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Dissertation

# SPECTRUM AND STORAGE CAPACITY MANAGEMENT USING NETWORK ECONOMICS AND OPTIMIZATION METHODS

by

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# Abstract

The last few years we are witnessing an unprecedented growth in user demand for high speed communication and ubiquitous network access. Moreover, novel communication paradigms and applications create an ever increasing volume of data traffic that must be transported in a timely and cost efficient fashion. This impressive transformation of communication networks poses new challenges for their design and management that cannot be addressed with traditional methods of resource over-provisioning and technology upgrading. At the same time, nowadays, it is clear that critical and valuable network resources, such as spectrum and in-network storage, are either underutilized or not adequately exploited. These observations indicate that there is still much space for improvement and call for innovative approaches in network design and network resource management. In this thesis, we propose market-based mechanisms and optimization methods in order to increase utilization of critical network resources and improve the performance of networks in terms of data transfer capability. These mechanisms are of paramount importance for contemporary networks which are often plagued by the lack of coordination and the egotistic behavior of their constituent nodes.

First we focus on *spectrum management*, a resource that is scarce and at the same time remains underutilized in large extent. A prominent proposed solution for this problem is the reform of spectrum allocation policy and the deployment of dynamic spectrum (DS) markets. In these markets, spectrum will be freely traded as a commodity and very often hierarchical structures among the interacting entities will emerge. We study these multi-layer markets and propose a mechanism that increases the allocative efficiency of spectrum. We consider the representative scenario that arises when a governmental agency sells spectrum channels to Primary Operators (POs) who subsequently resell them to Secondary Operators (SOs) through auctions in monopolistic markets. We show that this hierarchical scheme does not ensure a socially efficient spectrum allocation which is aimed by the agency, due to lack of coordination among the entities in different layers and the selfish revenue-maximizing strategy of POs. In order to reconcile these opposing objectives, we propose a pricing-based incentive mechanism, which aligns the actions of the POs with the objective of the agency, and thus leads to system performance improvement in terms of social welfare. The suggested mechanism constitutes a method for regulation which is proved to be of crucial importance for the emergins hierarchical spectrum markets.

Next, we relax the assumption of monopolistic markets and analyze the *competition of wireless services providers* over a common pool of users. We show that lack of information about the actual network capacity and egotistic strategies of operators may induce revenue reduction for the latter or efficiency loss for the market. We assume that users have a reservation utility or, equivalently, an alternative option to satisfy their communication needs. The operators must satisfy these minimum requirements in order to attract clients.

We model the interaction of users as an evolutionary game and the competition among the operators as a non cooperative pricing game. We prove that the equilibriums of both games depend on the reservation utility and the amount of spectrum the operators have at their disposal. Accordingly, we consider the scenario where a regulating agency is able to intervene in the market by tuning these parameters. Different regulators may have different objectives and criteria. We adopt a mechanism design perspective, analyze the various possible regulation methods and discuss their requirements, implications and impact on the welfare of the market and the revenue of the operators.

Apart from the hierarchical spectrum allocation through monopolistic or oligopolistic markets, another crucial aspect of these dynamic spectrum management schemes is that they aim to facilitate *spectrum exchange* among different entities in the same layer. For example, each secondary operator may lease or even exchange its channels to other operators. Similarly, users will be able to exchange bandwidth in order to satisfy their dynamic needs. In this setting, each entity is both a resource provider and a resource consumer. Moreover, these two roles are intertwined since they both presume the consumption of the entity's scarce spectrum resource. We model this problem and propose a dynamic pricing scheme that clears the market and ensures the maximization of social welfare. Through the adoption of proper pricing and allocation rules, selfish behavior is deterred and the actual needs of the market entities are revealed and satisfied.

Market-based network management methods provide incentives to network entities for increasing network resource utilization. Equally important for improving network performance is the exploitation of resources which until now have not been incorporated in the network design. A prominent example is *in-network storage*, a resource that nowadays is very cheap, compared to bandwidth, and available at large scale. We advocate that storage can be used to improve the data transfer capability of dynamic networks, i.e. networks with time varying link capacities, and we identify the conditions under which this improvement is realizable. The basic idea is to temporarily store data at nodes when network conditions (e.g. link capacities) are not favorable for data transmission. We use the technique of time-expanded graphs in order to map the dynamic networks to equivalent static networks. First, we introduce a method that increases the min-cut of the network by adding storage to certain nodes. Next, we propose the conjunction of storage control with routing. We define the max-flow problem in the time-expanded graph and solve it by using a distributed algorithm. The solution constitutes the optimal joint storage control and routing (JSR) policy. Finally, we discuss the importance of available information about current and future network state on performance of the proposed algorithms.

# Related Publications and Unpublished Reports

Some of the ideas presented in this thesis appear in the following publications and technical reports:

1. I. Koutsopoulos, and G. Iosifidis, Distributed Resource Allocation Algorithms for Peer-to-peer Networks, *ACM Valuetools*, Athens, 2008.

2. G. Iosifidis and I. Koutsopoulos, Double Auction Mechanisms for Resource Allocation in Autonomous Networks, *IEEE Journal on Selected Areas in Communications, Recent Advances on Autonomic Networks*, vol.28, no.1, pp.95-102, Jan. 2010.

3. I. Koutsopoulos, and G. Iosifidis, A Framework for Distributed Bandwidth Allocation in Peer-to-peer Networks, *Elsevier Performance Evaluation Journal*, vol.67, no.4, pp.285-298, April 2010.

4. I. Koutsopoulos, and G. Iosifidis, Auction Mechanisms for Network Resource Allocation, *Proceedings of WiOpt/RAWNET*, France, 2010.

5. G. Iosifidis, I. Koutsopoulos and G. Smaragdakis, The Impact of Storage Capacity on End-to-end Delay in Dynamic Networks, *Proceedings of IEEE INFOCOM*, Shanghai, 2011.

6. G. Iosifidis, and I. Koutsopoulos, Challenges in Auction Theory Driven Spectrum Management, *IEEE Communications Magazine*, Vol. 49, No. 8 Aug. 2011.

7. O. Korcak, T. Alpcan, and G. Iosifidis, Collusion of Operators in Wireless Spectrum Markets, *Proceedings of WiOpt*, Germany, 2012.

8. G. Iosifidis, A. K. Chorppath, T. Alpcan and I. Koutsopoulos, Incentive Mechanisms for Hierarchical Spectrum Markets, *Technical Report.* Available online: `http://arxiv.org/abs/1111.4350`

9. O. Korcak, G. Iosifidis, T. Alpcan and I. Koutsopoulos, Competition and Regulation in Wireless Services Markets, *Technical Report.* Available online: `http://arxiv.org/abs/1112.2437`

10. G. Iosifidis, and I. Koutsopoulos, Auction Theory Driven Market Mechanisms for Dynamic Spectrum Management, book chapter in *Mechanisms and Games for Dynamic Spectrum Allocation, Cambridge University Press*, 2013, Editors: T. Alpcan, H. Boche, M. Honig, H. V. Poor, *under preparation.*

# Contents

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# List of Abbreviations

| | | |
|---|---|---|
| AP | .............. | Access Point |
| CDA | .............. | Continuous Double Auctions |
| CO | .............. | Controller |
| DS Markets | .............. | Dynamic Spectrum Markets |
| DTN | .............. | Delay Tolerant Networks |
| ESS | .............. | Evolutionary Stable Strategies |
| FCC | .............. | Federal Communications Commission (USA) |
| FIP | .............. | Finite Improvement Path |
| IP | .............. | Internet Protocol |
| ISP | .............. | Internet Service Provider |
| MF | .............. | Maximum Flow |
| MNO | .............. | Mobile Network Operator |
| MVNO | .............. | Mobile Virtual Network Operator |
| NE | .............. | Nash Equilibrium |
| ODE | .............. | Ordinary Differential Equations |
| OfCom | .............. | The Office of Communications (UK) |
| PO | .............. | Primary Operator |
| PU | .............. | Primary User |
| P2P | .............. | Peer-to-Peer |
| SSA | .............. | Sponsored Search Auctions |
| SO | .............. | Secondary Operator |
| SU | .............. | Secondary User |
| WiFi | .............. | Wireless Fidelity |
| WSP | .............. | Wireless Service Provider |
| VCG | .............. | Vickrey-Clarke-Groves |

# Chapter 1

# Introduction

## 1.1 Motivation

The last few years we are witnessing an unprecedented growth in user demand for high speed communication and ubiquitous network access. The population of users increases continuously and novel applications create an ever growing volume of data traffic that must be transported in a timely and cost efficient fashion through wireless networks. This impressive transformation of networks is manifested and highlighted in white papers released recently by major players of the communications market. Specifically, a report published by Ericsson in November of 2011, [89], predicts that mobile data traffic will grow 10-fold between 2011 and 2016, driven mainly by video, Figure 1·1. Additionally, mobile broadband subscriptions which grew by 60% the last year, are expected to grow from 900 million in 2011 to almost 5 billion in 2016. Interestingly, the report predicts that by 2016, 60% of this traffic will be generated in metro and urban populated areas. This means that data traffic will increase both in volume and in density in certain geographical areas. In a more impressive report released also in 2011, [83], Cisco predicted that global mobile data traffic will increase 26 times until 2016, Figure 1·2. There is clearly a worldwide and aggressively growing public demand for mobile communication and associated services.

These developments pose new challenges for communication networks that cannot be addressed solely by the traditional network resource over-provisioning and technology upgrading methods. Namely, wireless operators in order to cope with this increased demand, must obtain additional spectrum licences, upgrade from 3G to 4G technology and at the same time enhance their network backhaul capacity. This presumes costly investments both in equipment and in network resources. Nevertheless, overcoming the economic obstacles is not enough. Electromagnetic spectrum, which is the cornerstone of wireless networks, is a scarce and difficult to acquire resource, especially in populated geographic areas and spectrum bands of high demand. Even worse, in many cases the additional capacity of upgrading will most probably be outpaced by the ever growing data traffic. For example, according to the predictions of Cisco and Ericsson, 4G capacity will be totally absorbed in less than 4 years. Clearly, there is a need for a fundamental rethinking of network design and for innovative network management methods.

### The Need for Spectrum Policy Reform: Dynamic Spectrum Markets

The first step towards this new communication era is rethinking current spectrum management policies. Today, spectrum is managed by governmental agencies such as the Federal Communications Committee (FCC) in US, and the Office of Communications (Ofcom) in UK which allocate large scale - long term spectrum licences through auctions. However, many recent studies [88], [27], [26], revealed that geographical variations in the utilization of assigned spectrum ranges from 15% to 85%. Despite the increased demand for spectrum, significant amount of it remains idle and unexploited by legitimate owners. Clearly, the coarse and static spectrum management policy hampers the proliferation of wireless networks and services. A proposed solution for this problem is the reform of the spectrum allocation policy and the deployment of dynamic spectrum (DS) markets. It is believed that economic incentives will improve spectrum utilization and enable the satisfaction of users increasing demand.

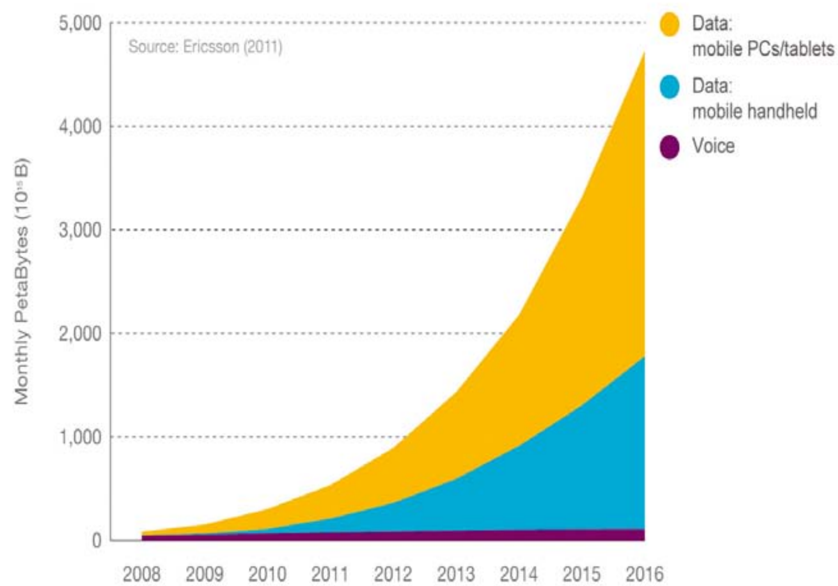



**Figure 1·1:** Ericsson Predicts that Mobile Data Traffic will increase 10 times by 2016 reaching almost 5 billion of broadband subscriptions.

The advent of spectrum brokers (e.g. *Spectrum Bridge* [97]), and the emergence of new business models where - for example - Mobile Virtual Network operators (MVNO) can sell wireless services without investing in costly spectrum licences, constitute the first steps towards this direction. Nevertheless, there is still much to be done and currently there is an ongoing discussion regarding the spectrum management policy of emerging dynamic spectrum markets, [85], [86]. According to the prevalent proposals, in these markets spectrum will be granted in different time scales and for various spatial ranges to operators or directly to users. Channel allocation and spectrum access will be either for exclusive long-term use (primary access) or for low cost secondary access. Regulators will organize auctions for selling spectrum licences to the so called primary operators (PO).

Apart from serving their clients, named primary users (PUs), the POs will additionally lease unused bandwidth to secondary operators (SOs). The latter will be able to serve secondary users (SUs) without the need to invest huge amounts of capital for licences, and also to exchange spectrum bands with each other in order to satisfy their dynamic needs. SUs will be able to form clusters and exchange communication services such as routing each others traffic. Certainly, in these markets there will be many different scenarios for spectrum and bandwidth allocation. The common denominator is the freedom of the various entities to trade and exchange spectrum at their own will.

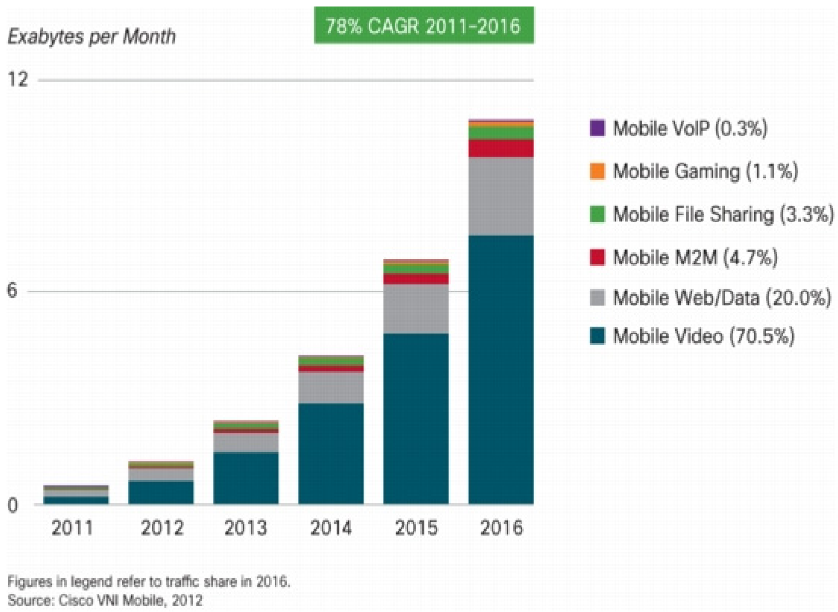



**Figure 1·2:** Cisco Predicts that Mobile Data Traffic will exhibit a compound annual growth rate (CAGR) of 78%, growing 26 times by 2016 and reaching 10.8 exabytes per month. Laptops and smartphones will lead traffic growth.

However, market-based spectrum management is not a panacea and should be carefully designed. For example, the liberalization of the spectrum market will give rise to hierarchical spectrum markets where spectrum will be allocated successively from governmental agencies to the POs and from the latter to the SOs. These schemes are very likely to result in inefficient spectrum allocation due to lack of coordination among the entities in the various layers and the conflict of their interests. Selfish revenue maximizing strategies and monopoly conditions will often bias and degrade the performance of these markets. Clearly, there is need for mechanisms that will alleviate theses problems and ensure the socially efficient allocation of spectrum.

Traditionally, sellers' competition has been employed as a method for leveraging the efficacy of a market. However, in communication markets, like mobile communication services market, this method may not yield the desirable results. Specifically, it is probable to have a fierce competition among operators that will yield decreased revenue for them, or market distortion phenomena such as collusion and price fixing that are detrimental for the users - clients. Again, a proper design of the market is necessitated so as to eliminate such problems. Finally, the particular characteristics of these communication markets such as the dual role of operators as spectrum consumers and spectrum sellers and the need for protocols that will enable real-time decentralized transactions among the various network entities (operators or users), require the development of novel market mechanisms.

### In-Network Storage: Cheaper than Bandwidth

Despite the recent renewed interest for node storage, [42], it is still a network resource that has not been adequately exploited in network design. Nowadays storage is cheap compared to bandwidth, [37], with a decreasing cost as depicted in Figure 1·3, and least space and power requirements. It can be used both in small portable devices and in large amounts located at central communication nodes of backbone networks. Therefore, it is of paramount importance to identify possible methods for enhancing the performance of a network in terms of data transfer capability by exploiting the storage capacity of its nodes. Specifically, we ask the following questions: **(i)** in which ways we can improve the performance of a network by using storage? **(ii)** under what conditions is such an improvement realizable? In this thesis, we analyze when and how the data transfer capability of a network is improved by adding storage at its nodes.

Consider, for example, an Access Point (AP) which transmits data to a mobile (or, simply, wireless) node, Figure 1·4. The storage capacity of the AP can be used to temporarily store data when the link conditions are not favorable for transmission. This way, data is stored closely to the intended receiver and delivered only when proper conditions are met. Failed transmissions and the respective energy consumption is avoided. Obviously, a policy like this improves the performance of the network since data is delivered faster than if it had been stored in the source node. Similarly, in-network storage can be used to improve the data transfer capability of backbone networks. In these networks, link available capacity often varies with time according to a predetermined pattern. A network designer can exploit this information and design proper data store and forward policies that increase the amount of data that can be conveyed within a certain time interval.

Clearly, compared to costly investments in link bandwidth, storage addition is an economical and hence attractive alternative option. Network operators (WSP or ISP) can use storage in order to upgrade their network in various ways. However, storage usage is beneficial only under certain conditions. Namely, exploitation of in-network storage is possible only in dynamic networks where the capacity of links varies with time. For these settings, we devise proper storage control strategies that increase the maximum amount of data the network is able to convey within a certain time interval. In order to realize this performance benefit, storage control must be considered in conjunction with routing. The resulting joint storage management and routing policies increase the performance of the network in a cost-efficient method. The proposed methodology constitutes a valuable tool for network operators and network designers.

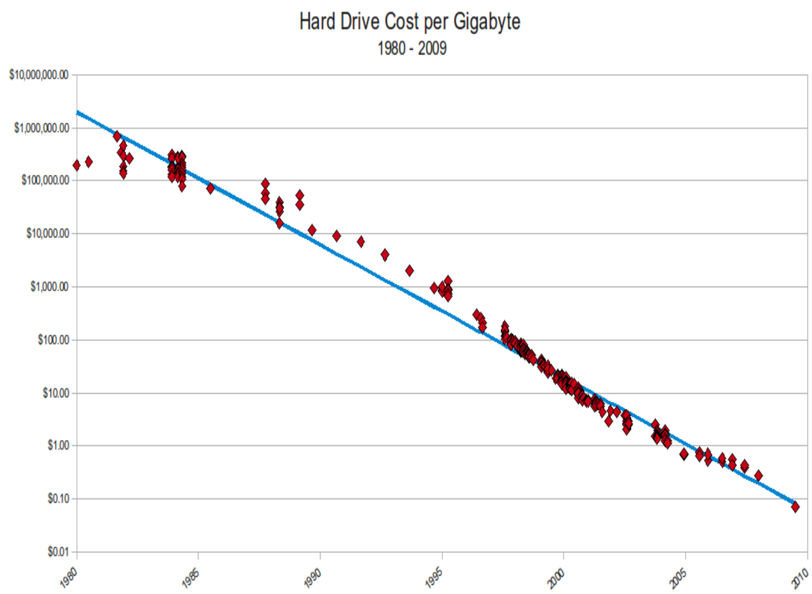


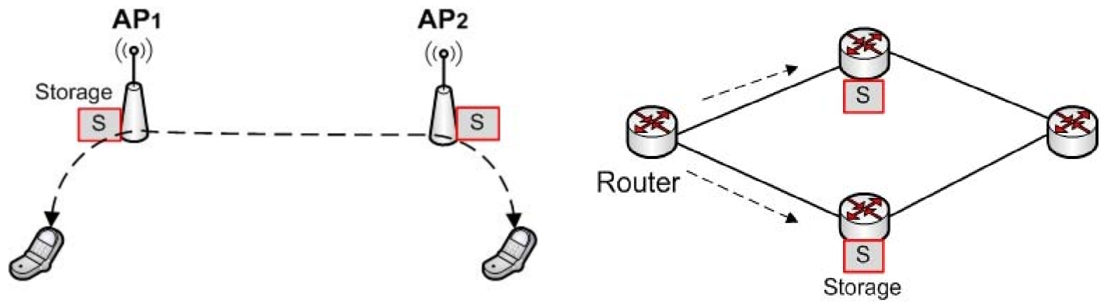

**Figure 1·3:** Evolution of hard drive cost over the last 30 years. A gigabyte cost 10.000 dollars in 1990, 10 dollars in 2000 and 0.1 dollars in 2010.

## 1.2 Synopsis

In this thesis, we propose market-based mechanisms and optimization methods in order to increase utilization of spectrum and exploit the potential of in-network storage. Our goal is to improve the performance of networks in terms of data transfer capability so as to meet the increasing user needs in a cost-affordable fashion. These mechanisms are of paramount importance for contemporary networks which are often plagued by the lack of coordination and the egotistic behavior of their constituent nodes. An overview of the thesis is presented in Figure 1·5.

We begin in **Chapter 2**, with a brief, yet self-contained, introduction in auction theory. Auction-based mechanisms are expected to play a key role in the design of economic-inspired spectrum allocation mechanisms. We identify the fundamental characteristics of auctions, such as the efficiency of the induced allocation of the auctioned items and the produced revenue for the auctioneer. We also highlight the different properties of the various auction algorithms and explain the criteria one should use to select the proper auction for each problem. This chapter introduces the basic auction concepts that are used in this thesis.

In **Chapter 3**, we focus on spectrum management and we study spectrum allocation mechanisms in hierarchical multi-layer markets. These markets are expected to proliferate in the near future according to the evolving spectrum policy reform proposals. We consider the scenario that arises when a governmental agency sells spectrum channels to Primary Operators (POs) who subsequently resell them to Secondary Operators (SOs) through




(a) Wireless network with storage in access points

(b) Backbone network with storage enhanced routers

**Figure 1·4:** Architectures of in-network storage.

auctions in monopolistic markets. We show that these markets do not ensure a socially efficient spectrum allocation which is aimed by the agency, due to lack of coordination among the entities in different layers and the inherently selfish revenue-maximizing strategy of POs. In order to reconcile these opposing objectives, we propose a pricing-based incentive mechanism, which aligns the strategy and the actions of the POs with the objective of the agency, and thus leads to improvement in terms of social welfare. A basic component of the proposed mechanism is a novel auction scheme which enables POs to allocate their spectrum by balancing their derived revenue and the welfare of the SOs. The suggested scheme constitutes a method for hierarchical market regulation which is proved to be of crucial importance especially for monopolistic spectrum markets.

Next, in **Chapter 4**, we relax the assumption of monopolistic markets and analyze the competition among a group of wireless services providers over a common pool of users-clients. A typical scenario is the competition of Primary Operators for attracting clients (primary users). We show that the lack of information about the actual network capacity and the egotistic strategy of operators may induce revenue reduction for the latter or efficiency loss for the market. We assume that users have a reservation utility or, equivalently, an alternative option to satisfy their communication needs. The operators must satisfy these minimum requirements in order to attract clients. This aspect is of particular interest today that users have many options to satisfy their communication needs.

We model the user interaction as an evolutionary game and the competition among the operators as a non cooperative pricing game. The outcomes of both games depend on the reservation utility and the amount of spectrum the operators have at their disposal. We express the market welfare and the revenue of the operators as functions of these two parameters. Accordingly, we consider the scenario where a regulating agency is able to intervene and change the outcome of the market by tuning these parameters. Different regulators may have different objectives and criteria according to which they intervene. We analyze the possible regulation methods and discuss their requirements, implications and impact on the market.

Apart from the hierarchical spectrum allocation through monopolistic or oligopolistic markets, another crucial aspect of these dynamic spectrum management schemes is that they aim to facilitate spectrum exchange among different entities in the same layer. For example, each secondary operator may lease or even exchange its channels to other operators. Similarly, users may exchange their leased spectrum or even create a cluster and route each other traffic in order to satisfy their communication needs in an ad hoc fashion. In these cases each entity is both a resource provider (seller) and a resource consumer (client). Moreover, these two roles are intertwined since they both presume the consumption of the entity's spectrum. In **Chapter 5**, we analyze this setting, and we propose a dynamic pricing scheme that clears the market and ensures the maximization of social welfare. Through the adoption of proper pricing and allocation rules, selfish behavior is deterred and the actual needs of the network entities (operators or users) are revealed and satisfied. The proposed algorithms can be implemented in a decentralized fashion which is a highly desirable property for contemporary networks.

Market-based network management schemes provide the necessary incentives to network entities for increasing network resource utilization. Equally important for improving network performance is the exploitation of resources which until now have not been adequately incorporated in network design methods. A prominent example is in-network storage, a resource that nowadays is cheap, compared to bandwidth, and with least space and power requirements. In **Chapter 6** we advocate that storage can be used to improve the data transfer capability of dynamic networks and we identify the conditions under which this improvement is realizable. We show that storage can reduce the need for link capacity and hence decrease the cost of network deployment. We use the methodology of time-expanded graphs and introduce a technique that iteratively increases the min-cut of the network by adding in-network storage capacity to certain nodes. This method dictates how much storage we must add in each node so as to achieve the maximum possible performance improvement. Additionally, we propose the conjunction of storage with link capacity and derive the joint storage control and routing (JSR) policy that maximizes network flow in the time-expanded graph. This policy ensures that the dynamic network will deliver the maximum possible amount of data within the specific time interval.

Finally, in **Chapter 7** we conclude our study and summarize the main findings of our work. We believe that the concepts and mechanisms proposed in this thesis can contribute to the better utilization of critical network resources such as spectrum and to the exploitation of in-network storage capacity. Additionally, we discuss possible future directions. The presented ideas can be also applied to similar network resource allocation problems, other than the discussed in this thesis.

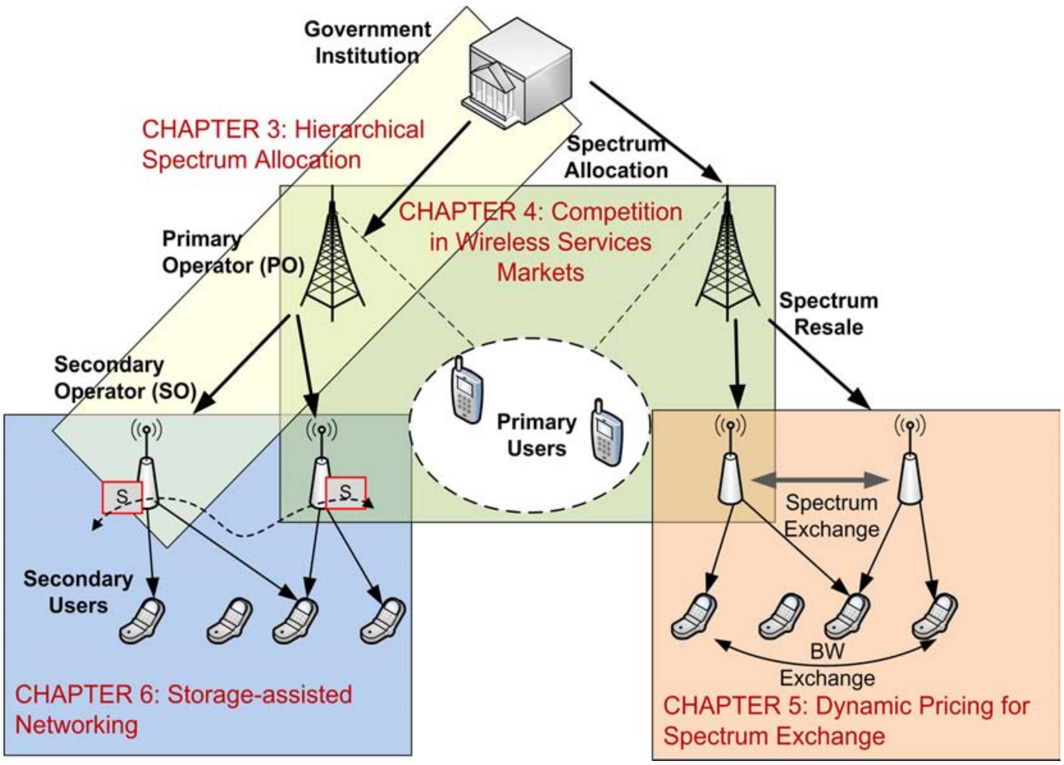



**Figure 1·5:** Spectrum allocation and network entities interaction in emerging dynamic spectrum markets: A state agency allocates spectrum channels to Primary Operators (POs) who compete to attract Primary Users. They also lease their idle spectrum to Secondary Operators. SOs serve their own clients, the Secondary Users, or exchange spectrum with each other. Finally, users are able to exchange bandwidth (BW) by routing each others traffic. Primary and secondary operators exploit in-network storage to increase their data transfer capability.

# Chapter 2

# Overview of Auction Mechanisms for Network Resource Allocation

## 2.1 Introduction

Nowadays, auctions are one of the most popular methods used both by private agencies and *governmental institutions* for selling a variety of items, ranging from antiques to wireless spectrum. The common characteristic in all these cases is that the seller does not know the valuation of the items to potential buyers. In these settings, traditional market mechanisms such as static pricing fail due to lack of this information. For example, if the seller sets very high prices for the items then these will probably remain unsold while low prices will yield low revenue. On the other hand, auctions lead to allocation of items to the buyers with the highest valuations and at the same time to substantial increase of the revenue of the seller. Moreover, auctions require minimum interaction among the sellers and the buyers since the latter simply have to declare their preferences about auctioned items.

While initially auctions were designed empirically, the last few decades game theory has been employed for their study. In 1961, Vickrey introduced the analysis of auctions as games of incomplete information [105]. In these games the players are the buyers who must select the appropriate bidding strategy in order to maximize their perceived utility, i.e. the value of the acquired items minus the payment to the seller. Each buyer is not aware of the valuation of the item for other buyers, and in some cases he cannot even observe their actions (bids). From this perspective, the auction design is a mechanism design problem where the seller-buyers interaction rules must be selected so as to ensure the desirable equilibrium. In most cases, the objective is to allocate the item to the buyer with the highest valuation. Nevertheless, the selection of the proper auction remains an intricate task. Therefore, more often than not, specialized entities, i.e. companies or organizations, undertake the task of designing and running the auctions on behalf of sellers, which are the actual owners of the auctioned goodies.

The last years the interest of communication and network engineers for auctions has substantially been increased. Today is apparent that auctions constitute a suitable mechanism for network resource allocation and network protocol design. Auctions require minimum signaling and information circulation and most importantly they are oblivious to the bidders utilities. Additionally, certain auction schemes can be implemented almost in real-time and decentralized fashion. At the same time, contemporary and emerging networks exhibit certain properties that render them a suitable field of auction theory application. First, future networks comprise diverse interacting *rational entities* with the natural propensity to solicit their own benefit and to strive to obtain maximum benefit from the network, while abstaining from any form of contribution to it. Entities are inclined towards misreporting local parameters that determine a socially optimal, global resource allocation regime. For example, they declare higher needs that real ones, in an effort to extract larger utility from apportioned resources. Such rational behaviors need to be understood through game theoretic models and tamed through mechanisms that deter selfishness and promote good-will cooperation and truthfulness.

Second, the need for *dynamic and decentralized resource allocation* is deemed more important than ever. Very often the management of the resources is to be realized autonomously without central brokers, and in response to rapid spatio-temporal variations in nodes requests. Third and more importantly, control decisions have to be taken with *partial or no knowledge* of parameters of the associated optimization problem. Perfect global network state information may be too costly, impractical or simply impossible or meaningless to obtain. Rapid traffic load changes, interference, and topology or channel quality variations (the latter when it comes to wireless networks) render it difficult for individual nodes to obtain full view even of their own derived utility for different resource allocation regimes. Privacy concerns may also discourage a node from reporting its utility. Careful deliberation is needed to design mechanisms capable of handling such situations as well.

## 2.2 Single-item Auctions

In the simplest form of auctions, there exist a set of buyers who bid to obtain one item and an auctioneer who collects these bids and decides which buyer will get the item and how much he will pay. The components of a typical auction are the allocation rule, the payment rule and the bidding rule, Figure 2·1. The first one determines the allocation of the auctioned item to buyers. Usually, the higher bidding buyer is the one that is awarded the item. The payment rule determines how much each bidder will pay. For example a winning bidder is charged with an amount equal to his bid or to the second highest bid. The basic difference of auctions compared to other similar mechanisms such as pricing schemes is that the allocation and the payment for each buyer depends not only on his bid, but also on bids of other buyers. The bidding rules define the machinery of the auction, i.e. what bids are allowed, whether the bids are sealed or revealed to all participants in the auction, or whether the bidders are able to update their offers in next rounds. Different combinations of these rules result in different auction schemes

Usually, each bidder knows only his own valuation of the item and not those of others. Knowledge of other bidders' valuations during the auction does not change the value of the item for a bidder. This model is one of *private values*. One of the most popular auction schemes is the open ascending price or *English* auction. The auctioneer starts by announcing a low price and keeps increasing it in small steps as long as there are at least two interested bidders. The auction stops if there is only one bidder. In another variant, bidders progressively increase bid offers. The auction ends when only one bidder

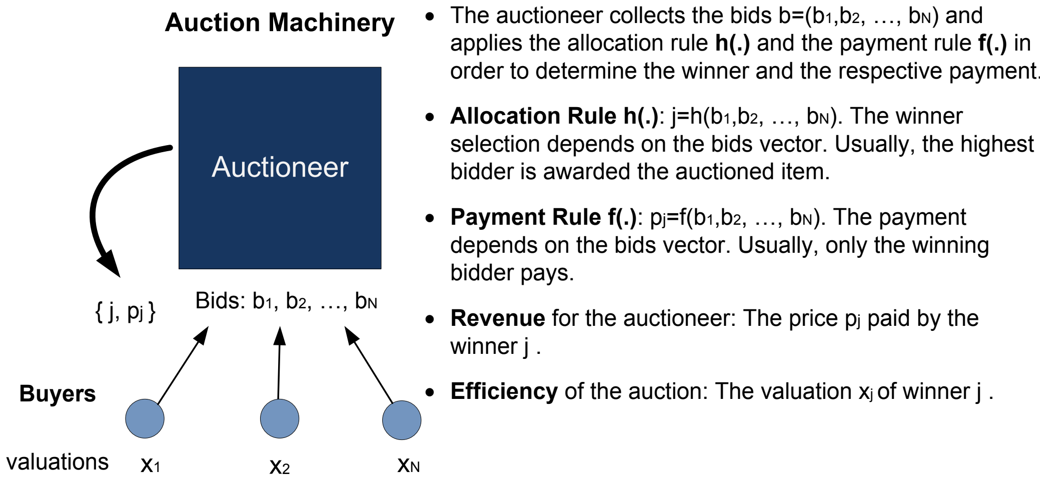



**Figure 2·1:** Auction machinery. The winner determination and the respective payment depend on the entire vector of bids. The revenue of the auctioneer is the payment of the winner. The efficiency of the auction is the total valuation of the allocated items for the winner(s).

remains. That bidder wins the item, and he pays an amount equal to the price at which the second-last bidder dropped out. On the other hand, in the *Dutch* auction, the auctioneer starts by announcing a very high price at which none of the buyers is interested. He then progressively lowers the price until some bidder declares he is interested. That bidder wins the item at that price. English and Dutch auctions are both open-bid auction schemes.

Consider now sealed-bid auctions in which a bidder does not see bids of others. In the sealed-bid *first price* auction, bidders submit bids in sealed envelopes. The buyer with the highest bid wins the item and pays the amount he bid. The sealed-bid *second price* (or Vickrey [105]) auction is similar, except that the highest bidder pays the second-highest bid. Under private values, the open Dutch auction is equivalent to the sealed-bid first price auction: bidding an amount in a first-price sealed-bid auction is the same as offering to buy at that amount in a Dutch auction provided the item is still available. An equivalence relationship holds also between the open English auction and the second-price sealed-bid auction [58].

The *Vickrey* auction has the desirable property that each bidder has no incentive not to bid its true valuation for the item. In other words, each bidder gains by truthfully declaring its true valuation for the item. To understand why this is true, consider bidder $i$ with item valuation $u_i$. Let $b$ denote the highest competing bid of others. Suppose first that $b < u_i$. If bidder $i$ bids $b_i = u_i$, he wins with net payoff $u_i - b > 0$. He does not want to bid $b_i > u_i$, as then he wins but with negative net payoff. On the other hand, if his bid $b_i < u_i$ he reduces the chances of winning the auction and does not affect his net payoff if he wins (which is again $u_i - b$). So on average he reduces expected payoff. Now suppose $b > u_i$. If $i$'s bid is $b_i < u_i$ he does not win (payoff zero), while if $b_i > u_i$ bidder $i$ may win the auction, but with net payoff $u_i - b < 0$. Thus again it is better to bid $b_i = u_i$ to lose the auction and have net payoff zero. Finally, if $b = u_i$, bidding $b_i = u_i$ does not make a difference from $b_i > u_i$ or $b_i < u_i$ (in all cases, net payoff is zero). Taking all into account, it is always to $i$'s benefit to bid $b_i = u_i$ regardless of the competing strategies.

### 2.2.1 Auction design objectives

A first meaningful criterion for performance assessment of auctions is allocative *efficiency*. For one item, this is equivalent to allocating it to the buyer who values it most. This instance arises when a governmental institution auctions a public good, and it is sought to allocate it to the most appropriate bidder. For multiple indivisible goods or one divisible good, efficiency is equivalent to maximizing social welfare incurred by the allocation. Apparently, efficient auctions presume the truthful bidding of buyers.

Another basic criterion is the incurred auctioneer revenue. The auction should be designed so as to increase competition, inducing bidders to participate and submit high bids and increasing expected price at which the item is sold. A well-known method to increase revenue is the adoption of a *reserve price*, namely a minimum (publicly announced) price at which the item is sold. One must counterbalance the risk of not selling the item with the higher payment if the item is sold, in order to compute the optimal reserve price that maximizes expected revenue [58, Ch.2.5]. However, one should keep in mind that any effort to maximize revenue may have undesired effect on allocation efficiency. In other words, maximizing auctioneer revenue and achieving high efficiency of the allocation may be conflicting objectives [72].

For many indivisible goods or one divisible good, *fairness* is another objective, which is related to certain properties of the vector of allocated quantities or the vector of obtained utilities. Other auction design objectives are promotion of truthful reporting of bidder valuations, bidder attraction, discouragement of collusion and simplicity of mechanism [57, Ch.3]. In the sequel we present some basic auction schemes and discuss their efficiency and produced revenue for the auctioneer.

### 2.2.2 Revenue and efficiency for some basic single-item auctions

In Figure 2·2 we present the machinery and the basic properties of three prevalent single-item auctions. We are interested in understanding what is the revenue and the efficiency ensured by each one of them. Assume that each bidder $i = 1, \dots, N$ has valuation $X_i$ for the item, where $X_i$ is a random variable with cumulative distribution function (c.d.f) $F(\cdot)$ and probability density function (p.d.f) $f(\cdot)$, which are common knowledge to all, together with number $N$. Valuations are independent random variables. Functions $F(x) = \Pr(X_i < x)$ and $f(x) = F'(x)$ are the same for all $i = 1, \dots, N$ and defined at some interval $[0, w]$. This model is called one of symmetric bidders. Notice though that in general c.d.f and p.d.f may vary for different bidders. Let $x_i$ denote the realization of each $X_i$ and let $b_i$ be the bid of $i$. Assume that bidders are risk-neutral (see section 2.2.C for a definition of risk-neutrality). Each bidder aims at optimizing its net payoff by adopting a bidding strategy $b_i(x_i)$, with $x_i \in [0, w]$.

