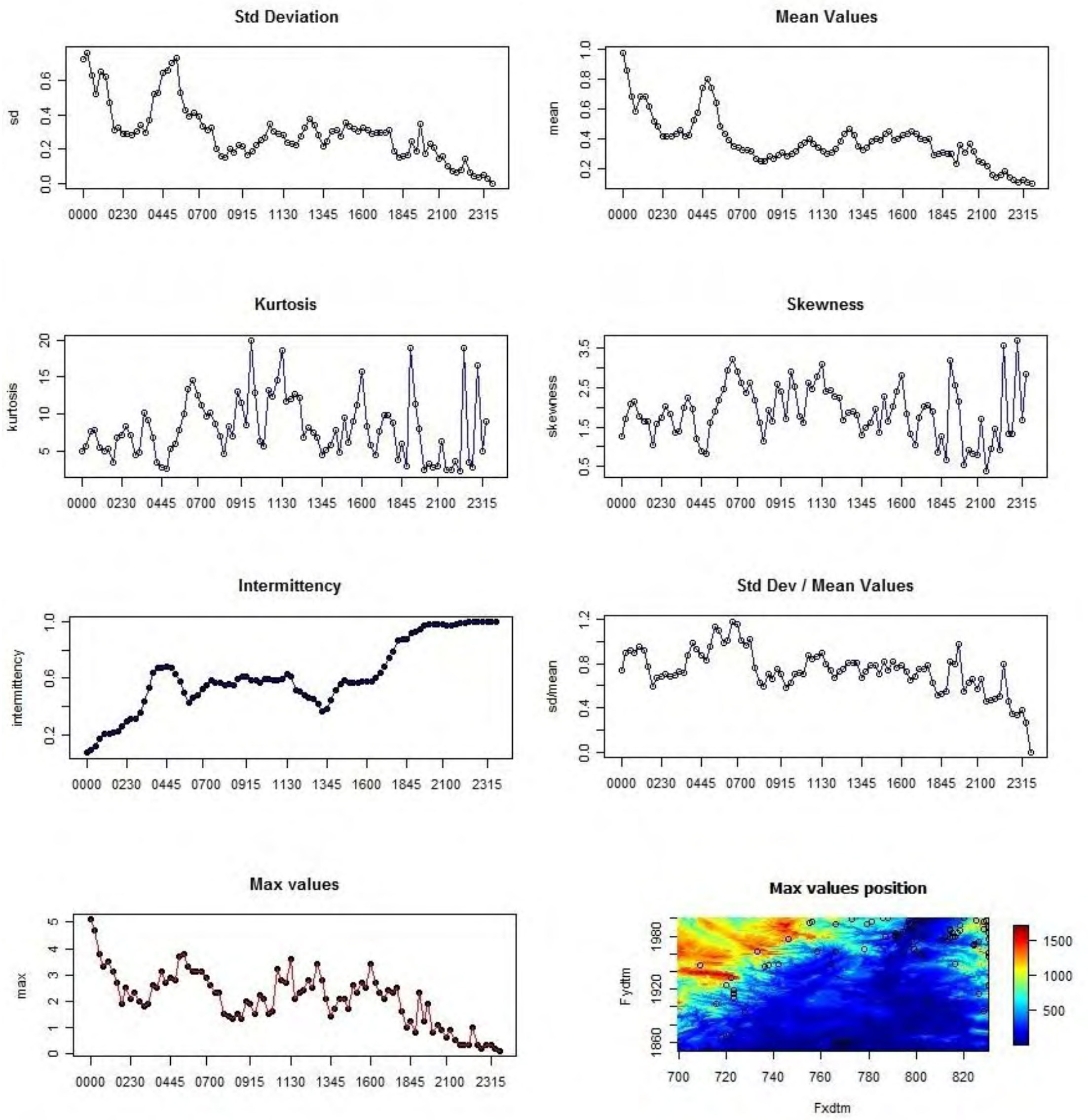

Figure 4. Statistical data from the examine event.

Now for the evolution of the rain, the statistics below let us separate three parts as we can see all diagrams Except of the max values position diagram; the first part is from 00:00 hours till 07:00 hours where we do have big oscillations at every parameter we mentioned, the second part is from 07:00 hours till 18:00 hours where we have small oscillations again at every parameter we mentioned and the third part is from 18:00 hours till 23:45 hours where we have descending diagrams in all mentioned parameters, except of the Intermittency which has ascending diagram and the skewness and kurtosis diagrams where at this third part we see bigger oscillations. We will see after at Section.4 if these characteristics are being confirmed in the cloud we analyse its rainfall structure.

Another thing that we notice on Fig.4 is that in max values positions diagram the max rainfall values are concentrated around the mountains on the North part of the map and many of these maxes inside the river valley on the North before the mountains narrowing. A few maxima can be seen at the bottom of the map where the ground elevation is too small; below 50m from the sea level. Surprisingly we have no max values over the plain with middle ground elevation (below 500m). It is of interest that are maxima located at the north of the Rhône Valley.

In Fig.5 we present the cumulative rainfall where we can see that the maximum of the cumulative rainfall is located in the north region. The rainfall is coming from North to South through the valley and Fig.5 shows that the maximum rain is located near Valence and Privas where the relief is certainly among the lowest. We have to note the great difference in rainfall amount; 25mm is certainly around the 50 percentile of the daily rainfall and 43mm more than the 75^{th} .

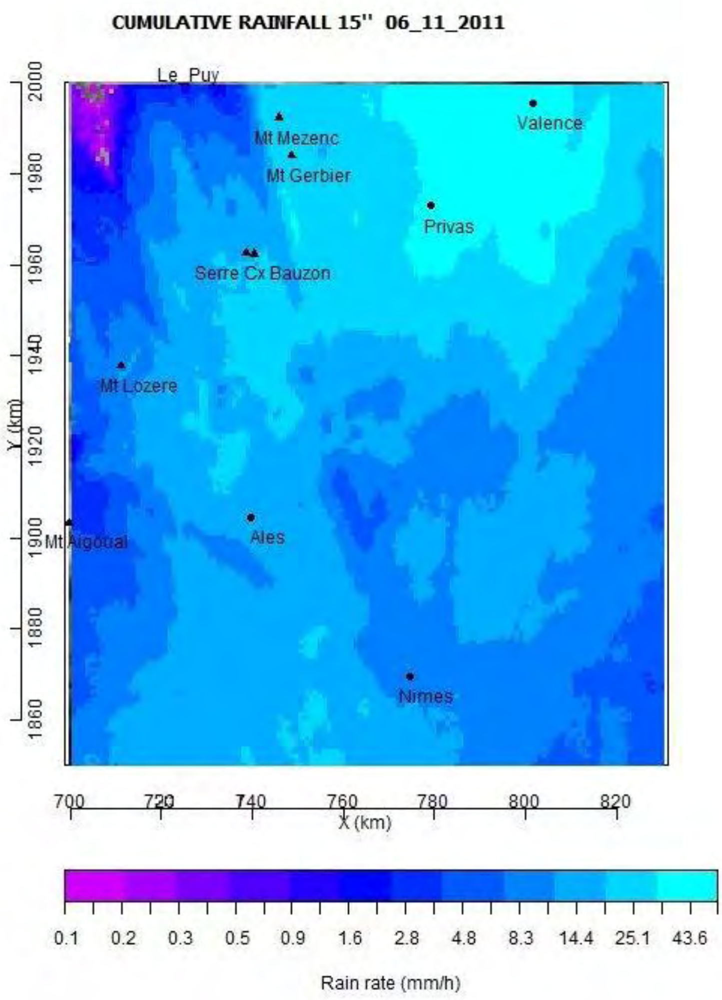

Figure 5. Cumulative rainfall at the examine area on November 06, 2011.

2.1 Local Analysis

In the following we decided to focus our analysis in the Rhône valley part of this region and more specifically to vertical cuts located at X=785km up to 810km as we can see in Fig.6. This choice was based on the fact that inside the valley we expect not to have significant influence from lateral winds.

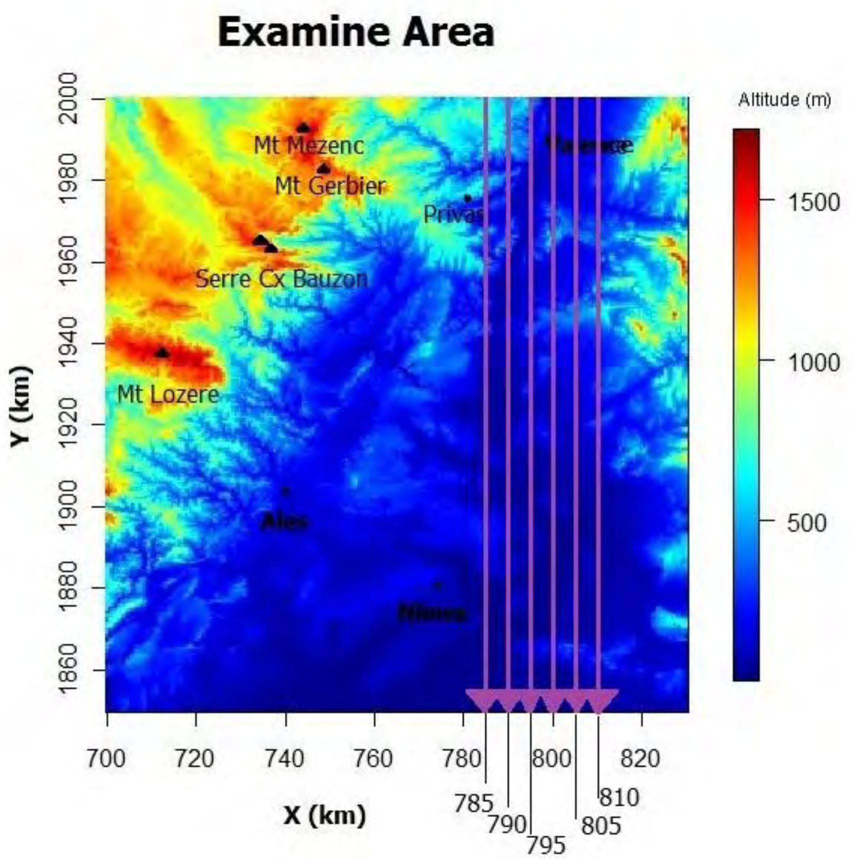

Figure 6. Cross sections along the examine area in Rhône river valley.

In this frame we examined the behaviour of time series at every pixel of the North-South cross section. For this purpose we used for the first time in rainfall structure analysis some tools from chaotic analysis like the Hurst Exponent and tools from other areas employed for the detection of change of behaviour like the Hjorth Mobility and Complexity. In the following section we discuss these methodologies and present some characteristic examples before presenting the application to the real data in Section 3.

3. HURST AND HJORTH THEORY

If we would like to explain in very short lines what are the statistical factors that we choose for analysis we could say that Hurst is a persistence factor, is “memory” of the returns sequence or a measure of long memory dependence. Hurst Exponent’s theory is presented in Section 3.1. For Hjorth Mobility; is showing how fast the variance varies and for Hjorth Complexity; is the mobility of the Mobility. Hjorth Mobility and Complexity theory is presented in Section 3.2. Academic examples about Hurst and Hjorth parameters are presented in Section 3.3.

3. 1. HURST EXPONENT

Hurst Exponent first used for Nile's river flood analysis. Its specific definition is; Long memory dependence or persistence in time series or spatial data is associated with power-law correlations and often referred to as Hurst effect [*Gneiting et al.*, 2004].

About Hurst *Clegg* [2006] mentions that while the Hurst parameter is perfectly well-defined mathematically, measuring it is problematic. The data must be measured at high lags/long frequencies where fewer reading are available. All estimators are vulnerable to trends in the data, periodicity in the data and other sources of corruption. Three techniques were tried to filter real-life traces in addition to make measurements purely in the raw data;

- Transform to log of original data (only appropriate if data is positive)
- Removal of mean and linear trend (this is, subtract the best fit line Y = a*X + b for constant a and b)
- Removal of high order best-fit polynomial of degree ten (the degree ten was chosen after higher degrees showed evidence of over fitting) [see for example *Clegg*, 2006].

Hurst Exponent is calculated using the following steps

Suppose that we have a time series h(t) with n measurements then:

-1. We note the value of the n measurements:

h(1), **h**(2),... **h**(n)

-2. Calculate **M** as the **M**ean of these levels:

M= (1/n) * [**h**(1)+**h**(2)+... +**h**(n)]

-3. Calculate the deviations of the mean:

x(1)= **h**(1) - **M**
x(2)= **h**(2) - **M**
....
x(n)= **h**(n) - **M**

-4. Then we calculate the sums:

Y(1)= **x**(1)
Y(2)= **x**(1) + **x**(2)
....
Y(n)= **x**(1) + **x**(2) + ...+ **x**(n)

-5. Let **R**(n) be the difference between the maximum and the minimum of the n-values that is called the Range.

R(n)= MAX[**Y**(n)] – MIN[**Y**(n)]

-6. Let **s**(n) be the standard deviation of the set of n **h**-values.

s(n) = STDEV[**h**(n)]

The R/s statistic is a well-known technique for estimating the Hurst parameter [*Clegg*, 2006]. According to this technique we have:

$$\mathbf{R}(n)/\mathbf{s}(n) \sim C\,\mathbf{n}^{\mathbf{H}}$$

where C is a positive, finite constant independent of n and **H** is the Hurst Exponent value.