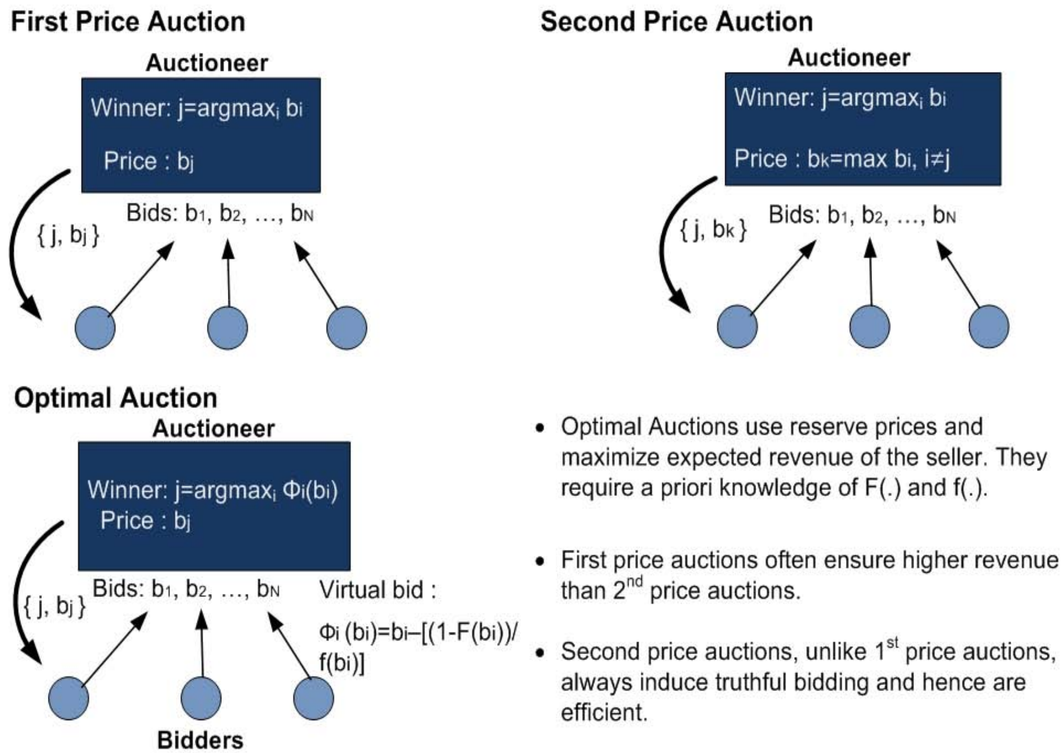



**Figure 2·2:** Comparison of first, second and optimal auction schemes.

### Second-price Auctions

The net payoff $U_i(\cdot)$ of bidder $i$ who participates in the auction with bid $b_i$ is:

$$U_i(x_i, b_i) = \begin{cases} x_i - \max_{j \neq i} b_j, & \text{if } b_i > \max_{j \neq i} b_j \\ 0, & \text{else}\,. \end{cases}$$

Second price auctions are always truthful, i.e. for each bidder bidding its actual valuation, i.e. $b_i(x_i) = x_i$ [58] is a dominant strategy. Therefore, these auctions always ensure the efficient allocation of auctioned items. Let us compute the expected payment by a bidder. Fix a winner, say $i$. Call $X = X_i$ its random valuation, and let $Y = \max_{j \neq i} X_j$ be the second highest valuation (therefore, bid) that will be paid by $i$. Denote by $G(\cdot)$ and $g(\cdot)$ the c.d.f and p.d.f of $Y$. Suppose $x$ is the winner valuation. We wish to compute the conditional p.d.f. of $Y$ given that $i$ wins, $g(y \,|\, Y < X, X = x)$. The conditional c.d.f. $G(y \,|\, x) = \Pr(Y \leq y \,|\, Y < X, X = x)$ is $\frac{\Pr(Y \leq y)}{\Pr(Y < x)}$ if $0 < y < x$, and it is 1, if $0 < x < y$. Then,

$$g(y \,|\, Y < X, X = x) = \frac{g(y)}{G(x)}\,,\; 0 < y < x\,. \tag{2.1}$$

The conditional expected payment given that $i$ wins is:

$$\mathbb{E}[Y \mid Y < X, X = x] = \frac{1}{G(x)} \int_0^x y g(y)\, dy\,.$$

The expected payment for valuation $X = x$ is therefore,

$$\mathbb{E}[Y \mid X = x] = \Pr(Y < x)\, \mathbb{E}[Y | Y < x] = \int_0^x y g(y)\, dy \tag{2.2}$$

with $g(y) = G'(y)$, $G(y) = F^{N-1}(y)$. One may also average over randomness of valuations to obtain the total average payment, $\mathbb{E}[Y] = \int_0^w \mathbb{E}[Y \mid X = x] f(x)\, dx$. Further, the total expected revenue of the seller is $N\mathbb{E}[Y]$.

### First-price Auctions

The net payoff of a bidder is:

$$U_i(x_i, b_i) = \begin{cases} x_i - b_i, & \text{if } b_i > \max_{j \neq i} b_j \\ 0, & \text{else}\,. \end{cases}$$

In [58, Proposition 2.2] it is shown that the optimal symmetric bidding strategy is $b(x) = \mathbb{E}[Y \mid Y < X, X = x]$, where $Y = \max_{j \neq i} X_j$ as before. The expected payment to the seller for given winner valuation $X = x$, is $\Pr(Y < x)\, \mathbb{E}[Y \mid Y < X, X = x]$. This is equal to the expected payment for the second-price auction in (2.2). The same holds for the expected revenue to the seller.

It can be shown that the total expected revenue is equal to the expectation of the second highest valuation for both the first- and the second-price auction. This is known as the *Revenue Equivalence* principle and holds only for risk-neutral bidders and seller, and independent private valuations. In general though, when these assumptions are relaxed the first price auction ensures higher revenue and with less risk for achieving it. However, the auction scheme that yields the maximum possible revenue for the seller is the optimal auction.

### Optimal Auction Mechanisms

Reserve prices increase the revenue of the auctioneer. Myerson, was the first one that systematically studied how they should be selected, [72]. He applied concepts from mechanism design and proposed the so-called *optimal auctions* which ensure the maximum expected revenue for selling one single item. For each bidder $i$ who submits a bid $b_i$, the auctioneer calculates the optimal reservation price by using $F_i(\cdot)$ and $f_i(\cdot)$. This price is subtracted from the actual submitted bid in order to calculate the virtual valuation (virtual bid) $\Phi_i(b_i)$:

$$\Phi_i(b_i) = b_i - \frac{1 - F_i(b_i)}{f_i(b_i)},\ i = 1, 2, \ldots, N \tag{2.3}$$

Given the virtual valuations of the bidders, an optimal auction simply allocates the item to the bidder with the maximum non-negative virtual bid. The winner pays the minimum bid that it is required to deem his virtual bid winning. Therefore, the net payoff for each bidder is:

$$U_i(x_i, b_i) = \begin{cases} x_i - \hat{b}_i, & \text{if } \Phi_i(\hat{b}_i) > \max_{j \neq i}\{\Phi_j(b_j),\, 0\} \\ 0, & \text{else}\,. \end{cases}$$

where:

$$\hat{b}_i = arg\min\{\Phi_i(\hat{b}_i) > \Phi_j(b_j) :\, \forall j \neq i\} \tag{2.4}$$

Although optimal auctions ensure truthful bidding, they may yield inefficient allocation for two reasons. First, if all virtual bids are negative the item remains unsold despite the existence of positive actual valuations. Moreover, in the case of asymmetric bidders, i.e. $F_i(\cdot) \neq F_j(\cdot)$ for $i \neq j$, it is probable that the highest virtual bid will not represent the highest actual valuation [58]. Obviously, there exist a tradeoff among allocative efficiency and revenue maximization in auctions.

### 2.2.3 Some auction classifications

#### Private versus interdependent values

In an auction with *private values*, each bidder knows its own valuation of the item, but he does not know those of other bidders. If a statistical model for valuations is used in an auction with private values, a bidder knows the probability distribution of his own valuation and of valuations of others. In any case, knowledge to a bidder about other bidders' valuations does not affect his own. In auctions of *interdependent values*, each bidder may have full or partial information about its own valuation of the item, however this valuation can be affected by information available to other bidders. A special case is the *common value* model, where the unknown valuation is common for all bidders.

#### Risk-averse versus risk-neutral seller and/or bidders

A seller (or bidder) is *risk-averse* if its utility function $U(\cdot)$ is concave. Assume that a seller runs the auction $K$ times. Say at the $i$-th time, the item is sold at price (bidder payment) $p_i$ and the utility to the seller is $U(p_i)$. Risk averseness in its simplest form means that:

$$\frac{1}{K}\sum_{i=1}^{K} U(p_i) \leq U\big(\frac{1}{K}\sum_{i=1}^{K} p_i\big)\,. \tag{2.5}$$

That is, the average utility from repeating the auction $N$ times (and possibly with different payments) is *less* than the total utility derived with the average payment at all auctions. Namely, payment variability around the mean payment reduces derived utility. A risk-averse seller prefers an auction with more balanced payments, even if this leads to smaller average payments. Similarly, a bidder is risk-averse if its average utility of the difference between valuation and the bid is less than the utility of the difference between the valuation and average bid. If $v$ is the item valuation (assumed to be fixed), $b_i$ are his bids at different auctions times $i$, and $U(\cdot)$ is the utility function, risk-averseness means,

$$\frac{1}{K}\sum_{i=1}^{K} U(v-b_i) \leq U\big(v-\frac{1}{N}\sum_{i=1}^{K} b_i\big)\,. \tag{2.6}$$

A risk-averse bidder prefers to have small average net gains (and thus to bid high on average), rather than having variable net gains (and thus bids). He prefers to win more frequently by bidding high even if his average net gain is smaller. On the other hand, a seller (or bidder) is *risk-neutral* if its utility function is linear. Then (2.5) and (2.6) hold with equality, and variability around the mean does not reduce utility. A first-price auction among risk-averse bidders leads to higher expected revenue for the seller than a second-price auction. First-price auctions are more preferable for risk-averse sellers as well [58, Ch.4.1].

## 2.3 Multiple-item Auctions

In multiple object auctions, multiple items are to be sold. These auctions are classified as *homogeneous* (or *multi-unit*) and *heterogeneous*, depending on whether items are units of the same good, or they are different goods. Homogeneous auctions may be *uniform-price* or *discriminatory-price* ones, depending on whether identical items are sold at the same price or not for different bidders. If items are auctioned one at a time as single-item auctions, the auction is called *sequential.* If all items are sold simultaneously, the auction is called *simultaneous.* Finally, auctions are *individual* if bidders can bid only at one item, and *combinatorial* if bids are allowed to combinations of items [21, Ch.14.2]. Here, we focus on homogeneous auctions.

### 2.3.1 Homogeneous sealed-bid Multi-unit auctions

Consider a simultaneous auction of $K$ identical items to $N$ bidders. Bidders submit bids for acquiring one or more items. Each bidder $i$ submits a bid vector $\mathbf{b}^i = (b_1^i, b_2^i, \ldots, b_K^i)$, such that $b_1^i \geq b_2^i \geq \ldots \geq b_K^i$, where $b_1^i$ is the amount $i$ is willing to pay for receiving one item, $b_2^i$ is the additional amount he is willing to pay for obtaining two units, and so on. Hence, the total amount that bidder $i$ is willing to pay for obtaining $M \leq K$ items is $\sum_{j=1}^{M} b_j^i$.

#### Discriminatory-price auction

In discriminatory-price auctions, the allocation is as follows. Bids $\mathbf{b}^i$, $i = 1, \ldots, N$ are ordered in decreasing order. The $K$ highest bids ($K_i$ of which refer to bidder $i$) are selected, and the $K$ items are allocated so that bidder $i$ obtains $K_i$ of them. Each bidder $i$ pays an amount equal to the sum of his bids that are deemed to be winning, $\sum_{j=1}^{K_i} b_j^i$. Each item is sold at different price.

#### Uniform-price auction

In uniform-price auctions, all $K$ items are sold at a single price (the market clearing price) so that the total demand is equal to total supply. First, the number of items $\tilde{K}_i$ that bidder $i$ wins is computed as follows. For each bidder $i$ with bid vector $\mathbf{b}^i$ (bids in decreasing order), let $\mathbf{c}^{-i}$ be the $K$-vector of competing bids for $i$. This is the vector of the highest $K$ bids out of the bids of bidders other than $i$, arranged in decreasing order. Bidder $i$ gets $\tilde{K}_i$ items if its highest bid exceeds the lowest of the competing ones, the second highest bid exceeds the second lowest of competing ones, and so on until the $\tilde{K}_i$-th highest bid, but this does not hold for the $(\tilde{K}_i + 1)$-th highest bid. The market clearing price turns out to be the highest losing bid over all bidders, $p = \max_i b^i_{\tilde{K}_i+1}$. Note that for $K = 1$, the uniform-price auction reduces to the second-price sealed-bid auction.

#### Vickrey auction

In the Vickrey sealed-bid multi-unit auction, the method to determine the number of items $\tilde{K}_i$ each bidder will obtain is the one above for uniform-price auction. A bidder who wins $\tilde{K}_i$ units pays the sum of the $\tilde{K}_i$ highest *losing* bids in $\mathbf{c}^{-i}$. These are found by removing winning bids of other bidders from $\mathbf{c}^{-i}$ and selecting the $\tilde{K}_i$ remaining ones. It can be seen that the amount that bidder $i$ pays is equal to the *externality* it causes to other bidders. The externality in this case is the additional amount that other bidders would pay in the allocation, had bidder $i$ been absent.

### 2.3.2 Auctions for a divisible resource

Multi-unit auction models also capture auctions of a single divisible good. Each bidder $i$ submits a continuous bid function $b_i(x)$ that indicates the amount he is willing to pay for resource amount $x$. Such a scenario is encountered in network resource sharing, where the good may be link bandwidth, power, energy or another type of resource.

An amount $C$ of divisible resource is to be allocated among $N$ users. Each user $i$ is characterized by a strictly concave, increasing, continuous differentiable utility function $U_i(\cdot)$ which is only privately known to him but unknown to the allocation controller. Let $x_i$ be the amount of good allocated to user $i$ and $\mathbf{x} = (x_1, \ldots, x_N)$ be an allocation vector. The social welfare maximization (SWM) problem is:

$$\max_{\mathbf{x}\geq\mathbf{0}} \sum_{i=1}^{N} U_i(x_i) \tag{2.7}$$

subject to:

$$\sum_{i=1}^{N} x_i \leq C\,. \tag{2.8}$$

If utility functions were known to the controller, the KKT conditions would give the necessary and sufficient conditions for the optimal allocation, $U_i'(x_i^*) = \lambda^*$ and $\lambda^*(\sum_i x_i^* - C) = 0$, where $\lambda^*$ is the optimal Lagrange multiplier for (2.8).

### Proportional Allocation Mechanism

Assume that the controller does not know utility functions $U_i(\cdot)$ but aims at socially optimal allocation. Consider the class of allocation mechanisms where each user submits a bid $b_i \geq 0$ for the amount he is willing to pay and is charged according to function $c(\cdot)$. The amount of allocated good, $x_i(b_i)$ is a function of their bid. Specifically, let $x_i(b_i) = b_i/\tilde{\lambda}$, where $\tilde{\lambda}$ is a price per unit of resource. We assume users are *price takers*, namely they do not consider the impact of their bid on the charge function $c(\cdot)$. It is reasonable to assume that each user is rational and casts his bid so as to maximize his net benefit, $U_i(x_i(b_i)) - c(b_i)$, namely his bid should satisfy:

$$U_i'(x_i^*)\frac{1}{\tilde{\lambda}} - c'(\tilde{b}_i) = 0\,. \tag{2.9}$$

Suppose the controller obtains bids $\tilde{b}_i$ and makes the allocation according to the solution of the following problem (P):

$$\max_{\mathbf{x}\geq\mathbf{0}} \sum_{i=1}^{N} \tilde{b}_i \log x_i\,, \tag{2.10}$$

subject to $\sum_{i=1}^N x_i \leq C$, and $x_i \geq 0$, $i = 1, \dots, N$. The KKT conditions for this problem give:

$$\frac{\tilde{b}_i}{\tilde{x}_i} = \tilde{\lambda}, \tag{2.11}$$

where $\tilde{\lambda}$, $\tilde{\mathbf{x}}$, is the optimal Lagrange multiplier and the optimal solution respectively of (P). The goal is to equalize the solutions of optimization problems (SWM) and (P). It turns out that if each user is charged according to $c(b_i) = b_i$, then from (2.9), (2.11) it is $\lambda^* = \tilde{\lambda}$ and $x_i^* = \tilde{x}_i$, which gives $\tilde{b}_i = x_i^* U'(x_i^*)$.

Since the optimal solution to (P) should satisfy $\sum_{i=1}^N \tilde{x}_i = C$, by using (2.11) we get $\tilde{\lambda} = \lambda^* = \frac{1}{C}\sum_{i=1}^N \tilde{b}_i$. This is the market clearing price, set by the controller. Furthermore,

$$\tilde{x}_i = \frac{\tilde{b}_i}{\sum_{i=1}^N \tilde{b}_i} C, \tag{2.12}$$

namely the allocated amount to each user is proportional to its bid [54]. Therefore, socially optimal resource allocation can be achieved by bidding (where each user's bid is a single number), and an appropriate charging scheme.

Kelly *et.al.* proposed this mechanism and showed that the problem above can be solved in a decentralized fashion [55]. The market clearing price $\lambda^{(n)}$ is iteratively computed at each step $n$ by the auctioneer according to a standard dual algorithm. Essentially, it is increased or decreased, depending on whether the instantaneous allocation exceeds $C$ or not. Then, each user adjusts its bid according to $U_i'\left(\frac{b_i}{\lambda^{(n)}}\right) = \lambda^{(n)}$. The dual price update together with the user response converges to the optimal solution of the network utility maximization problem. This algorithm is a distributed implementation of the bidding mechanism. The moral of the story is that for price-taking users, one-dimensional bids and appropriate charging lead to efficient allocation.

### Vickrey-Clark-Groves (VCG) mechanism

Consider now achieving an efficient allocation if users are *price-anticipating*, namely they strategically adapt their bid by taking into account its impact on the price so that they maximize net profit. In that case, a game interaction emerges with certain efficiency loss. The setup is the same as the one above, and each user chooses his bid to maximize the quantity:

$$U_i\Big(\frac{b_i}{\sum_{j=1}^N b_i}C\Big) - b_i\,. \tag{2.13}$$

Notice that now user $i$ explicitly understands that the price, $\frac{1}{C}\sum_{i=1}^N b_i$ depends also on its own bid $b_i$. A mechanism that guarantees an efficient allocation for selfish, price-anticipating users is the Vickrey-Clarke-Groves (VCG) mechanism [20],[38]. This is a generalization of the Vickrey mechanism for single item auctions. Here, the compromise is that the auctioneer requests each user to *reveal* its utility function. In the VCG mechanism, the amount charged to each user $i$ is the externality it causes to others. This is the total utility reduction caused by $i$ to all other users, and it is computed as follows. Let $\mathbf{x}^*$ be the optimal solution to (SWM) problem, and let $\bar{\mathbf{x}}$ be the optimal solution to the (SWM) problem *without* considering the effect of user $i$, namely to problem $\max_{\mathbf{x}} \sum_{j\neq i}^N U_j(x_j)$, such that $\sum_{j\neq i}^N x_j \leq C$. The charge to user $i$ is:

$$p_i = \sum_{j\neq i}^N U_j(\bar{x}_j) - \sum_{j\neq i}^N U_j(x_j^*)\,. \tag{2.14}$$

In the VCG mechanism, declaration of the true utility function $U_i(\cdot)$ is the best strategy for each user [94, Ch.6]. Namely, a user $i$ cannot do better by misreporting its utility function. To see this, observe that the net profit for a user $i$ that declares its true utility function is,

$$U_i(x_i^*) - p_i = \sum_{i=1}^N U_i(x_i^*) - \sum_{j\neq i} U_j(\bar{x}_j)\,. \tag{2.15}$$

Suppose now that user $i$ misreported its utility function and declared it as $\tilde{U}_i(\cdot)$ in an effort to get more profit. In that case, there would be a different solution (call it $\tilde{\mathbf{x}}$) to the (SWM) problem, and the profit of user $i$ would be

$$U_i(\tilde{x}_i) - \tilde{p}_i = \sum_{i=1}^N U_i(\tilde{x}_i) - \sum_{j\neq i} U_j(\bar{x}_j)\,. \tag{2.16}$$

If truthful reporting of utility were not optimal, (2.16) should exceed (2.15), which would mean $\sum_{i=1}^{N} U_i(\tilde{x}_i) > \sum_{i=1}^{N} U_i(x_i^*)$. This contradicts the fact that $\mathbf{x}^*$ is the optimal solution of the (SWM) problem. Thus, truthful reporting is optimal under VCG. The VCG mechanism leads to efficient allocation. Its clear drawback is that each user needs to submit to the auctioneer its entire utility function, namely an infinitely dimensional vector, which renders the mechanism quite complex and burdensome in terms of information exchange. Additionally, finding the optimal allocation and the VCG prices is a computationally difficult task. Especially if the auctioned items are discrete, non-homogeneous, the respective optimization problems become NP-hard.

### 2.3.3 Optimal Auctions

The single-item optimal auction introduced by Myerson, was later extended for one perfectly divisible item by Maskin, [69], and for the case of multiple homogeneous items by Branco, [12]. We will focus on the latter mechanism. In this scheme, the auctioneer needs again some initial information about the demand of bidders. Namely, the auctioneer needs to know the family of the bidders' utility functions $U(\alpha_i)$ and the distribution function $F(\cdot)$ of their types, $\alpha_i$, $i = 1, 2, \ldots, N$. The buyers submit a single bid $b_i$ to declare their types. The auctioneer combines this bid with the prior information and calculates the additional expected revenue he will receive by assigning a certain item, e.g. the $k^{th}$ item out of $K$, to a certain buyer, e.g. the $i^{th}$ buyer. This amount is known as the *contribution* of the bidder for buying the $k^{th}$ item, and is defined as:

$$\pi_k(b_i) = U_k(b_i) - \frac{dU_k(\alpha)}{d\alpha}\bigg|_{\alpha=b_i} \frac{1 - F(b_i)}{f(b_i)} \tag{2.17}$$

where $F(\cdot)$ and $f(\cdot)$ are the cdf and pdf of the buyers. If these contributions are monotonically strictly increasing in the types of the buyers and decreasing in the number of the auctioned items, then they satisfy the so-called *regularity conditions*, [12], and the auction is called *regular*. In this case the item allocation that maximizes the expected revenue of the seller can be easily derived using the following deterministic allocation and payment rules.

#### Optimal Auction Allocation Rule

The auctioneer calculates the contributions $\pi_k(b_i)$ of each buyer $i \in \mathcal{N}$ for all the auctioned items, $k = 1, 2, \ldots, K$ and selects the $K$ highest of them. In the sequel, he constructs the contribution vector $\mathbf{X}_K$ which has $K$ elements in decreasing order:

$$\mathbf{X}_K = (x_{(l)} : x_{(l)} > x_{(l+1)}, \, l = 1, \ldots, K) \tag{2.18}$$

Then, the auctioneer simply assigns each item $k = 1, \ldots, K$ to the respective $i^{th}$ bidder if $x_{(l)} = \pi_k(b_i)$

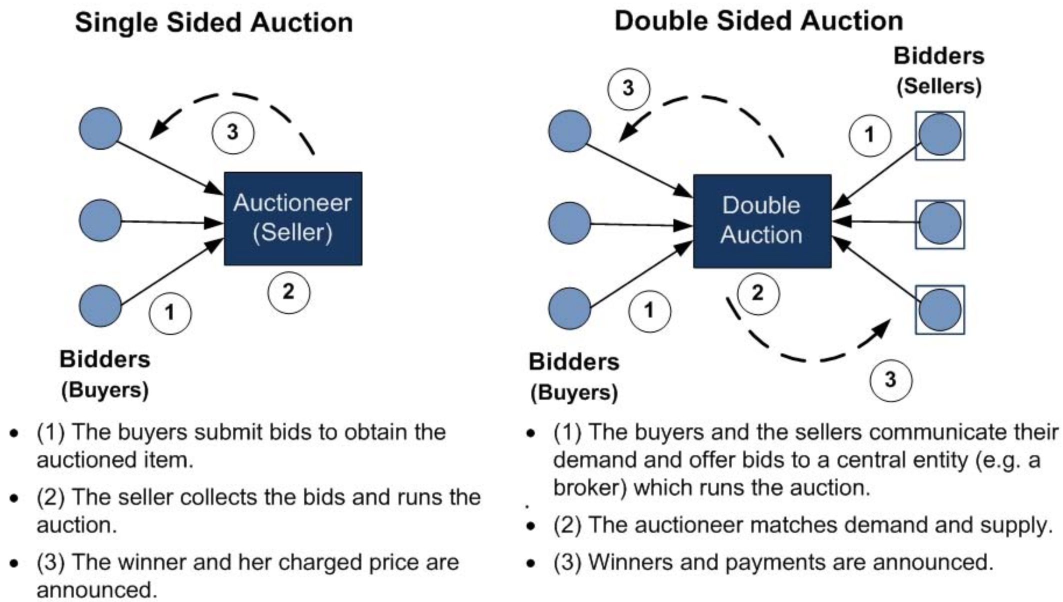



**Figure 2·3:** Single and double sided auctions. In single-sided auctions the auctioneer sells or asks for an item (procurement auctions). In double-sided auctions there exist many buyers and many sellers interacting concurrently. A centralized entity collects the offer and the ask-bids and runs the auction algorithm to determine the allocation of the items and the respective payments.

### Optimal Auction Payment Rule

The price that each bidder $i$ pays for receiving the $k^{th}$ item depends on the bids submitted by all the other bidders, $b_{-i}$. Let us denote with $z_k(b_{-i})$ the minimum bid that the $i^{th}$ buyer has to submit in order to acquire the $k^{th}$ item, [12]:

$$z_k(b_{-i}) = \inf\{\hat{\alpha}_i : \pi_k(\hat{\alpha}_i) \geq \max\{0, x_{(K+1)}\}\} \tag{2.19}$$

This means that in order to get the $k^{th}$ item the $i^{th}$ bidder has simply to submit a bid high enough to draft his contribution within the first $K$ elements of $\mathbf{X}_K$. The actual charged price for each item is equal to his valuation had he a type equal to this minimum bid. Hence the aggregate payment for all the items the $i^{th}$ bidder receives is:

$$h(b_i, b_{-i}) = \sum_k U_k(z_k(b_{-i})) \tag{2.20}$$

### 2.3.4 Double Auctions

In case there exist more than one sellers and there is lack of information both about demand and supply, it is required to employ double auction mechanisms. The sellers compete with each other in order to attract the buyers, while the latter are able to place bids to several sellers. These markets are usually cleared by an independent institution, that undertakes the role of the auctioneer. We depict the basic machinery of auctions in Figure 2·3. The buyers submit *ask bids*, revealing the amount of money they are willing to pay. Similarly, the sellers submit *offer bids* indicating the minimum offer they are willing to accept. The task of the auctioneer is to collect all the bids, determine winning sellers, allocate the items from sellers to buyers and compute the prices each seller must be paid and each buyer must be charged.

Designing a double auction is an intricate task and the related literature is quite restricted. One of the most prevalent double auction schemes is the McAfee double auction model, [70] that ensures efficient allocation of items in dominant bidders strategies. However, it is not always possible for the auctioneer to mach requests and offers and at the same time ensure the desirable property of truthful bidding. Double auctions can be asynchronous, also called Continuous Double Auctions (CDA), [21], or synchronous. In the former case, the ask and offer bids can be submitted or retracted anytime and unilaterally. On the contrary, in synchronized auctions, the submitted bids are binding and active until the market is cleared by the auctioneer. Double auctions are used extensively in stock and other commodities markets.

## 2.4 Sponsored Search Auctions

A particularly interesting class of auctions are the *sponsored search auctions* (SSA), or *keyword auctions*, which are used by web search engines for Internet advertising. In these auctions, bidders are the advertisers who wish to have the advertisement of their company appearing on a user's search results screen after the user types a related keyword. When they register their ad with the search engine, they provide keywords related to their ad. Following a keyword search by an Internet user, the system finds a set of ads with keywords that match the user query. Advertisements appear in the search results as a ranked list. The user clicks on an advertisement and it is taken to the advertiser's website. The advertiser then pays the search engine company for guiding the user to its web page. Advertisement positions (ranked slots) on search results are clearly of high importance to advertisements; the higher the ad is displayed on the list, the more probable it is that it will be clicked by the user, and the more likely it becomes that the advertiser will get some profit if the user buys the product or service. In SSAs, ranked advertisement positions are auctioned to advertisers. An advertisement is considered successful if a user clicks on the respective ad link. Advertisers pay an amount each time a user clicks on their ad.

The underlying feature of previously presented auctions is that there exist two parties, the auctioneer and the bidders, who determine the rules of the auction. Bidders cast their bids, and the auctioneer determines the allocation of items and the payment. Clearly, ad auctions are different in that the *auctioneer revenue and bidder payoff depend on a third entity*, the Internet user. This idea is similar to the well known *score auctions* where the bids are weighted with various parameters which are characterize the quality of the bidders. However, this is the first time that such schemes are applied so extensively. Google, Yahoo! and other search engines auction advertising positions using this class of auction mechanisms. Google [35] was the first to consider the dependency of position allocation and payment on user preferences. We are inspired by these auctions and use a similar mechanism in chapter 3.

Let us now give a brief overview of the SSAs machinery. Consider $N$ advertisers who bid for $K < N$ ad slots for a specific keyword. Let $u_i$ be the value of the ad for advertiser $i$, $i = 1, \ldots, N$. Let $b_i$ be the bid of advertiser $i$ and $p_i$ be the payment per click he will be charged. The auctioneer collects submitted bids and needs to decide which bidders will have their ads shown, in which order and the respective payments. Let $c_{ij}$ denote the probability that the ad of advertiser $i$ will be clicked by the user when in position $j$, $j = 1, \ldots, K$. This is also called click-through rate (CTR) and can be calculated by the search engine based on history statistics with various methods [73, Ch.3]. CTR depends on the ad of advertiser $i$ and the position $j$ and can be assumed to be $c_{ij} = \alpha_i \beta_j$, where $\alpha_i$ an ad-dependent parameter, the per-ad CTR (the ratio of number of clicks received by the ad over the number of times the ad was displayed). It is $\alpha_i = \sum_{j=1}^{K} c_{ij} y_{ij}$, where $y_{ij}$ is the probability that ad $i$ is displayed in position $j$. Also, $\beta_j$ is a position-dependent parameter, the per-position CTR. Higher ranked positions are more visible to users and attract more attention, so that $\beta_1 > \ldots > \beta_K$.

The auction goes as follows. Each advertiser $i$ chooses a bid $b_i$. Ads of advertisers appear in ad slots in decreasing order of their weighted bid, $b_i \alpha_i$. The advertiser in the $k$-th position, say with weighted bid $b_{(k)} \alpha_{(k)}$ pays a total amount equal to the weighted bid of the advertiser in the next position $k+1$, that is, total amount $b_{(k+1)} \alpha_{(k+1)}$. Hence, the amount paid per click is $p_{(k)} = b_{(k+1)} \alpha_{(k+1)} / \alpha_{(k)}$. The last ranked advertiser either pays a reserve price if $N < K$ or the amount of bid of the first omitted advertiser if $N > K$. This payment rule is a generalization of the one in Vickrey auction for one item, generalized to the setting where a set of ranked items are sold. Thus, the auction is often referred as *Generalized Second Price* (GSP) auction [25], [104]. The position allocation rule naturally ranks bidders in a decreasing order of expected revenues.

The probability that a user will click on an ad is a key factor to consider. Otherwise, less attractive ads will be displayed, and small revenue for the auctioneer will be incurred. Assume that bidders are risk-neutral. The net payoff for advertiser $i$ when his ad is displayed in position $j$ is $c_{ij}(u_i - p_i)$, where $p_i$ is the payment per click. From the perspective of the auctioneer, the problem is to find the position allocation that maximizes expected revenue [73]:

$$\max_{\mathbf{X}} \sum_{i=1}^{N} \sum_{j=1}^{K} p_i c_{ij} x_{ij} \tag{2.21}$$

subject to $\sum_{j=1}^{K} x_{ij} \leq 1$, for $i = 1, \ldots, N$, and $\sum_{i=1}^{N} x_{ij} = 1$ for $j = 1, \ldots, K$, where $\mathbf{X}$ is the $N \times K$ assignment matrix with $x_{ij} = 1$ if the $i$-th advertiser is allotted position $j$, and $x_{ij} = 0$ otherwise. If the auctioneer wants to maximize efficiency, he solves the assignment problem:

$$\max_{\mathbf{x}} \sum_{i=1}^{N} \sum_{j=1}^{K} u_i c_{ij} x_{ij} \tag{2.22}$$

subject to the allocation constraints above. The auctioneer does not know valuations $u_i$ and uses submitted bids, which may differ from valuations. For both revenue maximization and allocation efficiency, it is crucial to consider CTRs. The allocation of a slot does not generate utility for the advertiser and revenue for the auctioneer unless the ad link is clicked.

# Chapter 3

# Incentive Mechanisms for Hierarchical Spectrum Markets

## 3.1 Background

In the emerging dynamic spectrum markets, spectrum will be allocated from governmental agencies in a finer spatio-temporal scale to the interested buyers, the so-called primary operators (POs), [85], and more importantly, the POs will be able to lease their idle channels to secondary operators (SOs), [86], who serve fewer users in smaller areas, Figure 3·1. This 2-layer hierarchical spectrum allocation is expected to increase spectrum utilization and already several related business models exist in the market. For example, Spectrum Bridge, a company founded in 2007 offers a market place where spectrum licence holders are able to lease or resell their idle channels, [97]. Also, Mobile Virtual Network Operators (MVNO), which first appeared in UK in 1999, lease spectrum (or/and network infrastructure) from Mobile Network operators (MNO) and use it to provide communication services to their clients (users). Nevertheless, the market-based solution for the spectrum scarcity problem is not a panacea and should be carefully employed.

In these hierarchical markets, the objective of the agency, which we call hereafter *controller* (CO), is to allocate the spectrum efficiently, i.e. to maximize the aggregate social welfare from its use. However, this objective cannot be achieved because of the following reasons: (i) the **coordination problem**, and the (ii) **objective misalignment problem**. The first problem emerges when the CO assigns the spectrum to the POs without taking into account or even knowing the needs of the SOs (secondary demand). The second problem arises due to the inherently selfish behavior of POs who resell their spectrum in order to maximize their revenue. Clearly, this strategy contradicts the goal of the controller.

In this chapter we study the spectrum allocation in these hierarchical markets and propose an *incentive mechanism* that enhances their performance by addressing the above two issues. The mechanism is deployed by the controller who acts as *regulator* and incentivizes the POs to redistribute their spectrum in a socially aware fashion. We consider a basic setting depicted in Figure 3·1, where each PO is a monopolist and has a certain clientele of SOs. Monopolies are expected to arise often in these markets because the POs obtain the exclusive spectrum use rights for certain areas or because they collude and act effectively as one single seller. First, we analyze the performance of the unregulated hierarchical market, i.e. when there is no incentive mechanism, and we show that it results in an undesirable equilibrium. The spectrum allocation from the CO to the POs and from the POs to the

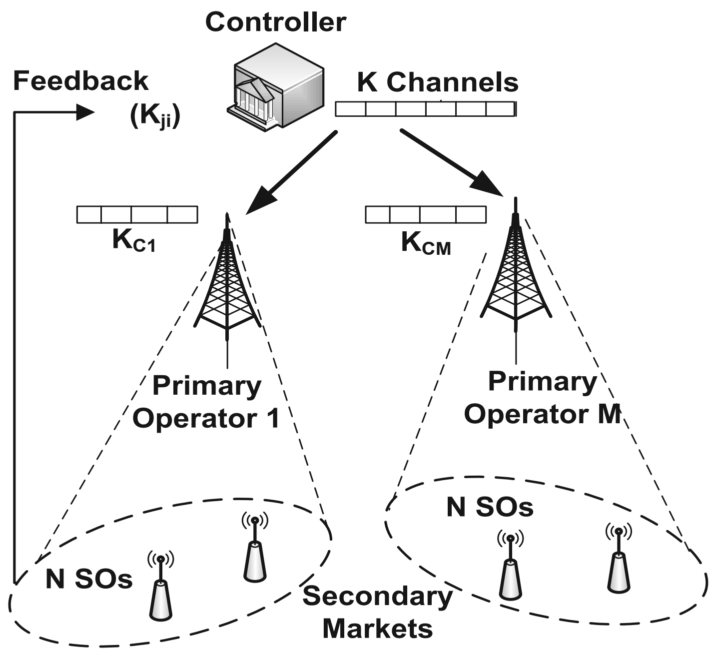



**Figure 3·1:** The system consists of one Controller which has at his disposal $K$ identical channels. There exist $M$ primary operators which ask for spectrum. Each PO $j$ acquires $K_{cj}$ channels and uses $K_{j0}$ of them to satisfy the needs of his own users and resells $K_{ji}$ channels to each SO $i$ in the underneath secondary market. There are $N$ SOs in the monopoly market under each PO which provide feedback to the CO for the decisions of the POs.

SOs is accomplished through auction-based mechanisms since there is lack of information about the spectrum demand. Namely, the CO uses an efficient auction such as the VCG auction, [58], while the POs employ an optimal auction, [72], which maximizes the expected revenue of the seller but induces efficiency loss, [1], [69].

Accordingly, we propose an incentive mechanism based on which the CO charges each PO in proportion to the inefficiency that is caused by his spectrum redistribution decisions. This way, the POs are induced to allocate their spectrum using a new auction scheme which produces less revenue for them but more welfare for the SOs. This is a *novel multi-item auction mechanism* where the objective of the auctioneer is a linear combination of his revenue and the valuations of the bidders. The balance between the objective of the POs and the SOs is tuned by a scalar parameter which is determined by the CO and captures his regulation policy. Finally, we apply our mechanism to dynamic spectrum markets where the CO-PO and the PO-SO interactions are realized in different time scale. Although in this case the coordination problem is inherently unsolvable, the proposed scheme still improves the performance of the market by aligning the decisions of the POs with the objective of the CO.

### 3.1.1 Motivating Example

We start with a simple example in order to motivate our study and present the intuition behind our mechanism. Consider the 2-layer spectrum market depicted in Figure 3·2. The Controller has 3 channels at his disposal which are initially allocated to the POs. We assume that each operator, PO or SO, is interested only in one channel. The POs and SOs valuations for acquiring one channel are: $\{V_1, V_2, U_1, U_2, U_3, U_4\} = \{8, 9, 3, 5, 2, 8\}$. Each PO allocates the channels he acquires through a Vickrey auction. The maximum revenue the $PO_1$ can accrue is equal to 3 units from reselling one channel to $SO_2$. Similarly, the maximum revenue of $PO_2$ for reselling one channel to $SO_4$ is 2 units. Therefore, the bids of $PO_1$ and $PO_2$ for the first 2 channels are $b_1 = \{8, 3\}$ and $b_2 = \{9, 2\}$ respectively.

The CO organizes a truthful auction and allocates each channel to the PO with the highest respective bid. $PO_1$ receives 2 channels, one of which is resold to $SO_2$, and $PO_2$ 1 channel which is used for his own needs. The final channel allocation is depicted in Figure 3·2 marked with red-coloured squares (each square represents one channel) and yields a social welfare of $SW = V_1 + V_2 + U_2 = 8 + 5 + 9 = 24$ units. Clearly, this is not an efficient allocation since $SO_4$, who does not receive a channel, has a higher channel valuation ($U_4 = 8$ units) from $SO_2$ ($U_2 = 5$ units). This efficiency loss is induced by the strategy of the POs who bid in accordance with their anticipated revenue and not with respect to the actual spectrum needs of their secondary markets. This results in an inefficient allocation of the channels in the first stage: $PO_1$ gets more channels than $PO_2$ although the latter has higher secondary demand. It is important to emphasize that, even if the channels are assigned efficiently in the first stage, still the selfish, revenue-maximizing allocation strategy of the POs, induces efficiency loss as we will explain in the next sections.

Let us now give the intuition behind the proposed mechanism. Assume that the controller is willing to increase the efficiency of the secondary market. Specifically, the CO decides to reimburse the POs in proportion to the welfare of the secondary market that is produced by their channel re-allocation decisions. This way, the CO expects to motivate the POs to reassign the channels efficiently. In this case, each PO receives for each channel that he resells the price paid by the SO and, additionally, the reimbursement from the CO. That is, the CO transfers to each PO, $\beta U_i$ monetary units, with $\beta > 0$, for every channel he allocates to each $SO_i$. This pricing scheme affects the bidding strategy of the POs and the outcome of the hierarchical allocation. Specifically, for the example of Figure 3·2 with $\beta = 0.4$, if $PO_1$ allocates a channel to $SO_2$ he will receive a payment of 3 units from $SO_2$ and a reimbursement of $5 \times 0.4 = 2$ units from CO. Hence, the updated bid of $PO_1$ is $b_1 = \{8, 5\}$. Similarly, the updated bid of $PO_2$ is $b_2 = \{9, 5.2\}$. Therefore, the CO allocates 2 channels to $PO_2$ and 1 channel to $PO_1$ which is the efficient channel allocation.