If we take the log of the above equation we do have:

$$\text{Log}(\mathbf{R}(n)/\mathbf{s}(n)) = \log C + \mathbf{H}*\log\mathbf{n}$$

then **H** is the slope of **R**/**s** versus **n** in a log-log diagram.

A value of H=0.5 corresponds to a random walk, 0.5<H<1 corresponds to a a persistent signal, i.e. large (small) values are followed by large (small) while 0<H<0.5 correspond to an anti-persistent signal.

Now we repeat the above scheme for all measurements, each time generating a point into our chart and this gives us the Fig. 8. At this example we calculated points up to n=992. Thus we have a time series with 992 values. The calculation part is in the following Fig. 7.

Example 2										
	h( )	M	x( )	Y( )	R	s	n	R/S	log(n)	log(R/s)
1	0									
2	0	0	0	0	0	0	2	-	0.301	-
3	0	0	0	0	0	0	3	-	0.477	-
4	0	0	0	0	0	0	4	-	0.602	-
5	0	0	0	0	0	0	5	-	0.699	-
6	0	0	0	0	0	0	6	-	0.778	-
7	0	0	0	0	0	0	7	-	0.845	-
8	0	0	0	0	0	0	8	-	0.903	-
9	0	0	0	0	0	0	9	-	0.954	-
10	1	0.100	0.900	0.900	0.900	0.316	10.000	2.846	1.000	0.454
11	1	0.182	0.818	1.718	1.718	0.405	11.000	4.247	1.041	0.628
12	1	0.250	0.750	2.468	2.468	0.452	12.000	5.457	1.079	0.737
13	1	0.308	0.692	3.160	3.160	0.480	13.000	6.579	1.114	0.818
14	1	0.357	0.643	3.803	3.803	0.497	14.000	7.649	1.146	0.884
15	1	0.400	0.600	4.403	4.403	0.507	15.000	8.684	1.176	0.939
...	...	...	...	...	...	...	...	...	...	...
988	0	0.401	-0.401	106.918	120.171	0.490	988.000	245.092	2.995	2.389
989	0	0.400	-0.400	106.517	120.171	0.490	989.000	245.133	2.995	2.389
990	0	0.400	-0.400	106.117	120.171	0.490	990.000	245.175	2.996	2.389
991	0	0.400	-0.400	105.718	120.171	0.490	991.000	245.216	2.996	2.390
992	0	0.399	-0.399	105.319	120.171	0.490	992.000	245.258	2.997	2.390

Figure7. Hurst calculation of Example 1

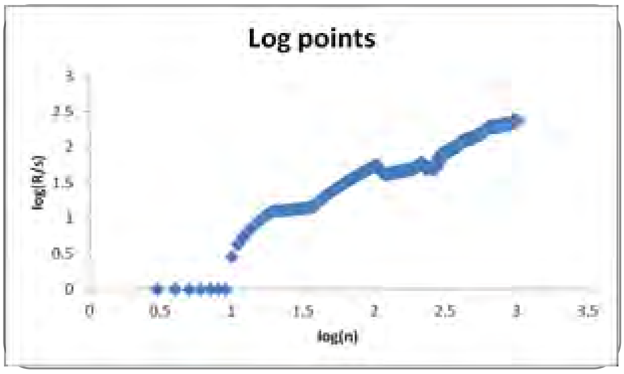

Figure.8

Now we choose linear fitting for the Hurst Exponent line. Thus we have the following Fig. 9. The linear region contains more than 870 points from 994 points in total.

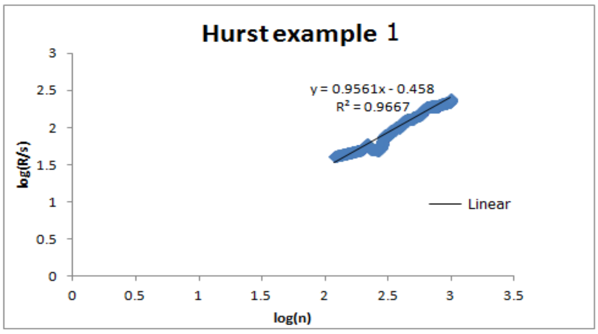

Figure.9

The Hurst exponent is the slope is H=0,9561.

3. 2. HJORTH MOBILITY & HJORTH COMPLEXITY

Hjorth coefficients first used in Electroencephalography and Clinical Neurophysiology [*Hjorth*, 1970]. The various notions are presented briefly below.

Mobility is defined as the square root of the ratio between the variances of the first derivative and the amplitude. Since these quantities are equally dependent on the mean amplitude, the ratio will be dependent on the curve shape only and in such a way that it measures the relative average slope [see for example *Balestrassi et al.*, 2011].

Complexity: This parameter is dimensionless and derived as the ratio between the mobility of the first derivative of the nonlinear time series and the mobility of the nonlinear time series [*Balestrassi et al.*, 2011].

How do we calculate the Hjorth Mobility.

On the same example 1 (Section 3.1) we need the standard deviation from all h(1), h(2) ..., h(n) , we have σ_a = STDEV[h(n)]. This can also be seen on the f(t) diagram. From the first derivative diagram we calculate the σ_d value. Actually σ_d = STDEV[dh(n) / dn] i.e. is the standard deviation from all dh(1)/dn, dh(2)/dn, ..., dh(n)/dn values.

$$\text{Hjorth Mobility} = \frac{\sigma_d}{\sigma_a}$$

How do we calculate the Hjorth Complexity

We keep the values of σ_a and σ_d from Hjorth Mobility. Now we need one more value which comes from the second derivative diagram and from that we calculate the σ_{dd} value. Actually σ_{dd} = STDEV[$d^2h(n) / dn^2$] i.e. is the standard deviation from all $d^2h(1)/dn^2$, $d^2h(2)/dn^2$, ..., $d^2h(n)/dn^2$ values.

$$\text{Hjorth Complexity} = \frac{\frac{\sigma_{dd}}{\sigma_d}}{\frac{\sigma_d}{\sigma_a}}$$

In order to have a better understanding a schematic diagram presenting the definitions of mobility and complexity is reproduced from *Hjorth* [1970] in Figure 10.

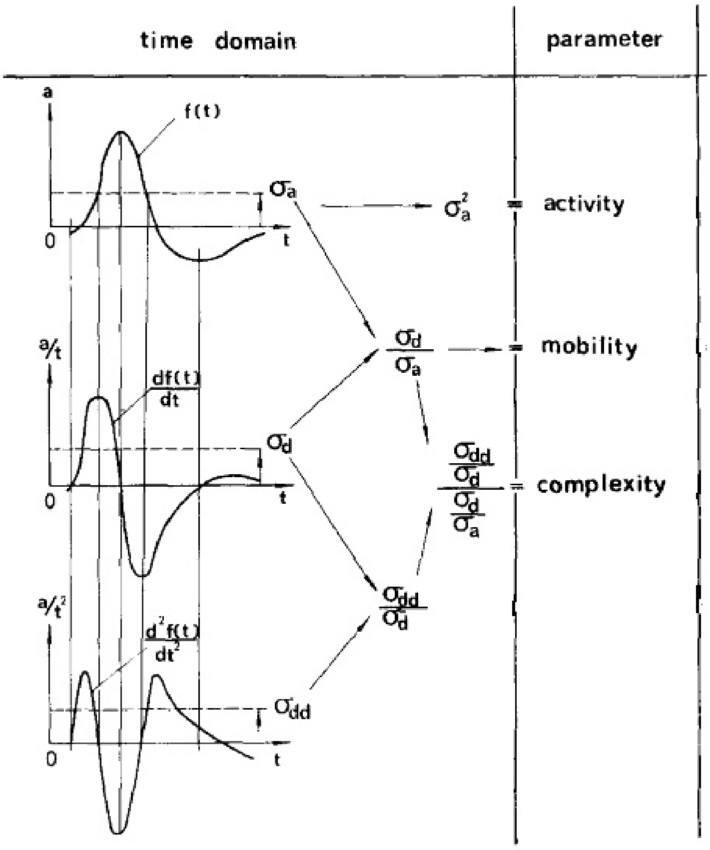

Figure 10. Schematic representation of Hjorth Parameters [Hjorth 1970].

As far as the quantity activity that is mentioned in Fig.10 infact activity is quantified by means of amlitude variance, which has the neccesary additive property to allow integration of different observations during time into one representative figure [*Hjorth* 1970].

3.3. Hurst and Hjorth parameters on Academic Examples

In this section we present some representative examples of Hurst and Hjorth parameters calculations before applying these methods to rainfall time series. The examples include intermittent and periodic signals. In the case of intermittent signals the value varies from zero to one, and the duration of non zero values is at least of a duration of ten consecutive time steps. In the case of periodic signals we examined the effect of the period, amplitude of oscillation as well as of the time step.

We must note here that the case of zero intermittency (mentioned in the following as example 1 in our examples) would correspond to constant value. However in these cases the

Hurst and Hjorth parameters cannot be defined as one can see from equations on Section 3 since we have zero-valued denominators.

3.3.1 Intermittent Signals

-Example 1. At Fig.11 A, B & C we see the time series, its first derivative and second derivative as a function of time which are used for the calculation of the Hjorth Mobility and Complexity. At Fig. 11 E we present also the plot from which the Hurst exponent is calculated. Parameter values can be seen on the table between Fig.11 D.

Now about the Intermittency, is calculated as the ratio of total zero values divided with the total number of all values. Furthermore, in all Intermittent signals examples, intermittency is counted like this way. At this example Intermittency is of 60%.

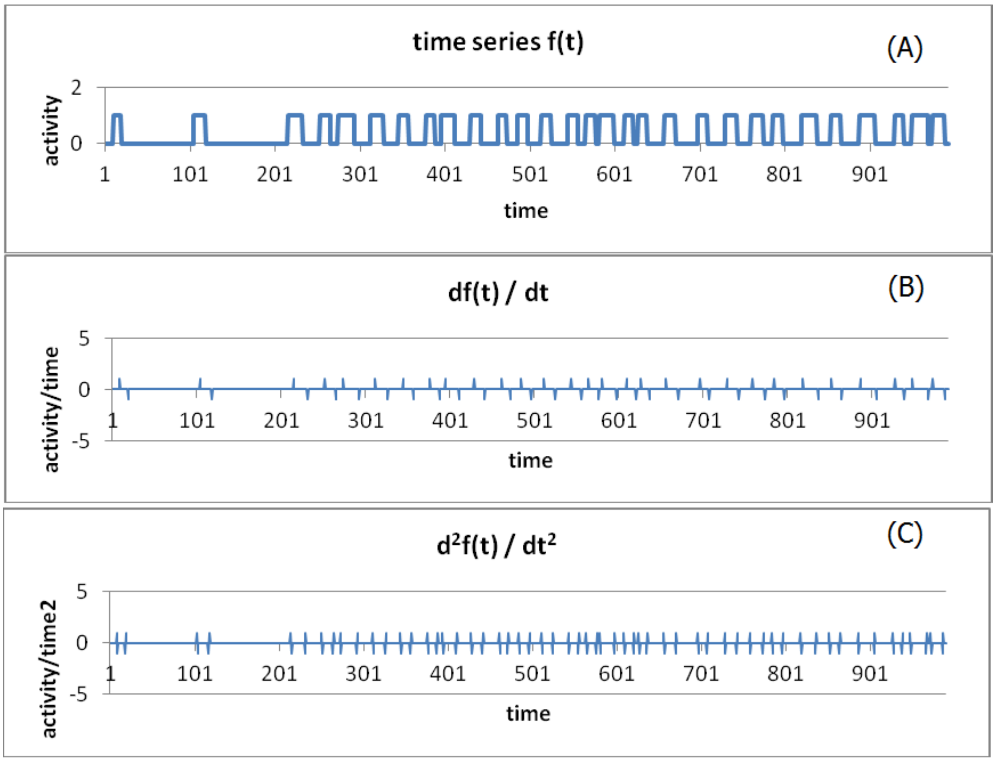

Figure.11 (A) time series (B) first derivative and (C) second derivative as a function of time

Hurst = 0,9561	Hjorth Mob = 0,494	Hjorth Complex = 2,8643

Figure.11 (D) Parameters values for Example 1

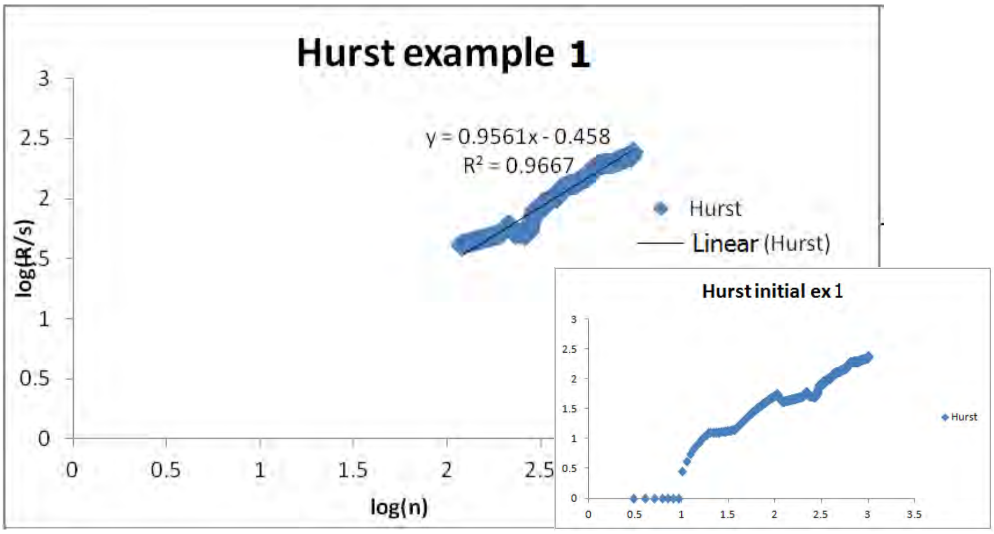

Figure.11 (E) Hurst Exponent plot.

We should mention that as the Hurst Exponent characterizes the power law relationship between R/s and n, we consider only the linear portion of the empirical data on the log(R/s)-log(n) diagram.

-Example 2. At Fig.12 we present time series as before but with intermittency 47%. Now at Fig.12 A, B & C we see the time series, its first derivative and second derivative as a function of time which are used for the calculation of the Hjorth Mobility and Complexity. At Fig. 12 E we present also the plot from which the Hurst exponent is calculated. Parameter values can be seen on the table Fig.12 D.

Intermittency is being calculated as in example 1; it is the ratio of total zero values divided with the total number of all values. At examples 3 and 4 intermittency is counted with the same way.

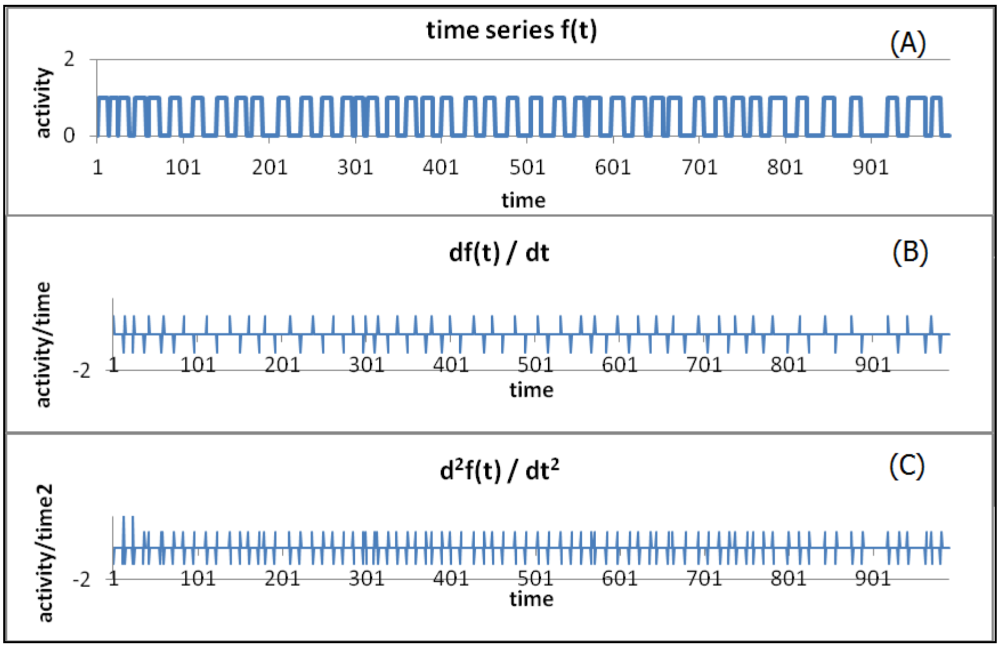

Figure.12 (A) time series (B) first derivative and (C) second derivative as a function of time

Hurst = 0,426	Hjorth Mob = 0,5833	Hjorth Complex = 2,4474

Figure.12 (D) Parameters values for Example 2

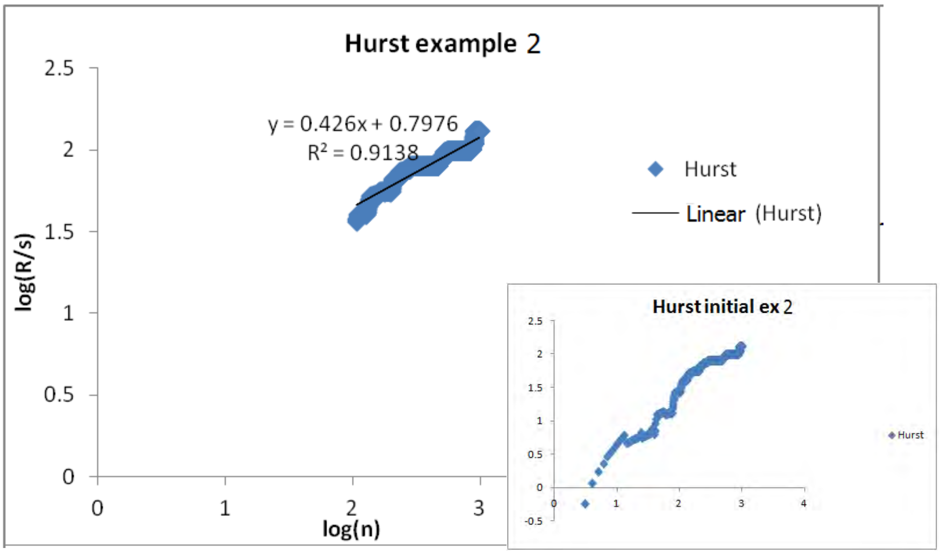

Figure.12 (E) Hurst Exponent plot

-Example 3. At Fig.13 we present time series as before but with intermittency 40%. Now at Fig.13 A, B & C we see the time series, its first derivative and second derivative as a function of time which are used for the calculation of the Hjorth Mobility and Complexity. At Fig. 13 E we present also the plot from which the Hurst exponent is calculated. Parameter values can be seen on the table Fig.13 D.

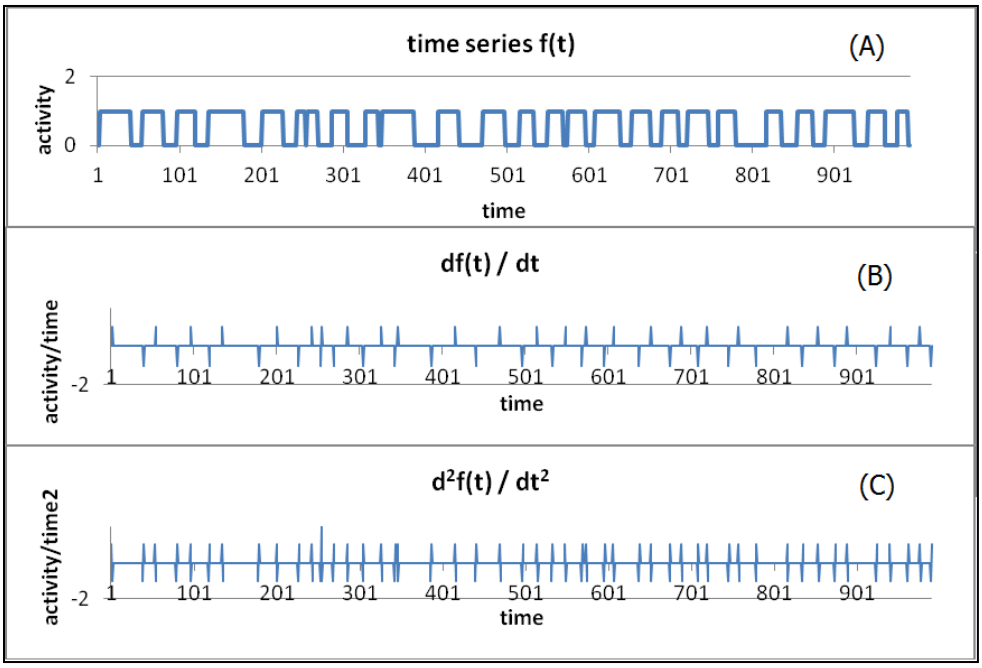

Figure.13 (A) time series (B) first derivative and (C) second derivative as a function of time

Hurst = 0,5305	Hjorth Mob = 0,4587	Hjorth Complex = 3,1156

Figure.13 (D) Parameters values for Example 3

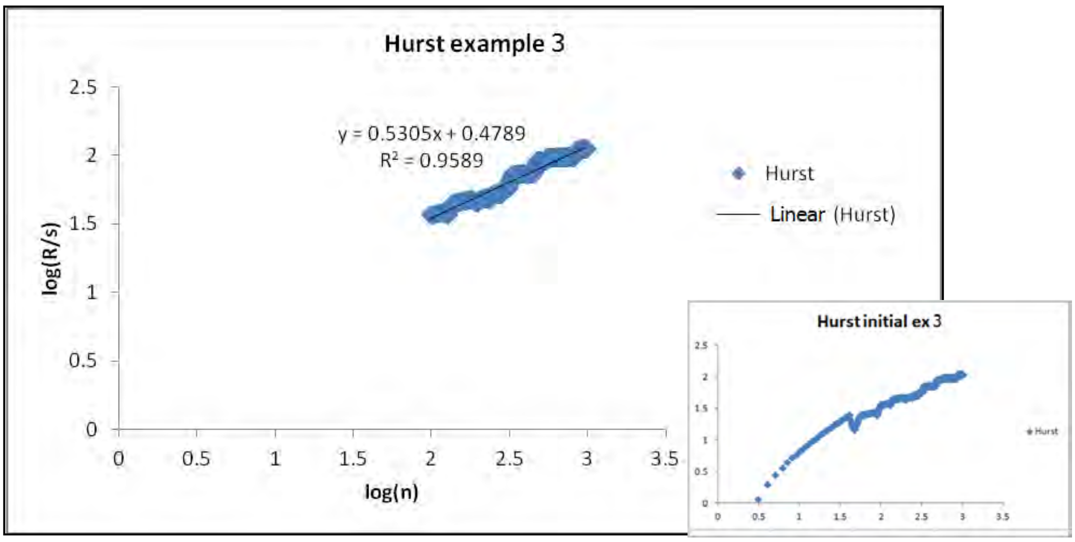

Figure.13 (E) Hurst Exponent plot

-**Example 4.** At Fig.14 we present time series as before but with intermittency 31%. Now in Fig.14 A, B & C we see the time series, its first derivative and second derivative as a function of time which are used for the calculation of the Hjorth Mobility and Complexity. At Fig. 14 E we present also the plot from which the Hurst exponent is calculated. Parameter values can be seen on the table Fig.14 D.

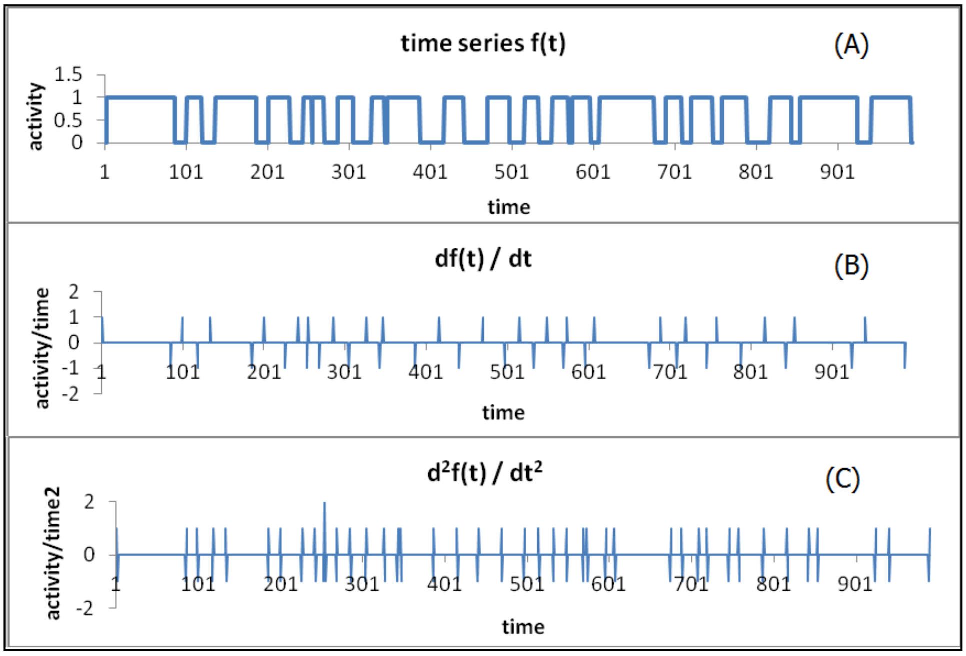

Figure.14 (A) time series (B) first derivative and (C) second derivative as a function of time

Hurst = 0,6418	Hjorth Mob = 0,4438	Hjorth Complex = 3,2263

Figure.14 (D) Parameters values for Example 4

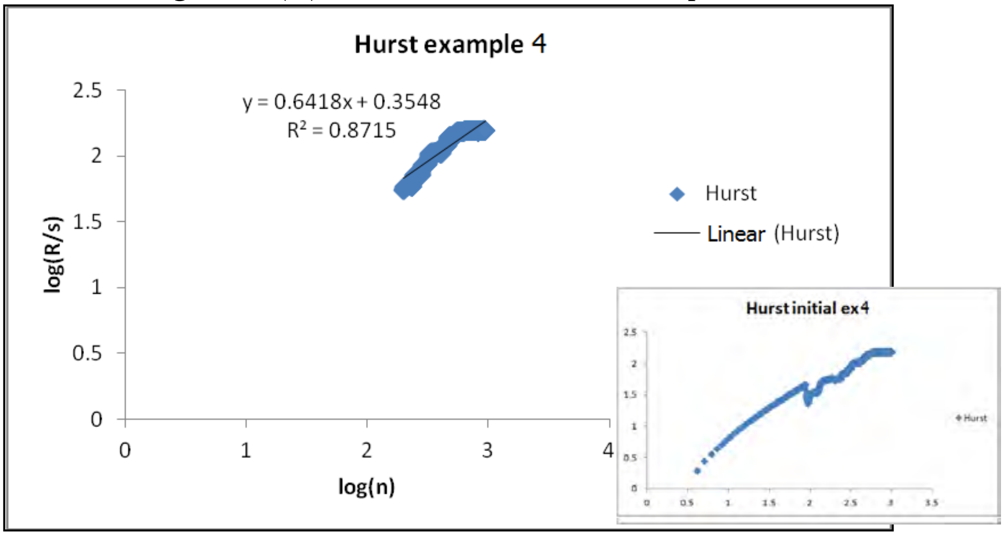

Figure.14 (E) Hurst Exponent plot

3.3.2 PERIODIC SIGNALS

-Example 5 . At Fig.15 we present a periodic time series with characteristics Period= 2,75sec / Amplitude= 0,1m / Time step= 0,25sec. Now at Fig.15 A, B & C we see the time series, its first derivative and second derivative as a function of time which are used for the calculation of the Hjorth Mobility and Complexity. At Fig. 15 E we present also the plot from which the Hurst exponent is calculated. Values can be seen on the table Fig.15 D.