### 3.1.2 Related Work and Contribution

Primary operators are considered revenue maximizing entities and hence they are expected to use an optimal auction mechanism since this yields the maximum expected revenue. Optimal auctions were introduced by Myerson [72] for single item allocation and

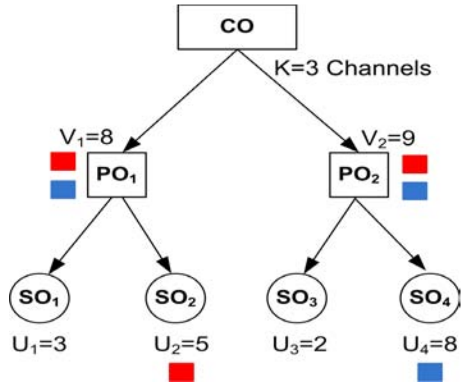



**Figure 3·2:** A 2-layer spectrum market where the CO has $K = 3$ channels. Each operator, PO or SO, has a single-unit demand with valuation: $\{V_1, V_2, U_1, U_2, U_3, U_4\} = \{8, 9, 3, 7, 2, 8\}$. First, the POs organize a second-price auction where the SOs reveal their actual valuations. Then the CO runs a second price auction where POs bid w.r.t their valuation for the channel and the expected revenue from their secondary market. This scheme yields an inefficient spectrum allocation that is marked with the red colored squares (each one represents a spectrum unit). The allocation that maximizes the social welfare is marked with the blue-coloured squares.

extended later for multiple items, [12], or divisible resource, [69]. They ensure the highest expected revenue for the auctioneer, compared to any other type of auction, but they induce efficiency loss, [1], [58]: it is not guaranteed that the auctioned items will be allocated to the bidders with the highest valuations. On the other hand, VCG auctions constitute the best option for achieving an efficient allocation under a variety of settings and assumptions. However, they often exhibit a high computational complexity that makes their implementation an extremely difficult -if not impossible- task. Additionally, they are not always budget balanced: the sum of the payments does not sum up to zero. This means that the auctioneer will need to inject additional money into the market. One should take this issues into consideration before deciding to employ a VCG-based mechanism, [67]. Nevertheless, VCG auctions have the following desirable properties:**(i)** they are as budget balanced as any efficient mechanism can be, **(ii)** they are weakly budget balanced, i.e. the sum of payments is positive, if the *no-single agent effect* condition is satisfied, **(iii)** they produce the highest revenue among all other efficient auctions, [58], **(iv)** their complexity can be reduced significantly under certain assumptions, e.g. if the auctioned items are homogeneous.

The interaction of primary and secondary operators is usually modeled as a monopoly market. For example, in [47] the authors consider a setting where each primary license holder sells his idle spectrum channels to a set of secondary users and show that the optimal auction yields higher profit but results in inefficient allocation. A similar monopolistic setting is considered in [32] and [36]. In [19], a multiple-item optimal auction is used by a primary service provider to allocate his channels to a set of secondary service providers while satisfying at the same time his own needs. It can be argued that even in oligopoly spectrum markets is highly probable that the PO - SO interaction will result in spectrum allocation that is not efficient from the perspective of the controller, [21]. All these works analyze exclusively the primary - secondary operators interactions without taking into account the hierarchical structure of the spectrum markets.

This hierarchical aspect is studied in [78] where the authors consider a multi-layer spectrum market and present a mechanism to match the demand and the spectrum supply of the interrelated spectrum markets in the different layers. Similar models have been considered in [24] and [53] where the buyer demand is considered known. However, in these studies there is no misalignment among the objectives of the various entities (operators, users, etc) since they all maximize the revenue or the efficiency of the allocation. Hierarchical auctions have been also studied for general network resource allocation problems, [100], [101]. It is explained that due to the different objectives among the $1^{st}$-layer auctioneer (initial owner of the auctioned items) and the intermediaries, the overall resource allocation is either inefficient or untruthful. In [9], the authors study a 2-layer market for bandwidth allocation in wired networks and draw similar conclusions. Moreover, they propose a solution that is based on ascending auctions. A prerequisite for the efficient allocation of bandwidth is that either the payment rule of the lower-level auction is selected by the social planner ($1^{st}$ layer auctioneer) or that the lower level market is an oligopoly, i.e. buyers are allowed to submit bids to all auctioneers.

For the problem under consideration, the entities in the different layers have conflicting interests and there is lack of information about the actual demand in each layer. The intermediaries ($2^{nd}$ layer auctioneers) have (self-) valuations for the spectrum, and are allowed to select the auction scheme that yields for them the maximum possible revenue. The controller (social planner) does not issue strict regulatory rules, e.g. does not impose the payment scheme of the lower level market. Instead, he employs a proper pricing strategy which, with a minimum feedback from the lower level bidders, ensures that the objectives of the primary operators will be aligned with his goal for efficient channel allocation. The proposed mechanism can be implemented in a single round. Our work is inspired by the sponsored search (keyword) auction mechanisms, [75], which assign the search engines advertising slots by taking into account the feedback from the *clickers*. Similar concepts can be used for the allocation of spectrum as we suggested in [43]. Here, we take a further step towards this direction by giving a detailed methodology.

In summary, the contributions of this work are the following: **(i)** we analyze the unregulated hierarchical spectrum allocation and show that it is inefficient, **(ii)** we present an incentive mechanism that motivates the POs to increase the efficiency of their spectrum redistribution, **(iii)** we introduce the $\beta$-optimal auction which achieves a balance between the revenue of the seller (optimality) and the welfare of the buyers (efficiency). This is a new mechanism that can be used also for the allocation of similar communication resources (bandwidth, transmission power, etc), **(iv)** we discuss how this incentive mechanism can be applied to dynamic markets, where the CO-POs and the PO-SOs interactions are realized in different time scale, and we show that it improves their efficiency.

The rest of the chapter is organized as follows. In Section 3.2 we introduce the system model and in Section 3.3 we analyze the hierarchical spectrum allocation without the intervention of the controller. This analysis helps us to describe the incentive mechanism and assess its efficacy in Section 3.4. Finally in Section 3.5 we apply our mechanism to more dynamic spectrum markets. We present our numerical study in section 3.6 and conclude in Section 3.7.

## 3.2 System Model

We consider a three-layer hierarchical spectrum market with one *controller* (CO) on top of the hierarchy, a set $\mathcal{M} = \{1, 2, \ldots, M\}$ of *primary operators* (POs) in the second layer and a set $\mathcal{N} = \{1, 2, \ldots, N\}$ of *secondary operators* (SOs) that lie in the third layer under each PO, as it is shown in Figure 3·3. There exists a set $\mathcal{K} = \{1, 2, \ldots, K\}$ of identical spectrum channels which the CO allocates to the $M$ primary operators. Accordingly, each PO redistributes the channels he acquired among himself and the $N$ SOs that lie in his secondary market. The objective of the POs is to incur maximum revenue from reselling the spectrum while satisfying their own needs.

The perceived utility of each operator (PO or SO) for acquiring a channel is represented by a scalar value. Following the law of diminishing marginal returns we consider that each additional channel has smaller value for the operator. Different operators may have different spectrum needs and hence different channel valuations. For example, an operator with many clients will have very high channel valuations. Also, the POs have in general higher valuations than the SOs since they serve more users. We summarize these different characteristics of the operators with a real-valued parameter which we call the *type* of the operator, [58], [19]. Notice that, our system model is general and satisfies the basic assumptions and requirements of many different settings, [32], [78], [24], [9].

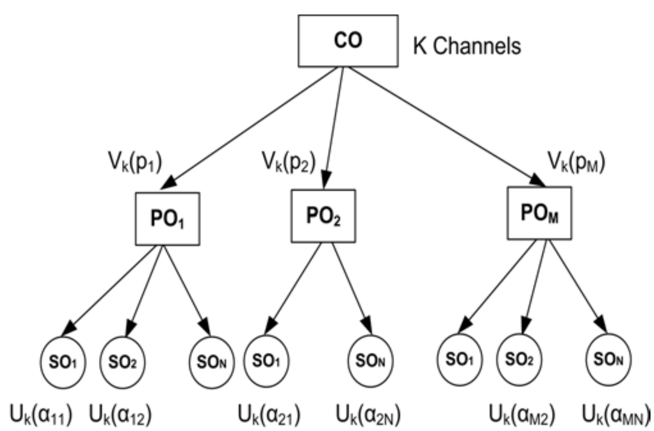



**Figure 3·3:** System Model. The CO has $K$ channels which allocates to $M$ POs. Each PO leases his idle channels to $N$ SOs. $V_k(p_j)$ is the valuation of PO $j$ for the $k^{th}$ channel and $U_k(\alpha_{ji})$ the respective valuation of the SO $i$ under PO $j$.

**Secondary Operators:** In detail, consider a SO $i \in \mathcal{N}$, in the secondary market of PO $j \in \mathcal{M}$, with $k-1$ channels at his disposal. His valuation for acquiring one more channel, i.e. the $k^{th}$ channel, is $U_k(\alpha_{ji}) \in \mathcal{R}^+$ which is assumed to be positive, monotonically increasing and differentiable function of parameter $\alpha_{ji}$. This is the *type* of the SO and represents his spectrum needs. The SOs types are independent random variables (i.r.v.), $\alpha_{ji} \in \mathcal{A} = (0, A_{max})$, $A_{max} \in R^+$, drawn from the same distribution function $F(\cdot)$ with finite density $f(\cdot)$ on $\mathcal{A}$. We denote $\alpha_j = (\alpha_{j1}, \alpha_{j2}, \ldots, \alpha_{jN})$ the vector of SOs types for the secondary market under PO $j \in \mathcal{M}$. We use this notation so as to distinguish the SOs in the different secondary markets. Notice that the type of each SO does not depend on the respective PO $j$. We assume that it is: $U_1(\alpha_{ji}) \geq U_2(\alpha_{ji}) \geq \ldots \geq U_K(\alpha_{ji}) \geq 0$, for each $\alpha_{ji} \in \mathcal{A}$, $i \in \mathcal{N}$, $j \in \mathcal{M}$. The SO $i$ pays for the channels an amount of money that is determined by the respective PO $j$.

**Primary Operators:** Each PO $j \in \mathcal{M}$ receives $K_{cj}$ channels from the CO at a cost of $Q(K_{cj})$ monetary units and decides how many he will reserve for his own needs, $K_{j0}$, and how many he will allocate to each one of the $N$ SOs at his secondary market, $\mathbf{K}_j = (K_{ji} : i \in \mathcal{N})$. We assume that the valuation of the PO for using the $k^{th}$ additional channel is $V_k(p_j) \in \mathcal{R}^+$ which belongs to a known family of functions $V_k(\cdot)$ and is parameterized by the private i.r. variable $p_j \in \mathcal{P} = (0, P_{max})$, $P_{max} \in R^+$ that is drawn from the same distribution function $G(\cdot)$. In analogy with $\alpha_{ji}$, $p_j$ is the type of the PO and models his spectrum needs. The valuation functions are considered positive, monotonically increasing and continuously differentiable w.r.t. the type $p_j$: $V_1(p_j) \geq \ldots \geq V_K(p_j) \geq 0$. The benefit of the PO from reselling his spectrum to the respective secondary market is given by the revenue component $H(\mathbf{K}_j, \alpha_j)$ which depends on the number of leased channels and the charged prices. Notice that different POs may accrue different revenue either because they sell different number of channels or because they have different demand (types of SOs) in the respective secondary market. We define the combined valuation - revenue objective of each PO $j \in \mathcal{M}$ as follows:

$$J(p_j, \alpha_j, K_{j0}, \mathbf{K}_j) = \sum_{k=1}^{K_{j0}} V_k(p_j) + H(\mathbf{K}_j, \alpha_j) \tag{3.1}$$

**Controller:** The goal of the controller is to increase the spectrum utilization and the efficiency of the hierarchical spectrum market. Therefore he acts as regulator and deploys an incentive mechanism to induce a channel allocation that maximizes a balanced sum of the POs' combined objectives and the valuations of the SOs:

$$C(\beta) = \sum_{j=1}^{M} [J(p_j, \alpha_j, K_{j0}, \mathbf{K}_j) + \beta \sum_{i=1}^{N} \sum_{k=1}^{K_{ji}} U_k(\alpha_{ji})] \tag{3.2}$$

where $\beta \in R^+$ is defined by the CO and determines this balance. Obviously, as $\beta$ increases, the allocation of spectrum will favor the SOs. Notice that the objective of the CO incorporates both the channel valuation of the POs and their revenue components, since the latter are the their motivation for reallocating the spectrum.

## 3.3 Unregulated Hierarchical Spectrum Allocation

We begin our study with the unregulated hierarchical spectrum allocation and show that it yields inefficient channel allocation. That is, the channels are not assigned to operators, primary or secondary, with the highest channel valuations. First, notice that the timing of the channel allocation in the different layers affects the outcome. The hierarchical channel allocation will definitely yield an inefficient result if the CO is not aware of (or does not take into account) the secondary demand when he allocates the channels to the POs. In other words, the POs should first learn the demand in their respective market and then ask for spectrum. Second, even if there is full information about the POs and SOs demand, the efficient allocation is not ensured since the POs will redistribute their channels so as to maximize their revenue and not the efficiency of the secondary market. In the sequel, we analyze these issues and prove that the unregulated spectrum allocation induces efficiency loss. The presented model and analysis is used in the next section in order to introduce our mechanism and assess its efficacy.

### 3.3.1 Second Stage: SOs - PO Interaction

The POs must first elicit the hidden information about the spectrum demand in their secondary market. Therefore, they organize a proper auction. In this auction, each PO $j \in \mathcal{M}$ aims to find the optimal allocation, $(\mathbf{K}_j^*, K_{j0}^*)$, of his $K_{cj}$ channels that maximizes his combined objective given by eq. (3.1). This allocation is derived from the solution of the **PO Spectrum Allocation Problem**, $(\mathbf{P_{po}})$:

$$\max_{\mathbf{K}_j, K_{j0}} J(p_j, \alpha_j, K_{j0}, \mathbf{K}_j) \tag{3.3}$$

s.t.

$$K_{j0} + \sum_{i=1}^{N} K_{ji} \le K_{cj},\ K_{ji},\ K_{j0} \in \{0, 1, 2, \dots, K_{cj}\} \tag{3.4}$$

Notice that, initially the POs do not know how many channels they will receive from the CO. Therefore, they determine the channel allocation as if they had all the channels, i.e. they solve problem $\mathbf{P_{po}}$ for $K_{cj} = K$. This way, the POs learn the entire secondary demand, i.e. how much each SO values each additional channel and hence how much he is willing to pay for it.

We assume that every PO has only partial information about the underneath secondary market. He knows the family of the valuation functions of the SOs, $U_k(\alpha)$, $k \in \mathcal{K}$, and their types distribution function $F(\cdot)$ but not their actual types. To elicit this missing information the PO runs an *optimal auction* where each one of the $N$ SOs submits a bid, $b_i \in \mathcal{A}$ in order to declare his type $\alpha_{ji}$. The PO collects the bids, $\mathbf{b} = (b_i : i \in \mathcal{N})$, and determines the allocated spectrum and the respective payment for each bidder. Here, the seller (PO) is also interested in the auctioned items and hence he compares his expected revenue from selling a channel with the valuation for using it, $V_k(\cdot)$, before he decides if he will allocate it to a SO or reserve it, [72],[19].

The maximization of the expected revenue, $(\mathbf{P_{po}})$, can be transformed to a deterministic channel allocation problem. Let us first define the additional expected revenue the PO incurs for assigning the $k^{th}$ channel to SO $i \in \mathcal{N}$. In auction theory, [12], this is known as the *contribution* of the bidder (here the SOs) and is defined as:

$$\pi_k(b_i) = U_k(b_i) - \frac{dU_k(\alpha)}{d\alpha}\Big|_{\alpha=b_i} \frac{1-F(b_i)}{f(b_i)} \tag{3.5}$$

where $F(\cdot)$ and $f(\cdot)$ are the cdf and pdf of the SOs. If these contributions are monotonically strictly increasing in the types of the SOs and decreasing in the number of channels, then they satisfy the so-called *regularity conditions*, [12], and the auction problem $\mathbf{P_{po}}$ is called *regular*. In this case the channel allocation that maximizes the combined objective of the PO $j$ can be easily derived using the following deterministic allocation and payment rules.

### PO Optimal Auction Allocation Rule

The auctioneer (PO $j$) calculates the contributions $\pi_k(b_i)$ of each SO $i \in \mathcal{N}$ for all the auctioned channels, $k = 1, \ldots, K_{cj}$, and selects the $K_{cj}$ highest of them. In the sequel, he compares these $K_{cj}$ contributions with his own valuations for the channels and constructs the contribution-valuation vector $\mathbf{X}_j$ which has $K_{cj}$ elements in decreasing order:

$$\mathbf{X}_j = (x_{(l)} : x_{(l)} > x_{(l+1)}, \, l = 1, \ldots, K_{cj}) \tag{3.6}$$

Then, the PO simply assigns each channel $l = 1, \ldots, K_{cj}$ to the respective $i^{th}$ SO if $x_{(l)} = \pi_k(b_i)$ or he reserves it for himself if $x_{(l)} = V_k(p_j)$. For example, for a PO with 4 channels and two SOs bidders, a possible instance of $X_j$ is $X_j = (V_1(p_j), \pi_1(b_1), \pi_1(b_2), V_2(p_j))$ which means that the PO will reserve 2 channels for himself and assign one to each SO.

### PO Optimal Auction Payment Rule

The price that each SO $i$ pays for receiving the $k^{th}$ spectrum channel depends on the bids submitted by all the other SOs, $b_{-i} = (b_n : n \in \mathcal{N} \setminus \{i\})$. Namely, let us denote with $z_k(b_{-i})$ the minimum bid that the $i^{th}$ SO has to submit in order to acquire the $k^{th}$ channel, [12]:

$$z_k(\mathbf{b}_{-i}) = \inf\{\hat{\alpha}_{ji} \in \mathcal{A} : \pi_k(\hat{\alpha}_{ji}) \geq \max\{0, x_{(K_{cj}+1)}\}\} \tag{3.7}$$

This means that in order to get the $k^{th}$ item the $i^{th}$ SO has simply to submit a bid high enough to draft his contribution within the first $K_{cj}$ elements of $\mathbf{X}_j$. The actual charged price for each channel is equal to his valuation had he a type equal to this minimum bid. Hence the aggregate payment for the SO is:

$$h(b_i, \mathbf{b}_{-i}) = \sum_{k=1}^{K_{ji}(b_i, \mathbf{b}_{-i})} U_k(z_k(\mathbf{b}_{-i})) \tag{3.8}$$

This payment rule is an extension of the original rule introduced in [72] and [12] and has been used also for the case that the seller has valuation for the auctioned items in [19].

Hence, each SO $i \in \mathcal{N}$ bids according to the **SO Bidding Problem**, ($\mathbf{P_{so}}$):

$$b_i^* = \arg\max_{b_i}\{ \sum_{k=1}^{K_{ji}(b_i,\mathbf{b}_{-i})} U_k(\alpha_{ji}) - h(b_i, \mathbf{b}_{-i})\} \tag{3.9}$$

Due to the payment and the respective monotonic allocation rule, the auction mechanism is incentive compatible and individual rational, [47], [75], hence $b_i^* = \alpha_{ji}$, $\forall i \in \mathcal{N}$.

### 3.3.2 First Stage: POs - CO Interaction

After learning the demand in their secondary spectrum markets, the POs ask the CO for spectrum. The controller determines the channel distribution, $\mathbf{K}_c = (\{\mathbf{K}_j, K_{j0}\} : j = 1, 2, \ldots, M)$ and the payment $Q(K_{cj})$ by each PO, where $K_{cj}$ are the total channels the PO $j$ receives, i.e. $K_{cj} = K_{j0} + \sum_{i=1}^{N} K_{ji}$. We assume that the CO knows the family of the valuation functions of POs, $V_k(\cdot)$, $k \in \mathcal{K}$ and of SOs, $U_k(\cdot)$, $k \in \mathcal{K}$, but not their exact types ($p_j$ and $\alpha_{ji}$ respectively), [58], [75]. Therefore the CO, in order to elicit this information, runs a Vickrey-Clarke-Groove (VCG) auction which is known to be efficient under a variety of assumptions, [58]. Every PO $j \in \mathcal{M}$ submits a vector bid $\mathbf{r}_j \in \mathcal{R}^{(N+1)}$, in order to declare his own type and the types of the SOs in his market. We assume that the first component of this vector $\mathbf{r}_j(1)$ represents the type of the PO, and the next $N$ components the types of the SOs in the respective secondary market. The CO collects these bids, $\mathbf{r} = (\mathbf{r}_j : j = 1, 2, \ldots, M)$, and finds the channel allocation that maximizes the aggregate combined objective of all the POs by solving the **CO Spectrum Allocation Problem,** ($\mathbf{P_{co}}$):

$$\max_{\mathbf{K}_c} \sum_{j=1}^{M} J(\mathbf{r}_j, K_{j0}, \mathbf{K}_j) \tag{3.10}$$

s.t.

$$\sum_{j=1}^{M} K_{cj} \leq K,\ \ K_{cj} \in \{0, 1, 2, \ldots, K\} \tag{3.11}$$

$$K_{cj} = K_{j0} + \sum_{j=1}^{N} K_{ji},\ j = 1, 2, \ldots, M \tag{3.12}$$

One simple method to find the solution $\mathbf{K}_c^*$ of problem $\mathbf{P_{co}}$, is to sort in decreasing order the valuations $V_k(\mathbf{r}_j(1))$ of all POs, $j = 1, 2, \ldots, M$, and the contributions $\pi_k(\mathbf{r}_j(i))$ $i = 1, 2, \ldots N$ of their SO clients, for each channel $k = 1, 2, \ldots K$. Then the CO allocates each channel to the operator with the highest valuation (for POs) or contribution (for SOs). Each PO gets the channels for himself and the underneath secondary market. Clearly, the number of channels the PO $j$ receives depends both on his own bid $\mathbf{r}_j$ and the bids of the other POs, $\mathbf{r}_{-j} = (r_m : m \in \mathcal{M} \setminus \{j\})$, i.e. $K_{cj}(\mathbf{r}_j, \mathbf{r}_{-j})$.

The payment imposed to each PO, according to the VCG payment rule [58], is equal to the externality he creates to the other POs:

$$Q(\mathbf{r}_j, \mathbf{r}_{-j}) = \sum_{m \neq j}^{M} J(\mathbf{r}_m, \tilde{K}^*_{m0}, \tilde{\mathbf{K}}^*_m) - \sum_{m \neq j}^{M} J(\mathbf{r}_m, K^*_{m0}, \mathbf{K}^*_m) \tag{3.13}$$

where $(K^*_{m0}, \mathbf{K}^*_m)$ are the channels allocated to each PO $m \in \mathcal{M} \backslash \{j\}$ and the respective SO market according to the solution of problem $(\mathbf{P_{co}})$, and $(\tilde{K}^*_{m0}, \tilde{\mathbf{K}}^*_m)$ the allocated channels when PO $j$ does not participate in the auction, i.e. when $\mathbf{r}_j = \mathbf{0}$.

In this auction, the POs determine their bid by solving the following **PO Bidding Problem**, $(\mathbf{P^b_{po}})$ :

$$\mathbf{r}^*_j = \arg\max_{\mathbf{r}_j} \{J(p_j, \alpha_j, K^*_{j0}(\mathbf{r}_j, \mathbf{r}_{-j}), \mathbf{K}^*_j(\mathbf{r}_j, \mathbf{r}_{-j})) - Q(\mathbf{r}_j, \mathbf{r}_{-j})\} \tag{3.14}$$

Since VCG auctions are incentive compatible, [58], each PO $j \in \mathcal{M}$ will reveal his actual type, $\mathbf{r}^*_j(1) = p_j$, and the true types of his SOs $\mathbf{r}^*_j(i+1) = \alpha_{ji}$, $i = 1, 2, \ldots, N$ which he learned in the first stage of this hierarchical spectrum allocation.

### 3.3.3 Inefficiency of the Unregulated Hierarchical Allocation

From the previous analysis it is evident that the main reason that renders inefficient this hierarchical spectrum allocation is the objectives misalignment problem. The POs act so as to maximize their valuation and expected revenue while the CO would like to increase the allocative efficiency of the channels. The efficiency loss induced by each PO $j \in \mathcal{M}$ can be easily calculated numerically for a given secondary demand $\alpha_j = (\alpha_{j1}, \alpha_{j2}, \ldots, \alpha_{jN})$ and number of channels $K_{cj}$, if we simply compare the contributions $\pi_k(\alpha_{ji})$ of the SOs with their valuations $U_k(\alpha_{ji})$. Clearly, POs allocate their channels to the SOs that pay higher and not to those with the highest valuations. Moreover, a PO may reserve a channel for himself, although there is a SO with a higher valuation for it, if selling it does not yield high enough revenue.

Additionally, due to this misalignment of the objectives, the CO fails to allocate the channels efficiently in the first stage. That is, the CO may allocate too many channels to a PO who has low secondary demand and less channels to a PO with higher secondary demand. Although that the controller, through the VCG auction he runs, learns the actual demand of each secondary market, he cannot allocate the channels efficiently. Notice that if the CO decides to maximize another function, e.g. the sum of POs and SOs valuations, and not the combined objectives of the POs, then the auction would not be incentive compatible anymore. The POs are free to select their objective function (revenue maximization) and the CO has to comply with this and run an auction with the same objective. In conclusion, in order to increase the efficiency of the hierarchical spectrum allocation, the controller must induce the POs to change their objective and at the same time to retrieve the hidden information about POs and SOs demand. This can be accomplished through a scheme that combines a pricing and an auction mechanism.

## 3.4 Regulated Hierarchical Spectrum Allocation

In this section we build upon the previous analysis and introduce our incentive mechanism. First, we explain the basic idea of the mechanism and the difficulties that the controller encounters in applying it. Next we introduce the $\beta$-optimal auction which is required in order to enable the POs to balance their revenue and the efficiency of their spectrum redistribution. Finally, we discuss the efficacy of the mechanism and its requirements.

### 3.4.1 Incentive Mechanism $\mathcal{M}_R$

The goal of the controller is to induce the channel allocation $\mathbf{K}_c^{\beta} = \{\{K_{j0}^*, \mathbf{K}_j^*\} : j = 1, 2, \ldots, M\}$ for each PO $j \in \mathcal{M}$ and the respective secondary market that maximizes his objective $C(\beta)$, given by eq. (3.2). This allocation stems from the solution of the **CO Balanced Spectrum Allocation Problem,** $(\mathbf{P_{co}^{bal}})$:

$$\max_{\mathbf{K}_c} \sum_{j=1}^{M} \left[ J(p_j, \alpha_j, K_{j0}, \mathbf{K}_j) + \beta \sum_{i=1}^{N} \sum_{k=1}^{K_{ji}} U_k(\alpha_{ji}) \right] \tag{3.15}$$

s.t.

$$\sum_{j=1}^{M} (K_{j0} + \sum_{i=1}^{N} K_{ji}) \leq K_c,\ K_{j0}, K_{ji} \in \{0, 1, 2, \ldots, K_c\} \tag{3.16}$$

parameter $\beta \in R^+$ is determined by the CO and defines implicitly the revenue of the POs and the welfare of the SOs.

The difficulties the controller encounters to achieve his goal are: **(i)** the CO is not aware of the types of the POs, $p_j$, $j \in \mathcal{M}$, **(ii)** he does not know the types of the SOs in each secondary market, $a_{ji}$, $i \in \mathcal{N}$, $j \in \mathcal{M}$ and **(iii)** he cannot directly dictate the POs how to redistribute the channels they acquired nor he can observe how they did allocated them. In economic terms, conditions **(i)** and **(ii)** capture the *hidden information asymmetry*, [21], of the spectrum market which means that the controller is not aware of the actual needs of the operators. Similarly, condition **(iii)** describes the *hidden action asymmetry*, which exists in the market because the CO is not aware of the actions of the POs. The introduced incentive mechanism, which we call *Mechanism* $\mathcal{M}_{\mathcal{R}}$, eliminates these asymmetries and achieves the desirable spectrum allocation.

The proposed scheme is based on pricing and the underlying idea is that the controller creates a coupling between the spectrum allocation decisions of the POs and their cost for acquiring the spectrum in order to bias their revenue maximizing strategy. Namely, we suggest that the CO should reimburse the PO $j \in \mathcal{M}$ with the following price:

$$L_j(\alpha_j, \mathbf{K}_j, \beta) = \beta \sum_{i=1}^{N} \sum_{k=1}^{K_{ji}} U_k(\alpha_{ji}) \tag{3.17}$$

This modifies the objective function of the PO as follows:

$$J_R(p_j, \alpha_j, K_{j0}, \mathbf{K}_j, \beta) = J(p_j, \alpha_j, K_{j0}, \mathbf{K}_j) + \beta \sum_{i=1}^{N} \sum_{k=1}^{K_{ji}} U_k(\alpha_{ji}) \tag{3.18}$$

$J_R(\cdot)$ is the regulated new combined objective of each PO which depends on parameter $\beta$ and is aligned with the balanced objective of the CO, eq. (3.2).

### 3.4.2 The $\beta$-Optimal Auction Mechanism

Each PO maximizes $J_R(\cdot)$ by solving a new allocation problem $\mathbf{P}_{\mathbf{po}}^{\beta}$ which differs from the respective $\mathbf{P}_{\mathbf{po}}$ problem in the objective function that is given now by eq. (3.18). Since the types of the SOs are unknown, the primary operator runs again an auction to elicit this hidden information. However, this is neither an efficient nor an optimal auction and hence he cannot employ any of the known auction schemes. To address this problem, we introduce a new multi-item auction mechanism, the **$\beta$-optimal** auction, which ensures the maximization of the balanced objective defined in eq. (3.18). This mechanism is similar to the optimal auction discussed in section 3.3.1 with the difference that the allocation rule is biased by parameter $\beta$. This modification affects the allocation of the channels and results in reduced payments from the bidders to the auctioneer and improved efficiency in channels allocation. The combination of optimal and efficient auctions has been also suggested in [65] for single item allocation where the authors proposed an efficient auction with a lower bound on the seller's revenue.

Let us now explain the rationale and machinery of the $\beta$-optimal auction. First we define the $\beta$-contribution for each SO $i \in \mathcal{N}$ under a certain PO $j \in \mathcal{M}$, as follows:

$$\pi_k^{\beta}(b_i) = (1+\beta)U_k(b_i) - \frac{dU_k(\alpha)}{d\alpha}\bigg|_{\alpha=b_i} \frac{1-F(b_i)}{f(b_i)} \tag{3.19}$$

Since $\beta \geq 0$ it is $\pi_k^{\beta}(\alpha_{ji}) \geq \pi_k(\alpha_{ji})$ for all the SOs and all the channels. Moreover, notice that if the initial *contributions* satisfy the *regularity* conditions, [12], then the $\beta$-contributions will also satisfy them, and hence problem $\mathbf{P}_{\mathbf{po}}^{\beta}$ will be regular. Therefore, we are able again to derive deterministic channel allocation and payment rules.

**$\beta$-Optimal Auction Allocation Rule:** Similarly to the allocation rule of the optimal auction, the $j^{th}$ PO calculates the $\pi_k^{\beta}(b_i)$ for all SOs $i \in \mathcal{N}$ and all channels $k = 1, \ldots, K_{cj}$ and compares them with his own valuations in order to construct the contribution-valuation vector $\mathbf{X}_j^{\beta}$:

$$\mathbf{X}_j^{\beta} = (x_{(l)}^{\beta} : x_{(l)}^{\beta} > x_{(l+1)}^{\beta}, \, l = 1, \ldots, K_{cj}^{\beta}) \tag{3.20}$$

Using $\mathbf{X}_j^{\beta}$, the PO allocates his channels to the respective $K_{cj}$ highest contributions and valuations. The resulting channel allocation $(K_{j0}^{\beta}, \mathbf{K}_j^{\beta})$ solves problem $\mathbf{P}_{\mathbf{po}}^{\beta}$ and maximizes the new objective $J_R(p_j, \alpha_j, K_{j0}^{\beta}, \mathbf{K}_j^{\beta}, \beta)$. Again, this allocation rule is monotone increasing in the types of the SOs.

**$\beta$-Optimal Auction Payment Rule:** The payment rule changes in order to comply with the new allocation rule. Namely, the minimum bid that the $i^{th}$ SO needs to submit in order to acquire the $k^{th}$ channel is:

$$z_k^{\beta}(b_{-i}) = \inf\{\hat{\alpha}_{ji} \in \mathcal{A} : \pi_k^{\beta}(\hat{\alpha}_{ji}) \geq \max\{0, x^{\beta}_{(K^{\beta}_{cj}+1)}\}\} \tag{3.21}$$

and, similarly to the previous mechanism, the total payment for this SO is :

$$h^{\beta}(b_i, b_{-i}) = \sum_{k=1}^{K^{\beta}_{ji}(b_i,b_{-i})} U_k(z_k^{\beta}(b_{-i})) \tag{3.22}$$

Under this new auction mechanism, each SO $i \in \mathcal{N}$ selects his bid so as to maximize his new payoff, (**SO $\beta$-Bidding Problem, $\mathbf{P^{\beta}_{so}}$**):

$$b_i^* = \arg\max_{b_i}\{ \sum_{k=1}^{K^{\beta}_{ji}(b_i,b_{-i})} U_k(\alpha_{ji}) - h^{\beta}(b_i, b_{-i})\} \tag{3.23}$$

This new auction mechanism improves the efficiency of the POs - SOs interaction and at the same time retains the required properties of the optimal auctions as we explain with the following proposition.

**Proposition 1.** *The $\beta$-optimal auction mechanism preserves the incentive compatibility and the individual rationality properties of optimal multi-unit auction introduced in [12].*

**Proof:**

We focus on PO $j \in \mathcal{M}$ with $K_{cj}$ channels. We denote $s_{ik}$ the probability of SO $i$ for receiving the $k^{th}$ channel which depends on the types of all the SOs. Additionally, $c_i(\alpha_{ji})$ is the payment of each SO $i$ for all the channels he acquired. *Definition* 2 and *Lemma* 1 in [12] give the necessary conditions for the structure of the bidders (SOs) valuation functions in order to ensure the (IC) and (IR) properties. These conditions hold independently of the objective of the auctioneer (PO) and hence they are not affected by the incorporation of the linear term of the SOs valuation.

The objective of the PO w.r.t. the expected types of the SOs is:

$$\begin{aligned} E_{\mathcal{A}}[J_R(\cdot)] = \sum_{i=1}^{N} E_{\mathcal{A}}[c_i(\alpha_{ji})] + \beta \sum_{i=1}^{N} E_{\mathcal{A}}[\sum_{k=1}^{K_{cj}} U_k(\alpha_{ji}) s_{ik}] + \\ + E_{\mathcal{A}}[\sum_{k=1}^{K_{cj}} V_k(p_j)(1 - \sum_{i=1}^{N} s_{ik})] \end{aligned}$$

The first term is the payment by the SOs, the second is the pricing and the third the valuation for the channels that are not sold. After some algebraic manipulations and following the proof of *Proposition* 1 in [12], we get:

$$E_{\mathcal{A}}[J_R(\cdot)] = \sum_{i=1}^{N} E_{\mathcal{A}}\Bigg[\sum_{k=1}^{K_{cj}}[(1+\beta)U_k(\alpha_{ji}) - V_k(p_j) - \frac{dU_k(\alpha_{ji})}{d\alpha}$$
$$\frac{1-F(\alpha_{ji})}{f(\alpha_{ji})}]s_{ik}\Bigg] - \sum_{i=1}^{N}[\sum_{k=1}^{K_{cj}} U_k(0) - c_i(0)] + E_{\mathcal{A}}[\sum_{k=1}^{K_{cj}} V_k(p_j)]$$

Using the necessary (IC) and (IR) conditions from *Lemma* 1 in [12], it stems that the $\beta$-optimal payment rule is given again by equation (10) of [12]:

$$c_i^*(\alpha_{ji}) = E_{\mathcal{A}}\Bigg[\sum_{k=1}^{K_{cj}} U_k(\alpha_{ji})s_{ik} - \int_0^{\alpha_{ji}} \frac{dU_k(\alpha)}{d\alpha}s_{ik}d\alpha\Bigg] \tag{3.24}$$

where the probabilities of allocation are selected so as to maximize the new objective of the auctioneer (instead of revenue only maximization as in [12]). The optimal payment rule is the one that yields zero payment and zero channel allocation for SOs with zero type.

If the problem is *regular* then the payment is as we described in section 3.4 and the first term in the PO's objective is maximized by using the $\beta$-optimal allocation rule. This can be easily derived following the proof of the respective *Proposition* 2 in [12]. Notice that if the original respective problem in [12] is *regular* then also this modified problem is *regular*. Apparently, the inclusion of the SOs buyers valuations does not affect the monotonicity of the allocation rule nor the critical value property of the payment rule, [75]. Hence, the modified auction is still truthful. ■

This new type of auction yields a more efficient allocation than the typical optimal auction of Myerson, [72] as the following proposition states.

**Proposition 2.** *The $\beta$-optimal auction is more efficient than the optimal auction.*

**Proof:** In $\beta$-optimal auction, the allocation of items is accomplished with regard to the modified contributions $\pi_k^{\beta}(\cdot)$ which are larger than the respective contributions of the optimal auction $\pi_k(\cdot)$. Notice that it holds $U_k(\cdot) \geq \pi_k^{\beta}(\cdot) \geq \pi_k(\cdot)$, for all $k = 1, 2, \ldots, K$. This means that the $\beta$-optimal auction induces a channel allocation that is more close to the efficient allocation that is produced if the auctioneer considers the actual valuations $U_k(\cdot)$ of the bidders and not their contributions $\pi_k(\cdot)$. ■

#### An Alternative Interpretation of $\beta$-Optimal Auction

Before we proceed, we will present here an alternative interpretation of the $\beta$-optimal auction which will justify its improved efficiency through a simple graph. The authors of [14] employ monopoly pricing theory to analyze the optimal auction mechanism of Myerson, [72]. We will adopt this methodology here in order to explain why and how the proposed regulation method increases the efficiency of the $\beta$-optimal auction. For simplicity, we will consider the single-item case.

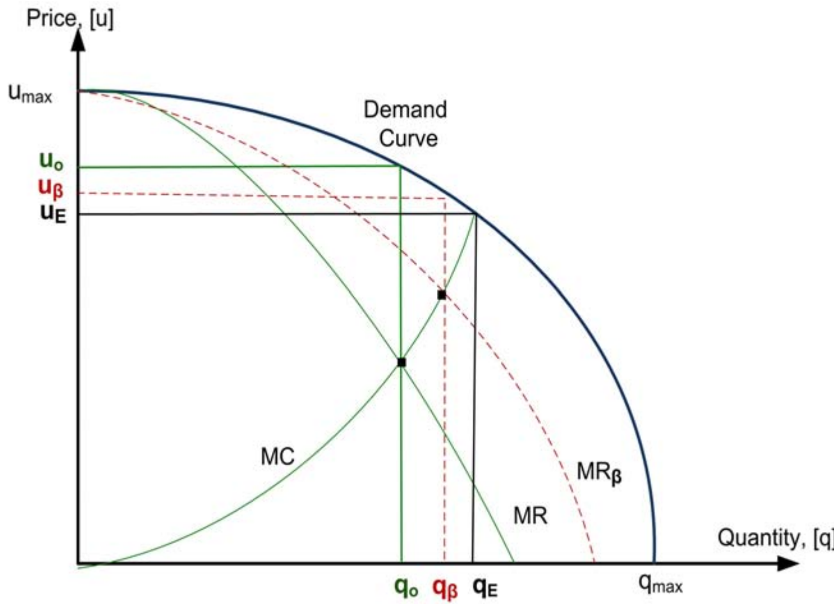



**Figure 3·4:** The pricing changes the MR and the MC curves and increases the efficiency of the $\beta$-optimal auction allocation.

We consider a simple setting where one item is sold to one buyer. The buyer has a valuation of $u_i$ units for the item which follows a cumulative distribution function $F(u_i)$. Therefore, the probability $q$ that the buyer's valuation is greater than a certain value $u$ is $q = P(u_i \geq u) = 1 - F(u_i)$. The buyer is willing to buy the item if it is offered at a price equal or lower than $u$. Equivalently, in a different scenario where the sold item is perfectly divisible, we can interpret $P(u_i \geq u)$ as the (normalized) quantity the buyer is willing to buy, when its per-unit price is $u$. Now we can plot the inverse demand curve which provides the probability (or the quantity) that the item will be sold if its price is $u$ units. For each price we have a different probability (quantity, respectively). In Figure 3·4 we depict the demand curve for the buyer. In the same figure we plot and superimpose the marginal revenue curve. The latter stems from the first order derivative of the revenue of the seller:

$$MR(u) = \frac{\mathrm{d(qu)}}{\mathrm{dq}} = u - \frac{1 - F(u)}{f(u)} \tag{3.25}$$

and models the seller's profit for each additional sold unit of the item. Finally, we superimpose also the marginal cost $MC(u)$ curve which represents the cost of the seller for selling the item (in various prices) or, equivalently, the value the item has for the seller.

According to the monopoly pricing theory, the optimal price to sell the item is the one that yields $MC(u^*) = MR(u^*)$. We denote the *optimal price* $u_o$ and the respective quantity (or probability) with $q_o$ in Figure 3·4. Assume now that the auctioneer would like to allocate the item efficiently. In this case, the price the seller would select corresponds

**Algorithm 1** (Mechanism $\mathcal{M}_R$)

**1st Stage:** *Channel Allocation ($\beta$ is announced by the CO).*

(**1.1:**) Each SO $i$ bids to the respective PO, according to problem $\mathbf{P}_{\mathbf{so}}^{\beta}$, eq. (3.23).

(**1.2:**) Each PO collects the bids from the SOs, and bids to the CO by solving $\mathbf{P}_{\mathbf{po}}^{\mathbf{b}_{\beta}}$, eq. (3.27). This bid reveals his own type and the types of the SOs in the underneath secondary market.

(**1.3:**) The CO solves problem $\mathbf{P}_{\mathbf{co}}^{\mathbf{bal}}$, eq. (3.15)-(3.16), and allocates $K_{cj}^{\beta} = K_{j0}^{\beta} + \sum_{i=1}^{N} K_{ji}^{\beta}$ channels to each PO $j \in \mathcal{M}$.

(**1.4:**) Each PO $j \in \mathcal{M}$ redistributes his channels according to the *$\beta$-Optimal Allocation* Rule, $X_j^{\beta}$.

**2nd Stage:** *Payments.*

(**2.1:**) Each SO $i$ pays the respective PO an amount of $h^{\beta}(b_i, b_{-i})$ monetary units, according to eq. (3.22).

(**2.2:**) Each SO $i$ reveals to the CO the allocation decisions of the respective PO (*feedback* for $K_{ji}^{\beta}$).

(**2.3:**) The CO collects the feedback and for every PO $j \in \mathcal{M}$ calculates the reimbursement $L_j(\alpha_j, \mathbf{K}_j, \beta)$, eq. (3.17), and the total price $\Lambda_j$ the PO has to pay:

$$\Lambda_j = Q_R(\mathbf{r}_j, \mathbf{r}_{-j}) - L_j(\alpha_j, \mathbf{K}_j(\mathbf{r}_j, \mathbf{r}_{-j}), \beta)$$

to the intersection of the $MC$-curve with the demand curve. We denote with $u_E$ and $q_E$ the efficient price and quantity respectively. Now, one can directly identify the efficiency loss due to the revenue maximizing strategy of the seller. This loss is the area below the demand curve and above the $MC$-curve that corresponds to the $x$-axis interval of $[q_o, q_E]$.

The incentive mechanism $\mathcal{M}_R$ modifies the $MR$ curve of the seller and hence changes the optimal selling price. Let us denote with $MR_{\beta}$ the modified marginal revenue curve. It is:

$$MR_{\beta} = u - \frac{1 - F(u)}{f(u)} + \beta u = (1+\beta)u - \frac{1 - F(u)}{f(u)} \tag{3.26}$$

with $\beta > 0$. This means that the $MR_{\beta}$ curve lies above the respective $MR$ curve and hence yields lower prices for each given quantity. In other words, the incentive mechanism decreases the reservation price for the sold item. Therefore, the new optimal price $u_{\beta}$ that is obtained by solving $MR_{\beta} = MC$, incurs efficiency loss which corresponds to the area below the demand curve (and above the $MC$ curve) for the interval $[q_{\beta}, q_E]$, which is less than the efficiency loss of the optimal auction. Finally, it is apparent that if the value of $\beta$ is very large, then the $\beta$-optimal auction may yield a price that is even lower than the price of the efficient auction, $u_{\beta} < u_E$.

### 3.4.3 Efficacy and Requirements of Mechanism $\mathcal{M}_R$

The pricing that is imposed by the CO, eq. (3.17), not only bias the channel distribution strategy of the POs, but also changes their bidding strategy. Namely, each PO $j \in \mathcal{M}$ after

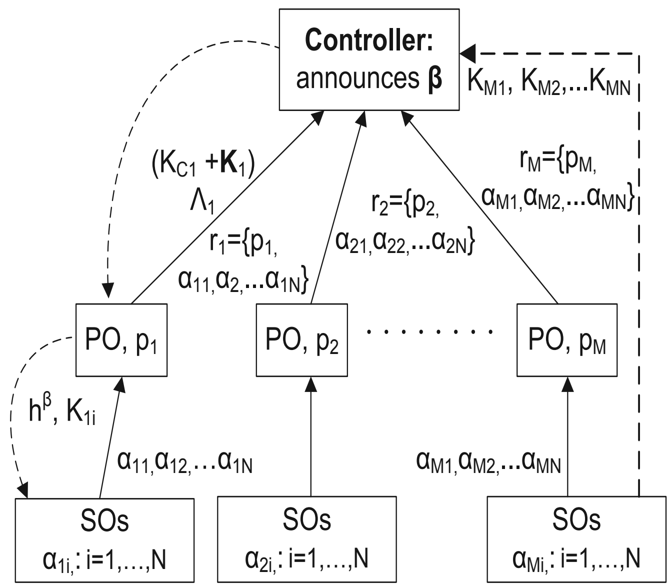



**Figure 3·5:** The machinery of incentive mechanism $\mathcal{M}_R$. The circulated information, bids and channel allocation among the SOs, POs and the Controller is depicted. The feedback can be provided from the SOs to the CO directly, or be inferred using other means.

receiving the bids of his SOs, determines his optimal bid by solving the **PO $\beta$-Bidding Problem,** $(\mathbf{P}_{\mathbf{po}}^{\mathbf{b}_\beta})$:

$$\mathbf{r}_j^* = \arg\max\{J_R(p_j, \alpha_j, K_{j0}^\beta(\mathbf{r}_j, \mathbf{r}_{-j}), \mathbf{K}_j^\beta(\mathbf{r}_j, \mathbf{r}_{-j})) - Q_R(r_j, r_{-j})\} \tag{3.27}$$

where $Q_R(\cdot)$ is the new price charged by the controller when he employs mechanism $\mathcal{M}_R$. Specifically, the CO determines the channel allocation by solving problem $\mathbf{P}_{\mathbf{co}}^{\mathbf{bal}}$, eq. (3.15) - (3.16), and calculates the new VCG prices as follows:

$$Q_R = \sum_{m\neq j}^{M} J_R(r_m, \tilde{K}_{m0}^\beta, \tilde{\mathbf{K}}_m^\beta) - \sum_{m\neq j}^{M} J_R(r_m, K_{m0}^\beta, \mathbf{K}_m^\beta) \tag{3.28}$$

Again, the number of channels allocated to each PO $m$, $(K_{m0}^\beta, \mathbf{K}_m^\beta)$, depends on bids submitted by all the POs. Also, $(\tilde{K}_{m0}^\beta, \tilde{\mathbf{K}}_m^\beta)$ is the channel allocation when $\mathbf{r}_j = \mathbf{0}$. Therefore, the POs are induced to bid truthfully, $\mathbf{r}_j(1)^* = p_j$, $\mathbf{r}_j(i+1) = \alpha_{ji}$, for $j = 1, 2, \ldots M$, $i = 1, 2, \ldots, N$. The improvement in the allocative efficiency under the $\beta$-optimal auction can be also realized by considering the inequality $U_k(\alpha_{ji}) \geq \pi_k^\beta(b_i) \geq \pi_k(b_i)$ which holds for all the POs and SOs. This means that each SO has to pay less in order to draft his *contribu-*

*tion* within the winning bids. We summarize mechanism $\mathcal{M}_R$ in Algorithm **1** and Figure 3·5.

In order to calculate the prices $L_j(\cdot)$, the CO needs to know the actual types of the SOs in the respective secondary market and the amount of spectrum that is allocated to them by the PO. The SOs types are truthfully revealed by the POs due to the VCG auction. However, the CO needs to verify that indeed the POs allocate the channels according to this scheme. There are many different methods and scenarios about how the CO can acquire this information. First, the SOs may directly provide it through a feedback loop, Figure 3·5. Equivalently, the CO may be able to observe the interaction of the SOs with the respective PO. Finally, the CO may be able to observe how many channels the POs reserve for them and this way infer how many they reallocate to their clients (secondary operators).

Since the controller is on top of this hierarchy and manages the spectrum, we can easily consider many similar methods that will allow him to receive direct or indirect feedback about the SO - PO interaction. Clearly, this assumption is necessary in order to increase the efficiency of the market. As it was made clear from the previous analysis, if the CO cannot impose the transaction rules of the lower level market and at the same time cannot observe the channel allocation decisions of POs, then there does not exist a realizable method for increasing the allocative efficiency. Another basic assumption is that POs do not collude with their SO clients in order to strategize and deceive the controller. Finally, we assume that the SOs bid to the POs before the latter ask for spectrum. If we relax this assumption, the coordination problem is by default unsolvable. However, even in this case our mechanism still improves the hierarchical spectrum allocation by addressing the objectives misalignment problem. This issue is discussed in the next section, in the context of the dynamic spectrum markets where it is more prevalent.

### Complexity and Budget Balance of $\mathcal{M}_R$

We now discuss the requirements of the proposed mechanism and specifically the computational and communication cost, as well as the budget balance property. Mechanism $\mathcal{M}_R$ is based on a VCG auction. The last years VCG auctions have been extensively proposed for the design of network protocols. These mechanisms ensure incentive compatibility and individual rationality for the players under a variety of settings and assumptions. Nevertheless, as the authors of [67] explain, VCG-based mechanisms have certain drawbacks. First, they entail an extremely high communication cost because the bidders must communicate their entire utility (valuation) functions. Second, the allocation decisions of the auctioneer and the determination of the prices presume the solution of problems which are of high computational complexity and very often are NP-hard. Finally, VCG mechanisms may not be budget balanced. This means that the sum of payments and reimbursements may not sum up to zero and hence an entity (usually the auctioneer) must inject additional money to the market. Fortunately, in our setting, none of these problems arises.

In the problem under consideration, the class of the valuation functions is common knowledge and the bidders are single-minded, i.e. scalar-parameterized. Therefore, the only information that must be circulated is the scalar bids of the buyers in each layer.

Namely, the SOs communicate a scalar parameter to the POs and each PO a vector with $N+1$ scalar parameters representing his own type and the types of his SO clients. Hence, the communication burden is low. Also, the auctioned channels are identical (homogeneous) and therefore the aggregate utility of each operator depends only on the total number of the channels he receives. This assumption facilitates the computation of the optimal channel allocation which is determined simply by ordering the channel valuations of the POs and the contributions of the SOs and selecting the $K$ highest of them. The channel allocation and the price determination optimization problems are not NP-hard.

The VCG auction organized by the CO satisfies the *no single-agent effect* and hence it is weak budget balanced, i.e. the payments to the auctioneer are equal to or greater than zero. Specifically, the no single-agent effect condition states that any one player can be removed from an optimal system-wide solution (allocation of items) without having a negative effect on the best choice available to the remaining players (bidders). This condition holds in auctions with only buyers, as long as all buyers have free disposal such that they have at least as much value for more items than less items. Obviously, in the problem we study, this condition holds because POs and SOs ask for as many channels as possible. Therefore the VCG payments from the POs to the controller are positive.

Finally, we make the following remarks for the efficacy of mechanism $\mathcal{M}_R$:

- The hierarchical channel allocation is more efficient under mechanism $\mathcal{M}_R$. Namely, the channel allocation $\mathbf{K}_c^{\beta}$ that stems from the solution of problem $\mathbf{P}_{\mathbf{CO}}^{\mathbf{bal}}$ is more efficient than the initial, unregulated, channel allocation $\mathbf{K}_c^{*}$. Notice however that for large values of the regulation parameter $\beta$, the allocation can be inefficient favoring, this time, the SOs.

- The total *net payoff* of the SOs, i.e. the aggregate valuation of channels assigned to SOs minus the total payment to POs, increases because:

  1. The SOs in every secondary market pay less than or equal money to the respective PO:

  $$\sum_{i=1}^{N}\sum_{k=1}^{K_{ji}^{\beta}} U_k(z_k^{\beta}(\mathbf{b}_{-i})) \leq \sum_{i=1}^{N}\sum_{k=1}^{K_{ji}^{*}} U_k(z_k(\mathbf{b}_{-i})),\ j=1,2,\ldots,M \qquad (3.29)$$

     To make this clear, notice that the reimbursement by the CO does not change the relative order of the contributions of the SOs. If for two SOs $n,m \in \mathcal{N}$ it holds $\pi_k(\alpha_{jn}) < \pi_k(\alpha_{jm})$, then it will also be $\pi_k^{\beta}(\alpha_{jn}) < \pi_k^{\beta}(\alpha_{jm})$. However, since $\pi_k^{\beta}(\cdot) > \pi_k(\cdot)$, SOs have to pay less money in order to outbid the channel valuation of the respective PO.

  2. The SOs receive in total at least as many channels as they receive without $\mathcal{M}_R$: $\sum_{j=1}^{M}\sum_{i=1}^{N} K_{ji}^{\beta} \geq \sum_{j=1}^{M}\sum_{i=1}^{N} K_{ji}^{*}$. Therefore, the social welfare in all secondary markets increases under $\mathcal{M}_R$.

- Auctions in both layers are incentive compatible and weak budget balanced. However, the revenue of the CO (who is considered a social planner and not a strategic player) decreases due to the reimbursement $\sum_{j=1}^{M} L_j(\cdot)$.

- The sum of the combined benefit of the POs increases under $\mathcal{M}_R$:

$$\sum_{j=1}^{M} J_R(p_j, \alpha_j, K_{j0}^{\beta}, \mathbf{K}_j^{\beta}, \beta) \geq \sum_{j=1}^{M} J(p_j, \alpha_j, K_{j0}^{*}, \mathbf{K}_j^{*}) \tag{3.30}$$

  Additionally since the upper layer auction organized by the CO is incentive compatible and individual rational, the net payoff of each PO, $J_R(\cdot) - Q_R(\cdot)$, is greater (or equal) than zero. However, we cannot directly compare this amount with the respective net payoff, $J(\cdot) - Q(\cdot)$, before employing mechanism $\mathcal{M}_R$.

## 3.5 Regulation in Dynamic Spectrum Markets

Until now, we ignored the dynamic aspect of the problem in order to facilitate the analysis and we focused on the novel balanced auction scheme. That is, we implicitly assumed that the interaction of the CO with the POs, and the interactions of the latter with the SOs are performed in the same time scale. This is a realistic assumption since the current suggestions about the spectrum policy reform advocate a more fine grained spatio-temporal management by the regulators, [85]. Nevertheless, the proposed mechanism $\mathcal{M}_R$ can be extended for the case where the CO-POs and POs-SOs interactions are realized in different time scales.

Assume that the time is slotted and divided in time periods, $\mathcal{I} = 1, 2, \ldots$, where each period is further divided in $T$ time slots, $t = 1, 2, \ldots, T$. The CO determines his $K_c$ channels allocation in the beginning of each period while the POs redistribute them in every slot. The CO $\mathbf{P_{co}^{bal}}$ problem for this setting is related to the spectrum allocation for all the $T$ slots within each period:

$$\max_{\{K_{j0}^t, \mathbf{K}_j^t\}} \sum_{t=1}^{T} \sum_{j=1}^{M} \left[ J(\mathbf{p}_j, \alpha^t, K_{j0}^t, \mathbf{K}_j^t) + \beta \sum_{i=1}^{N} \sum_{k=1}^{K_{ji}^t} U_k(\alpha_i^t) \right] \tag{3.31}$$

s.t.

$$\sum_{j=1}^{M} [K_{j0}^t + \sum_{i=1}^{N} K_{ji}^t] \leq K_c, \; t = 1, \ldots, T \tag{3.32}$$

where we have marked with the superscript $t$ the variables that change in each slot. Obviously the CO cannot allocate the spectrum optimally to the POs for the entire period since he is not aware of the future demands of the SOs. Additionally, even if the CO had this information, he could not determine the allocated channels to the POs, $K_{cj}$, once in each period since these should be adapted to the dynamic secondary demand, $K_{ji}^t$. Apparently, the **coordination problem** cannot be solved optimally in this setting.

Nevertheless, the CO is still able to solve the **objectives misalignment problem** and induce the POs to allocate their spectrum more efficiently. Assume that the CO-POs interaction is accomplished either without taking into account the secondary demand as in Section 3.3 or by considering the average demand of the SOs, $\bar{\alpha}_i$. This will result in a certain suboptimal channel allocation $\bar{K}_c = \{\bar{K}_{cj} : j \in \mathcal{M}\}$. Then, in each slot as the SOs demand will be realized, they will bid to the POs and at the same time the CO will receive feedback (directly or indirectly) about their needs. This way, the controller will be able to determine the prices $L_j(\cdot)$ for each PO $j \in \mathcal{M}$ at the end of the entire time period:

$$L_j^T(\alpha, \mathbf{K}_j^T, \beta) = \sum_{t=1}^{T} \beta \sum_{i=1}^{N} \sum_{k=1}^{K_{ji}^t} U_k(\alpha_i^t) \tag{3.33}$$

where $\mathbf{K}_j^T = \{\mathbf{K}_j^t : t = 1, \ldots, T\}$ is the $M \times T$ matrix of allocated channels. Obviously, this subsequent pricing at the end of each time period will induce the POs to allocate their spectrum by solving problem $\mathbf{P}_{\mathbf{po}}^{\beta}$ and maximizing eq. (3.18), instead of problem $\mathbf{P}_{\mathbf{po}}$, eq. (3.3)-(3.4), in each time slot. Therefore, the efficiency loss will be reduced.

## 3.6 Numerical Results

In order to obtain insights about the proposed mechanism $\mathcal{M}_R$, we simulate a representative three-layer hierarchical market with one CO, $M = 2$ POs and $N = 10$ SOs under each PO. We assume that the POs valuation functions for the $k^{th}$ channel are $V_k(p_j) = p_j/k$, where the types $p_j$ are uniformly distributed in the interval $[5, 6]$. Similarly, the SOs valuations are $U_k(\alpha_{ji}) = 0.1\alpha_{ji}/k$, and their types follow a uniform distribution $F(x) = x/4$ on the interval $(0, 4]$. The SOs contributions are $\pi_k(\alpha_{ji}) = (0.2\alpha_{ji} - 0.4)/k$ and the respective $\beta$-contributions are $\pi_k^{\beta}(\alpha_{ji}) = [(0.2 + \beta)\alpha_{ji} - 0.4]/k$. For each random realization of the SOs and POs types, the results are averaged over 40 runs in order to capture the variance on the spectrum demand.

For our study we use as a benchmark the *efficient* channel allocation to the SOs. This allocation corresponds to the hypothetical scenario where the CO would be able to assign directly the channels to both the POs and the SOs and maximize the aggregate spectrum valuations. In the upper plot of Figure 3·6 we show that in hierarchical unregulated market the number of total channels assigned to the SOs is less than the channels in the *efficient* allocation. Mechanism $\mathcal{M}_R$ with $\beta = 0.1$ reduces this difference and increases the SOs channels. Notice that the number of SOs channels is stil less than in the *efficient* allocation scenario, since the goal of the CO is the combined revenue-efficiency balanced allocation.

In the same Figure we show that the number of channels assigned to SOs vary with the value of $\beta$. Namely, when $\beta = 0$ the allocation is identical with the unregulated case while for $\beta \approx 0.35$ it reaches the *efficient* allocation. Notice that for larger values of $\beta > 0.37$ the SOs receive even more channels. This means that the CO favors the SOs too much and render the channel allocation inefficient. The impact of $\beta$ is depicted also in the lower plot of Figure, 3·7 where we see that for large values the improvement in the aggregate valuation of the POs and SOs becomes negative. For this plot, the number of SOs is $N = 20$ and the system welfare is maximized for $\beta = 0.1$. If $\beta$ is further increased, the welfare improvement decreases and eventually becomes negative. Finally, in the sequel we present a simple numerical example revealing the inefficiency of the unregulated spectrum allocation and the improvement by our mechanism.

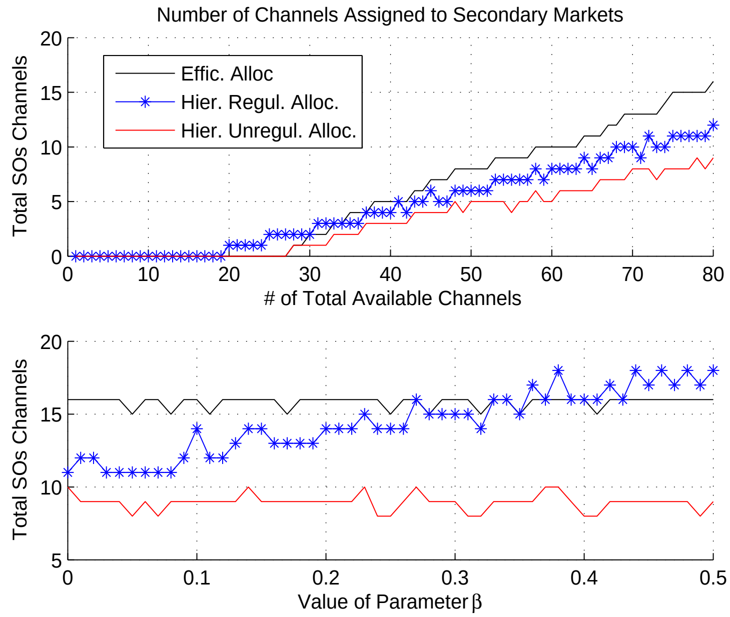



**Figure 3·6:** *Upper Plot*: For $\beta = 0.1$ the regulation mechanism $\mathcal{M}_R$ increases the number of channels assigned to the SOs. *Lower Plot*: The SOs receive more channels for larger values of $\beta$ ($K_c = 80$).

### 3.6.1 Numerical Example

Finally, we provide a simple numerical example to further explain the proposed mechanism. Consider a market where the CO has 12 channels, there are 2 POs and 2 SOs under each PO. The SOs types are drawn from a uniform cdf $F(x) = x/2$ in the interval $(0, 2]$ and their valuation for the $k^{th}$ channel is $U_k(\alpha) = \frac{\alpha}{k}$. The respective valuations of the POs are $V_k(p) = \frac{3*p}{k}$. We assume that $p_1 = 1$ with $\alpha_1 = 1.2$ and $\alpha_2 = 1.5$ and $p_2 = 1.2$ with $\alpha_3 = 1.3$ and $\alpha_4 = 1.4$. The contributions of the SOs are $\pi_k(\alpha) = \frac{2\alpha-2}{k}$. If the channel allocation is accomplished with the unregulated hierarchical method then in the first stage the CO allocates the channels to the highest valuations of the POs and these redistribute them comparing their own valuations with the contributions of the SOs in their market. This results in $Ch_{po} = 10$ channels allocated to the POs and $Ch_{so} = 2$ assigned channels to the SOs.

If however, the POs were socially aware and considered the valuations of the SOs (instead of their contributions) then the channel allocation would be $[Ch_{po}, Ch_{so}] = [8, 4]$. Finally, even this allocation is not the most efficient because in the first stage the secondary demand has not be considered. If for example the CO was able to allocate directly the channels w.r.t. the POs and SOs valuations, then the allocation would result in $[Ch_{po}, Ch_{so}] = [7, 5]$. Now, assume that we use the the proposed mechanism $\mathcal{M}_R$, with $\beta = 0.2$. In this case, the number of assigned channels will be $Ch_{po} = 9$ and $Ch_{so} = 3$, i.e.

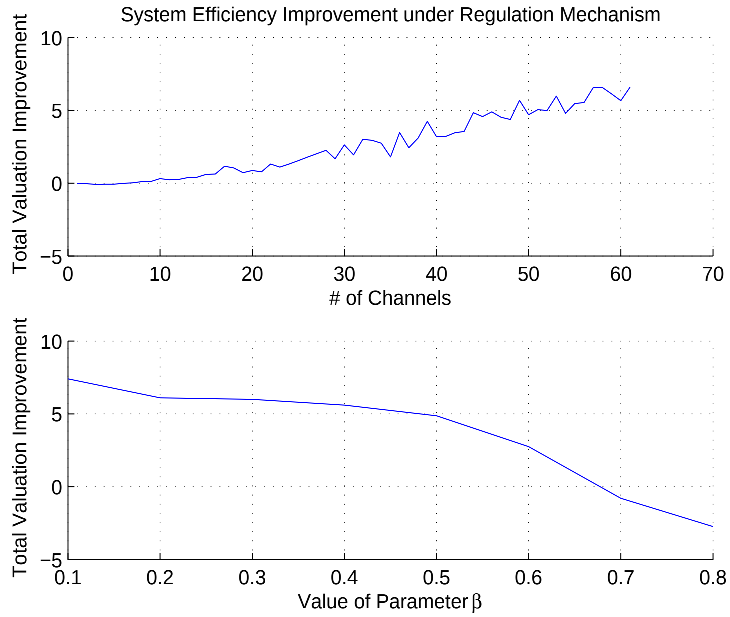



**Figure 3·7:** *Upper Plot*: The aggregate network efficiency (POs and SOs valuations) increases with the $\mathcal{M}_R$, $\beta = 0.1$, $N = 20$, $K_c = 1 : 60$. *Lower Plot*: For large values of $\beta$ the network efficiency decreases since the SOs are favored more than the POs.

increased by 1 for the SOs. Apparently for large values of $\beta$ the allocation will favor the SOs and the revenue of the POs will decrease.

## 3.7 Conclusions

We analytically proved that the emerging hierarchical spectrum markets will fail to allocate channels efficiently. Namely, primary operators who act as intermediaries, are expected to reallocate the channels with the objective to maximize their revenue and not the efficiency of the secondary markets. In order to solve this problem, we proposed an incentive mechanism that can be used by the controller so as to regulate the interaction between the primary and secondary operators and to induce a new market equilibrium. This equilibrium depends on a scalar parameter which is defined by the controller and determines the efficiency of the secondary markets by adjusting the number of channels allocated to the SOs. The mechanism is based on a novel auction scheme which has a revenue-welfare balanced objective.

# Chapter 4

# Competition and Regulation in Wireless Services Markets

## 4.1 Background

Consider a city where 3 commercial operators (companies) and one municipal operator offer WiFi Internet access to the citizens (users). The companies charge for their services and offer better rates than the municipal WiFi service which however is given gratis. Users with high needs will select one of the companies. However, if they are charged with high prices, or served with low rates, a portion of them will eventually migrate to the municipal network. In other words, the municipal service constitutes an alternative choice for the users and therefore sets the minimum requirements which the commercial providers should satisfy. Apparently, the existence of the municipal network affects both the user decisions and the operators pricing policy. In different settings, the minimum requirement can be an inherent characteristic of the users as for example a lower bound on transmission rate for a particular application, an upper bound on the price they are willing to pay or certain combinations of both of these parameters. Again, the operators can attract the users only if they offer more appealing services and prices.

In this chapter, we consider a general wireless communication services market where a set of *operators*, compete to sell their services to a common large pool of *users*. We assume that users have minimum requirements or alternative options to satisfy their needs which we model by introducing the reservation utility $U_0$, [2]. Users select an operator only if the offered service and the charged price ensure utility higher than $U_0$. We analyze the users strategy for selecting operator and the price competition among the operators under this constraint. We find that the market outcome depends on $U_0$ and on the amount of spectrum each operator has at his disposal $W$. Accordingly, we consider the existence of a regulating agency who is interested in affecting the market and enforcing a more desirable outcome, by tuning either $W$ or $U_0$. For example, consider the municipal WiFi provider who is actually able to set $U_0$ and bias the competition among the commercial providers. This is of crucial importance since in many cases the competition of operators may yield inefficient allocation of the network resources, [2] or even reduced revenue for them, [41]. We introduce a rigorous framework that allows us to analyze the various methods through which the regulator can intervene and affect the market outcome according to his objective.

Our model captures many different settings such as a WiFi market in a city, a mobile/cell-phone market in a country or even a secondary spectrum market where primary users lease

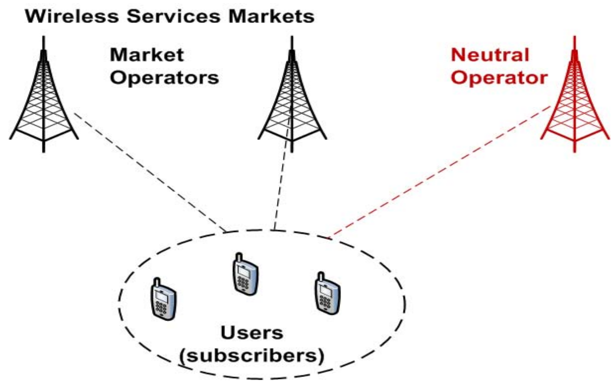



**Figure 4·1:** The market consists of a set of operators competing over a common pool of users. Each user selects one of the market operators or opts to abstain from the market and be associated with the neutral operator. The latter models the alternative out-of-the-market users option or their minimum service-price requirements.

their spectrum to secondary users. In order to make our study more realistic, we adopt a macroscopic perspective and analyze the interaction of the operators and users in a large time scale, for large population of users, and under limited information. The operators are not aware of the users specific needs and the latter cannot predict in advance the exact level of service they will receive. Each operator has a total resource at his disposal (e.g. the aggregate service rate) which is on average equally allocated to his subscribers, [2], [74]. This is due to the various network management and load balancing techniques that the operators employ, or because of the specific protocol that is used, [31]. Each user selects the operator that will provide the optimal combination of service quality and price. Apparently, the decision of each user affects the utility of the other users. We model this interdependency as an evolutionary game, [91] the stationary point of which represents the users distribution among the operators and depends on the charged prices. This gives rise to a non cooperative price competition game among the operators who strive to maximize their profits.

Central to our analysis is the concept or the *neutral operator* $P_0$ which provides to the users a constant and given utility of $U_0$ units. The $P_0$ can be a special kind of operator, like the municipal WiFi provider in the example above, or it can simply model the user choice to abstain from the market. This way, we can directly calculate how many users are served by the market and how many abstain from it and select $P_0$. Moreover, $P_0$ allows us to introduce the role of a regulating agency who can intervene and bias the market outcome through the service $U_0$. We show that $P_0$ can be used to increase the revenue of the operators or the efficiency of the market. In some cases, both of these metrics can be simultaneously improved at a cost which is incurred by the regulator. Alternatively, the outcome of the market can be regulated by changing the amount of spectrum each operator has at his disposal. Different regulating methods give different results and entail different

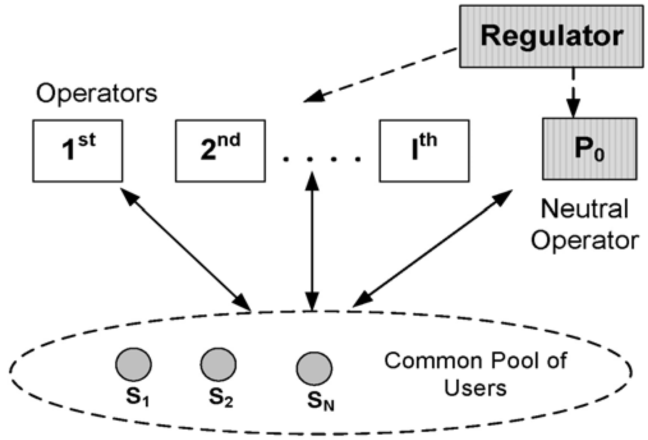



**Figure 4·2:** The oligopoly market consists of $I$ operators and $N$ users (S). Each user is associated with one operator at each specific time slot. Every operator $i = 1, 2, \ldots, I$ can serve more than one users at a certain time slot. The users that fail to satisfy their minimum requirements, $U_i \leq U_0$, $\forall i \in \mathcal{I}$, abstain from the market and select the neutral operator $P_0$.

cost for the regulator.

### 4.1.1 Related Work and Contribution

The competition of sellers for attracting buyers has been studied extensively in the context of network economics, [21], [93], both for the Internet and more recently for wireless systems. In many cases, the competition results in undesirable outcome. For example, in [2] the authors consider an oligopoly communication market and show that it yields inefficient resource allocation for the users. From a different perspective it is explained in [41], that selfish pricing strategies may also decrease the revenue of the sellers-providers. In these cases, the strategy of each node (buyer) affects the performance of the other nodes by increasing the delay of the services they receive, [2] (*effective cost*) or, equivalently, decreasing the resource the provider allocates to them, [74] (*delivered price*). This equal-resource sharing assumption represents many different access schemes and protocols (TDMA, CSMA/CA, etc), [31].

More recently, the competition of operators in wireless services markets has been studied in [74], [106], [79], [77], [68]. The users can be charged either with a usage-based pricing scheme, [106], or on a per-subscription basis, [79], [68]. We adopt the latter approach since it is more representative of the current wireless communication systems. We assume that users may migrate (churn) from one operator to the other, [68], and we use evolutionary game theory (EVGT) to model this process, [76]. This allows us to capture many realistic aspects and to analyze the interaction of very large population of users under limited information. The motivation for using EVGT in such systems is very nicely discussed in

[77]. Due to the existence of the *neutral operator*, the user strategy is updated through a hybrid scheme based on imitation and direct selection of $P_0$. We define a new revision protocol to capture this aspect and we derive the respective system dynamic equations.

Although the regulation has been discussed in context of networks, [21], it remains largely unexplored. Some recent work [64], [84] study how a regulator or an *intervention device* may affect a non-cooperative game among a set of players (e.g. operators). However, these works do not consider hierarchical systems, with large populations and limited information. Our contribution can be summarized as follows: **(i)** we model the wireless service market using an evolutionary game where the users employ a new hybrid revision protocol, based both on imitation and direct selection of a specific choice, namely the $P_0$. We derive the differential equations that describe the evolution of this system and find the stationary points, **(ii)** we define the price competition game for $I$ operators and the particular case that users have minimum requirements, or equivalently, alternative choices/offers, **(iii)** we prove that this is a Potential game and we analytically find the Nash equilibria, **(iv)** we introduce the concept of the neutral operator who represents the system/state regulator or the minimum users requirement, and **(v)** we discuss different regulation methods and analyze their efficacy, implications and the resources that are required for their implementation.

The rest of this chapter is organized as follows. In Section 4.2 we introduce the system model and in Section 4.3 we analyze the dynamics of the users interaction and find the stationary point of the market. In Section 4.4 we define and solve the price competition game among the operators and in Section 4.5 we discuss the relation between the revenue of the operators and the efficiency of the market and their dependency on the system parameters. Accordingly, we analyze various regulation methods for different regulation objectives and give related numerical examples. We conclude in Section 4.6.

## 4.2 System Model

We consider a wireless service market (hereafter referred to as a *market*) with a very large set of users $\mathcal{N} = (1, 2, \dots, N)$ and a set of operators $\mathcal{I} = (1, 2, \dots, I)$, which is depicted in Figure 4·2. We assume a time slotted operation. Each user cannot be served by more than one operator simultaneously. However, users can switch in each slot $t$ between operators or even they can opt to refrain and not purchase services from anyone of the $I$ operators. The net utility perceived by each user who is served by operator $i$ in time slot $t$ is:

$$U_i(W_i, n_i(t), \lambda_i) = V_i(W_i, n_i(t)) - \lambda_i \tag{4.1}$$

where $n_i(t)$ are the users served by this specific operator in slot $t$, $W_i$ the total spectrum at his disposal, and $\lambda_i$ the charged price. In order to describe the market operation we introduce the users vector $\mathbf{x}(t) = (x_1(t), x_2(t), \dots, x_I(t), x_0(t))$, where the $i^{th}$ component $x_i(t) = n_i(t)/N$ represents the portion of users that have selected operator $i \in \mathcal{I}$. Additionally, with $x_0(t) = n_0(t)/N$ we denote the portion of users that have selected neither of the $I$ operators. We assume that the number of users $N$ is very large, $N >> 1$ and therefore the variable $x_i(t) = n_i(t)/N$ is considered continuous. In other words, we assume that there exist a continuum of users partitioned among the different operators.

### Valuation function

The function $V_i(\cdot)$ represents the value of the offered service for each user associated with operator $i \in \mathcal{I}$. Users are considered homogeneous: all the users served by a certain operator are charged the same price and perceive the same utility. We consider the following particular valuation function:

$$V_i(W_i, x_i(t)) = \log \frac{W_i}{N x_i(t)},\ x_i(t) > 0 \tag{4.2}$$

Since $N$ is given, we use $x_i(t)$ instead of $n_i(t)$. This function has the following desirable properties: **(i)** the valuation for each user decreases with the total number of served users by the specific operator due to congestion, **(ii)** increases with the amount of available spectrum $W_i$, and **(iii)** it is a concave function and therefore captures the saturation of the user satisfaction as the allocated resource increases, i.e. it satisfies the principle of diminishing marginal returns, [21].

A basic assumption in our model is that users served by the same operator are allocated an equal amount of resource. We want to stress that this assumption captures many different settings in wireline, [2], or wireless networks, [74], [31], [79], [77], [17]. Some examples where the equal resource sharing assumption holds are the following:

- **FDMA - TDMA:** If the operator uses a multiple access scheme like Frequency Division Multiple Access (FDMA) or Time Division Multiple Access (TDMA), then the equal resource sharing assumption holds by default, [31]. The users served by a certain operator receive an equal share of his total available spectrum or an equal time share of the operator's channel.

- **CSMA/CA:** A similar result holds for the Carrier Sense Multiple Access scheme with Collision Avoidance, [8], that is used in IEEE 802.11 protocols. Users trying to access the channel receive an equal share of it and achieve - on average - the same transmission rate. Additionally, as it was shown in [15], even if the radio transmitters are controlled by selfish users, they can achieve this fair resource sharing.

- **Random access of multiple channels:** Even in more complicated access schemes as in the case, for example, where many different users iteratively select the least congested channel among a set of available channels, it is proved that each user receives asymptotically an equal share of the channel bandwidth, [17].

Additionally, the macroscopic perspective and the large time scale that we consider in this problem , ensure that spatiotemporal variations in the quality of the offered services will be smoothed out due to load balancing and other similar network management techniques that the operators employ. Therefore, users of each operator are treated in equal terms.

### Neutral Operator

Variable $x_0(t)$ represents the portion of users that do not select anyone of the $I$ operators. Namely, a user in each time slot $t$ is willing to pay operator $i \in \mathcal{I}$ only if the offered utility $U_i(W_i, x_i(t), \lambda_i)$ is greater than a threshold $U_0 \geq 0$. If all operators fail to satisfy this minimum requirement then the user abstains from the market and is associated with the *Neutral Operator* $P_0$, Figure 4·2. In other words, $P_0$ represents the choice of selecting neither of the $I$ operators and receiving utility of $U_0$ units. Technically, as it will be shown in the sequel, the inclusion of $P_0$ affects both the user decision process for selecting operator and the competition among the operators.

From a modeling perspective, the neutral operator may be used to represent different realistic aspects of the wireless service market. First, $P_0$ can be an actual operator owned by the state, as the public/municipal WiFi provider we considered in the introductory example. In this case, through the gratis $U_0$ service, the state intervenes and regulates the market as we will explain in Section 4.5. Additionally, $U_0$ can be indirectly imposed by the state (the regulator) through certain rules such as the minimum amount of spectrum/rate per user. Finally, it can represent the users reluctancy to pay very high prices for poor QoS, similarly to the individual rationality constraint in mechanism design. We take these realistic aspects into account and moreover, by using $x_0(t)$, we find precisely how many users are not satisfied by the market of the $I$ operators.

Unlike the valuation $V_i(\cdot)$ of the service offered by each operator $i \in \mathcal{I}$, $U_0$ is considered constant. When $U_0$ represents users minimum requirements or respective restrictions imposed by regulatory rules, this assumption follows directly and actually is imperative. In case $U_0$ models the service offered by the neutral operator (e.g. the municipal WiFi network), the constant value of $U_0$ means that it is independent of the number of users and hence non-congestible. We follow this assumption for the following two reasons:**(i)** $U_0$ is a free of charge service which in general is low and hence can be ensured for a large number of users. **(ii)** The state agency (i.e. the regulator) who provides $U_0$, is able to increase his resource in order to ensure a constant value for $U_0$. As we will explain in next sections, this latter aspect captures the cost of regulation, i.e. the cost of serving users through the neutral operator. Finally, notice that our model can be easily extended for the case that $U_0$ is a congestible service.

### Revenue

Each operator $i \in \mathcal{I}$ determines the price $\lambda_i \in R^+$ that he will charge to his clients. The decisions of the operators are realized in a different time scale than the decisions of the users. Namely, each operator $i$ determines his price in the beginning of each time epoch $\mathcal{T}$ which consists of $T$ slots, while users update their operator association decision in each slot. Let us define the price vector $\lambda = (\lambda_i : i = 1, 2, \ldots, I)$ and the vector of the $I-1$ prices of operators other than $i$ as $\lambda_{-i} = (\lambda_j : j \in \mathcal{I} \backslash i)$. We assume that $T$ is large enough so that for each price vector $\lambda$ set at the beginning of an epoch, the market of the users reaches a stationary point - if such a point is attainable - during this epoch. The objective of each operator $i \in \mathcal{I}$ is to maximize his revenue during each epoch $\mathcal{T}$:

$$R_i(x_i(t), \lambda_i) = \lambda_i x_i(t) N \tag{4.3}$$

In these markets there are no service level agreements (SLAs) or any other type of QoS guarantees and hence the operators are willing to admit and serve as many users as it is required to achieve their goal.

## 4.3 User Strategy and Market Dynamics

### 4.3.1 Evolutionary Game $\mathcal{G}_{\mathcal{U}}$ among Users

In order to select the optimal operator that maximizes eq. (4.1), each user must be aware of all system parameters, i.e. the spectrum $W_i$, the number of served users $n_i$ and the charged price $\lambda_i$ for each $i \in \mathcal{I}$. However, in realistic settings this information will not be available in advance. Given these restrictions and the large number of users, we model their interaction and the operator selection process by defining an evolutionary game, $\mathcal{G}_{\mathcal{U}}$, as follows:

- Players: the set of the $N$ users, $\mathcal{N} = (1, 2, \ldots, N)$.
- Strategies: each user selects a certain operator $i \in \mathcal{I}$ or the neutral operator $P_0$.
- Population State: the users distribution over the $I$ operators and the neutral operator, $\mathbf{x}(t) = (x_1(t), x_2(t), \ldots, x_I(t), x_0(t))$.
- Payoff: the user's net utility $U_i(W_i, x_i(t), \lambda_i)$ when he selects operator $i \in \mathcal{I}$, or $U_0$ when he selects $P_0$.