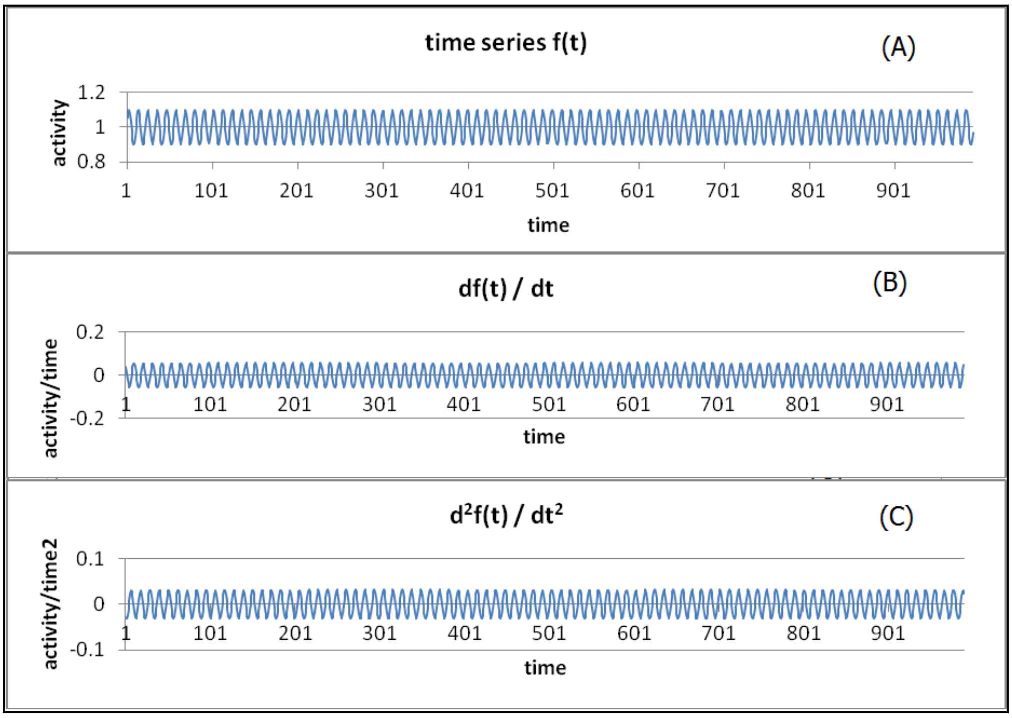

Figure.15 (A) time series (B) first derivative and (C) second derivative as a function of time

Hurst = 0,1673	Hjorth Mob = 0,5616	Hjorth Complex = 1,0025

Figure.15 (D) Parameters values for Example 5

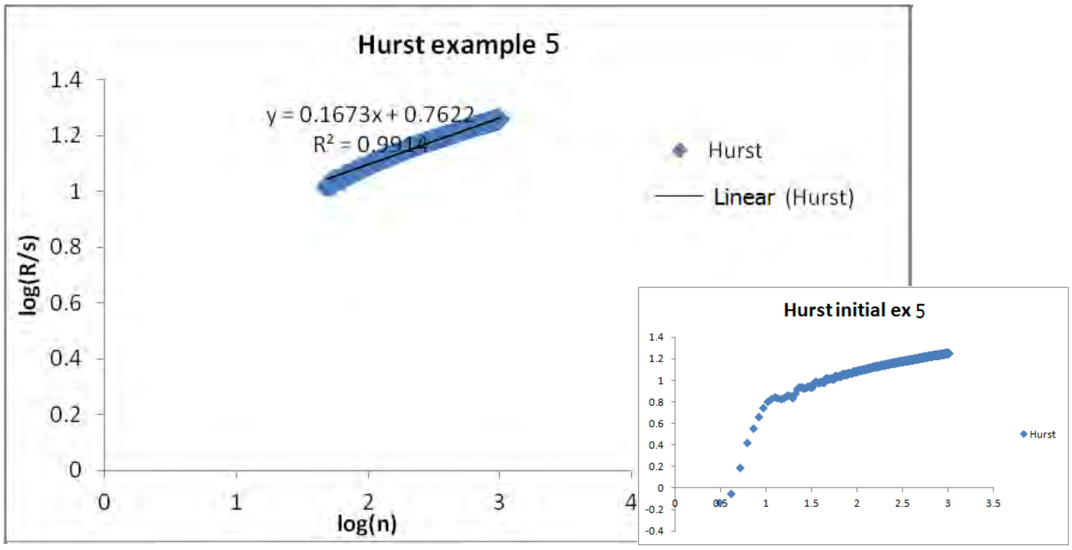

Figure.15 (E) Hurst Exponent plot

-**Example 6**. At Fig.16 we present a periodic time series as before but with characteristics Period= 2,75sec / Amplitude= 0,033m / Time step= 0,25sec. This means that in comparison with example 5 now we have smaller amplitude, bigger standard deviation and bigger variability. Now at Fig.16 A, B & C we see the time series, its first derivative and second derivative as a function of time which are used for the calculation of the Hjorth Mobility and Complexity. At Fig. 16 E we present also the plot from which the Hurst exponent is calculated. Values can be seen on the table Fig.16 D.

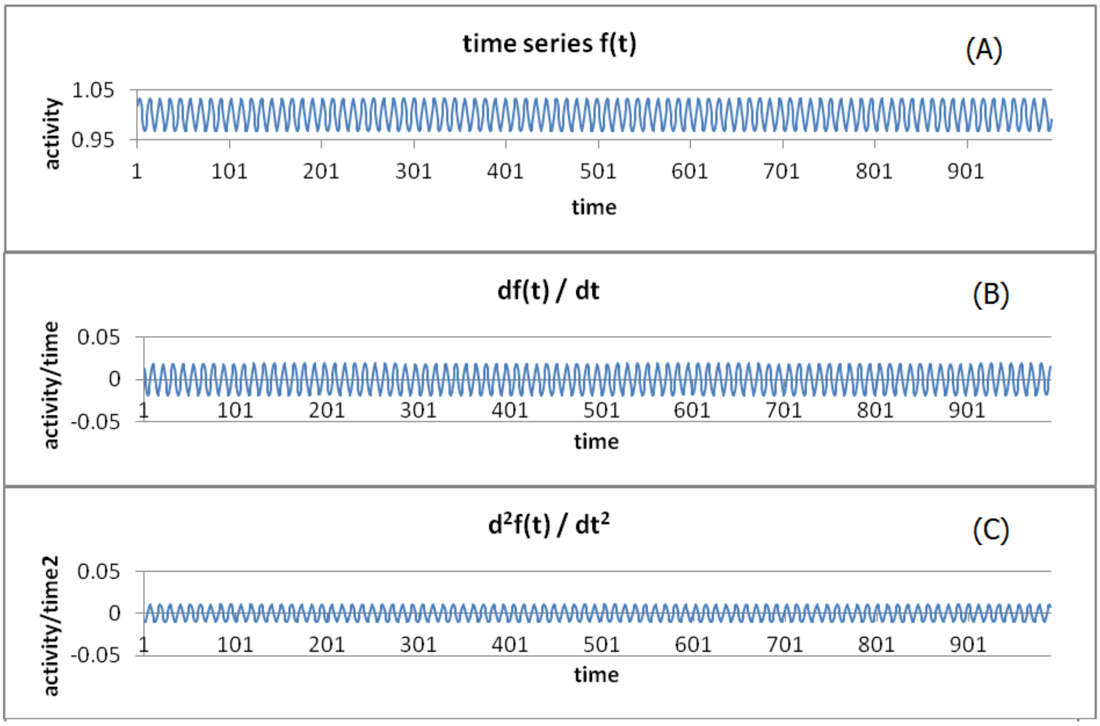

Figure.16 (A) time series (B) first derivative and (C) second derivative as a function of time.

Husrt =0,1673	Hjorth Mob = 0,5616	Hjorth Complex = 1,0025

Figure.16 (D) Parameters values for Example 6

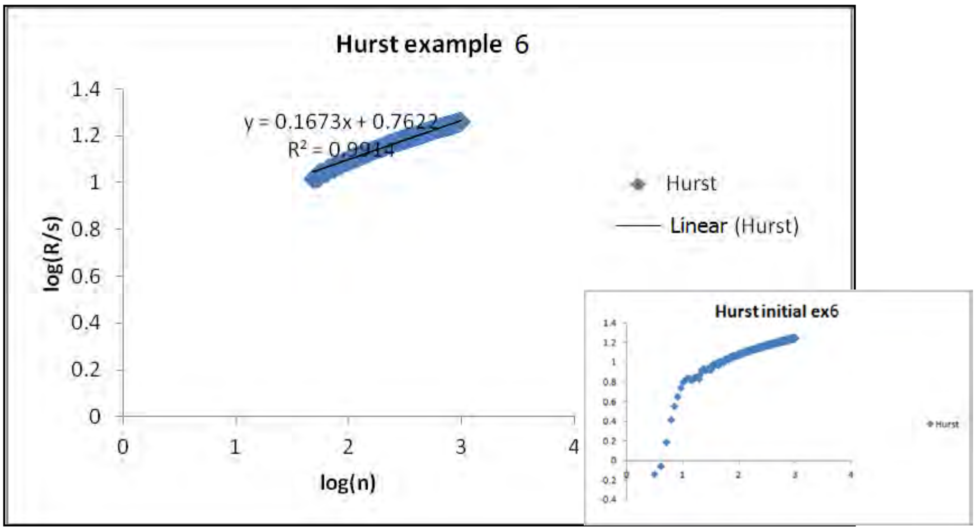

Figure.16 (E) Hurst Exponent plot

-Example 7. At Fig.17 we present a periodic time series as before but with characteristics Period= 2,75sec / Amplitude= 0,5m / Time step= 0,25sec. This means that in comparison with example 5 and 6 now we have bigger amplitude. Now at Fig.17 A, B & C we see the time series, its first derivative and second derivative as a function of time which are used for the calculation of the Hjorth Mobility and Complexity. At Fig. 17 E we present also the plot from which the Hurst exponent is calculated. Values can be seen on the table Fig.17 D.

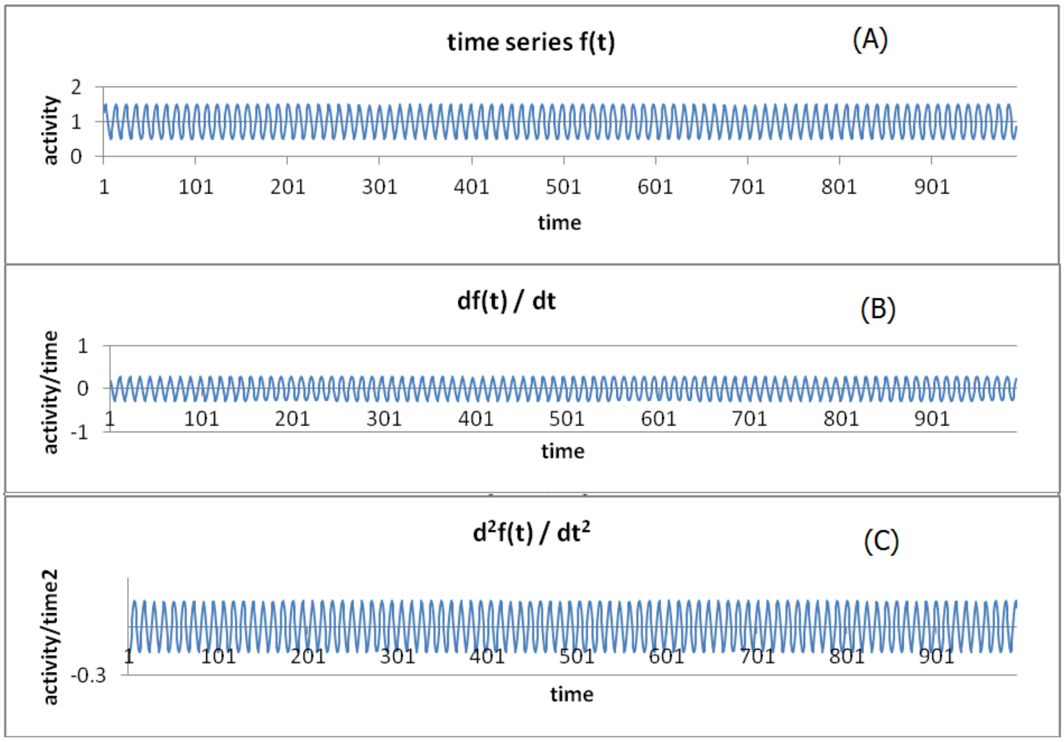

Figure.17 (A) time series (B) first derivative and (C) second derivative as a function of time

Hurst = 0,1673	Hjorth Mob = 0,5616	Hjorth Complex = 1,0025

Figure.17 (D) Parameters values for Example 7

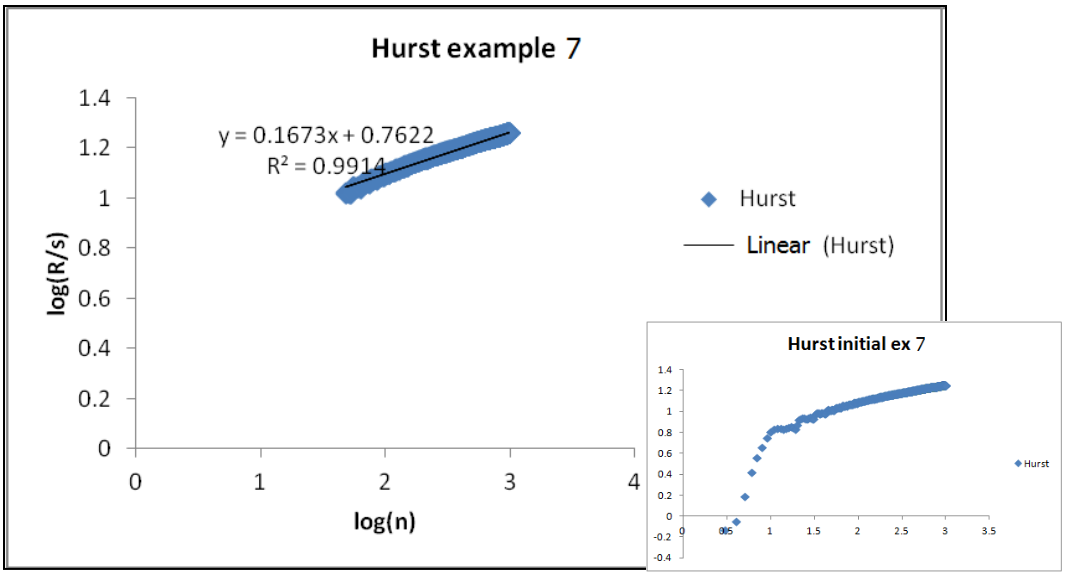

Figure.17 (E) Hurst Exponent plot.

-**Example 8**. At Fig.18 we present a periodic time series as before but with characteristics Period= 11,8sec / Amplitude= 0,1m / Time step= 0,25sec. This means that in comparison with example 5, 6 and 7 now we have bigger period. Now at Fig.18 A, B & C we see the time series, its first derivative and second derivative as a function of time which are used for the calculation of the Hjorth Mobility and Complexity. At Fig. 18 E we present also the plot from which the Hurst exponent is calculated. Values can be seen on the table Fig.18 D.

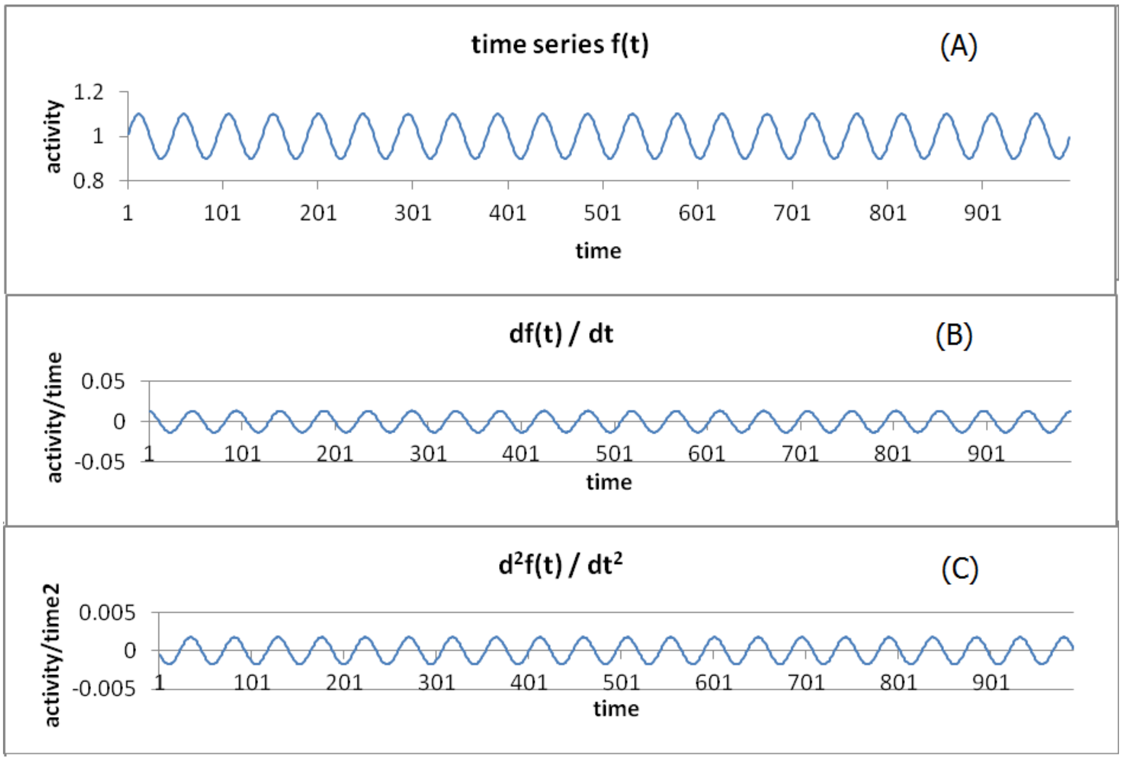

Figure.18 (A) time series (B) first derivative and (C) second derivative as a function of time

Hurst = 0,2149	Hjorth Mob = 0,1327	Hjorth Complex = 1,003

Figure.18 (D) Parameters values for Example 8

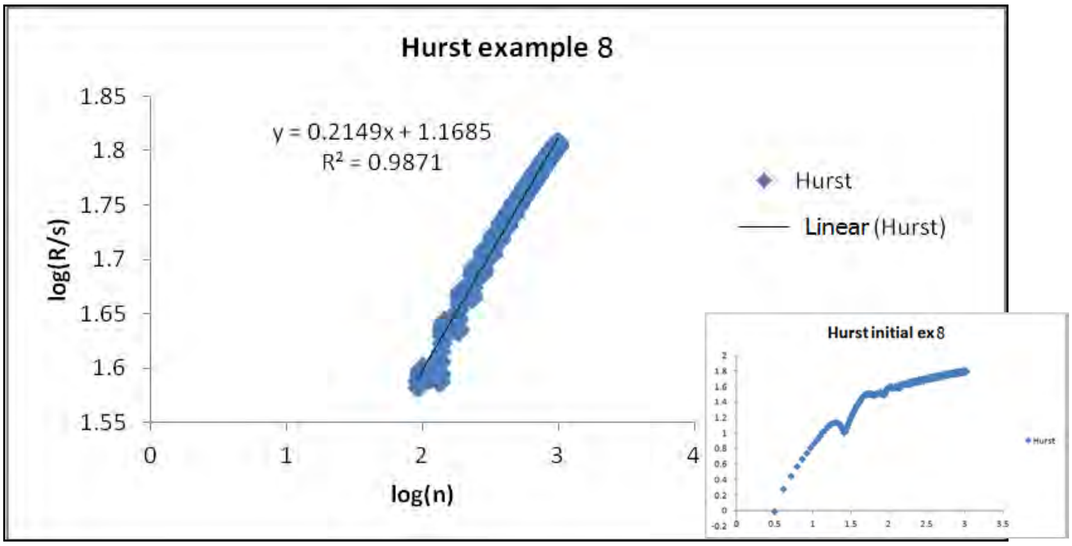

Figure.18 (E) Hurst Exponent plot

-Example 9. At Fig.19 we present a periodic time series as before but with characteristics Period= 2,75sec / Amplitude= 0,1m / Time step= 0,5sec. We notice here that in comparison with example 5 and 6 now we have bigger time step. Now at Fig.19 A, B & C we see the time series, its first derivative and second derivative as a function of time which are used for the calculation of the Hjorth Mobility and Complexity. At Fig. 19 E we present also the plot from which the Hurst exponent is calculated. Values can be seen on the table Fig.19 D.

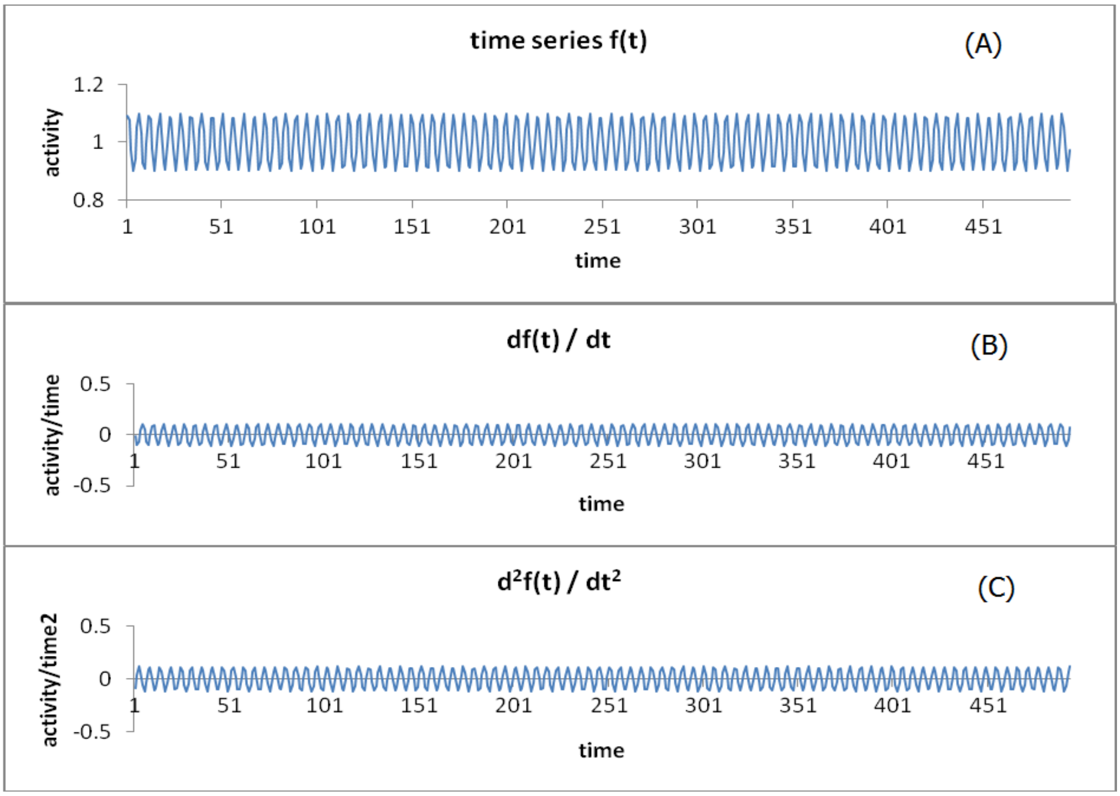

Figure.19 (A) time series (B) first derivative and (C) second derivative as a function of time

Hurst = 0,1604	Hjorth Mob = 1,0779	Hjorth Complex = 1,0021

Figure.19 (D) Parameters values for Example 9

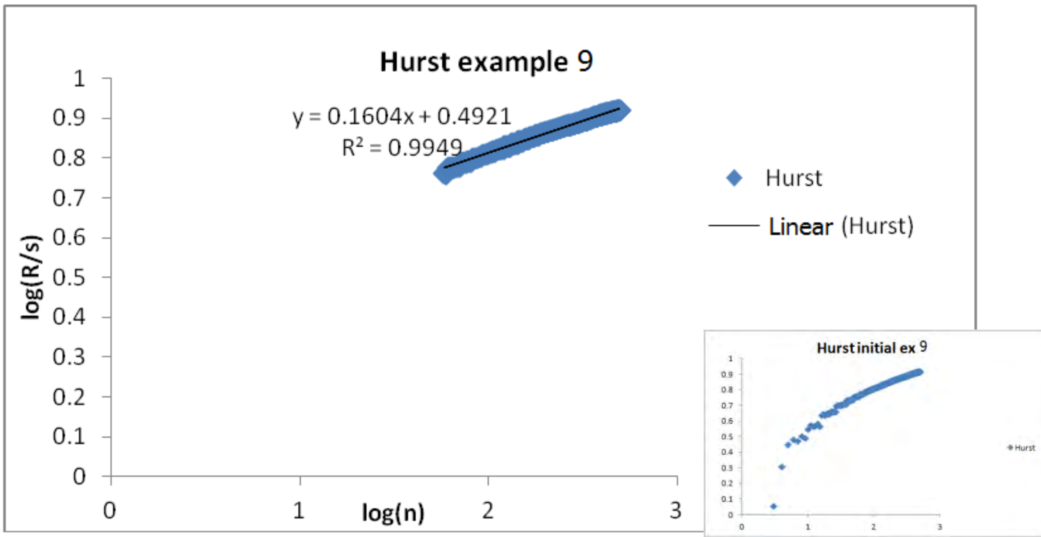

Figure.19 (E) Hurst Exponent plot

3.3.3 SOME CONCLUSIONS

So for the Hurst Exponent we could notice:

-Period and Hurst are varying in the same way (for periodic event).