To facilitate our analysis we make the following assumptions:

- **Assumption 1**: The number of users $N$ is very large, $N >> 1$ and therefore the variable $x_i(t) = n_i(t)/N$ is considered continuous.
- **Assumption 2**: The initial distribution of users over the $I$ operators is non zero: $x_i(0) > 0$, $\forall i \in \mathcal{I}$. It directly follows that $x_0(0) < 1$.

In the sequel we explain how each user selects his strategy under this limited information and what is the outcome of this game.

### 4.3.2 User Strategy Update

A basic component of every evolutionary game is the *revision protocol*, [91]. It captures the dynamics of the interaction among the users and describes in detail the process according to which a player iteratively updates his strategy. There exist many different options for the revision protocol, depending on the modeling assumptions of the specific problem. These assumptions are mainly related to how sophisticated, informed and rational are the players. On the one extreme, fully rational and informed players update their choices according to a best response strategy like in the typical (non-evolutionary) strategic games. This means that players make a *direct selection* of the best available strategy. On the other extreme, players follow an imitation strategy. In this case a player $(A)$ selects randomly another player $(B)$ and if the utility of the latter is higher, $(A)$ imitates his strategy with a probability that is proportional to the anticipated utility improvement. This modeling option is suitable for imperfectly informed players, or players with bounded rationality who update their strategy based on a better (instead of best) response strategy. Between these two extremes, there are many different options. For example, a player may update his strategy with a *hybrid* protocol based partially on imitation and on direct selection, [91].

In this work, we assume that each user updates his strategy by a special type of hybrid revision protocol which is a combination of imitation of other users associated with operators from the set $\mathcal{I}$ (market operators) and direct selection of the neutral operator $P_0$. The imitation component captures the lack of information users have at their disposal about the market. On the other hand, each user is aware of the exact value of $U_0$ and hence this choice is always available through direct selection. Notice that the considered revision protocol is not a typical hybrid protocol since the direct selection is related only to the selection of $P_0$ and not to the other operators.

In detail, the proposed revision protocol can be described by the following actions that each user may take in each slot $t$:

1. A user associated with an operator $i \in \mathcal{I}$, selects randomly another user who is associated with an operator $j \in \mathcal{I}$, $j \neq i$, and if $U_j > U_i$ imitates his strategy with a probability that is proportional to the difference $(U_j - U_i)$.

2. A user associated with the neutral operator $P_0$, selects randomly another user associated with operator $j \in \mathcal{I}$ and if $U_j > U_0$, imitates his strategy with a probability that is proportional to the difference $(U_j - U_0)$.

3. A user associated with operator $i \in \mathcal{I}$ selects the neutral operator $P_0$ with probability that is proportional to the difference $(U_0 - U_i)$.

Options 1 and 2, stem from the replicator dynamics introduced by Taylor and Jonker in [103] and are based on imitation of users with better strategies. On the other hand, option 3 is based on direct selection of better strategies, known also as pairwise dynamics, introduced by Smith in [96].

After defining the revision protocol, we can calculate the rate at which users switch from one strategy (operator) to another strategy (operator). In particular, the switch rate of users migrating from operator $i$ to operator $j \in \mathcal{I} \setminus i$ in time slot $t$, is:

$$\rho_{ij}(t) = x_j(t)[U_j(t) - U_i(t)]_+ \tag{4.4}$$

where $x_j(t)$ is the portion of users already associated with operator $j$. For simplicity, we express the user utilities as a function with a single argument, the time $t$. Additionally, the users switch rate from operator $i$ to neutral operator $P_0$, is:

$$\rho_{i0}(t) = \gamma[U_0 - U_i(t)]_+ \tag{4.5}$$

Notice the difference between imitation and direct selection [91]. Instead of multiplying the utilities difference with the population $x_0(t)$, we use a constant multiplier $\gamma \in R$. This is due to the model assumption that switching to the neutral operator is not accomplished through imitation and hence does not depend on the portion of users already been associated with

$P_0$. The probabilistic aspect captures the bounded rationality, the inertia of the users and other similar realistic aspects of these markets. Finally, the switch rate of users leaving $P_0$ and returning to the market (option 2) is:

$$\rho_{0i}(t) = x_i(t)[U_i(t) - U_0]_+ \tag{4.6}$$

Variables $\rho_{ij}$, $\rho_{i0}$ and $\rho_{0i}$ represent the rates at which users migrate from one operator to another, including the neutral operator $P_0$. It is interesting to notice that if these rates are normalized properly, they can be interpreted as the probabilities with which users update their operator selection strategy. This approach is discussed in [91]. In the sequel we use these rates to derive the ordinary differential equations (ODE) that describe the evolution of the population of users.

### 4.3.3 Market Stationary Points

The new type of hybrid revision protocol introduced above, results in user market dynamics that cannot be expressed with the known differential equations of replicator dynamics or other similar scheme, [91]. In Section 4.A.1 of the Appendix we prove that the mean dynamics of the system are:

$$\begin{aligned} \frac{dx_i(t)}{dt} &= x_i(t)[U_i(t) - U_{avg}(t) - x_0(t)(U_i(t) - U_0) \\ &- \gamma(U_0 - U_i(t))_+ + x_0(t)(U_i(t) - U_0)_+], \forall i \in \mathcal{I} \end{aligned} \tag{4.7}$$

where $U_{avg}(t) = \sum_{i\in\mathcal{I}} x_i(t)U_i(t)$ is the average utility of the market in each slot $t$. The user population associated with $P_0$ is:

$$\frac{dx_0(t)}{dt} = x_0 \sum_{i\in\mathcal{I}^+} x_i(U_0 - U_i) + \gamma \sum_{j\in\mathcal{I}^-} x_j(U_0 - U_j) \tag{4.8}$$

where $\mathcal{I}^+$ is the subset of operators offering utility $U_i(t) > U_0$, and $\mathcal{I}^-$ is the subset of operators offering utility $U_i(t) < U_0$, at slot $t$.

The important thing is that despite its different evolution, as we prove in Section 4.A.2, this system has the same stationary points as the systems that are described by the classical replicator dynamic equations:

$$\dot{x}_i(t) = 0 \Rightarrow x_i(t)[U_i(t) - U_{avg}(t)] = 0, \forall i \in \mathcal{I} \tag{4.9}$$

and

$$\dot{x}_0(t) = 0 \Rightarrow x_0(t)[U_0 - U_{avg}(t)] = 0 \tag{4.10}$$

The user state vector $\mathbf{x}^*$ and the respective user utility $U_i^*$, $i \in \mathcal{I}$, that satisfy these stationary conditions can be summarized in the following 3 cases:

- **Case A**: $x_i^*$, $x_0^* > 0$ and $U_i^* = U_0$, $i \in \mathcal{I}$.

- **Case B**: $x_i^*,\, x_j^* > 0,\, x_0^* = 0$ and $U_i^* = U_j^*$, with $U_i^*,\, U_j^* > U_0, \forall\, i,\, j \in \mathcal{I}$.
- **Case C**: $x_i^*,\, x_j^* > 0,\, x_0^* = 0$ and $U_i^* = U_j^* = U_0,\ \forall\, i,\, j \in \mathcal{I}$.

Case $A$ corresponds to the scenario where all operators offer to their clients net utility which is equal to the value of the service offered by the neutral operator. On the other hand, in case $B$ the market operators offer higher utility than the neutral operator and hence all users are served by the market. Finally, in case $C$, the $I$ operators offer marginal services, i.e. equal to $U_0$, but they have attracted all the users.

It is interesting to compare the above results with the Wardrop model and the Wardrop equilibrium, [107]. The market stationary points for **Case A** and **Case C** satisfy the *Wardrop first principle* and yield an equilibrium where the available strategy options ("operators" in our problem) result in equal utility for the players ("users"). However, this does not hold for **Case B** where operators other than $P_0$ offer higher utility. This emerges due to the fact that the alternative option (or reservation utility) is non-congestible, i.e. independent of $x_0$. The evolutionary game allows us to provide a richer model than the typical Wardrop model and more importantly to capture the users interaction and dynamics.

Before calculating the stationary point $\mathbf{x}^*$ for each case, and in order to facilitate our analysis, we define the scalar parameter $\alpha_i = W_i/(Ne^{U_0})$ for each operator $i \in \mathcal{I}$ and the respective vector $\alpha = (\alpha_i\, :\, i = 1, 2, \ldots, I)$. As it will be explained in the sequel, these parameters determine the operators and users interaction and will help us to explain the role of the regulator. We can find the stationary points for **Case A** by using equation $U_i(W_i, x_i^*, \lambda_i) = U_0$ and imposing the constraint $x_0^* > 0$. Apparently, the state vector $\mathbf{x}^*$ depends on the price vector $\lambda$. Therefore, we define the set of all possible **Case A** stationary points, $X_A$, as follows (see Section 4.A.2 for details):

$$X_A = \left\{ x_i^* = \alpha_i e^{-\lambda_i}, \forall i \in \mathcal{I}, x_0^* = 1 - \sum_{i=1}^{I} \alpha_i e^{-\lambda_i} \,:\, \lambda \in \Lambda_A \right\} \tag{4.11}$$

where $\Lambda_A$ is the set of prices for which a stationary point in $X_A$ is attainable, i.e. for which it holds $x_0^* > 0$:

$$\Lambda_A = \left\{ (\lambda_1, \lambda_2, \ldots, \lambda_I) \,:\, \sum_{i=1}^{I} \alpha_i e^{-\lambda_i} < 1 \right\} \tag{4.12}$$

Recall that due to the very large number of users, we consider $x_i$ a continuous variable.

Similarly, for **Case B**, we calculate the stationary points by using the set of equations $U_i(W_i, x_i^*, \lambda_i) = U_j(W_j, x_j^*, \lambda_j),\, \forall\, i, j \in \mathcal{I}$:

$$X_B = \left\{ x_i^* = \frac{\alpha_i}{e^{\lambda_i} \sum_{j=1}^{I} \alpha_j e^{-\lambda_j}}, \forall i \in \mathcal{I}, x_0^* = 0 \,:\, \lambda \in \Lambda_B \right\} \tag{4.13}$$

**Table 4.1:** Wireless Service Market Stationary Points.

| | $X_A$ | $X_B$ | $X_C$ |
|---|---|---|---|
| $x_i^*$ | $\alpha_i e^{-\lambda_i}$ | $\frac{\alpha_i}{e^{\lambda_i}\sum_{j=1}^{I}\alpha_j e^{-\lambda_j}}$ | $\alpha_i e^{-\lambda_i}$ |
| $x_0^*$ | $1-\sum_{i=1}^{I}\alpha_i e^{-\lambda_i}$ | 0 | 0 |
| Cond. | $\lambda \in \Lambda_A$ | $\lambda \in \Lambda_B$ | $\lambda \in \Lambda_C$ |

where $\Lambda_B$ is the set of prices for which a stationary point in $X_B$ is feasible, i.e. $U_i^* > U_0$:

$$\Lambda_B = \left\{(\lambda_1, \lambda_2, \ldots, \lambda_I) \,:\, \sum_{i=1}^{I} \alpha_i e^{-\lambda_i} > 1\right\} \tag{4.14}$$

Finally, the stationary points for the **Case C** solution must satisfy the constraint $\sum_{i=1}^{I}\alpha_i e^{-\lambda_i} = 1$ which yields:

$$X_C = \left\{x_i^* = \alpha_i e^{-\lambda_i}, \forall\, i \in \mathcal{I}, x_0^* = 0 :\ \lambda \in \Lambda_C\right\} \tag{4.15}$$

with

$$\Lambda_C = \left\{(\lambda_1, \lambda_2, \ldots, \lambda_I) \,:\, \sum_{i=1}^{I} \alpha_i e^{-\lambda_i} = 1\right\} \tag{4.16}$$

Notice that the stationary point sets $X_A$, $X_B$ and $X_C$ and the respective price sets, $\Lambda_A$, $\Lambda_B$, and $\Lambda_C$ depend on the vector $\alpha$. These results are summarized in Table 4.1. For each operators price profile $\lambda$, the evolutionary game admits a unique stationary point $\mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_I^*, x_0^*)$ which belongs in the respective set $X_A$, $X_B$, or $X_C$. The utility of the users is equal to $U_0$ for the **Case A** and **Case C**, while for **Case B** it depends on $\lambda$.

### Stability of Stationary Points

Now that we found the stationary points of the hybrid revision protocol, it is important to characterize their stability. We prove in the sequel that these points are Evolutionary Stable Strategies (ESS) and hence they are locally asymptotically stable, i.e. stable within a limited region. ESS and replicator dynamics are the two concepts used for studying evolutionary games. Unlike the replicator dynamics, ESS is a static concept which requires that the strategy of players in the equilibrium is stable when it is invaded by a small population of players playing a different strategy, [30]. When the players population is homogeneous, as we assumed in our model, an ESS is stable in the replicator dynamic, but not every stable steady state is an ESS. Additionally, every ESS is Nash, and hence ESS is a refinement of the Nash equilibrium.

Let us first give a simple definition of the ESS, tailored to our system model. Assume that the users market has reached the stationary state described by vector $\mathbf{x}^*$. Suppose now that a small portion $\epsilon > 0$ of the users population deviates from their decision in the stationary state (i.e. selects another operator) and selects another operator $j \in \mathcal{I}$ or the neutral operator. This yields a new distribution of users which we denote by $\mathbf{x}_\epsilon =$

$(x_1^\epsilon, x_2^\epsilon, \ldots, x_I^\epsilon, x_0^\epsilon)$. We say that $\mathbf{x}^*$ is an ESS if (i) users that deviate from $\mathbf{x}^*$ receive lower utility in the new system state $\mathbf{x}_\epsilon$ or, (ii) the utility of the deviating users in $\mathbf{x}_\epsilon$ is the same as in the previous state $\mathbf{x}^*$, but the utility of the legitimate users (those insisting in their initial decisions) is higher in $\mathbf{x}_\epsilon$ than in $\mathbf{x}^*$. In both cases, the deviating users worsen their obtained utility. The stationary points derived above satisfy these conditions and hence they are ESS.

In detail, assume that the system has a stationary point $\mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_I^*, x_0^*) \in X_B$, with $x_0^* = 0$. Suppose that a user who is associated with operator $i \in \mathcal{I}$ deviates and selects another operator $j \in \mathcal{I}$. In this case, the population of users in operator $i$ decreases, $x_i^\epsilon < x_i^*$ and the population of users in operator $j$ increases, $x_j^\epsilon > x_j^*$. Initially, these two operators offered identical utility, $U_i(W_i, x_i^*, \lambda_i) = U_j(W_j, x_j^*, \lambda_j)$ but after the decision of the deviating user it becomes $U_i(W_i, x_i^\epsilon, \lambda_i) > U_j(W_j, x_j^\epsilon, \lambda_j)$. Clearly, the deviating user obtains less utility and hence there is no incentive to deviate. Similarly, if a user deviates and selects the neutral operator, he will receive reduced utility since when $\mathbf{x}^* \in X_B$, it is $U_i^* > U_0$, $\forall i \in \mathcal{I}$.

Assume now that the system attains a stationary point $\mathbf{x}^* \in X_A$. Similarly to the previous analysis, it is straightforward that a user who deviates from $\mathbf{x}^*$ and moves from an operator $i \in \mathcal{I}$ to another operator $j \in \mathcal{I}$ will decrease his utility. If the user migrates to the neutral operator, his utility will not be reduced because $U_0$ is constant (non-congestible). However, in this case, the users that will insist in their initial choice of operator $i$ will now receive higher utility due to the move of the deviator. Due to the ESS definition and specifically according to Smith's second condition, [95], this is not a preferable choice for the deviator and hence $\mathbf{x}^* \in X_B$ is an ESS.

Finally, when $\mathbf{x}^* \in X_C$, user deviation from a market operator $i \in \mathcal{I}$ to another market operator $j \in \mathcal{I}$ or to $P_0$ is not beneficial for the deviator, either because it decreases his utility or because it increases the utility of other users. In conclusion, the stationary points of the proposed revision protocol are ESS equilibriums and hence locally stable.

## 4.4 Price Competition Among Operators

In the previous section we analyzed the stationary points of users interaction and showed that they depend on the prices selected by the operators. Each operator anticipates the users strategy and chooses accordingly for each epoch $\mathcal{T}$ the price that maximizes his revenue. This gives rise to a non-cooperative price competition game $\mathcal{G}_\mathcal{P}$ among the operators that is played in the beginning of each time epoch $\mathcal{T}$. We assume that operators are aware of the parameters of the users market and also know the values of parameters $\alpha_i$, $i \in \mathcal{I}$ and $U_0$. Specifically, we model the operators competition as a static simultaneous move normal form game of complete information, following the Bertrand competition model [21]. We are interested not only in finding the Nash equilibriums (NE) of this game but also to understand if and how the game converges to them.

We prove that $\mathcal{G}_\mathcal{P}$ is a potential game and hence if it is played in many rounds and operators choose their prices based on the previous prices of the other operators, the game converges to a NE. In other words, we analyze the dynamics induced by the repeated play of the same game assuming that operators follow simple myopic rules. We show that the equilibrium of the competition game depends on vector $\alpha$ and the value of $U_0$. For certain combinations of these parameters, the game admits a unique equilibrium while for other combinations, it reaches one of the infinitely many equilibriums depending on the initial prices.

### 4.4.1 Price Competition Game $\mathcal{G}_{\mathcal{P}}$

Before analyzing this game, it is important to emphasize that the revenue function depends on the price vector $\lambda$. In particular, using equation (4.3), we can calculate the revenue of operator $i$ when $\lambda \in \Lambda_A$, when $\lambda \in \Lambda_B$, and when $\lambda \in \Lambda_C$, denoted as $R_i^A(\cdot)$, $R_i^B(\cdot)$ and $R_i^C(\cdot)$ respectively:

$$R_i^A(\lambda_i) = \alpha_i \lambda_i N e^{-\lambda_i}, \; R_i^B(\lambda_i, \lambda_{-i}) = \frac{\alpha_i \lambda_i N}{e^{\lambda_i} \sum_{i=1}^{I} \alpha_i e^{-\lambda_i}}, \; R_i^C(\lambda_i) = \alpha_i \lambda_i N e^{-\lambda_i} \tag{4.17}$$

$R_i^A(\cdot)$ and $R_i^C(\cdot)$ depend only on the price selected by operator $i$, while $R_i^B(\cdot)$ depends on the entire price vector $\lambda$. However, in all cases, the price set ($\Lambda_A$, $\Lambda_B$ or $\Lambda_C$) to which the price vector $\lambda = (\lambda_i, \lambda_{-i})$ belongs, is determined jointly by all the $I$ operators.

Let us now define the non-cooperative **Pricing Game** among the $I$ operators, $\mathcal{G}_{\mathcal{P}} = (\mathcal{I}, \{\lambda_i\}, \{R_i\})$:

- The set of *Players* is the set of the $I$ operators $\mathcal{I} = (1, 2, \ldots, I)$.
- The *strategy space* of each player $i$ is its price $\lambda_i \in [0, \lambda_{max}]$, $\lambda_{max} \in \mathcal{R}^+$, and the strategy profile is the price vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_I)$ of the operators.
- The payoff function of each player is his revenue $R_i : (\lambda_i, \lambda_{-i}) \rightarrow \mathcal{R}$, where $R_i \triangleq R_i^A$ or $R_i^B$ or $R_i^C$.

The particular characteristic of this game is that each operator has 2 different payoff functions depending on the price profile. Despite this characteristic, the payoff function is continuous and quasi-concave as we prove in the Appendix, Section 4.B.1. In the sequel, we analyze the best response of each operator which constitutes a reaction curve to the prices set by the other operators. The equilibrium of the game $\mathcal{G}_{\mathcal{P}}$ is the intersection of the reaction curves of the operators.

### 4.4.2 Best Response Strategy of Operators

The best response of each operator $i$, $\lambda_i^*$, to the prices selected by the other $I-1$ operators, $\lambda_{-i}$, depends on the users market stationary point. Notice that for certain $\lambda_{-i}$, operator $i$ may be able to select a price such that $(\lambda_i, \lambda_{-i})$ belongs to any price set ($\Lambda_A$, $\Lambda_B$ or $\Lambda_C$) while for some $\lambda_{-i}$ the operator choice will be restricted in two or even a single price set.

**Best Response when $\lambda \in \Lambda_A$:** If the $I-1$ operators $j \in \mathcal{I} \setminus i$ select such prices, $\lambda_{-i}$, that the market stationary point is $\mathbf{x}^* \in X_A$, then operator $i$ finds the price $\lambda_i^*$ that maximizes his revenue $R_i^A(\cdot)$ by solving the following constrained optimization problem $(\mathbf{P}_i^A)$:

$$\max_{\lambda_i \geq 0} \alpha_i \lambda_i N e^{-\lambda_i} \tag{4.18}$$

s.t.

$$\sum_{j=1}^{I} \alpha_j e^{-\lambda_j} < 1 \tag{4.19}$$

The objective function of this problem is quasi-concave, [11]. However, the constraint defines an open set and hence uniqueness of optimal solution is not ensured. To overcome this obstacle we substitute constraint eq. (4.19) with the closed set:

$$\lambda_i \geq \log \frac{\alpha_i}{1 - \sum_{j \neq i} \alpha_j e^{-\lambda_j}} + \epsilon \tag{4.20}$$

where $\epsilon > 0$ is an arbitrary small constant number. This inequality stems from eq. (4.19) by solving for $\lambda_i$ and adding $\epsilon$. It does not affect the problem definition and formulation nor the obtained results since, as we will prove in the sequel, operators do not select a price in the lower bound of the constraint set. After this transformation the problem has a unique optimal solution which is equal to the solution of the respective unconstrained problem, $\lambda_i^* = 1$, if $(1, \lambda_{-i}) \in \Lambda_A$.

**Best Response when $\lambda \in \Lambda_B$:** Similarly, when $\lambda_{-i}$ is such that operator $i$ can select a price $\lambda_i^*$ with $(\lambda_i^*, \lambda_{-i}) \in \Lambda_B$, then his revenue is given by the function $R_i^B(\cdot)$ and is maximized by the solution of problem $(\mathbf{P}_i^B)$:

$$\max_{\lambda_i \geq 0} \frac{\lambda_i \alpha_i N}{e^{\lambda_i} \sum_{j \in \mathcal{I}} \alpha_j e^{-\lambda_j}} \tag{4.21}$$

s.t.

$$\sum_{j=1}^{I} \alpha_j e^{-\lambda_j} > 1 \tag{4.22}$$

This is also a concave problem which would have a unique solution if the constraint set was closed and compact. Again, we substitute the constraint with the (almost) equivalent inequality:

$$\lambda_i \leq \log \frac{\alpha_i}{1 - \sum_{j \neq i} \alpha_j e^{-\lambda_j}} - \epsilon \tag{4.23}$$

Now, the problem has a unique solution which coincides with the solution of the respective unconstrained problem, denoted $\mu_i^*$, if $(\mu_i^*, \lambda_{-i}) \in \Lambda_B$ as we explain in detail in Section 4.B.2.

**Best Response when $\lambda \in \Lambda_C$:** In this special case, the price of each operator $i$ is directly determined by the prices that the other operators have selected. Namely, given the vector $\lambda_{-i}$, each operator $i$ has only one feasible solution (otherwise $\lambda$ does not belong to $\Lambda_C$):

$$\lambda_i^* = \log \frac{\alpha_i}{1 - \sum_{j \neq i} \alpha_j e^{-\lambda_j}} \tag{4.24}$$

Whether each operator $i$ will agree and adopt this price or not, depends on the respective accrued revenue $R_i^C(\lambda_i^*, \lambda_{-i})$.

We can summarize the best response price strategy of each operator $i \in \mathcal{I}$, by defining his revenue function as follows:

$$R_i(\lambda_i, \lambda_{-i}, \alpha) = \begin{cases} \frac{\alpha_i \lambda_i N}{\sum_{j=1}^{I} \alpha_j e^{\lambda_i - \lambda_j}} & \text{if } \lambda_i < l_0, \\ \alpha_i \lambda_i N e^{-\lambda_i} & \text{if } \lambda_i \geq l_0. \end{cases} \tag{4.25}$$

where $l_0 = \log(\alpha_i / (1 - \sum_{j \neq i} \alpha_j e^{-\lambda_j}))$. Clearly, the optimal price $\lambda_i^*$ depends both on the prices of the other operators $\lambda_{-i}$ and on parameters $\alpha_i$, $i = 1, 2, \ldots, I$:

$$\lambda_i^* = arg \max_{\lambda_i} R_i(\lambda_i, \lambda_{-i}, \alpha) \tag{4.26}$$

where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_I)$. Clearly, each operator needs to know the vector $\alpha$ and to be able to observe the other operators prices in order to calculate his best response.

For each possible price vector $\lambda_{-i}$ of the $\mathcal{I} \setminus i$ operators, operator $i$ will solve all the above optimization problems and find the solution that yields the highest revenue. In Section 4.B.2 we prove that this results in the following best response strategy:

$$\lambda_i^*(\lambda_{-i}, \alpha) = \begin{cases} 1 & \text{if } (1, \lambda_{-i}) \in \Lambda_A, \\ \mu_i^* & \text{if } (\mu_i^*, \lambda_{-i}) \in \Lambda_B, \\ l_0 & \text{otherwise.} \end{cases} \tag{4.27}$$

These options are mutually exclusive. Moreover, if $\sum_{j \neq i} \alpha_j / e^{\lambda_j} \geq 1$, the only feasible response is $\lambda_i^* = \mu_i^*$. The dependence of $\lambda_i^*$ on parameters $\alpha_i = W_i/(Ne^{U_0})$, $i = 1, 2, \ldots, I$, has interesting implications and brings into the fore the role of the regulator. Finally, observe that the transformation of the constraint set of problems $\mathbf{P}_i^A$ and $\mathbf{P}_i^B$ did not affect the best response strategy of operator $i$ since he only selects the solution of the respective unconstrained problems.

### 4.4.3 Equilibrium Analysis of $\mathcal{G}_{\mathcal{P}}$

The price competition game $\mathcal{G}_{\mathcal{P}}$ is a finite ordinal potential game and therefore not only has pure Nash equilibria but also the players can reach them under any best response strategy. That is, if we consider that $\mathcal{G}_{\mathcal{P}}$ is played repeatedly by the operators who update their strategy with a myopic best response method, we can show that the convergence to the equilibriums is ensured under any finite improvement path (FIP), [71]. The potential function is:

$$\mathcal{P}(\lambda) = \begin{cases} \sum_{j=1}^{I}[\log\lambda_j - \lambda_j], & \text{if} \sum_{j=1}^{I}\alpha_j e^{-\lambda_j} \le 1, \\ \sum_{j=1}^{I}[\log\lambda_j - \lambda_j] - \log\left(\sum_{j=1}^{I}\alpha_j e^{-\lambda_j}\right), & \text{else.} \end{cases} \tag{4.28}$$

The detailed proof is given in Section 4.B.3. In order to find the NE we solve the system of equations (4.27), $i = 1, 2, \ldots, I$ and specifically we use the iterated dominance method (Section 4.B.4).

The outcome of the game $\mathcal{G}_{\mathcal{U}}$ affects the strategy of operators and therefore the outcome of the game $\mathcal{G}_{\mathcal{P}}$. A price vector $(\lambda_i^*, \lambda_{-i}^*)$ is an equilibrium of the game $\mathcal{G}_{\mathcal{P}}$, parameterized by the vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_I)$, if it satisfies:

$$R_i(\lambda_i^*, \lambda_{-i}^*, \alpha) \ge R_i(\lambda_i, \lambda_{-i}^*, \alpha), \forall\, i \in \mathcal{I},\ \forall \lambda_i \ge 0,\ \forall\, \mathbf{x}^* \in\ X_A \cup X_B \cup X_C \tag{4.29}$$

In order to simplify our study and focus on the results and implications of our analysis, we assume that all operators have the same amount of available spectrum $W_i = W$ and therefore it is also $\alpha_i = \alpha$, $\forall i \in \mathcal{I}$.

The equilibrium of the price competition game and subsequently the market stationary point $\mathbf{x}^*$, depend on the value of $\alpha$. These results are summarized in Table 4.2 and stem from the following Theorem:

**Theorem 4.4.1.** *The non-cooperative game $\mathcal{G}_{\mathcal{P}}$ where operators select their strategy in order to maximize their revenue, converges to one of the following pure Nash equilibria:*

- *If $\alpha \in A_1 = (0, e/I)$, there is a unique Nash Equilibrium $\lambda^* \in \Lambda_A$, with $\lambda^* = (\lambda_i^* = 1 : i = 1, 2, \ldots, I)$ and the respective unique market stationary point is $\mathbf{x}^* \in X_A$.*
- *If $\alpha \in A_3 = (e^{\frac{I}{I-1}}/I, \infty)$, there is a unique Nash Equilibrium $\lambda^* \in \Lambda_B$, with $\lambda^* = (\lambda_i^* = \frac{I}{I-1} : i = 1, 2, \ldots, I)$, which induces a unique respective market stationary point $\mathbf{x}^* \in X_B$.*
- *If $\alpha \in A_2 = [e/I, e^{\frac{I}{I-1}}/I]$, there exist infinitely many equilibria, $\lambda^* \in \Lambda_C$, and each one of them yields a respective market stationary point $\mathbf{x}^* \in X_C$.*

**Proof:** In Section 4.B.3 of the Appendix we provide the detailed proof according to which $\mathcal{G}_{\mathcal{P}}$ is a potential game and in Section 4.B.4 we use iterated strict dominance to find the Nash equilibrium $\lambda^*$ which depends on parameter $\alpha$. ■

In conclusion, $\mathcal{G}_{\mathcal{P}}$ is a non-cooperative game of complete information that attains certain pure Nash equilibriums (NE) which depend on parameters $\alpha_i$, $i = 1, 2, \ldots, I$. It is proved to be a potential game and hence the equilibriums can be reached if $\mathcal{G}_{\mathcal{P}}$ is played repeatedly and operators update their strategy by simple best response or other similar utility improvement methods. If $\alpha_i$ parameters are equal, i.e. $\alpha_i = \alpha$, $\forall i \in \mathcal{I}$, then the NE is unique for $\alpha \in A_1$ or $\alpha \in A_3$. For the case $\alpha \in A_2$, the reached equilibrium depends on the initial price vector.

**Table 4.2:** Equilibriums of $I$ operators competition for different values of $\alpha$.

| Prices/Rev. | $\alpha \in A_1$ | $\alpha \in A_2$ | $\alpha \in A_3$ |
|---|---|---|---|
| $\lambda_i^*$ | 1 | $\lambda_i \neq \lambda_j$ | $\frac{I}{I-1}$ |
| | | or $\lambda_i = \lambda_j = \log I\alpha$ | |
| $R_i^*$ | $\frac{\alpha N}{e}$ | $R_i \neq R_j$ | $\frac{N}{I-1}$ |
| | | or $R_i = R_j = \frac{N}{I} \log I\alpha$ | |
| $\mathbf{x}^*$ | $X_A$ | $X_C$ | $X_B$ |

## 4.5 Market Outcome and Regulation

The outcome of the users and operators interaction can be characterized by the following two fundamental criteria: the efficiency of the users market and the total revenue the operators accrue. We show that both of them depend on parameter $\alpha$ and we further explore the impact of $W$ and $U_0$ on them. Accordingly, we analyze the problem from a mechanism design perspective and explain how a regulator, as the municipal WiFi provider in the introductory example, can bias the market operation (outcome) by adjusting the value of $\alpha$. We consider different regulation methods and discuss their implications.

### 4.5.1 Market Outcome and Regulation Criteria

#### Market Efficiency

A market is efficient if the users enjoy high utilities in the stationary point. However, in certain scenarios, the services provided by the $P_0$ may impose an additional cost to the system (e.g. the cost of the municipal WiFi provider is borne by the citizens) and hence it would be preferable to have all the users served by the $I$ operators. Therefore, we use the following two metrics to characterize the efficiency of the market: **(i)** the aggregate utility ($U_{agg}$) of users in the stationary point $\mathbf{x}^*$, and **(ii)** the cost $J_0 = x_0 N U_0$ incurred by the neutral operator $P_0$ for serving the portion $x_0$ of the users. Both of these metrics depend on parameter $\alpha$ and hence on system parameters $W$ and $U_0$.

In detail:

- When $\alpha \in A_1 = (0, e/I)$, it is $\mathbf{x}^* \in X_A$, which means that a portion of users $x_0^* >$ selects $P_0$. The latter incurs cost of $J_0 = x_0^* N U_0$ units. All users receive utility of $U_0$ units and hence the aggregate utility is $U_{agg} = NU_0$.
- On the other hand, when $\alpha \in A_2 = [e/I, e^{I/(I-1)}/I]$, it is $\mathbf{x}^* \in X_C$. In this case, all users are served by the $I$ operators with marginal utility, i,e. $U_i^* = U_0$ for $i = 1, 2, \ldots, I$. There is no cost for $P_0$, i.e. $J_0 = 0$. Again, it is $U_{agg} = NU_0$ but unlike the previous case, there is no cost for $P_0$.
- Finally, if $\alpha \in A_3 = (e^{I/(I-1)}/I, \infty)$ it is $\mathbf{x}^* \in X_B$. All users are served by the $I$ operators, i.e. $x_0^* = 0$ and $J_0 = 0$, and receive high utilities $U_i^* > U_0$, $i = 1, 2, \ldots, I$. The welfare is higher in this case, i.e. $U_{agg} > NU_0$.

In summary, the aggregate utility of the users changes with $\alpha$ as follows:

$$U_{agg} = \begin{cases} NU_0, & \text{if } \alpha \in A_1 \cup A_2, \\ N(log(\frac{WI}{N}) - \frac{I}{I-1}), & \text{if } \alpha \in A_3. \end{cases} \tag{4.30}$$

It can be easily verified that $U_{agg}$ is a continuous function.

We have expressed $U_{agg}$ in terms of $W$ and $U_0$ in order to investigate the impact of the system parameters in the market. When $\alpha \in A_1 \cup A_2$, $U_{agg}$ increases with $U_0$ and is independent of the spectrum $W$. On the contrary, when $\alpha \in A_3$, $U_{agg}$ increases with $W$ and is independent of $U_0$. Notice that when the value of $\alpha$ changes from $A_1$ to interval $A_2$, $U_{agg}$ remains the same but the other metric of efficiency, the cost of neutral operator $J_0$, is improved:

$$J_0 = \begin{cases} \frac{\alpha INU_0}{e}, & \text{if } \alpha \in A_1, \\ 0, & \text{if } \alpha \in A_2 \cup A_3. \end{cases} \tag{4.31}$$

**Revenue of Operators**

When $\alpha$ lies in the interval $A_1$, the optimal prices are $\lambda_i^* = 1$, $\forall i \in \mathcal{I}$ and all the operators accrue the same revenue $R_i^* = \alpha N e^{-1} = We^{-(U_0+1)}$, which is proportional to $\alpha$, increases with the available spectrum $W$, decreases with $U_0$ and is independent of the number $N$ of users. In Figure 4·4 we depict the revenue of each operator for different values of $\alpha$, in a duopoly market. Notice that the revenue increases linearly with $\alpha \in (0, e/2)$.

When $\alpha \in A_2$, the competition of the operators may attain different equilibria, $\lambda^* \in \Lambda_C$, depending on the initial prices and on the sequence the operators update their prices. In Figure 4·5 we present the revenue of two operators (duopoly) at the equilibrium, for various initial prices and for $\alpha = e \in A_2$. Here we assume that the $1^{st}$ operator is able to set his price $\lambda_1(0)$ before the $2^{nd}$ operator. Also, in Figure 4·4 we illustrate the dependence of the revenue of the operators on the value of $\alpha$ when it lies in $A_2$, given that $\lambda_1(0) = 1.1$. For certain prices, e.g. when $\lambda_1(0) = \log 2\alpha$, both operators accrue the same revenue at the equilibrium, $R_1^* = R_2^* = \frac{N \log 2\alpha}{2}$.

If $\alpha \in A_3 = (e^{I/(I-1)}, \infty)$ all operators set their prices to $\lambda_i^* = I/(I-1)$ and get $R_i^* = N/(I-1)$ units, as shown in Table 4.2. Figure 4·6 depicts the competition of two operators and the convergence to the respective Nash equilibria for $\alpha = e^3 \in A_3$. We assume that both operators have selected prices $\lambda_1(0) = \lambda_2(0) = \log 2\alpha \approx 3.7$. However, this price vector does not constitute a NE and hence an operator (e.g. the $1^{st}$) can temporarily increase his revenue by decreasing his price to $\lambda_1 = 3$. Accordingly, the other operator ($2^{nd}$) will react by reducing his price to $\lambda_2 = 2.5$. Gradually, the competition of the operators will converge to the NE where both of them will set $\lambda_1^* = \lambda_2^* = 2/(2-1) = 2$ and will have revenue $R_1^* = R_2^* = 1$. Interestingly, the revenue of both operators in the equilibrium is lower than their initial revenue when they did not compete. Finally, notice that, unlike the aggregate utility $U_{agg}$, the revenue of the operators depends only on $\alpha = W/(Ne^{U_0})$ and not the specific values of $W$ and $U_0$.

Before we proceed, let us summarize the above results:

- If $\alpha \in A_1 = (0, e/I)$, it is $R_i^* = \alpha N e^{-1} = We^{-(U_0+1)}$, $i = 1, 2, \ldots, I$. Operators

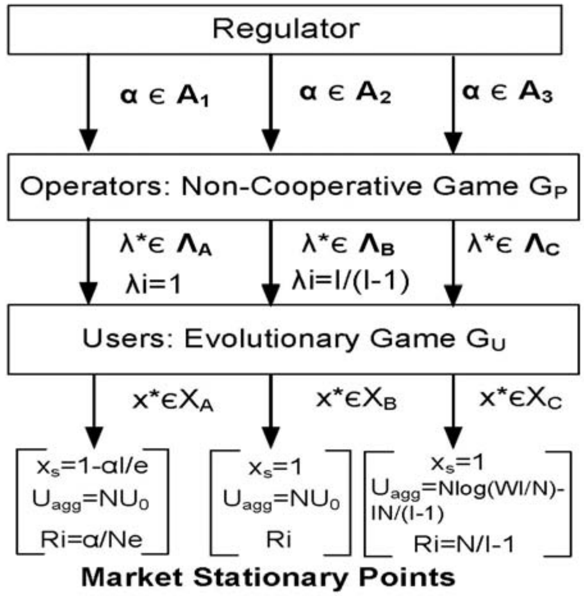



**Figure 4·3:** The regulator selects parameter $\alpha$, the operators compete and select the respective optimal prices $\lambda_i^*$, and then, the users are divided among the operators.

receive equal revenue which is **(i)** proportional to $W$, **(ii)** inversely proportional to $U_0$ and **(iii)** independent of the number of users $N$.

- If $\alpha \in A_2 = [e/I, \frac{e^{I/(I-1)}}{I})$, $R_i^*$ depends on the initial prices operators select. In the particular case that a single operator $i$ sets first his price $\lambda_i$ so as to be $\lambda_i(0) = \log I\alpha$, then all operators obtain finally equal revenue $R_i^* = \frac{N \log I\alpha}{I}$.
- If $\alpha \in A_3 = [\frac{e^{I/(I-1)}}{I}, \infty)$, it is $R_i^* = \frac{N}{I-1}$. Operators receive equal revenue which is **(i)** proportional to $N$, **(ii)** independent of $U_0$ and $W$.

### 4.5.2 Regulation of the Wireless Service Market

Since both the market efficiency and the operator revenue depend on $\alpha$ and system parameters $W$ and $U_0$, a regulating agency can act as a *mechanism designer* and steer the outcome of the market in a more desirable equilibrium according to his objective. This can be achieved by determining directly or indirectly (e.g. through pricing) the amount of spectrum $W$ each operator has at his disposal, or by intervening in the market and setting the value $U_0$ as the example with the municipal WiFi Internet provider. This process is depicted in Figure 4·3.

### Regulating to Increase Market Efficiency

First, we highlight the impact of parameters $W$ and $U_0$ on the efficiency metrics. This is of crucial importance because tuning $W$ or $U_0$ has different implications for the regulator and the market. For example, as it is explained below, the regulator can achieve the same level of market efficiency either by selling more spectrum to operators, e.g. by decreasing the spectrum price, or by allocating more spectrum to the neutral operator:

- *Assume that $U_0$ is fixed.* As the allocated spectrum $W$ to each operator increases, aggregate utility $U_{agg}$ remains constant until parameter $\alpha$ increases up to $\alpha \geq \frac{e^{I/(I-1)}}{I}$. When $\alpha \in A_3$, $U_{agg}$ is log-proportional to $W$. Also, the cost $J_0$ increases with $W$, as long as $\alpha \in A_1$, and becomes zero for larger values of $\alpha$.

- *Assume that $W$ is fixed.* $U_{agg}$ increases with $U_0$ as long as $\alpha \in A_1 \cup A_2$. For larger values of $\alpha$, $U_{agg}$ does not depend directly on $U_0$. Additionally, the cost $J_0$ increases with $U_0$ as long as $\alpha \in A_1$ while for larger values of $\alpha$ it becomes zero.