Period	Hurst	
11,6	0,2149	Example 8
2,9	0,1673	Example 5

Figure.20 Period and Hurst Exponent

Fig.20 confirms the fact that the Period change between Example 8 and Example 5 from 11,6 value to 2,9 respectively means lowering in Hurst from 0,2149 to 0,1673 respectively between the two examples.

-The amplitude doesn't seem to affect Hurst exponent (for periodic event).

Amplitude	Hurst	
0,1	0,1673	Example 5
0,033	0,1673	Example 6
0,5	0,1673	Example 7

Figure.21 Amplitude and Hurst Exponent

Indeed the amplitude change that we see above (Fig.21) at Example 5, 6 and 7 doesn't affect Hurst value. This is explainable because the Hurst is calculated from the difference between the maximum and minimum of sums undulation points.

-The time step doesn't change Hurst exponent, we observe only small value difference (for periodic event). Despite the Time step change (50% reduction) in Fig.22 that takes place from Example 9 to Example 5, the Hurst value is almost equal between the two examples.

;

Time step	Hurst	
0,5	0,1604	Example 9
0,25	0,1673	Example 5

Figure.22 Time step and Hurst Exponent

-For the Intermittency is a matter of alterations sequence concerning Hurst exponent.

Intermittency	Hurst	
60%	0,9561	Example 1
47%	0,4260	Example 2
40%	0,5305	Example 3
31%	0,6418	Example 4

Figure.23 Intermittency and Hurst Exponent.

In fact the Intermittency as we see at Fig.23 above is changing a lot at many examples but this is not clearly seen on the Hurst values. The first two examples show a varying in the same way relationship between Intermittency and Hurst but the last two examples (Example 3 and 4) shows that this is not a rule.

- Standard Deviation.

Standard Deviation	Hurst	
0,07076	0,1673	Example 5
0,02359	0,1673	Example 6

Figure.24 Standard deviation and Hurst Exponent.

The variation of standard deviation doesn't change Hurst exponent (for periodic event). Actually Fig.24 above shows that Example 5 and 6 have different Standard Deviation values but both have the same Hurst value. This happens because in Hurst calculation we count the difference between the maximum and minimum of sums undulation points, thus the Hurst is the same but the amplitude –as seen before- and standard deviation are different.

For the Hjorth parameters we could summarize the following:

-The period is varying inverse to Hjorth Mobility while Hjorth Complexity is not changing (for periodic event). The last remark can be supported by Fig.25 below, where we see that Example 8 and Example 5 have more than three times different period while the Hjoth Complexity is not affected and the Hjorth Mobility is varying inverse to the period change.

Period	Hjorth Mobility	Hjorth Complexity	
11,6	0,1327	1,003	Example 8
2,75	0,5616	1,003	Example 5

Figure.25 Period with Hjorth Mobility and Complexity.

-The amplitude change doesn’t effect Hjorth Mobility and Hjorth Complexity (for periodic event). It is obvious on the Fig.26 above that the amplitude reduction from Example 5 to Example 6 doesn’t appeal on both Hjorth Mobility and Hjorth Complexity values. The same for the amplitude increase from Example 5 to Example 7. This is also acceptable because both Hjorth coefficients are coming from standard deviation ratios.

Amplitude	Hjorth Mobility	Hjorth Complexity	
0,1	0,5616	1,0025	Example 5
0,033	0,5616	1,0025	Example 6
0,5	0,5616	1,0025	Example 7

Figure.26 Amplitude with Hjorth Mobility and Complexity.

-The Time step is varying in the same way to Hjorth Mobility while Hjorth Complexity doesn’t change significantly for periodic event as we see at Fig.27. The Time step lowering from Example 9 to Example 5 can be seen also to the lowering in the same way of the Hjorth Moblity; on the other hand the Hjorth Complexity factor seems to remain unaffected.

Time step	Hjorth Mobility	Hjorth Complexity	
0,5	1,0779	1,0021	Example 9
0,25	0,5616	1,0025	Example 5

Figure 27. Time step with Hjorth Mobility and Complexity.

-Intermittency is a matter of alterations sequence concerning Hjorth Mobility and Hjorth Complexity. As we could see in Fig.28 below the big or low intermittency doesn't give a clear message of how Hjorth Mobility and Complexity values respectively will behave.

;

Intermittency	Hjorth Mobility	Hjorth Complexity	
60%	0,4940	2,8643	Example 1
47%	0,5833	2,4477	Example 2
40%	0,4587	3,1156	Example 3
31%	0,4438	3,2263	Example 4

Figure 28. Intermittency with Hjorth Mobility and Complexity.

4. THE APPLICATION OF LINEAR AND NON-LINEAR ANALYSIS IN THE RHONE VALLEY CASE

At this section we took rainfall time series data in vertical cross sections of our Examine area; the Rhône valley at Fig.29. These cross sections were done as we can see from X=785km till X=810km. As we notice they were done above Rhone river valley and they have 5km difference from each other.

For each image pixel we calculate Hurst and Hjorth coefficients for the corresponding rainfall time series. The Hurst Exponent analysis is the non-linear while the Hjorth coefficients analysis is the linear one. Thus we have all following spatial Figures (30 till 32). We take into account that the rain is coming from the North along with the average wind.

We decided to focus on the North part of Rhone river valley (from 2000km till 1920km, Fig.29) as it includes mountains that create steady perturbations than focus on the South part of Rhone river valley (from 1920km till 1850km) as it includes open valley where the North-South wind often interacts with East-West circulations.

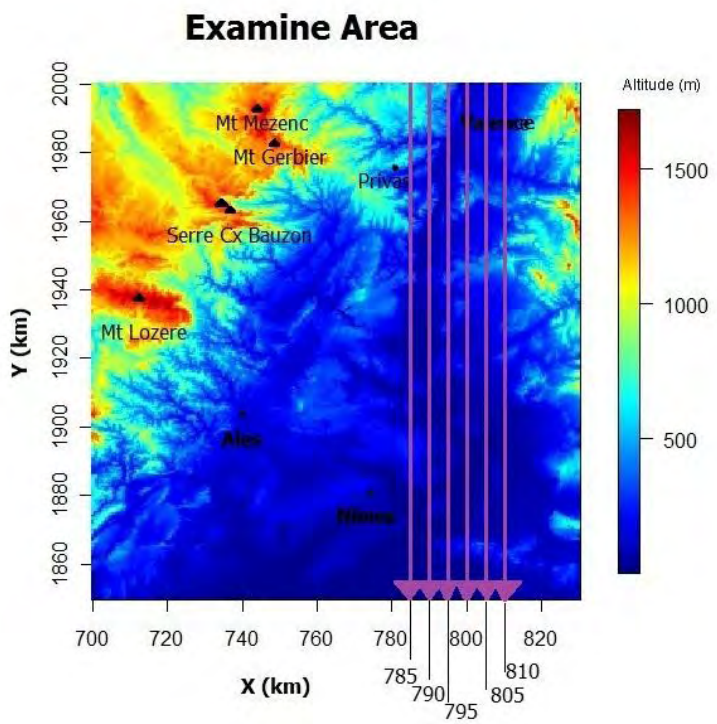

Figure 29. Cross sections of study along the examine area in Rhône river valley.

At Fig.30 a we notice that based on the Hurst Exponent results we can distinguish three different behaviour areas. The first one is on the North at Y=1977km where Hurst reaches maximum values in two parts. The second area is a flat area (Y=1965km till Y=1952km) where Hurst has small values. The third area is from Y=1945km till Y=1935km where Hurst reaches maximum values. These three areas are at the same vertical line between X=790km till X=810km. We must mention that between these limits there are mountains that seem to block persistence on the first and third area. That is why the second area has small Hurst exponent values and thus smaller persistence than the first one.

Another thing that is worth mentioning is that the mountain areas at X=815km and X=820km on the right part of our examine region has the lowest persistence from all other regions of this examine area. So Hurst exponent changes while crossing a mountain region. We will see if we can make a conclusion afterwards.

Hjorth Complexity (Fig.30c) almost behaves in the same way like Hurst exponent and presents variations at the same characteristic locations. The second and third area present large and small values respectively. This means that high complexity corresponds to low persistence and the opposite. High complexity means abrupt changes in time series. The persistence here is an indicator of chaotic behaviour.

The above areas seen on Hurst Exponent diagram can be also seen at the Hjorth Mobility diagram Fig.30b. More speciffically the second area and the third one have small and high values respectively.

Our analysis shows that the Hurst exponent and Hjorth parameters seem to follow the relief variations; the points that the parameters changes coincides with the ground elevation change.

These measures seem to be more sensible to the elevation variation than the cumulative rainfall. Thus they may be used in the analysis of spatiotemporal phenomena since they can take into account the temporal behaviour in various spatial positions.

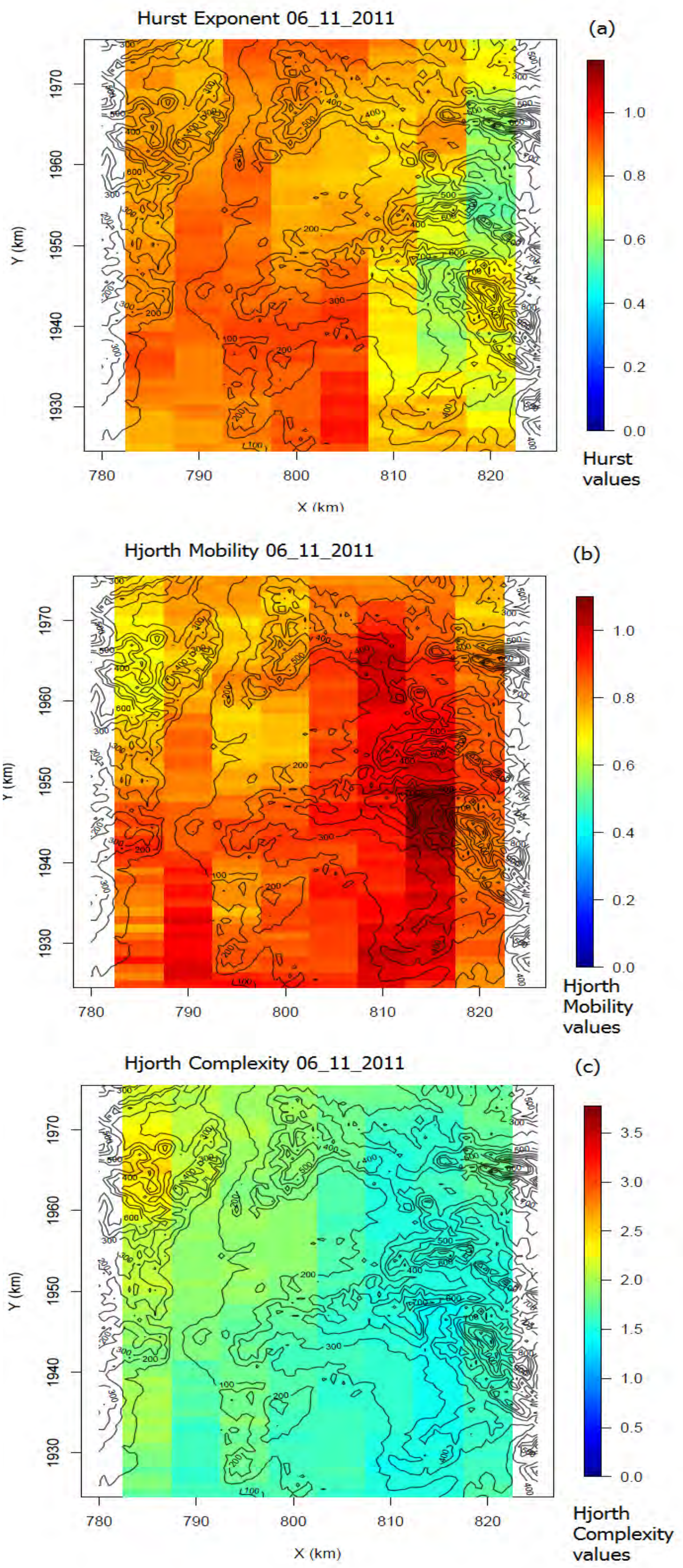

Figure.30 North Rhône river valley (a) Hurst Exponent map, (b) Hjorth Mobility map and (c) Hjorth complexity map.