Let us now give a specific scenario for regulation. Assume that initially $\alpha \in A_1 = (0, e/I)$. Hence, a portion of users is not served by anyone of the $I$ operators, $x_0^* > 1$ and all the users receive utility equal to $U_0$. The regulator can improve the market efficiency, i.e. increase $U_{agg}$ and decrease $J_0$, by increasing the value of $\alpha$. This can be achieved either by increasing $W$ or decreasing $U_0$. Let us assume that the regulator selects the first method. For example, he can change the price of $W$ and allow the operators to acquire more spectrum. If $W$ is increased until $\alpha = e/I$, then the market stationary point $\mathbf{x}^*$ switches to $X_B$. In this case, all users are served by the market, $x_0^* = 0$, but they still receive only marginal utility, $U_{agg} = NU_0$. If the regulator provides even more spectrum $W$ to operators so as $\alpha > e^{I/(I-1)}/I$, then $x_0^* = 0$ and moreover the users perceive higher utility because $U_{agg}$ increases proportional to $\log W$, eq. (4.30). Obviously, the improvement in market efficiency comes at the cost (*opportunity cost*) of the additional spectrum the regulator must provide to operators.

On the other hand, the regulator may prefer to directly intervene in the market through $P_0$ and tune $U_0$. If $U_0$ decreases, the value of $x_0^*$ decreases and users return to the market (to the $I$ operators). The portion of users $x_0^*$ becomes zero when $\alpha = e/I$. This way, the cost of the regulator $J_0$ decreases (since $P_0$ serves less users) but at the same time the aggregate utility, $U_{agg} = NU_0$, is also reduced. Namely, $U_{agg}$ decreases linearly with $U_0$ until $\alpha = e^{I/(I-1)}/I$ and remains constant for larger values of $U_0$, eq. (4.30). Again, the decision of the regulator depends on his cost and on the efficiency he wants to achieve. In conclusion, depending on they system parameters ($N$, $W$, $I$) the efficiency of the market may be improved either by increasing the resources of operators (sell more spectrum) or by rendering highly competitive the services provided by the neutral operator $P_0$.

### Regulating for Revenue

As illustrated in Table 4.2, the revenue of the operators increases proportionally to $\alpha$ for $\alpha \in A_1$, and proportionally to $\log \alpha$ for $\alpha \in A_2$, while it remains constant when $\alpha \in A_3$. Notice that the revenue, unlike the market efficiency, depends on the value of $\alpha$ and not

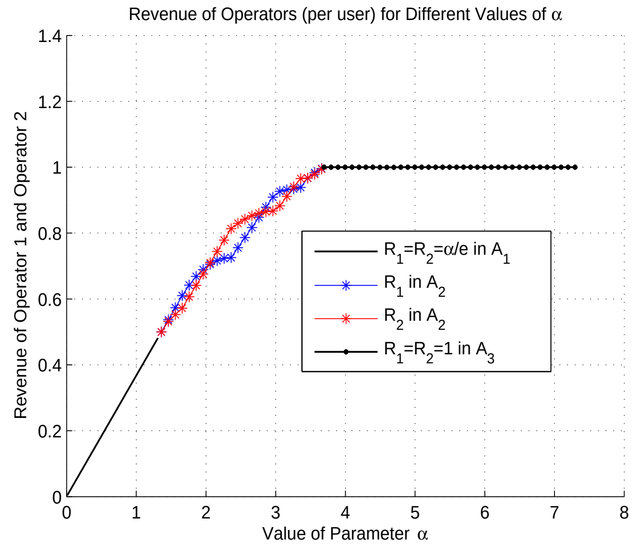



**Figure 4·4:** The outcome of the operator competition ($\mathcal{G}_{\mathcal{P}}$ equilibrium) for different values of parameter $\alpha$, i.e. in different intervals.

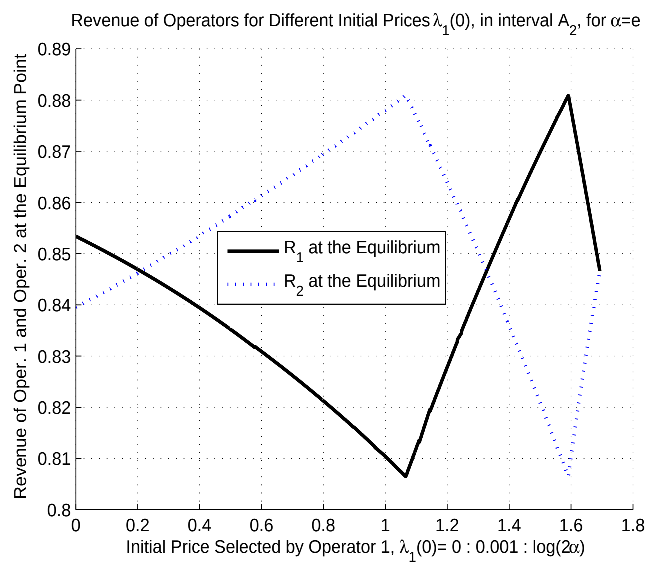



**Figure 4·5:** The outcome of the competition of two operators, with $\alpha = e \in A_2$ and $N = 1000$. Operator 1 is assumed to set his price $\lambda_1(0)$ first. $R_1^*$ and $R_2^*$ depend on $\lambda_1(0)$.

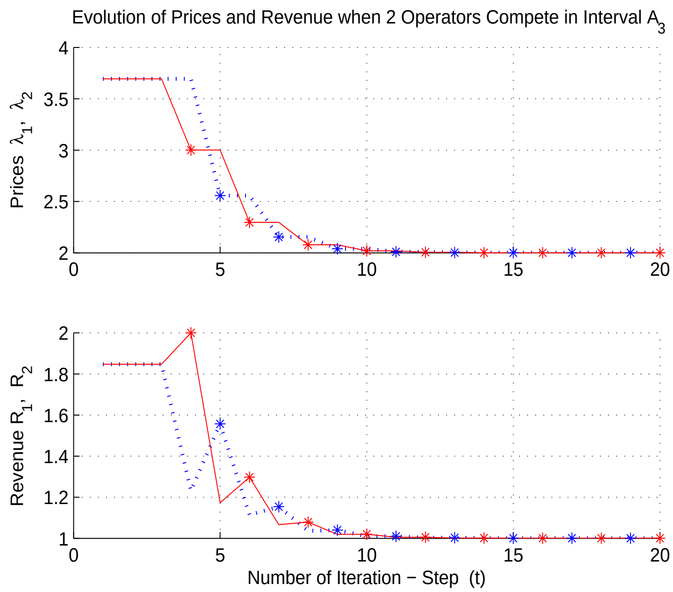



**Figure 4·6:** Evolution of operator competition for $\alpha = e^3 \in A_3$. The game is played repeatedly and operators updated myopically their price based on the previous strategy of the other operators.

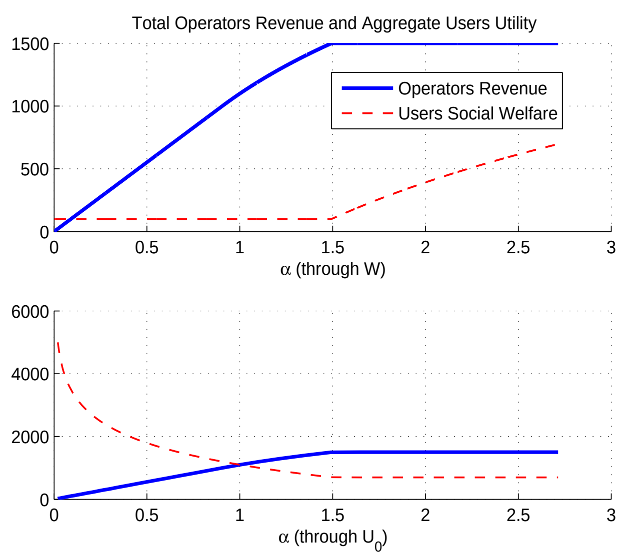



**Figure 4·7:** Total revenue of operators and aggregate utility of the users for different values of parameter $\alpha$. In the upper plot, the value of $\alpha$ changes through $W$ while in the lower plot it changes through the tuning of $U_0$.

on the specific combination of $W$ and $U_0$. These results are presented in Figure 4·7 for a market with $I = 3$ operators and $N = 1000$ users. In the upper plot, it is $U_0 = 0.1$ and the regulator increases the value of $\alpha$ by increasing $W$. The aggregate utility is constant and equal to $U_{agg} = NU_0 = 100$ for $\alpha < e^{3/(3-1)}/3 \approx 1.5$ while it increases proportionally to $\log W$ for $\alpha > 1.5$. Obviously, increasing the spectrum of operators improves both their revenue and the efficiency of the market.

In the lower plot, the spectrum at the disposal of each operator is constant, $W = 5000$, and the regulator increases the value of $\alpha$ by decreasing $U_0$. In this case, the total revenue increases but at the expense of market efficiency. When $\alpha \in A_1 \cup A_2 = (0, e^{1.5}/3]$, the aggregate utility $U_{agg}$ is reduced as $U_0$ decreases but for $\alpha > e^{1.5}/3$ it remains constant. Notice that for very small values of $\alpha$, $U_{agg}$ is large. However, this desirable result comes at a cost for the regulator. Namely, in this case only a small portion of users are served by the market, while the rest of them select $P_0$. Therefore, the incurred cost $J_0$ for the regulator is high.

Another interesting point in Figure 4·7 is the following. In the upper subplot, for $U_0 = 0.1$, the total operators revenue is $R_{tot} = 1500$ units and the aggregate utility is $U_{agg} = 100$, achieved by increasing the spectrum $W$ until $W = 1657.8$ units, which yields $\alpha = 1.5$. In the lower plot, the same total revenue is reached for $W = 5000$, and decreasing $U_0$ until $U_0 = 1.204$ units. In this case, the aggregate utility is $U_{agg} = 1204$ units. If, for example, the regulator is interesting only in maximizing the revenue of operators, then he would prefer the first method since it requires less spectrum and lower value for $U_0$.

## 4.6 Conclusions

In this chapter, we studied the operators price competition in a wireless services market where users have a certain reservation utility $U_0$. We modeled the users interaction as an evolutionary game and the competition of the operators as a non cooperative game of complete information. We proved that the latter is a potential game and hence has pure Nash equilibriums. The two games are realized in different time scale but they are interrelated. Additionally, both of them depend on the reservation utility $U_0$ and the amount of spectrum $W$ each operator has at his disposal. Accordingly, we considered a regulating agency and discussed how he can intervene and change the outcome of the market by tuning either $U_0$ or $W$. Various regulation methods yield different market outcomes and induce a different cost for the regulator.

## 4.A Analysis of the Evolutionary Game $\mathcal{G}_{\mathcal{U}}$

### 4.A.1 Derivation of Evolutionary Dynamics

Here, we derive the new differential equations that describe the evolution of the market of the users under the new introduced revision protocol. Recall that, the latter is described by the following equations:

$$\rho_{ij}(t) = x_j(t)[U_j(t) - U_i(t)]_+, \forall i, j \in \mathcal{I} \tag{4.32}$$

$$\rho_{i0}(t) = \gamma[U_0 - U_i(t)]_+, \forall i \in \mathcal{I} \tag{4.33}$$

$$\rho_{0i}(t) = x_i(t)[U_i(t) - U_0]_+, \forall i \in \mathcal{I} \tag{4.34}$$

where $\rho_{ij}(t)$ is the rate at which users associated with operator $i$ switch to operator $j$ in time slot $t$, $\rho_{i0}(t)$ is the switch rate from operator $i$ to neutral operator $P_0$ and $\rho_{0i}(t)$ the rate at which users return from $P_0$ to an operator $i \in \mathcal{I}$ in the market. The constant value $\gamma \in R^+$ represents the frequency of the direct selection.

For imitation-based revision protocols, the dynamics of the system can be described with the well-known replicator dynamics [91]. The hybrid revision protocol defined in equations (4.32), (4.33) and (4.34) is in part imitation-based ($\rho_{ij}(t)$ and $\rho_{0i}(t)$) and in part a probabilistic direct selection of the neutral operator ($\rho_{i0}(t)$). Therefore, the respective evolutionary dynamics of the system cannot be described by the replicator dynamic equations which correspond to the pure imitation mechanism. We have to stress that the hybrid protocol that we introduce, differs from the hybrid protocol in [91] in that users select directly only the neutral operator and not the other $I$ operators.

The portion of users $x_i$ who are associated with operator $i$ changes from time $t$ to the time $t + \delta t$, according to the following equation:

$$\begin{aligned} x_i(t+\delta t) &= x_i(t) - x_i(t)\delta t \sum_{j\neq 0} x_j(t)(U_j(t) - U_i(t))_+ - x_i(t)\delta t \gamma (U_0(t) - U_i(t))_+ \\ &+ \sum_{j=0}^{I} \delta t x_j(t) x_i(t)(U_i(t) - U_j(t))_+ \end{aligned} \tag{4.35}$$

for $\delta t \to 0$ we obtain the derivative:

$$\frac{dx_i(t)}{dt} = x_i(t)[\sum_{j\neq 0} x_j(t)U_i(t) - \sum_{j\neq 0} x_j(t)U_j(t) - \gamma(U_0 - U_i)_+ + x_0(t)(U_i - U_0)_+]$$

or, if we omit the time index and rewrite the equation:

$$\frac{dx_i(t)}{dt} = x_i[U_i - U_{avg} - x_0(U_i - U_0) - \gamma(U_0 - U_i)_+ + x_0(U_i - U_0)_+] \tag{4.36}$$

which can be analyzed in:

$$\frac{dx_i(t)}{dt} = x_i(U_i - U_{avg}),\ \forall i \in \mathcal{I}^+ \tag{4.37}$$

$$\frac{dx_j(t)}{dt} = x_j[U_j - U_{avg} - (\gamma - x_0)(U_0 - U_j)],\ \forall j \in \mathcal{I}^- \tag{4.38}$$

where $\mathcal{I}^+$ is the set of operators offering utility $U_i(t) \geq U_0$, and $\mathcal{I}^-$ is the set of operators offering utility $U_j(t) < U_0$.

The dynamics of the population $x_0$ can be derived in a similar way:

$$x_0(t+\delta t) = x_0(t) - x_0(t)\delta t \sum_{i\neq 0} x_i(t)(U_i - U_0)_+ + \sum_{i\neq 0} x_i(t)\delta t\gamma(U_0 - U_i)_+ \tag{4.39}$$

which can be written as:

$$\frac{dx_0(t)}{dt} = (x_0 \sum_{i\in I^+} x_i(U_0 - U_i) + \gamma \sum_{j\in\mathcal{I}^-} x_j(U_0 - U_j)) \tag{4.40}$$

Equations (4.37), (4.38) and (4.40) describe the evolutionary dynamics of game $\mathcal{G}_{\mathcal{U}}$.

### 4.A.2 Analysis of Stationary Points

Despite the different dynamics, the system reaches the same stationary points as if users where employing the typical imitation revision protocol. In detail, the market state vector at a fixed point, $\mathbf{x}^* = (x_i^*, x_j^*, x_0^*\colon \forall\, i \in \mathcal{I}^+, \forall\, j \in \mathcal{I}^-)$, can be found by the following set of equations:

$$\frac{dx_i(t)}{dt} = \frac{dx_j(t)}{dt} = \frac{dx_0(t)}{dt} = 0\ \forall i \in \mathcal{I}^+,\, j \in \mathcal{I}^- \tag{4.41}$$

**Lemma 4.A.1.** *The stationary points of the evolutionary dynamics defined in equations (4.37), (4.38) and (4.40) are identical to the stationary points of the ordinary replicator dynamics [91] given by:*

$$\dot{x}_i(t) = 0 \Rightarrow x_i(t)[U_i(t) - U_{avg}(t)] = 0,\ \forall\, i \in \mathcal{I} \tag{4.42}$$

*and*

$$\dot{x}_0(t) = 0 \Rightarrow x_0(t)[U_0 - U_{avg}(t)] = 0 \tag{4.43}$$

**Proof:** First we prove that, in any stationary point, $x_j^*$, $j \in \mathcal{I}^-$ should be equal to zero. We prove this claim by contradiction. Assume that $x_j^* > 0$. Since $U_{avg} \geq U_0 > U_j$, this implies that there should be at least one operator $i$ with $U_i > U_{avg}$ and $x_i^* > 0$. Therefore $(U_i - U_{avg})$ cannot be equal to zero $\forall i \in \mathcal{I}^+$, and $\dot{x}_i$ will be nonzero for at least one operator. Therefore (4.41) cannot be satisfied, if $x_j^* \neq 0$.

When $x_j = 0$, the evolutionary dynamics given by eq. (4.37), (4.38) and (4.40) reduce to ordinary replicator dynamics:

$$\dot{x}_i(t) = x_i(t)[U_i(t) - U_{avg}(t)] \; \forall i \in \mathcal{I}, \quad \dot{x}_0(t) = x_0(t)[U_0 - U_{avg}(t)] \tag{4.44}$$

Stationary points are identical to the stationary points of the typical replicator dynamics, [91]. ■

Due to this lemma, the stationary points for the users population associated with each operator $i \in \mathcal{I}$ should satisfy one of the following conditions: $(i)$ $x_i^* = 0$, or $(ii)$ $x_i^* > 0$ and $U_i^* = U_{avg}$. Similarly, for the neutral operator $P_0$, eq. (4.43), it must hold: $(i)$ $x_0^* = 0$ and $U_0 < U_{avg}$, $(ii)$ $x_0^* > 0$ and $U_0 = U_{avg}$ or $(iii)$ $x_0^* = 0$ and $U_0 = U_{avg}$. The case $x_i^* = 0$ implies zero revenue for the $i^{th}$ operator and hence case $(i)$ does not constitute a valid choice. Therefore, there exist in total 3 possible combinations (cases) that will satisfy the stationarity properties given by eq. (4.42) and (4.43):

- **Case A**: $x_i^*$, $x_0^* > 0$ and $U_i^* = U_0$, $i \in \mathcal{I}$.
- **Case B**: $x_i^*$, $x_j^* > 0$, $x_0^* = 0$ and $U_i^* = U_j^*$, with $U_i^*, U_j^* > U_0, \forall\, i,\, j \in \mathcal{I}$.
- **Case C**: $x_i^*$, $x_j^* > 0$, $x_0^* = 0$ and $U_i^* = U_j^* = U_0$, $\forall\, i,\, j \in \mathcal{I}$.

We find now the exact value of the market state vector at the equilibrium (stationary point) $\mathbf{x}^*$ for each case. First, we define for every operator $i \in \mathcal{I}$ the scalar parameter $\alpha_i = W_i/(Ne^{U_0})$ and the respective vector $\alpha = (\alpha_i \,:\, i \in \mathcal{I})$.

We can find the stationary points for **Case A** by using the equation $U_i(W_i, x_i^*, \lambda_i) = U_0$ and imposing the constraint $x_0^* > 0$:

$$U_i(W_i, x_i^*, \lambda_i) = \log \frac{W_i}{Nx_i^*} - \lambda_i = U_0 \Rightarrow x_i^* = \frac{W_i}{Ne^{\lambda_i + U_0}} = \alpha_i e^{-\lambda_i} \tag{4.45}$$

and

$$x_0^* > 0 \Rightarrow 1 - \sum_{i=1}^{I} \alpha_i e^{-\lambda_i} > 0 \Rightarrow \sum_{i=1}^{I} \alpha_i e^{-\lambda_i} < 1 \tag{4.46}$$

Apparently, the state vector $\mathbf{x}^*$ depends on the operators' price vector $\lambda$. Therefore, we define the set of all possible **Case A** stationary points, $X_A$, as follows:

$$X_A = \left\{ x_i^* = \alpha_i e^{-\lambda_i}, \forall i \in \mathcal{I}, x_0^* = 1 - \sum_{i=1}^{I} \alpha_i e^{-\lambda_i} \,:\, \lambda \in \Lambda_A \right\} \tag{4.47}$$

where $\Lambda_A$ is the set of prices for which a stationary point in $X_A$ is reachable, i.e. $x_0^* > 0$:

$$\Lambda_A = \left\{ (\lambda_1, \lambda_2, \ldots, \lambda_I) \,:\, \sum_{i=1}^{I} \alpha_i e^{-\lambda_i} < 1 \right\} \tag{4.48}$$

Similarly, for **Case B**, we calculate the stationary points by using the set of equations $U_i(W_i, x_i^*, \lambda_i) = U_j(W_j, x_j^*, \lambda_j)$, $\forall\, i, j \in \mathcal{I}$, which yields:

$$\log \frac{W_1}{Nx_1^*} - \lambda_1 = \log \frac{W_2}{Nx_2^*} - \lambda_2 = \ldots = \log \frac{W_i}{Nx_i^*} - \lambda_i \tag{4.49}$$

or, equivalently:

$$x_j^* = x_i^* \frac{e^{\lambda_i}\alpha_j}{e^{\lambda_j}\alpha_i} \quad \forall i, j \in \mathcal{I} \tag{4.50}$$

Moreover since $x_0^* = 0$ for **Case B**, the following holds:

$$\sum_{i \in \mathcal{I}} x_i^* = 1 \tag{4.51}$$

Using (4.50) and (4.51),

$$x_i^* = \frac{\alpha_i}{e^{\lambda_i} \sum_{j \in \mathcal{I}} \alpha_j e^{-\lambda_j}} \tag{4.52}$$

Additionally, $U_i > U_0$ implies that:

$$\log \frac{W_i}{Nx_i^*} - \lambda_i > U_0 \Rightarrow x_i^* < \alpha_i e^{-\lambda_i} \tag{4.53}$$

Using (4.51) and (4.53),

$$\sum_{i=1}^{I} x_i^* < \sum_{i=1}^{I} \alpha_i e^{-\lambda_i} \Rightarrow \sum_{i=1}^{I} \alpha_i e^{-\lambda_i} > 1 \tag{4.54}$$

Therefore, according to (4.52) and (4.54), we define the set of all possible **Case B** stationary points, $X_B$, as follows:

$$X_B = \left\{ x_i^* = \frac{\alpha_i}{e^{\lambda_i} \sum_{j=1}^{I} \alpha_j e^{-\lambda_j}}, \forall i \in \mathcal{I}, x_0^* = 0 : \lambda \in \Lambda_B \right\} \tag{4.55}$$

where $\Lambda_B$ is the set of prices for which a stationary point in $X_B$ is feasible, $U_i^* > U_0$:

$$\Lambda_B = \left\{ (\lambda_1, \lambda_2, \ldots, \lambda_I) : \sum_{i=1}^{I} \alpha_i e^{-\lambda_i} > 1 \right\} \tag{4.56}$$

Finally, the stationary points for the **Case C** solution must satisfy the following:

$$U_i = U_0, \quad x_0^* = 0 \tag{4.57}$$

which yields:

$$x_i^* = \alpha_i e^{-\lambda_i}, \quad \sum_{i=1}^{I} \alpha_i e^{-\lambda_i} = 1 \tag{4.58}$$

Therefore, we define the set of all possible **Case C** stationary points, $X_C$, as follows:

$$X_C = \left\{ x_i^* = \alpha_i e^{-\lambda_i}, \forall\, i \in \mathcal{I}, x_0^* = 0 :\ \lambda \in \Lambda_C \right\} \tag{4.59}$$

with

$$\Lambda_C = \left\{ (\lambda_1, \lambda_2, \dots, \lambda_I)\ :\ \sum_{i=1}^{I} \alpha_i e^{-\lambda_i} = 1 \right\} \tag{4.60}$$

## 4.B Analysis of the Pricing Game $\mathcal{G}_{\mathcal{P}}$

First, we show that the revenue of each operator $i \in \mathcal{I}$ is a continuous and a quasi-concave function. Secondly, we analyze best response pricing in game $\mathcal{G}_{\mathcal{P}}$. Then, we derive the Nash equilibriums (NEs) of the game using iterated strict dominance. Finally, we prove convergence to these equilibriums by showing that $\mathcal{G}_{\mathcal{P}}$ is a potential game.

### 4.B.1 Properties of the Revenue Function

The revenue function of each operator $i$ is given by the following equation:

$$R_i(\lambda_i, \lambda_{-i}) = \begin{cases} \dfrac{\alpha_i \lambda_i N}{e^{\lambda_i} \sum_{j=1}^{I} \alpha_j e^{-\lambda_j}} & \text{if } \lambda_i < l_0, \\ \alpha_i \lambda_i N e^{-\lambda_i} & \text{if } \lambda_i \geq l_0. \end{cases} \tag{4.61}$$

where $l_0 = \log(\alpha_i / (1 - \sum_{j \neq i} \alpha_j e^{-\lambda_j}))$.

Each component (for each case) is a positive function which is also *log-concave*. This means that it is a quasiconcave function and hence uniqueness of optimal solution is ensured for a proper constraint set. Namely, it is:

$$f_A(\lambda_i) = \log \alpha_i \lambda_i N e^{-\lambda_i} = \log \alpha_i \lambda_i N - \lambda_i \tag{4.62}$$

and

$$f_A(\lambda_i)^{(1)} = \frac{1}{\lambda_i} - 1 \Rightarrow f_A(\lambda_i)^{(2)} = \frac{-1}{\lambda_i^2} < 0 \tag{4.63}$$

Hence, $f_A(\cdot)$ which is the log-function of $R_i^A(\cdot)$, is concave which means that the later is log-concave and since it is $R_i^A(\lambda_i) > 0$, it is also quasi-concave. Similarly, for the other component of the revenue function:

$$f_B(\lambda_i, \lambda_{-i}) = \log \frac{\alpha_i \lambda_i N}{\alpha_i + \beta e^{\lambda_i}} = \log \alpha_i \lambda_i N - \log \alpha_i + \beta e^{\lambda_i} \tag{4.64}$$

where $\beta = \sum_{j \neq i} \alpha_j e^{-\lambda_i}$. The second derivative is:

$$f_B(\lambda_i, \lambda_{-i})^{(2)} = \frac{-1}{\lambda_i^2} - \frac{\alpha_i \beta e^{\lambda_i}}{(\alpha_i + \beta e^{\lambda_i})^2} < 0 \tag{4.65}$$

Hence, $R_i^B(\cdot)$ is also quasiconcave. Finally, it is easy to see that the function is continuous:

$$R_i^A(l_0, \lambda_{-i}) = R_i^B(l_0, \lambda_{-i}) = N(1-\beta)\log\frac{\alpha_i}{1-\beta} \tag{4.66}$$

### 4.B.2 Best Response Pricing in $\mathcal{G}_{\mathcal{P}}$

Each operator $i$ finds his best response price $\lambda_i^*$ for each price profile of the other $I-1$ operators by solving the following optimization problems. For the case the price vector belongs to the set $\Lambda_A$, $\lambda \in \Lambda_A$, $(\mathbf{P}_i^A)$:

$$\max_{\lambda_i \geq 0} \alpha_i \lambda_i N e^{-\lambda_i} \tag{4.67}$$

s.t.

$$\sum_{j=1}^{I} \alpha_j e^{-\lambda_j} < 1 \tag{4.68}$$

In order to ensure the uniqueness of the problem solution, we transform the constraint set to a closed and compact set as follows:

$$\lambda_i \geq \log\frac{\alpha_i}{1 - \sum_{j \neq i} \alpha_j e^{-\lambda_j}} + \epsilon \tag{4.69}$$

where $\epsilon > 0$ is an arbitrary small positive constant number. As we will show immediately this transformation of the constraint set does not affect the solution of the game. The problem now is quasi-concave with a closed and compact constraint set and hence it has a unique optimal solution, [11] which we denote $\lambda_i^A$ and it is:

$$\lambda_i^A = 1, \texttt{ or } \lambda_i^A = \log\frac{\alpha_i}{1 - \sum_{j \neq i} \alpha_j e^{-\lambda_j}} + \epsilon \tag{4.70}$$

The value $\lambda_i^A = 1$ is the optimal solution of the respective unconstrained problem, which yields optimal revenue $R_i^A = \alpha N/e$, and it is feasible if $\lambda = (1, \lambda_{-i}) \in \Lambda_A$. Otherwise, since $R_i^A(\cdot)$ is a decreasing function of $\lambda_i$, operator $i$ can only select the minimum price $\lambda_i^A$ such that $(\lambda_i^A, \lambda_{-i}) \in \Lambda_A$.

Similarly, when the price vector belongs to the set $\Lambda_B$, i.e. $\lambda \in \Lambda_B$, the revenue maximization problem for each operator $i \in \mathcal{I}$ $(\mathbf{P}_i^B)$ is:

$$\max_{\lambda_i \geq 0} \frac{\lambda_i \alpha_i N}{e^{\lambda_i} \sum_{j \in \mathcal{I}} \alpha_j e^{-\lambda_j}} \tag{4.71}$$

s.t.

$$\sum_{j\in\mathcal{I}} \alpha_j e^{-\lambda_j} > 1 \tag{4.72}$$

Similarly to the previous analysis, we transform the constraint set to a closed and compact set by using the following inequality:

$$\lambda_i \leq \log \frac{\alpha_i}{1-\sum_{j\neq i}\alpha_j e^{-\lambda_j}} - \epsilon \tag{4.73}$$

This is also a concave problem which has unique solution and can be either the optimal solution of the respective unconstrained problem, $\lambda_i^*$ if $(\lambda_i^*, \lambda_{-i}) \in \Lambda_B$, or the maximum price for which the price vector belongs to $\Lambda_B$ ($R_i^B(\cdot)$ increases with $\lambda_i$:

$$\lambda_i^B = \mu_i^*, \texttt{ or } \lambda_i^B = \log \frac{\alpha_i}{1-\sum_{j\neq i}\alpha_j e^{-\lambda_j}} - \epsilon \tag{4.74}$$

Finally, for the special case that $\lambda \in \Lambda_C$, the price of each operator $i$ is directly determined by the prices that the other operators have selected. Namely:

$$\lambda_i^C = \log \frac{\alpha_i}{1-\sum_{j\neq i}\alpha_j e^{-\lambda_j}} \tag{4.75}$$

Whether each operator $i$ will agree and adopt this price or not, depends on the respective accrued revenue, $R_i^C(\lambda_i^C, \lambda_{-i})$.

In the sequel, we examine and analyze jointly the solutions of the above optimization problems and derive the exact best response of the $i^{th}$ operator for each vector $\lambda_{-i}$ of the $I-1$ prices.

**Lemma 4.B.1.** *For each operator $i \in \mathcal{I}$, if $(1, \lambda_{-i}) \notin \Lambda_A$, then there is no best response price $\lambda_i^*$, such that $(\lambda_i^*, \lambda_{-i}) \in \Lambda_A$. That is, operator $i$ will not select $\Lambda_A$.*

**Proof:** Given that the price vector $\lambda \in \mathbf{\Lambda_A}$, best response price is:

$$\lambda_i^A = \begin{cases} 1 & \text{if } (1, \lambda_{-i}) \in \Lambda_A, \\ l_0 + \epsilon & \text{if } (1, \lambda_{-i}) \notin \Lambda_A. \end{cases} \tag{4.76}$$

where $l_0 = \lambda_i^C$ is the price operator $i$ selects when $\lambda \in \mathbf{\Lambda_C}$.

If $(1, \lambda_{-i}) \notin \Lambda_A$, then $l_0 + \epsilon > 1$. Otherwise price vector $(l_0 + \epsilon, \lambda_{-i})$ will not belong to $\Lambda_A$. Therefore, $R_i^A(\cdot)$ is a decreasing function at the point $\lambda_i = \lambda_0 + \epsilon$ due to quasi-concavity property. Therefore, if $(1, \lambda_{-i}) \notin \Lambda_A$, then $R_i^C(l_0) = R_i^A(l_0) > R_i^A(l_0 + \epsilon)$ which means that $\lambda_i^C$ always gives better response than $\lambda_i^A$. ■

**Lemma 4.B.2.** *Let us denote with $\mu_i^*$ the optimal solution of the unconstraint problem $P_i^B$. For each operator $i \in \mathcal{I}$, if $(\mu_i^*, \lambda_{-i}) \notin \Lambda_B$, then there is no best response price $\lambda_i^*$, such that $(\lambda_i^*, \lambda_{-i}) \in \Lambda_B$.*

**Proof:** Given that the price vector $\lambda \in \mathbf{\Lambda_B}$, best response price is:

$$\lambda_i^B = \begin{cases} \mu_i^* & \text{if } (\mu_i^*, \lambda_{-i}) \in \Lambda_B, \\ l_0 - \epsilon & \text{if } (\mu_i^*, \lambda_{-i}) \notin \Lambda_B. \end{cases} \tag{4.77}$$

and recall that $\lambda_i^C = l_0$. If $(\mu_i^*, \lambda_{-i}) \notin \Lambda_B$, then $l_0 - \epsilon < \mu_i^*$. Otherwise, the price vector $(l_0 - \epsilon, \lambda_{-i})$ cannot be in $\Lambda_B$. Therefore, $R_i^B(\cdot)$ is an increasing function at the point $\lambda_i = \lambda_0 - \epsilon$ due to quasi-concavity property. Therefore, if $(\mu_i^*, \lambda_{-i}) \notin \Lambda_B$, then $R_i^C(l_0) = R_i^B(l_0) > R_i^B(l_0 - \epsilon)$ which means that $\lambda_i^C$ always gives better response than $\lambda_i^B$. ∎

In other words, the previous two Lemmas state that the only eligible best response for each operator $i \in \mathcal{I}$ in the price sets $\Lambda_A$ and $\Lambda_B$ are prices $\lambda_i^* = 1$ and and $\lambda_i^* = \mu_i^*$ respectively.

**Theorem 4.B.3.** *The best response price of an operator $i$ is:*

$$\lambda_i^* = \begin{cases} 1, & \textit{if } (1, \lambda_{-i}) \in \Lambda_A, \\ \mu_i^*, & \textit{if } (\mu_i^*, \lambda_{-i}) \in \Lambda_B, \\ \lambda_i^C = l_0, & \textit{otherwise.} \end{cases} \tag{4.78}$$

**Proof:** First we prove that $(1, \lambda_{-i}) \in \Lambda_A$ and $(\mu_i^*, \lambda_{-i}) \in \Lambda_B$ cannot be true at the same time. Since $\mu_i^*$ is the optimal solution of unconstraint $R_i^B$:

$$\frac{dR_i^B(\lambda_i)}{d\lambda_i} = 0 \Rightarrow e^{\mu_i^*}(\mu_i^* - 1) = \frac{\alpha_i}{\sum_{j \neq i} \alpha_j e^{-\lambda_j}} \tag{4.79}$$

It is obvious that equation (4.79) can only hold when $\mu_i^* > 1$. Note that if $(\mu_i^*, \lambda_{-i}) \in \Lambda_B$, the vector $\lambda = (l, \lambda_{-i}) \in \Lambda_B$ for any price $l < \mu_i^*$. Hence, it should also hold that $\lambda = (1, \lambda_{-i}) \in \Lambda_B$. With a similar reasoning, when $(1, \lambda_{-i}) \in \Lambda_A$, $(l, \lambda_{-i}) \in \Lambda_A$ holds for any price $l > 1$ and therefore $(\mu_i^*, \lambda_{-i}) \in \Lambda_A$. Also, if $(1, \lambda_{-i}) \in \Lambda_A$, $\lambda_i^C$ cannot be a best response, because $R_i^A(1) > R_i^A(\lambda_i^C) = R_i^C(\lambda_i^C)$. Similarly, if $(\mu_i^*, \lambda_{-i}) \in \Lambda_B$, $\lambda_i^C$ is not a best response.

Finally, from Lemma 4.B.1 and Lemma 4.B.2, we can say that $\lambda_i^C$ dominates all other prices if $(1, \lambda_{-i}) \notin \Lambda_A$ and $(\mu_i^*, \lambda_{-i}) \notin \Lambda_B$ which concludes the proof. ∎

### 4.B.3 Existence and Convergence Analysis of Nash Equilibriums

In the previous section, we derived the best response strategy for each player of the game $\mathcal{G}_\mathcal{P}$. The next important steps are **(i)** to explore the existence of Nash Equilibriums (NE) for $\mathcal{G}_\mathcal{P}$, and **(ii)** to study if the convergence to them is guaranteed. In [71], it is proven that if the game can be modeled as a potential game, not only the existence of pure NEs are ensured, but also convergence to them is guaranteed under any finite improvement path. In other words, a potential game always converges to pure NE when the players adjust their strategies based on accumulated observations as game unfolds. In this section, we provide the necessary definitions for ordinal potential games, and we prove that game $\mathcal{G}_{\mathcal{P}}$ belongs in this class of games.

**Definition 4.B.4.** *A game $(\mathcal{I}, \lambda, \{R_i\})$ is an* ***ordinal potential game****, if there is a potential function $\mathcal{P} : [0, \lambda_{max}] \to \mathbb{R}$ such that the following condition holds:*

$$\operatorname{sgn}(\mathcal{P}(\lambda_i, \lambda_{-i}) - \mathcal{P}(\lambda_i', \lambda_{-i})) = \operatorname{sgn}(R_i(\lambda_i, \lambda_{-i}) - R_i(\lambda_i', \lambda_{-i}))\forall i \in \mathcal{I},\ \lambda_i, \lambda_i' \in [0, \lambda_{max}] \tag{4.80}$$

*where* sgn$(\cdot)$ *is the sign function.*

**Lemma 4.B.5.** *The game $\mathcal{G}_{\mathcal{P}}$ is an ordinal potential game.*

**Proof:** We define the potential function as:

$$\mathcal{P}(\lambda) = \begin{cases} \sum_{j=1}^{I} (\log \lambda_j - \lambda_j) & \text{if} \sum_{j=1}^{I} \alpha_j e^{-\lambda_j} \leq 1, \\ \sum_{j=1}^{I} (\log \lambda_j - \lambda_j) - \log \left(\sum_{j=1}^{I} \alpha_j e^{-\lambda_j}\right) & \text{if} \sum_{j=1}^{I} \alpha_i e^{-\lambda_j} > 1. \end{cases} \tag{4.81}$$

Therefore,

$$\mathcal{P}(\lambda_i, \lambda_{-i}) - \mathcal{P}(\lambda_i^{'}, \lambda_{-i}) = \begin{cases} \log \frac{\lambda_i e^{\lambda_i^{'}}}{\lambda_i^{'} e^{\lambda_i}} & \text{if}\, \lambda_i, \lambda_i^{'} \geq l_0 \\ \log \dfrac{\lambda_i e^{\lambda_i^{'}} (\alpha_i e^{-\lambda_i^{'}} + \sum_{j \neq i} \alpha_j e^{-\lambda_j})}{\lambda_i^{'} e^{\lambda_i} (\alpha_i e^{-\lambda_i} + \sum_{j \neq i} \alpha_j e^{-\lambda_j})} & \text{if}\, \lambda_i, \lambda_i^{'} < l_0 \\ \log \dfrac{\lambda_i e^{\lambda_i^{'}}}{\lambda_i^{'} e^{\lambda_i} (\alpha_i e^{-\lambda_i} + \sum_{j \neq i} \alpha_j e^{-\lambda_j})} & \text{if}\, \lambda_i < l_0,\, \lambda_i^{'} \geq l_0 \\ \log \dfrac{\lambda_i e^{\lambda_i^{'}} (\alpha_i e^{-\lambda_i^{'}} + \sum_{j \neq i} \alpha_j e^{-\lambda_j})}{e^{\lambda_i} \lambda_i^{'}} & \text{if}\, \lambda_i \geq l_0,\, \lambda_i^{'} < l_0 \end{cases} \tag{4.82}$$

where $l_0 = \log(\alpha_i / (1 - \sum_{j \neq i} (\alpha_j e^{-\lambda_j})))$. Moreover, using (4.61),

$$\log R_i(\lambda_i, \lambda_{-i}) = \begin{cases} \log \frac{\lambda_i}{e^{\lambda_i}} + \log \alpha_i N & \text{if } \lambda_i \geq l_0, \\ \log \dfrac{\lambda_i}{e^{\lambda_i} (\frac{\alpha_i}{e^{\lambda_i}} + \sum_{j \neq i} \frac{\alpha_j}{e^{\lambda_j}})} + \log \alpha_i N & \text{if } \lambda_i < l_0. \end{cases} \tag{4.83}$$

Now, it is straightforward to show that $\mathcal{P}(\lambda_i, \lambda_{-i}) - \mathcal{P}(\lambda_i', \lambda_{-i}) = \log R_i(\lambda_i, \lambda_{-i}) - \log R_i(\lambda_i', \lambda_{-i})$ for any operator $i \in \mathcal{I}$ and for any $\lambda_i, \lambda_i' \in [0, \lambda_{max}]$. Since $\log R_i(\lambda_i, \lambda_{-i}) - \log R_i(\lambda_i', \lambda_{-i})$ has always same sign as $R_i(\lambda_i, \lambda_{-i}) - R_i(\lambda_i', \lambda_{-i})$, condition given in (4.80) is satisfied, and game $\mathcal{G}_{\mathcal{P}}$ is an ordinal potential game. ∎

### 4.B.4 Detailed Analysis of Nash Equilibriums

In the previous section, we proved the existence of pure NE and convergence to them. In this section, we extend our analysis further in order to find these NEs. For the sake of simplicity, we consider the case where all the operators have same amount of available spectrum $W_i = W$ and hence $\alpha_i = \alpha$, $\forall i \in \mathcal{I}$.