If we move at X=795km the cross section at this point will give us the following Fig.31;

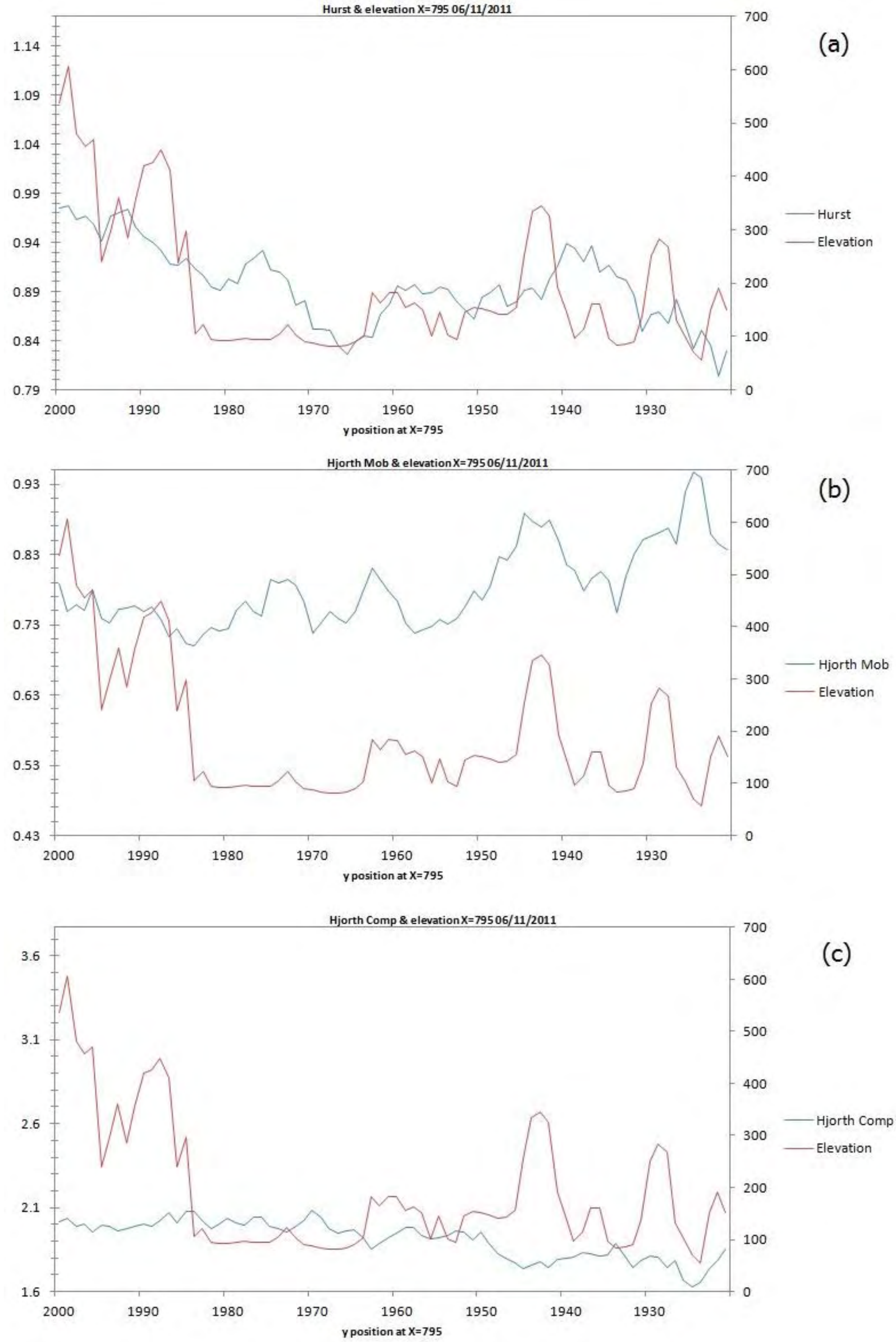

Figure.31 (a) Hurst Exponent and Elevation X=795km, (b) Hjorth Mobility and Elevation X=795km, (c) Hjorth Complexity and Elevation X=795km, we examine it from left to right; from 2000km to 1921km.

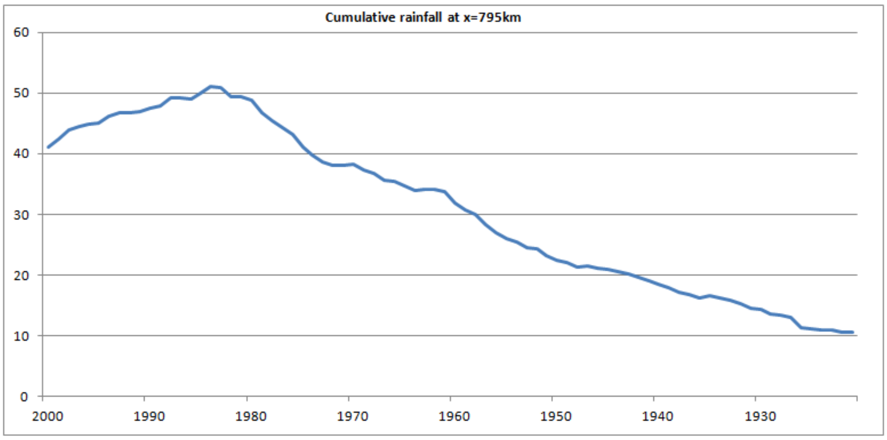

Figure.31 (d), Cumulative rainfall at X=795km,
we examine it from left to right; from 2000km to 1921km

Practically we examine all diagrams from Y=2000km till 1921km where we have a mountainous region as we explained before. Also if we start looking the Hurst Exponent diagram (Fig.31) from the beginning we notice that that its descending course interrupts an up rise at Y=1965km and at Y=1944km right after mountain crests. Even in Y=1931km we notice a smaller rise up than before and again this point coincides with a mountain crest.

The three points we mentioned above (Y=1965km, Y=1944km and Y=1931km) are points of big rise up also for the Hjorth Mobility diagrams(Fig.31 b) while for the Hjorth Complexity diagram (Fig.31 c) at these points we notice small descending course.

At the area between the two crests of Y=1944km and Y=1931km we see that Hurst exponent and Hjorth Mobility rise up stops and they are descending during all this part. On the contrary Hjorth Complexity is ascending during this part.

The rainfall intensity Fig.31 d doesn’t show any change at the mentioned three points (Y=1965km, Y=1944km and Y=1931km) where the rainfall intensity seems stable descending (we remind that we examine all Figures form left to right).

So we could say that the relief change has an influence on Hurst exponent and Hjorth parameters. Thus the relief and elevation change influence rainfall intensity parameters.

If we move at X=800km the cross section at this point will give us the following Fig.32.

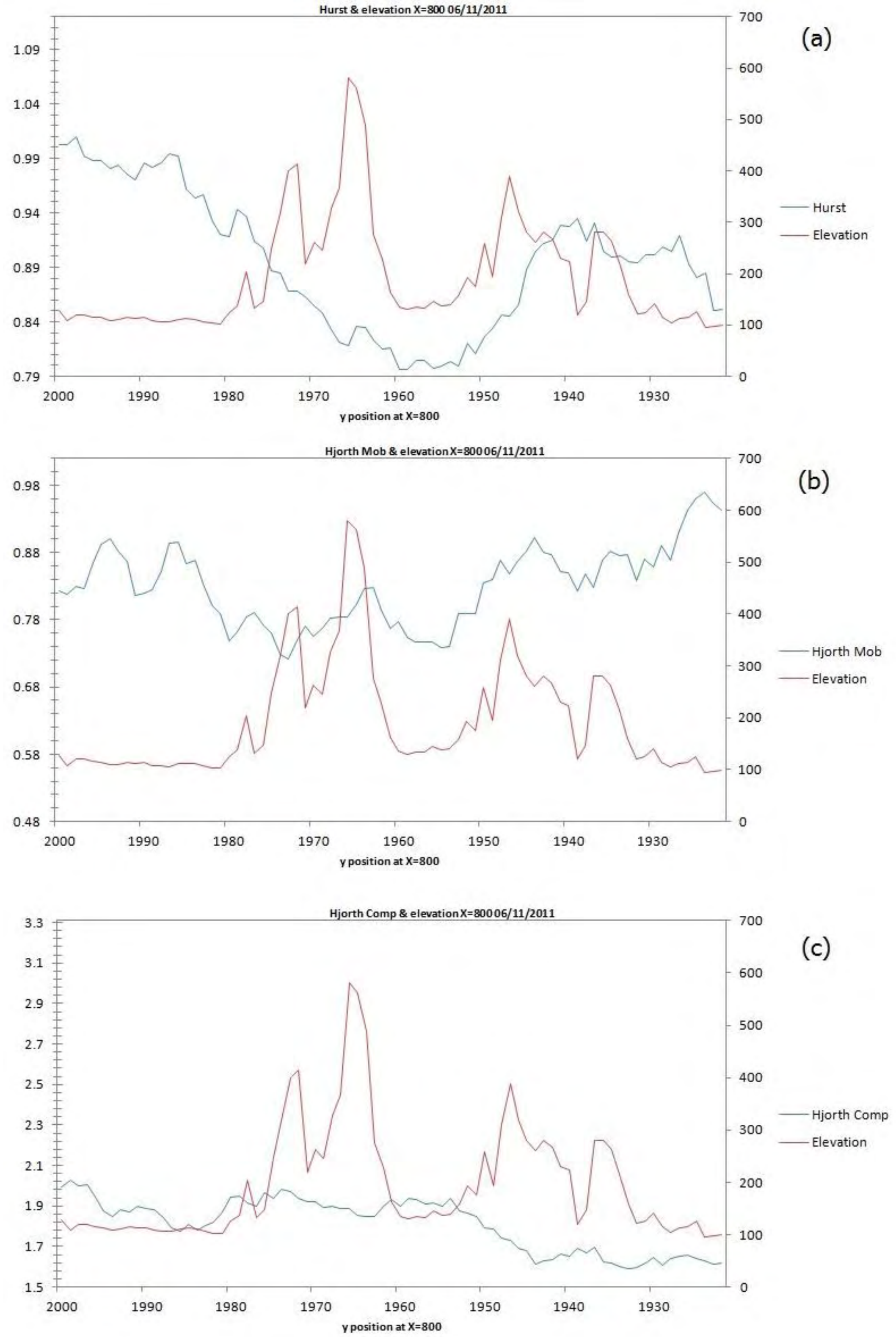

Figure.32 (a) Hurst Exponent and Elevation X=800km, (b) Hjorth Mobility and Elevation X=800km, (c) Hjorth Complexity and Elevation X=800km, we examine it from left to right; from 2000km to 1921km.

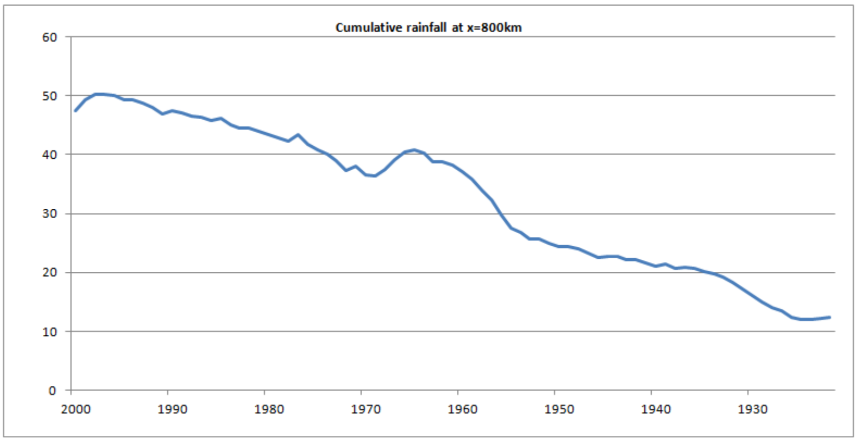

Figure.32 (d), Cumulative rainfall at X=800km,
we examine it from left to right; from 2000km to 1921km

At X=800km Hurst exponent diagram seems not to change from the first big mountain crest at Y=1977km and continuous its descending course while when meets the next mountain range at Y=1951km it starts ascending course. This ends when it meets the foothill area of Y=1941km till Y=1921km where is seems to be stable (with small oscillations) but for sure smaller values than the plane area in the beginning (Y=2000km till Y=1981km).

The two mountain ranges mentioned (Y=1977km and Y=1951km) are points where Hurst Mobility (Fig.32 b) curve rises up and still rises even upon the foothill area of Y=1941 till Y=1921km. The Hjorth Complexity (Fig.32 c) on the other hand seems to be stable at the first mountain range (Y=1977km) but starts descending course when cross the other one (Y=1951km).

What is interesting is that these plots (Fig.32a-c) confirm what we mentioned for Fig.30 as in the area between the two mountain crests Hurst exponent and Hjorth parameters are influenced strongly by the ground elevation; Hurst exponent reaches its lowest values, Hjorth Mobility low values and Hjorth Complexity its highest values. And for sure these values differ from the ones before and after these two mountain crests for all parameters. We remind here that our study is focused on the North part (Y=2000km till Y=1921km).

The cumulative rainfall (Fig.32d) is continuously descending at this examine area and makes a big uprising loop between Y=1967km and Y=1957km almost above the first big mountain range. Surprisingly this loop starts right after the first big mountain range. This is natural because of the turbulence around the valley (we mentioned in Fig.30 about high complexity in this valley between the mountain) that forcing for convection and consequently rainfalls.

These comments brings us close to reach a conclusion for the relationship between relief and elevation change with rainfall intensity parameters.

Another interesting figure is Fig.32 e where we can see the time series in characteristic points. We notice that the length of the time series is small i.e. 93 points and they contain many zeros. These fact make us be aware of using trends or other corruptions as we saw in Hurst theory in Section 3.1.

One fact in Fig.32e is that at almost all time series the rainfall intensity is strong the first 7 hours, is getting lower the coming 10 hours and from this point till the end is almost zero (no rain). The time series (Y=1993km Y=1980km and Y=1960km) that are on the North part of the map change from the rest because they show a continuous strong rainfall in the middle time part (between 07:00hours and 17:00) as in the first hours. This is natural because the maximum rainfall is concentrated on the North part of the valley (Fig.5). The time series of Fig.32 e confirm the statistical data that we demonstrated for the event on Section.2 Fig.4 where we saw three time parts with different behaviour for the statistical factors. Thus the observed rain here is coming from the same king of cloud we show in the statistical analysis.