Before starting our analysis, we rewrite constraint of set $\Lambda_A$ given in eq. (4.48) as follows:

$$\alpha \leq \frac{1}{\sum_{j\in\mathcal{I}} e^{-\lambda_j}} = \frac{H(\{e^{\lambda_j}|j \in \mathcal{I}\})}{I} \tag{4.84}$$

where $H(\cdot)$ is the harmonic mean function of the variables $(e^{\lambda_1}, e^{\lambda_2}, \ldots, e^{\lambda_I}) = (\{e^{\lambda_j}|j \in \mathcal{I}\})$:

$$H(\{e^{\lambda_j}|j \in \mathcal{I}\}) = \frac{I}{e^{-\lambda_1} + e^{-\lambda_2} + \ldots + e^{-\lambda_I}} \tag{4.85}$$

Therefore, if $\lambda \in \Lambda_A$, it is:

$$H(\{e^{\lambda_j}|j \in \mathcal{I}\}) \geq \alpha I \tag{4.86}$$

Similarly, according to (4.56), if $\lambda \in \Lambda_B$ then:

$$H(\{e^{\lambda_j}|j \in \mathcal{I}\}) \leq \alpha I \tag{4.87}$$

and finally, if $\lambda \in \Lambda_C$:

$$H(\{e^{\lambda_j}|j \in \mathcal{I}\}) = \alpha I \tag{4.88}$$

Next, we define a new variable, $h$ as the natural logarithm of the harmonic mean:

$$h = \log H(\{e^{\lambda_j}|j \in \mathcal{I}\}) \tag{4.89}$$

Note that, since $e^h$ is the harmonic mean of $\{e^{\lambda_j}|j \in \mathcal{I}\}$, we can say that one of the following should hold:

1. Every operator $i \in \mathcal{I}$ adopts the same price $\lambda_i = h$.
2. If one operator $j \in \mathcal{I}$ selects a price $\lambda_j < h$, then there must be at least one other operator $k \in \mathcal{I}$ who will adopt a price $\lambda_k > h$.

Additionally, we define the variable $h_{-i}$ which is similar to $h$ except that price of the $i^{th}$ operator is excluded. That is:

$$h_{-i} = \log(H(\{e^{\lambda_j}|j \in \mathcal{I} \setminus i\})) \tag{4.90}$$

It is obvious that if $\lambda_i > h$, then $h_{-i} < h$, if $\lambda_i < h$, then $h_{-i} > h$, and if $\lambda_i = h$, then $h_{-i} = h$.

**Lemma 4.B.6.** *If $\alpha \in A_1 = (0, e/I)$, there is a unique NE $\lambda^* \in \Lambda_A$, with $\lambda^* = (\lambda_i^* = 1 : i \in \mathcal{I})$*

**Proof:** First, we prove that the NE cannot be in $\Lambda_B$ or $\Lambda_C$ ($\lambda^* \notin \Lambda_B \cup \Lambda_C$) if $\alpha \in A_1 = (0, e/I)$. Notice that, when the price vector is not in $\Lambda_A$, $h \leq \log(\alpha I) < 1$ for given $\alpha$ values. Therefore there exists at least one operator with price less than one. Since $R_i^B$ is an increasing function between $\lambda_i \in (0,1)$, operators with $\lambda_i < 1$ would gain more revenue by unilaterally increasing their prices. Therefore $\lambda^*$ can only be in $\Lambda_A$. According to Theorem 4.B.3, given that the price vector is in $\Lambda_A$, optimal price for any operator $i$ can only be $\lambda_i^A = 1$ if $(1, \lambda_{-i}) \in \Lambda_A$. Since $\lambda^* = (\lambda_i^* = 1 : i \in \mathcal{I}) \in \Lambda_A$ when $\alpha \in A_1$, it is a feasible and unique solution. ■

**Lemma 4.B.7.** *$\mu_i^*$ is always between $\frac{I}{I-1}$ and $h_{-i}$*

**Proof:** We can rewrite equation (4.79) as follows:

$$e^{\mu_i^*}(\mu_i^* - 1) = \frac{e^{h_{-i}}}{I-1} \tag{4.91}$$

where $h_{-i}$ is defined in equation (4.90). Now, if $h_{-i} < \frac{I}{I-1}$, or equivalently if $h_{-i} - 1 < \frac{1}{I-1}$, then $\lambda_i^*$ should be greater than $h_{-i}$ in order to satisfy (4.91). Moreover, if $\lambda_i^* > h_{-i}$, then $\lambda_i^* - 1$ should be less than $\frac{1}{I-1}$ in order to satisfy (4.91). Therefore, $h_{-i} < \lambda_i^* < \frac{I}{I-1}$. Similarly, if $h_{-i} \geq \frac{I}{I-1}$, then $\frac{I}{I-1} \leq \lambda_i^* \leq h_{-i}$, which proves the lemma. ■

**Lemma 4.B.8.** *If $\alpha \in A_3 = (e^{I/(I-1)}, \infty)$, there is a unique NE $\lambda^* \in \Lambda_B$, with $\lambda^* = (\lambda_i^* = I/(I-1) : i \in \mathcal{I})$*

**Proof:** First we prove that there is no NE in $\Lambda_A$ if $\alpha \geq e/I$ (i.e. if $\alpha \in A_2 \cup A_3$). According to Theorem 4.B.3, optimal price for any operator $i$ can only be $\lambda_i^A = 1$ if $(1, \lambda_{-i}) \in \Lambda_A$. Otherwise $\lambda_i^C$ dominates $\lambda_i^A$. Since $\lambda^* = (\lambda_i^* = 1 : i \in \mathcal{I}) \notin \Lambda_A$ when $\alpha \in A_2 \cup A_3$, there is no NE in $\Lambda_A$.

Secondly, we prove that there is no NE in $\Lambda_C$ if $\alpha \in A_3 = (e^{I/(I-1)}, \infty)$. Recall that, when the price vector is in $\Lambda_C$, $h = \log(\alpha I) > I/(I-1)$, which means that there exists at least one operator with price $\lambda_i^C > I/(I-1)$ and $\lambda_i^C \geq h$. Remember that if $\lambda_i \geq h$, then $h_{-i} \leq h$, so $\lambda_i \geq h_{-i}$. Therefore, for an operator $i$, $\lambda_i^C$ is greater than both $h_{-i}$ and $I/(I-1)$. According to Theorem 4.B.3 and Lemma 4.B.7, when $(\mu_i^*, \lambda_{-i}) \in \Lambda_B$, best response price of operator $i$ is $\mu_i^*$ which is between $h_{-i}$ and $I/(I-1)$. This means that for at least one operator, $\lambda_i^C$ is greater than $\mu_i^*$, which implies that $(\mu_i^*, \lambda_{-i}) \in \Lambda_B$. This operator can increase his revenue by reducing his price to $\mu_i^*$. Therefore, there is no NE in $\lambda_C$ for the given $\alpha$ values, and we proved that the NE can only be in $\Lambda_B$.

Finally, we prove that the only NE is $\lambda^* = (\lambda_i^* = I/(I-1) : i \in \mathcal{I})$, if $\alpha \in A_3$. According to Lemma 4.B.7, $\mu_i^*$ is between $h_{-i}$ and $I/(I-1)$ for all operators. $h$ can be greater than or less than $I/(I-1)$. If $h \geq I/(I-1)$, unless all of the operators set their prices to $I/(I-1)$, there exists at least one operator $i$ with price $\lambda_i$ greater than both $h_{-i}$ and $I/(I-1)$. Hence, $\lambda_i$ is also greater than $\mu_i^*$ and this operator can increase his revenue by reducing his price to $\mu_i^*$. Similarly, if $h < I/(I-1)$, there exists at least one operator with price $\lambda_i$ less than both $h_{-i}$ and $I/(I-1)$. This operator can increase his revenue by increasing his price. If all the operators set their prices to $\lambda_i = I/(I-1)$, the price vector is in $\Lambda_B$ and none of the operators can increase his revenue by unilaterally changing his price. Therefore, the only NE is $\lambda^* = (\lambda_i^* = I/(I-1) : i \in \mathcal{I})$.

Hence the lemma is proved. ■

**Lemma 4.B.9.** *If $\alpha \in A_2 = [e/I, e^{I/(I-1)}]$, a NE can only be in $\Lambda_C$.*

**Proof:** In the proof of the Lemma 4.B.8, we showed that there is no NE in $\Lambda_A$, if $\alpha \in A_2$. We can also prove that there is no NE in $\Lambda_B$ for the given range of $\alpha$ values. If the price vector is in $\Lambda_B$ and $\alpha < \frac{e^{I/(I-1)}}{I}$, then $h < I/(I-1)$. Therefore, there is at least one operator with price $\lambda_i \leq h_{-i}$ and $\lambda_i < I/(I-1)$, who can increase his revenue by increasing his price. So we conclude that, if $\alpha \in A_2 = [e/I, e^{I/(I-1)}]$, there is no NE in $\Lambda_A$ or $\Lambda_B$. ∎

**Lemma 4.B.10.** *If $\alpha \in A_2 = [e/I, e^{I/(I-1)}]$, $\lambda^* \in \Lambda_C$ with $\lambda^* = (\lambda_i^* = \log(I\alpha) : i \in \mathcal{I})$ is a NE.*

**Proof:** When all the operators set the same price $\lambda_i^* = \log(I\alpha)$ and $\alpha \in A_2 = [e/I, e^{I/(I-1)}]$, $\log(I\alpha)$ is between 1 and $\mu_i^*$ for all operators (this can be verified through equation (4.79)). Therefore, for any operator i, $(1, \lambda_{-i}) \notin \Lambda_A$ and $(\mu_i^*, \lambda_{-i}) \notin \Lambda_B$. Then, according to Theorem 4.B.3, best response price is $\lambda_i^C$ which is equal to $\log(I\alpha)$. Hence, no operator can gain more revenue by unilaterally changing his price, and $\lambda^* = (\lambda_i^* = \log(I\alpha) : i \in \mathcal{I})$ is a NE. ∎

Finally we analyze the NE for boundary values of $A_2$, i.e. for $\alpha = e/I$ and $\alpha = e^{I/(I-1)}/I$. In the previous lemma, it is proven that $\lambda^* = (\lambda_i^* = \log(I\alpha) : i \in \mathcal{I})$ is a NE if $\alpha \in A_2$. We can also prove that it is the only NE for these boundary values. In Lemma 4.B.9, it is proven that any NE is in $\Lambda_C$ for these $\alpha$ values. So, when $\alpha = e/I$, $h = \log(I\alpha) = 1$, which means that unless all of the users set their prices to one, there exists some operators with $\lambda_i < 1$. These operators would gain more revenue by setting their prices to one. Hence, the only NE is $\lambda_i^* = \log(I\alpha) = 1$. Similarly, when $\alpha = e^{I/(I-1)}/I$, $h = \log(I\alpha) = I/(I-1)$. This means that unless all of the users set their prices to $I/(I-1)$, there exists some operators with $\lambda_i$ greater than both $h_{-i}$ and $M/(M-1)$. These operators would gain more revenue by reducing their prices. Hence, the only NE is $\lambda_i^* = \log(I\alpha) = I/(I-1)$.

We also show that when $\alpha \in (e/I, e^{I/(I-1)})$, there can be infinitely many NEs, all in $\Lambda_C$, via numerical simulations. For different initial price settings, the game converges to different NE.

**Theorem 4.B.11.** *The game $\mathcal{G}_\mathcal{P}$ attains a pure NE which depends on the value of parameter $\alpha$ as follows:*

- *If $\alpha \in A_1 = (0, e/I)$, there is a unique NE $\lambda^* \in \Lambda_A$, with $\lambda^* = (\lambda_i^* = 1 : i \in \mathcal{I})$ and respective market equilibrium $\mathbf{x}^* \in X_A$.*
- *If $\alpha \in A_3 = (e^{\frac{I}{I-1}}/I, \infty)$, there is a unique NE $\lambda^* \in \Lambda_B$, with $\lambda^* = (\lambda_i^* = \frac{I}{I-1} : i \in \mathcal{I})$, which induces a respective market equilibrium $\mathbf{x}^* \in X_B$.*
- *If $\alpha \in A_2 = [e/I, e^{\frac{I}{I-1}}/I]$, there exist infinitely many NEs, $\lambda^* \in \Lambda_C$, and each one of them yields a respective market stationary point $\mathbf{x}^* \in X_C$.*

**Proof:** Lemma 4.B.5 proves that the game $\mathcal{G}_{\mathcal{P}}$ is a finite ordinal potential game. Therefore, it always attains a pure NE. Lemma 4.B.6 proves the case for $\alpha \in A_1 = (0, e/I)$. Lemma 4.B.8 proves the case for $\alpha \in A_3 = (e^{\frac{I}{I-1}}/I, \infty)$. The case for $\alpha \in A_2 = [e/I, e^{\frac{I}{I-1}}/I]$ is proven in Lemma 4.B.9 and Lemma 4.B.10. ■

# Chapter 5

# Dynamic Pricing Mechanisms for Spectrum Markets

## 5.1 Background

A particular characteristic of the emerging dynamic spectrum markets is that they aim to facilitate spectrum exchange among peer network entities as it is depicted in Figure 5·1. Namely, in these markets, secondary operators will be able to directly interact and redistribute the spectrum they acquired from the primary operators, in a fine spatio-temporal scale, in order to satisfy their dynamic needs. Also, they will provide inter-connection services to each other and exchange traffic. Similarly, users will be able to route each others traffic so as to satisfy their communication needs in an ad hoc fashion. In detail, in this chapter we will consider the following three scenarios:

- *Secondary operators provide interconnection services*. Each SO delivers the traffic that originates from a user of another SO and is destined to a user within his own service range.
- *Users exchange bandwidth*. The users form an ad hoc network and each one of them, along with his own traffic, routes also the traffic of other users.
- *Secondary operators exchange their spectrum*. SOs exchange for certain time periods their leased spectrum in order to satisfy the highly dynamic communication needs of their users.

These networking examples have certain common properties that call for novel network management schemes and protocols. Specifically:

- There exist many resource (spectrum or bandwidth) buyers and sellers.
- Each entity is at the same time a buyer and seller, i.e. both consuming and providing resources.
- Moreover, these two roles are intertwined. For example, a secondary operator consumes his spectrum either for satisfying his own communication needs (acting as client) or for delivering the traffic of other SOs (acting as a server). That is, both roles presume the consumption of the same resource. The same holds for the users who utilize their bandwidth either for forwarding their own traffic (client role) or for routing the traffic of other users (server role).

- The interests of the different entities are very often conflicting. An operator would like to lease spectrum from another operator at the minimum possible price and sell his own spectrum at the highest price. Also, a user prefers other users to forward his traffic while he is not willing to spend his probably scarce capacity for serving them.

- The interactions and transactions among these entities should be realized in an almost real-time fashion.

- The entities most often interact and coordinate in a decentralized fashion, i.e. without the coordination of a central authority such as a network controller. Therefore all the required protocols and market clearing algorithms must be amenable to distributed execution.

- In some cases it is impossible to have a billing and charging infrastructure. For example, in an ad hoc network it is not very likely that users will be able to pay each other for the bandwidth they exchange.

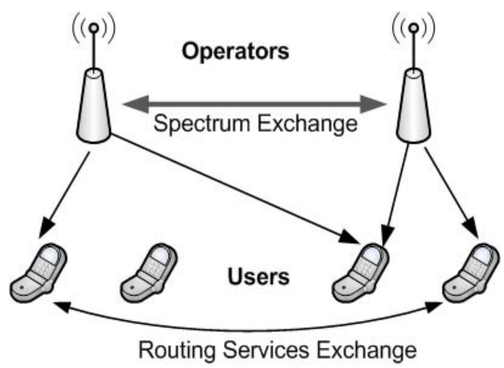



**Figure 5·1:** Operators and users exchange spectrum (or bandwidth) to satisfy their dynamic needs.

In this chapter, we introduce a dynamic pricing scheme to address the challenges above and ensure the successful operation of secondary spectrum markets. We model node interaction through market transactions. Each node (either user or SO) announces one separate bid for buying resource from any potential seller, and one bid for selling resource to any potential buyer. The bids represent the nodes willingness for acquiring or selling the resource. Sellers receive bids from buyers and need to decide how to allocate their resource and how much to charge for that. On the other hand, buyers receive resource offers and need to select the sellers to which they submit bids for resource request. The distinguishing characteristic of our model is that *each node in the system is simultaneously a seller and buyer*. These two roles are intertwined, and in fact, each one places limitations on the other. Each node must decide how much portion of the resource it will dedicate to serving others and to receiving service itself, so as to maximize its benefit. The latter is usually captured by a utility function which can include charging and reimbursement for resource exchanges.

Loosely speaking, this setup can be considered as a *Double Auction Market* due to the concurrent interaction of multiple buyers and sellers and the bidding process. However, we assume in this work that nodes, sellers or buyers, are price-takers. That is, nodes do not anticipate the impact of their bidding strategy to the announced prices. This assumption simplifies the analysis and allows us to derive a social welfare maximizing mechanism with low computational and communication requirements. Price-taking behavior is highly probable to appear in settings where the number of nodes is large, [90],[48], or when each node is not aware of the strategy space and the actions of other nodes. An interesting explanation about the intuition and the implications of assuming that nodes of a network are price-takers is given in [98]. Additionally, in the context of large-scale wired networks, such as the Internet, there exists a large volume of works, originating from the seminal paper by Kelly [54], that study mechanisms for bandwidth allocation among competing but price-taking nodes.

The proposed mechanism uses one-dimensional bidding and simple charging rules, in line with [55]. Our contributions are as follows: **(i)** we develop a dynamic pricing scheme for these markets that captures the double role of a node as resource contributor and consumer, (ii) we prove that there exist bidding and charging strategies that maximize social welfare and we explicitly compute them. Nodes are induced to follow these strategies, otherwise they get isolated by the network, (iii) we design a decentralized realization of the protocol that relies only on lightweight feedback from the network, through which nodes coordinate in a distributed fashion, (iv) we generalize our framework to optimization of a generic network objective, other than social welfare, (v) we also introduce a pricing method which does not need a charging infrastructure.

The chapter is organized as follows. In section 5.2 we present a literature overview and in section 5.3 we give the model and problem formulation. In section 5.4 we define the pricing mechanism for a conventional setting with a central controller, and then we describe the decentralized realization of it. In section 5.5 we generalize our method to achieve the optimization of a generic network objective. Section 5.6 contains numerical results and section 5.7 concludes our study.

## 5.2 Related Work

In single-sided markets, a central controller needs to allocate a divisible good among a set of buyers. Buyers submit bid signals to the controller in order to declare their willingness for acquiring the good. Accordingly, the controller computes an allocation and a payment for each buyer. The buyers can be either price takers or price anticipators, depending on whether they cast their bid without or with consideration of its impact on the subsequent price. The objective is to come up with an efficient allocation, namely one that maximizes the sum of user utilities, without explicit knowledge of their utility functions. For rational, price taking users, Kelly et.al. [55] showed that the problem above can be solved in a decentralized way, and [66] introduced an auction mechanism that entails an efficient allocation of capacity to network flows. Users submit bids, they receive resource amount that is proportional to their bid and they pay amount equal to their bid.

On the other hand, price anticipating users strategically adapt their bid by taking into account its impact on the price, and thus a game interaction emerges with certain efficiency loss [49]. A mechanism that guarantees an efficient allocation for selfish, price-anticipating nodes is the Vickrey-Clarke-Groves (VCG) mechanism [105]-[38]. In the VCG auction, a node is charged according to its externality, namely the induced reduction to the maximum total utility it causes to all other nodes. The allocation is performed according to the solution of the total utility maximization problem. However, VCG-based mechanisms have significant drawbacks, [67]. First, they exhibit high computational complexity since very often they are NP-hard. Second, they entail an intolerable communication cost because the bidders have to communicate their entire utility functions (demand functions). Finally, VCG mechanisms are not budget balanced, i.e. the sum of subsidies (paid to the sellers) exceeds the sum of charges (paid by the buyers) and hence there is need to inject money in the market.

Recent studies address the problem of the communication burden of VCG mechanisms by proposing a combination of VCG and proportional allocation methods [109], [50]. In these works, nodes submit one-dimensional bids and are charged according to the rules of the VCG scheme. Other single-sided auction methods where many buyers submit bids for the resource provided by one seller are [23], [92], where a two-dimensional bid (a per-unit price and the maximum amount of resource the user is willing to buy) is submitted. This bid corresponds to a specific class of utility functions. The charging is performed as in VCG auctions, and the allocation is according to the total utility maximization problem. The survey [94] provides an overview of the cases and objectives encountered in single-sided auctions. The case of many sellers bidding to meet the demand of a buyer is dual and admits similar results. Here, a basic distinction about sellers is whether they select charging prices to maximize their profit or satisfy a socially optimal objective [82].

Most of the works in single-sided auctions assume a single divisible resource that is to be allocated among candidate buyers. Some works generalize this mechanism to multiple divisible goods, where each user needs bandwidth on a set of links that constitutes its path [63], [10]. These works however do not consider the existence of many buyers and sellers at the same time. The framework of double-sided auction markets, where many buyers interact with many sellers addresses this scenario. The works [23] and [45] study double auction methods for link capacity allocation in networks. In [23], each link sells its bandwidth and each node bids for the allocation of bandwidth in a bundle of links. A central agent collects these bids and determines bandwidth allocations and payments by solving the social welfare maximization problem. These schemes are not directly applicable to decentralized settings such as networks since they require a central auctioneer. In [108], the authors present a double auction mechanism for routing protocols in mobile ad-hoc networks. Multiple source-destination pairs interact with a set of intermediate relay nodes in order to allocate their traffic in a cost-efficient way. The authors prove that this mechanism ensures node cooperation through proper payments.

The main innovation of our approach compared to these works is that it addresses the simultaneous double role of each node as resource seller and buyer, the fact that one role affects the other, and the objective of each node to bid for buying and offer for selling resource with any other node in a distributed fashion. Additionally, by assuming that nodes are price-takers, we manage to derive a mechanism with low computational complexity and reduced communication burden.

## 5.3 System Model

We consider a group of $N$ nodes that may represent a set of secondary wireless operators or a group of users that form a wireless ad hoc network. Hereafter, we use the term node for both SOs and users. Each node strives to maximize his own perceived satisfaction. We adhere to the scenario where a node may interact potentially with any other node in a full mesh topology. Depending on the specific network instance, a node may interact with a subset of the group. For example, in the ad hoc network scenario, each user interacts and cooperates only with his neighbors.

Node interactions entail service exchanges which are directly translated into consumption of respective amounts of resources. Node $i$ possesses a finite amount of resource of $C_i$ units that is available for provisioning to others or for satisfying its own needs, potentially from other nodes. Henceforth we use terms "resource" and "service" to refer to this exchange. For any pair of interacting nodes $i, j$ denote by $x_{ij}$ the amount of resource that node $i$ spends for satisfying its own needs through node $j$. Denote by $y_{ij}$ the amount of resource granted from $i$ to $j$, namely the amount that node $i$ uses to satisfy $j$'s needs. Clearly, node $i$ can satisfy its needs through node $j$ only if $j$ grants the corresponding amount of resource, namely it is $y_{ji} = x_{ij}$.

For the example of SOs interconnection, $x_{ij}$ is the amount of bandwidth that operator $i$ consumes for routing traffic that is destined to a user within the service area of operator $j$. On the other hand, $y_{ij}$ is the bandwidth that $i$ dedicates to serve traffic originating from SO $j$. This interaction of SOs is depicted in Figure 5·2. In wireless ad-hoc networks, $x_{ij}$ is the amount of bandwidth that node $i$ spends to forward its own traffic to node $j$ (which will then spent equal capacity $y_{ji}$ to forward it further), and $y_{ij}$ is the bandwidth $i$ dedicates to forwarding traffic of $j$. This process is shown in Figure 5·3. Finally, for the scenario of spectrum exchange among the SOs, $x_{ij}$ is the amount of spectrum operator $i$ asks from operator $j$ and $y_{ij}$ the spectrum that $i$ sells to $j$ in response to the request $x_{ji}$ of the latter. This scenario is depicted in Figure 5·4. Define vectors $\mathbf{x}_i = (x_{ij} : j = 1, \ldots, N)$, and $\mathbf{y}_i = (y_{ij} : j = 1, \ldots, N)$. Thus, network operation is represented by the $N \times N$ resource request and allocation matrices $\mathbf{X} = (\mathbf{x}_i : i = 1, ..., N)$ and $\mathbf{Y} = (\mathbf{y}_i : i = 1, ..., N)$. The amounts of resource that node $i$ uses for its own needs and for serving others' needs satisfy

$$\sum_{j=1}^{N} x_{ij} + \sum_{j=1}^{N} y_{ij} \leq C_i \,. \tag{5.1}$$

Each node $i$ is characterized by a utility function $J_i(\cdot)$. We assume that $J_i(\cdot)$ is separable, in the sense that $J_i(\mathbf{x}_i, \mathbf{y}_i) = \sum_{j \neq i} J_{ij}(x_{ij}, y_{ij})$, where $J_{ij}(\cdot)$ is the perceived utility of node $i$ due to its interaction with node $j$. This models the general case where node $i$ obtains different utility from different nodes $j$ due to the different importance or timeliness of service, or other properties such as the quality of the spectrum band, etc. Moreover,

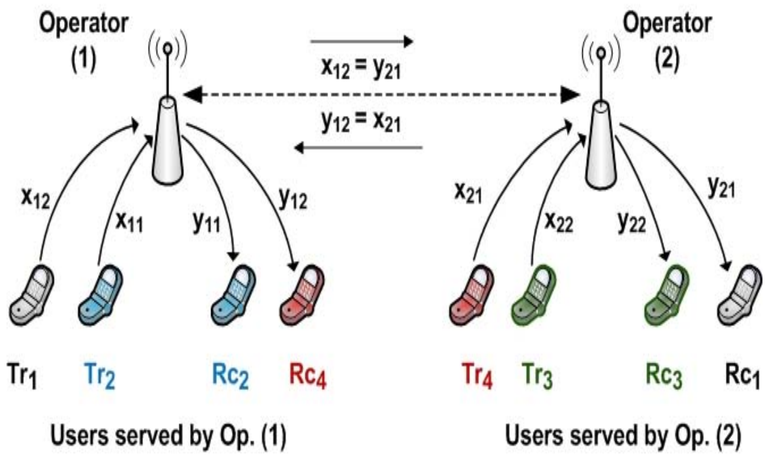



**Figure 5·2:** Two operators serve their own clients and exchange traffic: operator (1) consumes $x_{12}$ amount of his bandwidth $C_1$, to serve user 1 ($Tr_1$) who sends data to his pair-receiver node ($Rc_1$) who lies within the range of operator (2). The latter must also spend an equal amount of resource, $y_{21} = x_{12}$ for this connection. For each operator, the aggregate bandwidth used for the uplink and the aggregate bandwidth used for the downlink should not exceed his capacity: $x_{12}+x_{11}+y_{12}+y_{11} \leq C_1$, $x_{21}+x_{22}+y_{21}+y_{22} \leq C_2$. Operators are connected with links of high capacity (backbone network).

we assume that the utility function is further decomposed into two components: (i) one component for the client (buyer) side. Let $U_{ij}(x_{ij})$ be the *utility* of node $i$ from satisfying its own needs by using amount $x_{ij}$ through node $j$. Function $U_{ij}(\cdot)$ is differentiable, strictly concave, positive and increasing; (ii) one component for the server (seller) operation of the node. Let $W_{ij}(y_{ij})$ be the *cost* incurred if node $i$ *provides* resource amount $y_{ij}$ to node $j$. This is also a differentiable and strictly concave function. However, this function is negative and decreasing, since service provisioning results in consumption of the node resources. Additionally, for the case of SOs spectrum exchange, $W_{ij}(\cdot)$ captures the *opportunity cost* of operator $i$ for leasing his spectrum to operator $j$. Therefore, the utility function of node $i$ can be written as:

$$J_i(\mathbf{x}_i, \mathbf{y}_i) = \sum_{j=1}^{N} [U_{ij}(x_{ij}) + W_{ij}(y_{ij})] \,. \tag{5.2}$$

The most common and straightforward criterion that quantifies efficient operation of a group of nodes is the maximization of the sum of node utility functions, known as social welfare. The socially optimal operating point of the group is the solution to the social welfare problem (SWP):

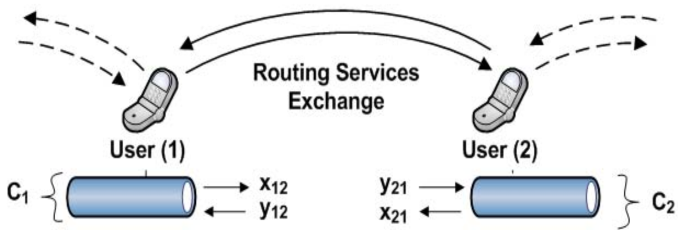



**Figure 5·3:** User (1) reserves $x_{12}$ portion of his capacity $C_1$ for sending data that will be further forwarded by user (2). Also, he uses the rest of his capacity for forwarding the data of user (2). Apparently, it must hold $x_{12} + y_{12} \leq C_1$, and $x_{21} + y_{21} \leq C_2$.

$$\max_{\mathbf{X},\mathbf{Y}} \sum_{i=1}^{N} \sum_{j=1}^{N} [U_{ij}(x_{ij}) + W_{ij}(y_{ij})] \tag{5.3}$$

subject to:

$$\sum_{j=1}^{N} x_{ij} + \sum_{j=1}^{N} y_{ij} \leq C_i,\ i = 1, \ldots, N, \tag{5.4}$$

$$x_{ij} = y_{ji},\ \forall\, i, j,\ \text{ with } i \neq j. \tag{5.5}$$

The SWP problem has a unique solution since the objective function is strictly concave and the constraint set is convex. We relax constraints and define the Lagrangian:

$$\begin{aligned} L &= \sum_{i=1}^{N} \sum_{j=1}^{N} [U_{ij}(x_{ij}) + W_{ij}(y_{ij})] - \sum_{i=1}^{N} \lambda_i (\sum_{j=1}^{N} x_{ij} + \\ &+ \sum_{j=1}^{N} y_{ij} - C_i) + \sum_{i=1}^{N} \sum_{j=1}^{N} r_{ij}(y_{ji} - x_{ij}) \end{aligned} \tag{5.6}$$

where $\boldsymbol{\lambda} = (\lambda_i \geq 0,\, i = 1, \ldots, N)$ is the vector of Lagrange multipliers (dual variables) corresponding to capacity constraints. Also $\mathbf{R} = (r_{ij} : i, j = 1, \ldots, N)$ is the $N \times N$ matrix of the Lagrange multipliers $r_{ij}$ corresponding to the equality constraints. The optimal primal variables $\mathbf{X}^*, \mathbf{Y}^*$ and dual variables $\boldsymbol{\lambda}^*, \mathbf{R}^*$ satisfy the Karush-Kuhn-Tucker (KKT)

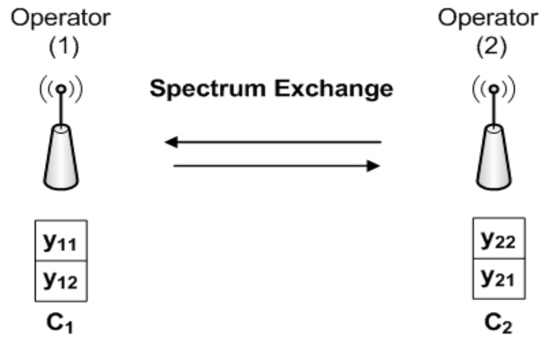



**Figure 5·4:** Operator (1) reserves $y_{11}$ units of his spectrum $C_1$ for serving his own needs and leases the residual spectrum of $y_{12}$ units, to operator (2), in response to the request $x_{21}$ of the latter. The aggregate spectrum constraint is $y_{11} + y_{12} \leq C_1$.

optimality conditions:

$$
\begin{aligned}
&(A1): \quad U_{ij}^{'}(x_{ij}^*) = \lambda_i^* + r_{ij}^*, \ \forall\, i,j, \ \text{ with } i \neq j,\\
&(A2): \quad W_{ij}^{'}(y_{ij}^*) = \lambda_i^* - r_{ji}^*, \ \forall\, i,j, \ \text{ with } i \neq j,\\
&(A3): \quad \lambda_i^*(\sum_{j=1}^{N} x_{ij}^* + \sum_{j=1}^{N} y_{ij}^* - C_i) = 0, \ i = 1,\ldots,N,\\
&(A4): \quad \sum_{j=1}^{N} x_{ij}^* + \sum_{j=1}^{N} y_{ij}^* \leq C_i, \ i = 1,\ldots,N,\\
&(A5): \quad x_{ij}^* = y_{ji}^* \ \forall\, i,j, \ \text{ with } i \neq j,\\
&(A6): \quad x_{ij}^*, y_{ij}^*, \lambda_i^* \geq 0, \ \forall\, i,j, \ \text{ with } i \neq j.
\end{aligned}
$$

The optimal solution of SWP, namely the operation point at which the efficiency of node interaction is maximized satisfies equations (A1)-(A6). However, the group consists of rational and selfish nodes whose interests are not aligned with the social objective. Selfish nodes act towards maximizing their own utility function, a strategy which clearly results in the degradation of the group operation. In the sequel, we present a dynamic pricing method to achieve the optimal operating point of the group in a distributed fashion in the presence of price-taking selfish nodes.

## 5.4 The Dynamic Pricing Mechanism

### 5.4.1 Rationale

We derive first a central agent aided algorithm to find an efficient resource allocation. Recall that we impose the requirement that node utility functions are private to each node. SWP cannot be solved by a single central agent due to lack of knowledge on node utilities. To overcome this difficulty, we propose a dynamic pricing mechanism which is inspired from the algorithm in [55]. Given that nodes are rational utility maximizers, this mechanism will incur an allocation that maximizes social welfare, i.e. it is the solution of SWP.

Each node $i$ submits buy (ask) bids $p_{ij}$ for receiving the available resource of node $j = 1, \ldots, N$, and sell (offer) bids $a_{ij}$ for granting its entire resource to each node $j$. These bids are collected by the central controller which subsequently determines (i) the resource allocation regimes $(\mathbf{X}, \mathbf{Y})$, (ii) the charging and reimbursement amounts, $h(p_{ij})$ and $l(a_{ij})$ respectively, for each pair of interacting nodes $i$ and $j$. By $h(\cdot)$ and $l(\cdot)$ we denote the charges and reimbursements as continuous functions of submitted buy and sell bids respectively. The controller resource allocation is derived from the solution of a certain optimization problem. The charging and reimbursement amounts are calculated through functions $h(\cdot)$ and $l(\cdot)$. The key challenge is to come up with the structure of optimization problem and functions $h(\cdot)$ and $l(\cdot)$ such that the resource allocation coincides with the optimal solution of SWP, defined by equations (A1)-(A6).

The controller anticipates rational behavior by nodes in the process of selecting their bids. Each bidder in turn knows the resource allocation problem and charging and reimbursement functions and attempts to find buy and sell bidding strategies that optimize its net benefit. That is, each node $i$ solves the following $2(N-1)$ problems (NODE problems):

$$\max_{p_{ij}} \{U_{ij}(x_{ij}(p_{ij})) - h(p_{ij})\}, \; j = 1, \ldots, N, \tag{5.7}$$

$$\max_{a_{ij}} \{W_{ij}(y_{ij}(a_{ij})) + l(a_{ij})\}, \; j = 1, \ldots, N. \tag{5.8}$$

Notice that $x_{ij}$ and $y_{ij}$ depend on respective bids, $p_{ij}$ and $a_{ij}$.

For the problem at hand, we propose that the controller should determine the optimal allocation from the solution of the following optimization problem (Network Controller Problem, NCP):

$$\max_{\mathbf{X},\mathbf{Y}} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} (p_{ij} \log x_{ij} - \frac{a_{ij}}{2} y_{ij}^2) \tag{5.9}$$

subject to $\sum_{j=1}^{N} x_{ij} + \sum_{j=1}^{N} y_{ij} \leq C_i$, for $i = 1, \ldots, N$ and $x_{ij} = y_{ji}$ for all $i, j \neq i$. Note that the objective function is selected such that it is strictly concave. The NCP problem has unique solution which satisfies KKT conditions:

$$(B1) : x_{ij}^* = \frac{p_{ij}}{\lambda_i^* + r_{ij}^*}, \quad (B2) : y_{ij}^* = \frac{-\lambda_i^* + r_{ji}^*}{a_{ij}}$$

$\forall\, i, j,\, i \neq j$, and four additional conditions, call them (B3)-(B6) which are identical to

(A3)-(A6). These are the optimal amounts of resource that each node $i$ should receive and provide according to the controller problem. They depend on node $i$ bids and on dual variables (and through them, on other nodes' bids). Note that the amount of resource $x_{ij}$ which is granted to $i$ is proportional to its bid $p_{ij}$. On the other hand, the resource $y_{ij}$ granted by node $i$ to $j$ is inversely proportional to the charging bid $a_{ij}$. Comparing the solutions of SWP and NCP, one can see that if nodes submit their bids as follows:

$$(G1) : p_{ij}^* = x_{ij}^* U_{ij}'(x_{ij}^*), \;\; (G2) : a_{ij}^* = \frac{-W_{ij}'(y_{ij}^*)}{y_{ij}^*},$$

then equations (B1)-(B6) are identical to equations (A1)-(A6). The bid expressions above hold if the charging and reimbursement functions are chosen as follows:

$$h(p_{ij}) = p_{ij}, \;\; l(a_{ij}) = \frac{(\lambda_i - r_{ji})^2}{a_{ij}}, \; \forall\, i, j, \;\; \text{with } i. \neq j. \tag{5.10}$$

Then, the proposed mechanism achieves the socially optimal solution. These charging and reimbursement rules are quite intuitive: each node as client (resource consumer) is charged exactly the amount he bid, namely the amount it declared it is willing to pay. On the other hand, its reimbursement is inversely proportional to the submitted sell bid. That is, nodes that submit high offers to sell the good finally receive less money. Implicit here is the assumption that nodes are price takers.

The mechanism is executed in successive rounds, each round $t$ with the following steps:

- Each node $i$ solves the NODE problems and calculates $p_{ij}^{(t)}$ and $a_{ij}^{(t)}$, separately for each node $j$ it interacts with.
- The central controller collects all bids and solves NCP. It then allocates the current optimal amounts of resource $x_{ij}^{(t)}$ and $y_{ij}^{(t)}$ and determines the charges $h(p_{ij}^{(t)})$ and reimbursements $l^{(t)}(a_{ij})$. Finally, it communicates the new Lagrange multipliers.

The bids calculated in each iteration round are not the final ones in (G1) and (G2). After each iteration, the solution of the NODE problem changes due to the updated Lagrange multipliers. Indeed, the NODE problem is solved by substituting variables $x_{ij}$ and $y_{ij}$ from the previous round and optimizing with respect to new bids $p_{ij}$ and $a_{ij}$. Calculation of derivatives at each step leads to equations $U_{ij}'(x_{ij}) = p_{ij}/x_{ij}$ and $W_{ij}'(y_{ij}) = -a_{ij} y_{ij}$, for the client and server operation of the node. These are fed to the controller, which then computes the new allocations.

The iterative procedure converges and the final bids equal to the *social optimal bids* $p_{ij}^*$ and $a_{ij}^*$. Therefore, nodes finally bid according to (G1) and (G2). This shows that the solution of NCP, together with node rationality (which induces nodes to optimize their strategy in a prescribed way), achieves the socially optimal point, at which nodes receive and provide resources according to the solution of SWP.

### 5.4.2 Decentralized Realization

We now provide a variant of the dynamic pricing mechanism that is realizable for clearing markets and resource exchange without presuming the presence of a central coordinator or a controller. For example, this scheme is very important for the management of ad hoc networks that are formed by the users. Controller tasks such as resource allocation, charging and reimbursement decisions are undertaken by nodes in a distributed way. These tasks are accomplished by solving the NCP problem in a decentralized fashion using dual decomposition for any given set of bids. We relax constraints and define the Lagrangian,

$$L(\mathbf{Y}, \mathbf{X}, \mathbf{R}, \boldsymbol{\lambda}) = \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} (p_{ij} \log x_{ij} - \frac{a_{ij}}{2} y_{ij}^2)$$

$$- \sum_{i=1}^{N} [\lambda_i (\sum_{j=1}^{N} x_{ij} + \sum_{j=1}^{N} y_{ij} - C_i) + \sum_{j=1}^{N} r_{ij}(y_{ji} - x_{ij})] \,, \tag{5.17}$$

where $\mathbf{R}$, $\boldsymbol{\lambda}$ are the Lagrange multipliers defined previously. By exploiting the separability properties of $L(\cdot)$, we can derive a distributed algorithm for the solution of this problem [11]. For NCP, this leads to the DP-NCP Algorithm 2.

Some implicit assumptions here are in order. First, communication of required information is synchronous. Secondly, nodes update and communicate multipliers as described above and do not strategize or manipulate them before their broadcast. Convergence of this class of algorithms is guaranteed if the objective function is differentiable and strictly concave, and the step size $s_t$ is properly selected [11]. Moreover, convergence can be verified through Lyapunov stability theory. Following the rationale of the proof in [82], for the Lyapunov function

$$V(\mathbf{R}, \boldsymbol{\lambda}) = \frac{1}{2} \sum_{i=1}^{N} (\lambda_i - \lambda_i^*)^2 + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (r_{ij} - r_{ij}^*)^2 \tag{5.18}$$

it can be shown that $dV/dt \leq 0$ by using complementary slackness, (B1) and (B2) and some algebra. Hence the proposed two-step mechanisms for the solution of SWP attain the optimal operation point $(\mathbf{Y}^*, \mathbf{X}^*)$.