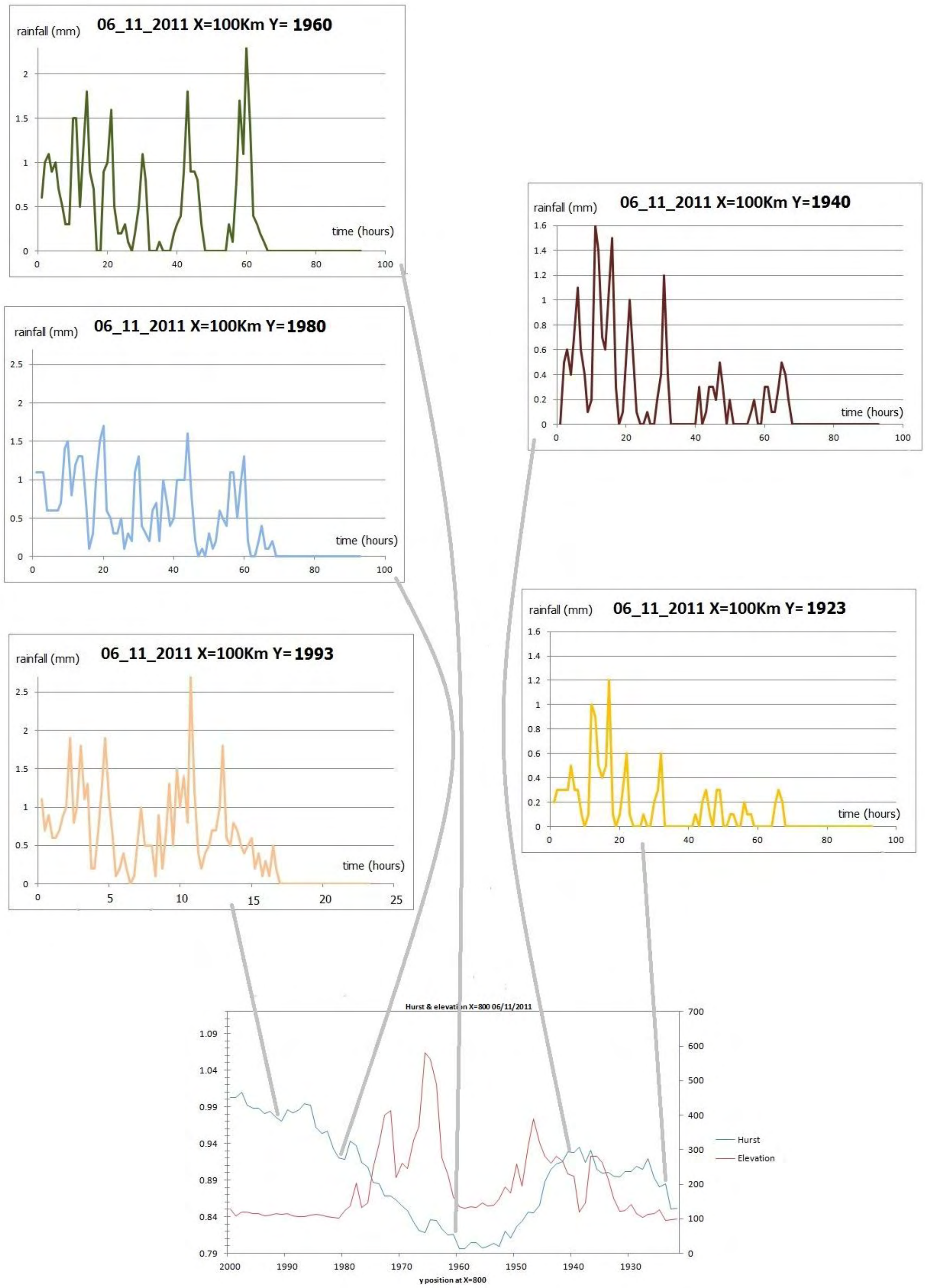

Figure.32 e, Time series at characteristic points for X=800km cross section.

At this point we will see the X=805km cross section;

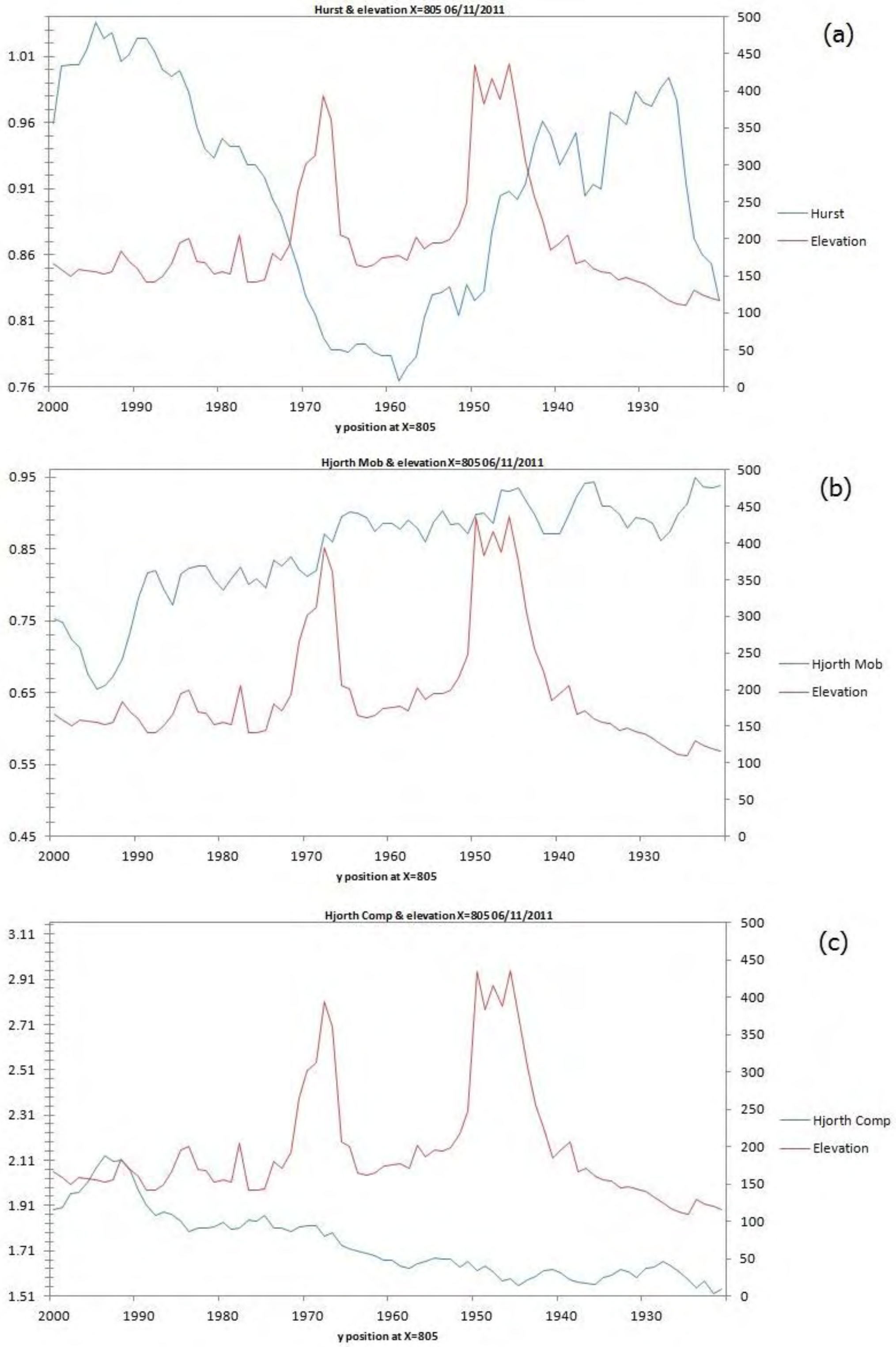

Figure.33 (a) Hurst Exponent and Elevation X=805km, (b) Hjorth Mobility and Elevation X=805km, (c) Hjorth Complexity and Elevation X=805km, we examine it from left to right; from 2000km to 1921km.

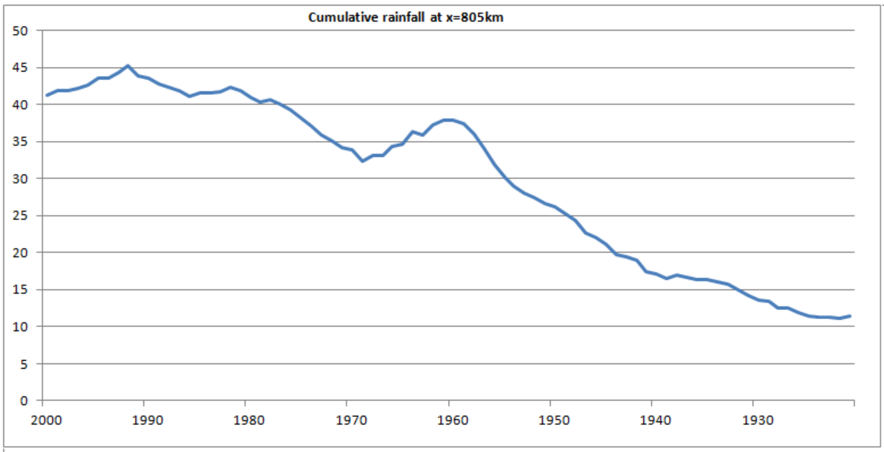

Figure.33 (d), Cumulative rainfall at X=805km,
we examine it from left to right; from 2000km to 1921km.

For the X=805km cross section we notice the same behaviour as in that of X=800km. Hurst Exponent (Fig.33a) doesn't change its descending course when crossing the first mountain range (Y=1973km) while in the second mountain range (Y=1950km) starts a big ascending, as we examine the figures from right to left. At the foothill area that follows (Y=1942km) the Hurst Exponent stays stable.

The two mountain ranges mentioned (Y=1973km and Y=1950km) are points where Hurst Mobility (Fig.33 b) curve rises up and still rises even upon the foothill area of Y=1942 till Y=1921km. The Hurst Complexity (Fig.33 c) on the other hand seems to be stable at the first mountain range (Y=1977km) but starts descending course when cross the other one (Y=1950km). The comments of this paragraph are on the same way as in X=800km.

The area between the mountain crests is where Hurst Exponent shows the lowest values in this event while the Hjorth mobility and Complexity show small increasing and decreasing curves respectively.

The cumulative rainfall (Fig.33 d) is stable descending at this examine area and makes a big uprising loop between Y=1970km and Y=1956km almost above the first big mountain range. As we explained in X=800km analysis this is natural due to turbulence presence in this area, between mountain ranges.

From all the above the three factors Hurst exponent, Hjorth Mobility and Hjorth Complexity seems to be influenced by the topography.

5. Discussion

Our results show clearly that inside the Rhone Valley region that we examined the Hurst exponent and Hjorth parameters resulting from the local analysis of the corresponding time series produced fro the event under study present a characteristic change in behavior over the regions examined which seem to be strongly related to the variation of the ground elevation and more specifically to the presence of the mountains. It turns out that there is an interaction of the cloud system with the mountain.

High values of the Hurst exponent (larger than 0.5) correspond to high persistence i.e. the system tends to maintain the values of rainfall. A decrease of the values, although they remain in general larger than 0.5, corresponds to the fact that there is a modification of the system dynamics leading to a certain loss of this persistence. An important variation on the ground elevation seems to perturb the cloud dynamics and the corresponding rainfall dynamics leading to the variation of the Hurst exponent observed here. The same seems to hold also for the Hjorth parameters which seem to be influenced by the ground elevation.

This idea is further supported by the fact that Hurst is related to system complexity (not to be confused with the Hjorth complexity) which is related to chaotic behavior of system and its fractal dimension through the relation D=2-H , where H is Hurst and D is fractal dimension [see for example *Gneiting et al.*, 2004]. The value of the fractal dimension concerns the minimum number of variables needed to describe the system under study. Thus a higher value of D corresponds to a more complex system. As we can understand an important variation of the Hurst exponent will lead to important variation of D. A high value of Hurst exponent would correspond to a system with smaller dimension thus less complex. As we have seen the Hurst exponent is higher in regions presenting less ground elevation variations, where one could reasonably think that the system behavior is less complex. However as we have seen the Hurst exponent reduced close to regions with important ground elevation variations leadings as a result to larger D i.e. more complex system behavior.

Finally we could say that there is common behaviour between the Hurst and Hjorth parameter diagrams that come from the rainfall measurements and the different examine elevation areas (plane, foothill and mountain). The loss of persistence means perturbation and this is due to the foothill appearance on the North to South average wind direction at all cases on 06/11/2011.

The conclusion is these parameters behaviour reveals that the foothill presence creates a perturbation in rainfall structure; because of the variation of height rather than the height itself.

Another interesting point is that these parameters i.e. Hurst exponent and Hjorth parameters seem to be promising for the identification of changes in the system behaviour, a fact that could be employed in the analysis of other environmental systems too.

6. FUTURE WORK

In the present study we analysed events (one of them is our examined one) where the rainfall was coming from North to South that almost all maximum rainfall values were concentrated around the mountains and a few only above the plain. This can be seen in Fig.3 and need to be investigated more.

The analysis using the Hurst exponent and Hjorth parameters could be expanded to other events in order to establish in a more clear way the dependence from ground elevations.

A point that needs further investigation is how works this interaction of the cloud system with the mountains and how precisely works such a physical mechanism.

Acknowledgements.
The author thanks both supervisors for their precious help and their strong will to help the author become a good engineer.

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