### 5.4.3 Pricing Without a Charging Infrastructure

The pricing mechanism above is realizable contingent on the assumption that nodes are able to charge and reimburse nodes for consuming and providing resources respectively. Otherwise, the NODE problem cannot include functions $h(\cdot)$ and $l(\cdot)$ and thus, nodes cannot submit the socially optimal bids. In some cases, such as ad hoc networks, an accounting infrastructure is rather difficult to be created, and thus we need to devise an alternative charging method. We propose the "absorption" of charging and reimbursement into resource allocation, in line with the rationale of [40]. That is, instead of computing charging and reimbursement functions as means for facilitating the social optimal solution, we apply a method based on equivalent *reduction* of provisioned services. Consider the net utility components of $i$ and $j$ due to their interaction:

$$
\begin{aligned}
NU_{ij}(y_{ij}, y_{ji}) &= U_{ij}(y_{ji} + W_{ij}(y_{ij}) + l(a_{ij}) - h(p_{ij}) \\
NU_{ji}(y_{ji}, y_{ij}) &= U_{ji}(y_{ij}) + W_{ji}(y_{ji}) + l(a_{ji}) - h(p_{ji})
\end{aligned}
\tag{5.19}
$$

At the end of node interaction, we have the following equalities due to the buy-sell transactions:

$$
l(a_{ij}) = h(p_{ji}), \;\; l(a_{ji}) = h(p_{ij}), \quad x_{ij} = y_{ji}, \;\; x_{ji} = y_{ij} \, . \tag{5.20}
$$

In order to avoid computing charges and reimbursements, each node should allocate amounts of resource different than what is dictated by the SWP to nodes it interacts with. In other words, it should provide resource amount to each node such that it covers the potential incurred charges and reimbursements. Namely, $j$ should allocate to $i$ resource $z_{ji}$ such that:

$$
U_{ij}(z_{ji}) = U_{ij}(y_{ji}) - h(p_{ij}) + l(a_{ij}) \, . \tag{5.21}
$$

This means that the amount of allocated resource from node $j$ to $i$ in the case of no charging infrastructure will be less than the corresponding amount in the case of charging infrastructure if $i$ is charged more by $j$ than its reimbursement for serving $j$, and it is increased otherwise. Consequently, nodes do not have to pay fees or get reimbursed. Thus, the net utility of node $i$ for interacting with $j$ becomes:

$$
NU_{ij}(z_{ij}, z_{ji}) = U_{ij}(z_{ji}) + W_{ij}(z_{ij}) \, . \tag{5.22}
$$

A basic issue is to consider the efficiency loss, if any, for the outcome of node interaction in the absence of a charging infrastructure. The difference between total system utility in the original setup with charging infrastructure and total utility in the operation point $\mathbf{Z} = (z_{ij} : i, j = 1, \ldots, N)$ in the case of absence of a charging infrastructure, is:

$$
\Delta U = \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} [U_{ij}(y_{ji}) + W_{ij}(y_{ij}) - U_{ij}(z_{ji}) - W_{ij}(z_{ij})] \, .
$$

If we substitute $U_{ij}(z_{ji})$ from (5.21), we get

$$
\Delta U = \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} [W_{ij}(y_{ij}) - l(a_{ij}) + h(p_{ij}) - W_{ij}(z_{ij})]
$$

where the quantity $z_{ij}$ is estimated based on the equation:

$$
U_{ji}(z_{ij}) = U_{ji}(y_{ij}) - h(p_{ji}) + l(a_{ji}) \, . \tag{5.23}
$$

Hence efficiency loss $\Delta U$ depends explicitly on the relation between functions $U_{ji}(\cdot)$ and $W_{ij}(\cdot)$. For example, in a symmetric case where these are the same in absolute value, the method does not entail efficiency loss. Nevertheless, in the general case the degradation will depend on the specific function components of nodes. Moreover, an additional requirement is that each node $i$ should know the utility functions of others in order to find new allocations $z_{ij}$. These limitations call for further study of the method. However, the method constitutes a significant first step towards eliminating the need for accounting and charging infrastructure. Furthermore, it can be directly implemented as the final stage of the DP-NCP algorithm. DP-NCP could be modified by adding a step at which each node $i$ computes quantities $z_{ij}, j = 1, \ldots, N$, and updates its initial resource allocation decisions accordingly.

## 5.5 Dynamic Pricing for a Generic Network Objective

Now we consider the case when the network objective is the optimization of a generic function, other than social welfare. For example, consider a group of nodes that agree to achieve load balancing or ensure some kind of fairness among them. This is different from the social welfare maximization problem. The key question is the following: given that nodes will bid myopically based on the maximization of their net benefits, according to the NODE problems, how can we engineer the dynamic pricing scheme such that this different objective is achieved? Namely, we would like to derive the appropriate resource allocation and charging and reimbursement strategies that guarantee optimal operation for the case of this new objective function.

Suppose the desirable objective is represented by the following maximization problem (Generic Network Problem, GNP):

$$\max_{\mathbf{X},\mathbf{Y}} F(\mathbf{X},\mathbf{Y}) \tag{5.24}$$

subject to $\sum_{j=1}^{N} x_{ij} + \sum_{j=1}^{N} y_{ij} \leq C_i$ for $i = 1, \ldots, N$ and $x_{ij} = y_{ji}$ for all $i, j \neq i$, where the objective function $F(\cdot)$ is the new network objective. We assume that $F(\cdot)$ is differentiable, strictly concave and separable, so that

$$F(\mathbf{X},\mathbf{Y}) = \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \left[ d_{ij}(x_{ij}) + g_{ij}(y_{ij}) \right] , \tag{5.25}$$

where $d_{ij}(\cdot) \neq U_{ij}(\cdot)$ denotes the pairwise obtained utility between nodes $i$ and $j$ corresponding to the new objective function $F(\cdot)$, and $g_{ij}(\cdot) \neq W_{ij}(\cdot)$ captures the cost corresponding to $F(\cdot)$. The solution of this problem is unique and similar to that of SWP except that, instead of (A1) and (A2), we have

$$\begin{aligned} (H1): &\quad d'_{ij}(x^*_{ij}) = \lambda^*_i + r^*_{ij}, \ \forall\ i, j, \ \text{with } i \neq j \\ (H2): &\quad g'_{ij}(y^*_{ij}) = \lambda^*_i - r^*_{ji}, \ \forall\ i, j, \ \text{with } i \neq j \, . \end{aligned}$$

Consider again the two-step distributed mechanism where nodes submit bids, and then a controller solves the NCP problem. The optimal allocations are then described by (B1)-(B6). The solution of NCP coincides with the solution of the new problem, GNP, if nodes submit bids as

$$p_{ij}^* = x_{ij}^* d_{ij}'(x_{ij}^*),\ a_{ij}^* = \frac{-g_{ij}'(y_{ij}^*)}{y_{ij}^*},\ i, j,\ \text{with } i \neq j\,, \tag{5.26}$$

and provided that nodes are charged and reimbursed as

$$h(p_{ij}) = \frac{U_{ij}(x_{ij})}{d_{ij}(x_{ij})} p_{ij}, \quad l(a_{ij}) = \frac{(\lambda_i - r_{ji})^2\, W_{ij}(y_{ij})}{a_{ij} g_{ij}(y_{ij})}\,.$$

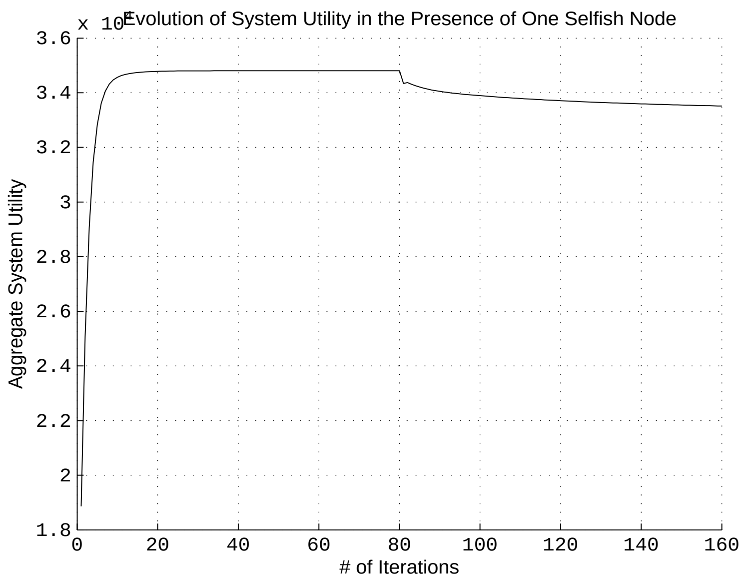



**Figure 5·5:** Evolution of system utility in the presence of one selfish node that takes effect at $t = 80$.

The intuition behind these rules is evident again. Node charges are raised if they act only towards maximization of their utility components $U_{ij}(\cdot)$ instead of $d_{ij}(\cdot)$. In addition, reimbursements are decreased if nodes attempt to minimize their cost components $W_{ij}(\cdot)$ without considering the desirable cost $g_{ij}(\cdot)$. For this setting, the NODE problem, eq. (5.7) and (5.8), solved by each node $i$ for its interaction with each other node $j$, results in equations (H1) and (H2) and yields the GNP optimal solution. A distributed algorithm similar in flavor to DP-NCP can be applied to circumvent the use of a central controller here as well.

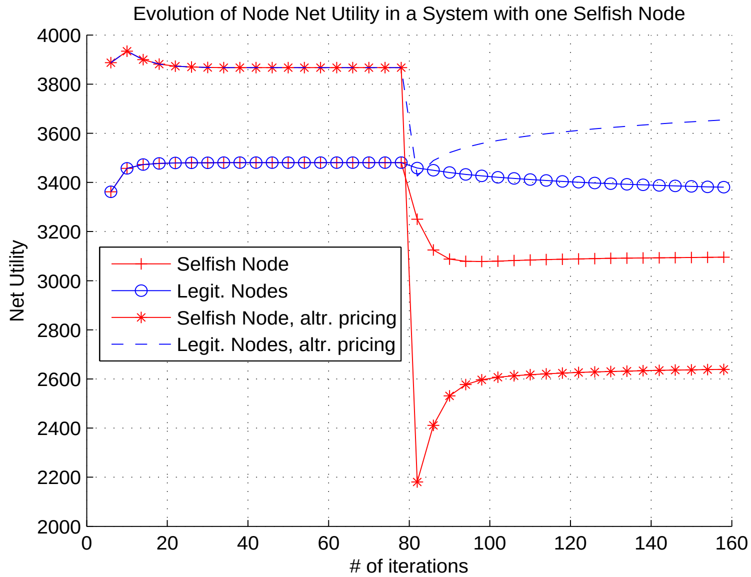



**Figure 5·6:** Utility of a legitimate and a selfish node for DP-NCP, and DP-NCP with alternative pricing.

## 5.6 Numerical Results

In order to validate our approach and the convergence of respective algorithms, we simulate the interactions of a small group of $N = 10$ nodes. Consider for example 10 SOs that exchange spectrum or bandwidth, i.e. routing services, or 10 users that create a group and route each other traffic. We focus on SWP and distributed algorithm DP-NCP. The simulation setup and conclusions apply for GNP as well. The basic attributes of each node $i$ are the available amount of resource, $C_i$ and its utility function $J_i(\cdot)$. For this setup, we choose $J_i(\mathbf{x}_i, \mathbf{y}_i) = \sum_{j=1}^{N}[\alpha_{ij} \log(x_{ij} + 1) - \beta_{ij} y_{ij}^2]$ which satisfies our assumptions for $J_i(\cdot)$. Parameters $\alpha_{ij}$ and $\beta_{ij}$ are used to capture different node profiles. Algorithm execution is assumed to be time slotted, and node interaction is synchronous.

In Figure 5·5 we depict the evolution of aggregate utility for a system which consists of 9 legitimate (i.e. cooperating) and one selfish node. All nodes have $C_i = C = 100$, and identical utility functions. The selfish node initially cooperates, but after $t = 80$, it decides to provide zero service to others. This results in a degradation of system performance, though this remains stable after initial reduction in overall utility. The evolution of utility of this selfish node is depicted in Figure 5·6. We depict results both for the DP-NCP algorithm under the assumption of a charging infrastructure and for the proposed alternative pricing method. In both cases, the response of legitimate nodes is immediate and leads to punishment of egotistic behavior by providing the selfish node with degraded service. This proves the desirable property of cooperation enforcement of our mechanism. In Figure 5·7, we depict the protocol tolerance for different numbers of selfish nodes. At time $t = 80$,

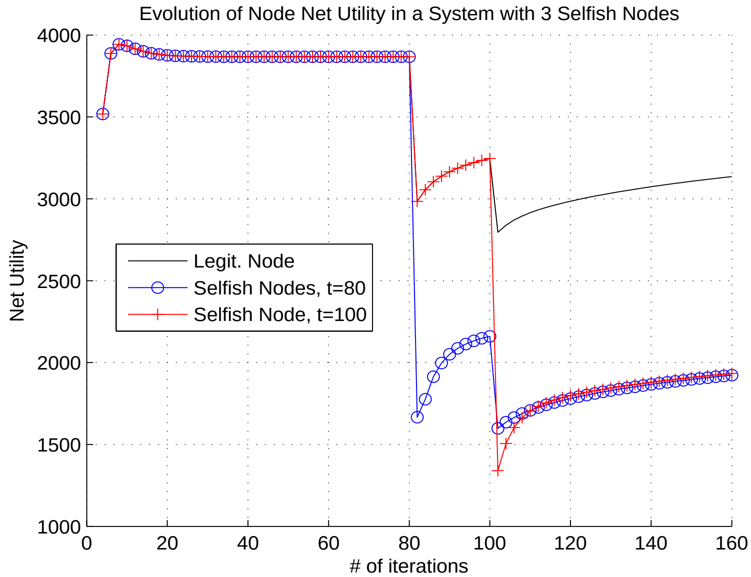



**Figure 5·7:** Node utility in a system with many selfish nodes taking effect at different times. Selfish nodes eventually get punished.

two nodes cease serving others, and at $t = 100$, one more does the same. We observe that selfish nodes are eventually punished by the system.

Finally, we demonstrate the benefit of our method in terms of dynamic adjustment of the server / client (seller / buyer) role of each node. In Figure 5·8 we compare the proposed dynamic scheme to an instance in which each node has statically allocated 30% of its resource to serving others and 70% to receiving service itself. The static allocation results in degraded utility for nodes (case 1). Furthermore, this degradation increases when the network setting is more complex and dynamic, as for example in the case of selfish nodes that stop serving others after $t = 80$ (case 2). These results hold independently of the static allocation to each role.

## 5.7 Conclusions

We introduced a framework for spectrum and bandwidth exchange in emerging wireless markets based on a novel dynamic pricing mechanism. The interacting entities (nodes) can be secondary operators who exchange spectrum or provide interconnection services to each other. They can also be users who route each others traffic in an ad hoc fashion. The novel attribute of the proposed mechanism is that it optimally captures the tradeoff of resource sharing between the resource provider and consumer roles of each node. We designed a decentralized realization that does not rely on existence of a central controller. We showed that, through proper dynamic remuneration and charging, rational price-taking nodes are

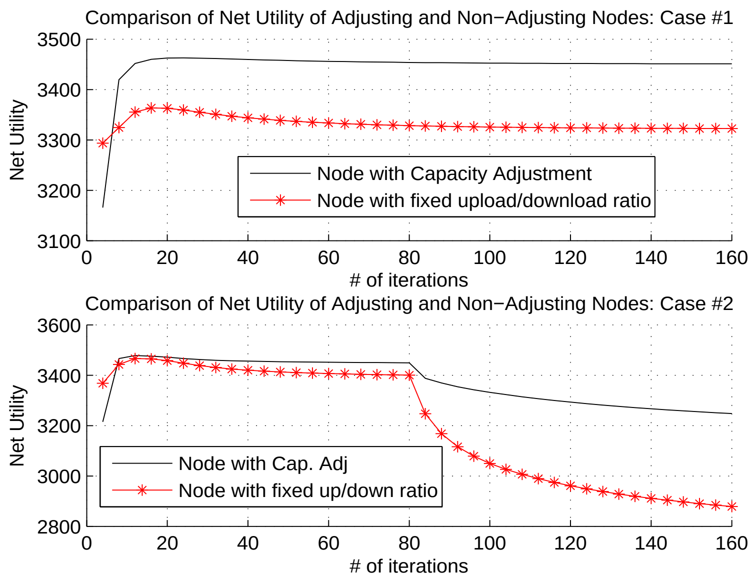



**Figure 5·8:** Improvement in performance due to dynamic resource allocation to the seller and buyer role, as opposed to static one.

induced to behave towards maximizing optimal social welfare. Furthermore, we extended the framework to one that does not rely on an accounting and charging infrastructure through design of an alternative scheme based on service reciprocation.

**Algorithm 2** (DP-NCP Algorithm)

**1st Step:** *Initialization.*
(**1.1:**) Set $t = 0$.
(**1.2:**) Set initial values $\lambda_i^{(0)} \geq 0$ and $\mathbf{R}^{(0)} = \alpha \mathbf{I}$, with $\mathbf{I}$ the unit matrix, $\alpha \in \mathbb{R}$.
(**1.3:**) Each node starts with arbitrary bids $p_{ij}^{(0)}, a_{ij}^{(0)}$, $j = 1, \dots, N$.
**2nd Step:** *Resource Allocation.*
(**2.1:**) At each stage $t > 0$, each node $i$ maximizes the Lagrangian $L(\cdot)$ w.r.t. its primal variables $x_{ij}$ and $y_{ij}$, for every $j$. This yields:

$$x_{ij}^{(t)} = \frac{p_{ij}^{(t-1)}}{\lambda_i^{(t-1)} + r_{ij}^{(t-1)}}, \;\; y_{ij}^{(t)} = \frac{r_{ji}^{(t-1)} - \lambda_i^{(t-1)}}{a_{ij}^{(t-1)}}\,. \tag{5.11}$$

.
**3rd Step:** *Charging and Reimbursement.*
(**3.1:**) Each node $i$ charges and reimburses others according to the scheme:

$$h(p_{ij}^{(t-1)}) = p_{ij}^{(t-1)} = x_{ij}^{(t)}[\lambda_i^{(t-1)} + r_{ij}^{(t-1)}], \tag{5.12}$$

$$l(a_{ij}^{(t-1)}) = \frac{[\lambda_i^{(t-1)} - r_{ji}^{(t-1)}]^2}{a_{ij}^{(t-1)}} = y_{ij}^{(t)}[r_{ji}^{(t-1)} - \lambda_i^{(t-1)}]. \tag{5.13}$$

.
**4th Step:** *Lagrange Multiplier Update.*
(**4.1:**) Each node $i$ minimizes $L(\cdot)$ w.r.t. $\lambda_i$ and $\{r_{ij}\}$, $j = 1, \dots, N$ using gradient update:

$$r_{ij}^{(t)} = r_{ij}^{(t-1)} - s_t(y_{ji}^{(t)} - x_{ij}^{(t)}), \forall j, \;\; i \neq j, \tag{5.14}$$

$$\lambda_i^{(t)} = [\lambda_i^{(t-1)} + s_t(\sum_{j=1}^{N} x_{ij}^{(t)} + \sum_{j=1}^{N} y_{ij}^{(t)} - C_i)]^+ \tag{5.15}$$

with $x^+ = x$ if $x > 0$, otherwise $x = 0$.
(**4.2:**) The new multipliers are communicated to the group.
**5th Step:** *Bid Update.*
(**5.1:**) Each node $i$ updates its buy and sell bids according to:

$$p_{ij}^{(t)} = x_{ij}^{(t)} U_{ij}'(x_{ij}^{(t)}), \;\; a_{ij}^{(t)} = \frac{-W_{ij}'(y_{ij}^{(t)})}{y_{ij}^{(t)}}\,. \tag{5.16}$$

.
**6th Step:** *Chek termination condition.*
(**6.1:**) Set $t \leftarrow t + 1$.
(**6.2:**)If there exist one or more primal or dual updated variables that do not coincide with their respective instances in the previous time slot, go to **Step 1**. Otherwise **exit**.

# Chapter 6

# Storage Capacity Control Policies for Time-Varying Networks

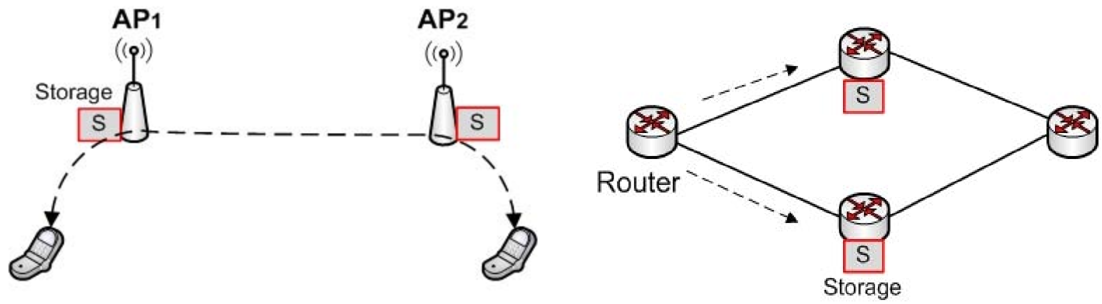
## 6.1 Background

In order to cope with the increasing user demand for ubiquitous and high speed network access, wireless operators are gradually upgrading their networks to $4G$ and acquire more spectrum licences. However, according to predictions of key players of the communications industry like Ericsson and Cisco, [89] and [83], these advances are expected to be compensated by the ever growing user traffic. Clearly, apart from these traditional methods of resource over-provisioning and technology upgrading, there is a need for novel network design and network management methods. Specifically, it is imperative operators to explore the possible benefits from using in-network storage.

Nowadays storage is cheap compared to bandwidth, [4], and with least space and power requirements. Hence, it can be used both in small portable devices and in large amounts located at central communication nodes of backbone networks. In light of these observations, it is challenging to study the performance benefits of storage in terms of the amount of data that can be conveyed to the destination and characterize the conditions under which these benefits are enlarged. Two examples of in-network storage assisted networking are depicted in Figure 6·1. Storage can be used in Access Points (APs) of wireless networks for temporarily storing data and transmitting it when the wireless link state is favorable. Also, large-scale storage can be used in backbone networks in order to exploit the time-diversity of link capacity variation and increase the end-to-end data transfer capability of the network.

### 6.1.1 Motivating Example

Let us first present a simple example to motivate the benefit of using in-network storage in networks with time-varying link capacity, i.e. dynamic networks. Consider the linear network of Figure 6·2 and assume time slotted operation with slot duration $T_0$. The link capacity, measured in packets/sec, change every other slot $t$ according to a periodic pattern, i.e. $C_{AB}(t) = D$, $C_{BC}(t) = 1$ and $C_{AB}(t+2) = 1$, $C_{BC}(t+2) = D$, $D > 1$. In between the 2 slots the link capacities remain constant. Transmission delay over each link is equal to the slot duration $T_0$. In this setting, we ask the question: How much time is required to convey an amount of $D$ packets of data from node $A$ to node $C$ if **(i)** the intermediate node $B$ has no storage capacity, (ii) node $B$ has storage capacity of $S_B > D$ packets? The




(a) Wireless network with storage in access points

(b) Backbone network with storage enhanced routers

**Figure 6·1:** Architectures of in-network storage in wired and wireless networks.

answer is straightforward, yet illuminating. In the first case, node $A$ can push in each time slot $t$ only as much data as node $B$ is able to forward in the immediately next time slot $(t+1)$, i.e. $C_{BC}(t+1)T_0$. Hence, for the link capacity variation above the required time for data transfer is $(D+1)$ slots. However, when $S_B > D$, node $A$ pushes up to $C_{AB}(t)T_0$ data and the excess amount that cannot be immediately routed to destination $C$, i.e. $(C_{AB}(t) - C_{BC}(t+1))T_0$ is stored at node $B$. In the subsequent slot $(t+2)$, when $C_{BC}$ is high, stored data along with the new arrived data from $A$ is delivered to the sink. Therefore, in this case the required time for the delivery of $D$ units of data is only 2 time slots.

Clearly, the use of storage reduces significantly the incurred end-to-end delay for the data transfer. From a different perspective, storage increases the maximum amount of data that the network can deliver within a certain time interval. Notice that, in order to achieve the same performance without storage use, we would have to increase the capacity of either one of the two links up to $(D-1)$ units. In other words, in this example node storage is actually used as a special type of link capacity and augments the end-to-end data transfer capability of the network.

However, it can be inferred from the previous observations that the benefit from storage utilization depends on relative link capacity values. Namely, link capacities should vary with time and, moreover, capacity variation patterns of different links should differ. Link capacity variations are common in contemporary information networks. For example, the capacity of wireless links often varies due to channel impairments. Additionally, in backbone networks, the available link capacity for an operator varies due to variations in the traffic of flows that correspond to other operators that traverse the same link. Such traffic variations are beyond the control of the operator.

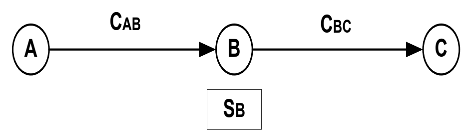



**Figure 6·2:** A tandem (linear) network of 3 nodes. Node $A$ is the source, node $C$ the sink and $S_B$ is the available storage capacity at node $B$.

### 6.1.2 Contribution

We consider dynamic networks and analyze the impact of in-network storage on the maximum amount of data that can be transferred from source to destination within a certain time interval. We use the technique of time-expanded graphs, [29] which map the time evolution of dynamic networks to ordinary graphs. In these graphs, storage is modeled by defining a special type of link connecting different time instances of the same node. We focus on the minimum cut (min-cut) of the expanded graphs. This represents the upper bound of the data amount that can be transferred in the dynamic network for the given time period. Under certain conditions, this bound can be increased by utilizing storage links. We are interested in identifying these conditions and devise the optimal storage control policy which determines the amount of data that must be stored at each node in every time slot, and the amount of data to be forwarded to the next nodes.

We continue with formulating the max flow problem on the time-expanded graph and derive the optimal policies in terms of routing and storage capacity control. Like in common graphs, max flow problem can be solved in polynomial time using the $\epsilon$-relaxation method which is amenable to distributed implementation. This last feature leads to *distributed* joint storage control and routing (JSR) max-flow policies. In summary, our contributions are as follows: **(i)** we identify the benefits of storage capacity and the conditions under which this benefit is realizable, **(ii)** we extend our study in general networks and provide a formulation based on expanded graphs for finding the storage enhanced min-cut and the optimal storage allocation policy, **(iii)** we propose the conjunction of storage control with routing and define the joint storage control and routing max-flow problem for a single commodity, and **(iv)** we provide a distributed method for its solution.

The rest of the chapter is organized as follows. In Section 6.2 we discuss related works, in Section 6.3 we analyze the performance of linear networks with storage capable nodes and in Section 6.4 we introduce the optimal storage control policy for general networks. In Section 6.5 we study joint storage control and routing policies that achieve flow maximization in dynamic networks and present an algorithm for their distributed implementation. In Section 6.6 we discuss the impact of the available information about current and future network state on performance of the introduced algorithms. Finally, in 6.7 we present numerical results that verify our analysis and in section 6.8 we conclude our study.

## 6.2 Related Work

Storage has been considered in wireless networks in the context of Delay Tolerant Networks (DTN) [46], in order to alleviate intermittent connectivity problems between the source and the destination. In these networks often there exists no permanent end-to-end path and therefore traditional routing algorithms fail. Hence, various Store and Forward (SnF) policies are employed to circumvent the challenge above. Data is stored at intermediate nodes and is transmitted whenever required links are available. Apparently, available information about current and future state of the network determines the performance of these SnF strategies, [51]. A particularly interesting work is [33] where the authors study flow optimization in DTNs. The problem is formulated as a utility maximization problem and is solved through Lagrange dual decomposition. However, in this class of problems the objective is to guarantee delivery of packets and storage is used as a strategy whenever routing is not possible. On the contrary, we utilize in-network storage in order to reduce data transfer delay.

In a similar context, the transfer of delay tolerant bulk data was introduced in [62] and [59]. The authors consider tandem (linear) wired networks where intermediate nodes have storage capability. The objective is to achieve the transmission of large amounts of data with minimum monetary cost under certain pricing schemes. The method is extended in [18] for general network graphs with time varying but a priori known capacity variation pattern. Node storage varies with time in terms of capacity and cost. It is explained that through the combination of time-expanded graphs and flow optimization techniques, a centralized solution provides the optimal (minimum-cost) transfer of data. In [61] this methodology is employed for the design of *NetStitcher*, a centralized system that exploits unutilized bandwidth of backbone networks in order to achieve low cost transfer of large data files among different sites of datacenters.

In more abstract modeling terms, the problem of minimum delay routing in networks with node storage is considered in [80] and [81]. The work [80] studies a single commodity network with time-varying capacities and presents a centralized algorithm that yields the *earliest arrival* flow for a time period of $T$ slots. This flow maximizes the amount of data that reaches the sink for every $\tau$, $\tau \in (0, T)$ and hence can be viewed as a minimum delay flow. It is assumed that the capacity variation patterns of the network links are a priori known. This information is used to construct the time-expanded graph. The requirement for future knowledge is relaxed in [81] where the data transfer delay for each link of the network is a random variable. The network is described by a set of stochastic processes, one for each link, with known state space and empirically calculated state transition probabilities. The objective is to find the shortest path for the delivery of a packet to the destination. It is an online problem which, in general, the authors prove that is intractable.

Recently, in-network storage utilization has gained renewed interest. In [34], a novel wireless network architecture of distributed caching is used to satisfy delay-sensitive user requests while consuming low backhaul bandwidth. Additionally, storage is proved to play a central role in the novel paradigm of Content Centric Networking (CCN), [44]. In CCN, data caching is of paramount importance and therefore in-network storage affects the content availability and the network performance in terms of content retrieval delay, [16]. Finally, other recent network architectures like the publish/subscribe system for Internet-scale content distribution [22] make use of in-network storage. All these different classes of application scenarios reveal the important role of node storage and further motivate the analytical study of storage-assisted networking.

Finally, other works have also studied flow algorithms in dynamic networks with storage-capable nodes, [3] and references therein. Among them [28] presents an interesting negative result according to which node storage does not improve minimum cost flows over time. However, this result refers to networks with constant link capacities over time. The above works do not explicitly study the impact of node storage, i.e. in-network storage, on the network performance. On the contrary, in this chapter we identify the conditions that render storage use beneficial, in terms of delay reduction, and at the same time propose methods for achieving this improvement. We focus on flow-level storage control policies where storage capacity is orders of magnitute larger than caches or buffers. Until now, storage has been considered mainly in networks where delay was not the main performance criterion, i.e. delay tolerant networks. Instead, here we analyze how storage can reduce incurred end-to-end delay. The main idea is to store data when network conditions are not favorable and transmit it latter. This way, we exploit the diversity in the link capacity variation patterns of a network and increase its data transfer capability. Storage is considered an additional resource that should be managed efficiently in conjunction with link capacity.

## 6.3 Impact of Storage Capacity in Linear Networks

We start from linear networks where routing is fixed, from a node to its next one towards the destination. In this case, storage benefit in terms of delay improvement can be directly calculated for arbitrary link capacity variation patterns. For example, consider again the three-node network of Figure 6·2, where link capacity changes every two time slots. We pose the following questions: (i) what is the incurred delay to deliver an amount of $D$ data units (packets) to the destination? and (ii) how much data can we deliver from node $A$ to node $C$ in time period of $T$ time slots, if node $B$ has storage capacity?

When node $B$ has zero storage capacity, the network end-to-end capacity, i.e. the rate at which data can be transferred end-to-end at every time slot $t$, due to the flow conservation constraint is:

$$C_{AC}(t) = \min\{C_{AB}(t), C_{BC}(t+1)\} \tag{6.1}$$

We denote with $\mathbf{C}_{AB} = (C_{AB}(t) : t = 1, 2, \ldots, T)$ and $\mathbf{C}_{BC} = (C_{BC}(t) : t = 1, 2, \ldots, T)$ the capacity variation vectors of links $(A, B)$ and $(B, C)$ respectively. Hence, the required time for the transfer of $D$ units of data from node $A$ to node $C$ is $MT_0$ seconds, where $M$ is the minimum integer for which holds:

$$\sum_{t=1}^{M} C_{AC}(t)T_0 \geq D \tag{6.2}$$

Also, the amount of data transferred in $T$ time slots is:

$$D = \sum_{t=1}^{T-2} C_{AC}(t) T_0 \tag{6.3}$$

Notice that, since we assumed that transfer delay is equal to the slot duration $T_0$, it takes two slots for each packet to reach node $C$. Hence, data should be send from the source node $A$ before slot $(T-2)$ in order to reach the destination within the $T$ slots.

When node $B$ has storage capacity of $S_B$ data packets, the network end-to-end capacity, denoted $C_{AC}^{S_B}$, changes and can be significantly increased under certain conditions. Specifically, $C_{AC}^{S_B}(t)$ is the minimum of the data $X_B(t+1)$ that is available on node $B$ at $t$, and the capacity of the last hop link $C_{BC}(t+1)$:

$$C_{AC}^{S_B}(t) = \min\{X_B(t+1), C_{BC}(t+1)\} \tag{6.4}$$

Let us explain the intuition behind this expression. On the one hand, the maximum possible amount of data that can be delivered at node $C$ at slot $(t+2)$ is upper bounded by the capacity of link $(B,C)$ at $(t+1)$. On the other hand, if $C_{BC}(t+1)$ is very large, the data that can be delivered is upper bounded by the data that is available at node $B$ at the previous slot $(t+1)$. The latter consists of the data that has been transmitted from node $A$ in the exactly previous slot, i.e. the instantaneous capacity $C_{AB}(t)$, and the data that has been accumulated until slot $t$ in node $B$, $Y_B(t)$:

$$X_B(t+1) = C_{AB}(t) + \frac{Y_B(t+1)}{T_0} \tag{6.5}$$

where $Y_B(t)$ is always nonnegative and upper bounded by node $B$ maximum storage $S_B > 0$:

$$Y_B(t+1) = \min\{S_B, \max\{Y_B(t) + [C_{AB}(t) - C_{BC}(t+1)]T_0,\, 0\}\} \tag{6.6}$$

is and it is assumed that $Y_B(0) = 0$. Therefore in this case the end-to-end capacity of the three-nodes network is $C_{AC}^{S_B}(t) \geq C_{AC}(t)$ for every time slot $t$.

This means that the use of storage at intermediate node $B$ significantly improves both the delay and the end-to-end data transfer rate. The exact improvement that comes with storage at node $B$ depends on the relative variation pattern of link capacities $\mathbf{C}_{AB}$ and $\mathbf{C}_{BC}$. The more diverse the capacity value sequences are, the larger is the benefit from the storage usage. For instance, the benefit from $S_B$ is enlarged when the capacity variation patterns of the two successive links is such that a high capacity at one slot at link $(A,B)$ is followed by a high capacity at link $(B,C)$ at the next slot. Obviously, there exist various scenarios for which in-network storage use is beneficial and all of them are related to the link capacity variation patterns. In order to quantify the variability in capacity patterns of links $(A,B)$ and $(B,C)$ for time period $T$ we define a metric that we name *Dissimilarity Index* $L(\mathbf{C}_{AB}, \mathbf{C}_{BC})$. This metric, which is a function of the capacity variation patterns, measures the aggregate amount of data that can be temporarily stored in node $B$ and delivered subsequently in node $C$ within the time period $T$, assuming that there is no

**Table 6.1:** Example for 3-node Network, $T_0 = 1$, $L = 16$, $S_B = 30$

| Slot $t$ | $C_{AB}$ $(p/sec)$ | $C_{BC}$ $(p/sec)$ | $C_{AC}$ $(p/sec)$ | $D$ $(p)$ | $Y_B$ $(p)$ | $C^S_{AC}$ $(p/s)$ | $D^S$ $(p)$ |
|---|---|---|---|---|---|---|---|
| 1 | 10 | 6 | 2 | 0 | 0 | 2 | 0 |
| 2 | 12 | 2 | 0 | 2 | 8 | 0 | 2 |
| 3 | 14 | 0 | 10 | 2 | 20 | 10 | 2 |
| 4 | 2 | 10 | 2 | 12 | 24 | 12 | 12 |
| 5 | 2 | 12 | 2 | 14 | 14 | 8 | 24 |
| 6 | 4 | 10 | 4 | 16 | 6 | 4 | 32 |
| 7 | 6 | 14 | - | **20** | 0 | 6 | **36** |

storage space constraint.

We first define the amount of data $u(t)$ that is delivered from node $A$ to node $B$, in each time slot $t$, but cannot be immediately forwarded to node $C$ due to the low capacity of link $(B, C)$:

$$u(t) = \max\{[C_{AB}(t) - C_{BC}(t+1)]T_0, 0\}$$

Similarly, we denote with $h(t)$ the additional amount of data that can be forwarded to node $C$, along with the data that arrive in $B$ in the current slot:

$$h(t) = \max\{[C_{BC}(t+1) - C_{AB}(t)]T_0, 0\}$$

This is actually the amount of data that, if it was previously stored in node $B$, in the current slot $t$ can be restored and forwarded to the destination node. We can now define the dissimilarity index:

$$L(\mathbf{C}_{AB}, \mathbf{C}_{BC}) = \sum_{t=1}^{T} \min\left\{h(t), \max\{\sum_{n=1}^{t-1}[u(n) - h(n+1)], 0\}\right\} \tag{6.7}$$

This parameter actually leads to conditions under which storage is beneficial. When $L = 0$, storage does not increase the end-to-end data transfer rate. The dissimilarity index has zero value when the capacities of the 2 links have equal values in successive slots, $C_{AB}(t) = C_{BC}(t+1)$ or if link $(A, B)$ has always lower capacity than link $(B, C)$, i.e. $C_{AB}(t) < C_{BC}(t+1)$, $\forall t$. In this case there is no need to utilize intermediate storage at node $B$ since all data that is transmitted from node $A$ can be immediately, i.e. after 2 slots, delivered to the destination node $C$. Moreover, even if the link capacity variation patterns are such that data is accumulated in node $B$, it may be impossible to push it further if link $(B, C)$ is always the network bottleneck, i.e. $C_{BC}(t+1) < C_{AB}(t)$, $\forall t$. In Table $I$ we present an example for this network and demonstrate the benefit of storage at node $B$. Capacities are measured in packets/sec $(p/s)$ while data and storage in packets $(p)$. Packets are assumed to be of equal length. Similar results hold for linear networks with more than three nodes.

From the analysis above, we infer that it is possible to exploit the diverse evolution of

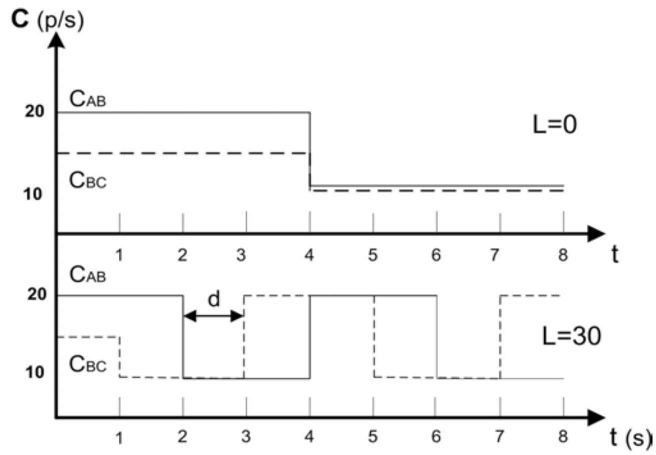



**Figure 6·3:** Dissimilarity Index, $L$, for two different capacity variation patterns. The bottom plot shows the case where links are as diverse as possible in terms of their time variation, reaching upper and lower bounds in an antisymmetric fashion. Parameter $d$ is the transmission delay, i.e. the time a data packet needs to traverse a link connecting two successive nodes.

link capacities and use intermediate node storage to augment the amount of data transferred in a certain time horizon. Notice that in linear networks storage policy is straightforward. Each node accepts all incoming traffic and stores the excess data that it cannot forward in the current time slot so as to exploit possible capacity increase in the subsequent slots. The more the storage capacity is at intermediate nodes, the more we benefit from its use. However, determining the storage control policy in general graphs is a non-trivial task and requires the knowledge of the capacity evolution for all network links. In next section we consider general networks with known capacity variation patterns and provide a method for deriving the optimal storage control policy.

## 6.4 Storage Capacity Allocation for General Networks

In certain cases it is possible to know in advance or predict with good precision the future values of link capacities. For example, small networks with predictable capacity evolution such as networks of satellites [80] fall within this class. Another scenario is networks with constant link capacities but periodic time varying traffic patterns where we attempt to exploit residual capacity [62], [61]. We provide a method for devising the optimal storage allocation policy, i.e. the policy that leads to maximum possible network performance improvement from storage. Recall that the performance of a network, in terms of data transfer capability, is upper bounded by the capacity $C(Q_{min})$ of the minimum cut $Q_{min}$ of its graph, [29]. This represents the maximum flow that can be delivered from the source to the destination node. Hence, for a time period of $T$ units, the maximum amount of delivered data is $D = C(Q_{min})T$. Obviously, by increasing the capacity of the minimum cut, we increase the maximum amount of data that can be transferred to the destination.

Consider a directed network graph $G = (V, E)$ with a set $V$ of $N = |V|$ nodes and a set $E$ of links. The network is dynamic, i.e. every link capacity $C_{kl}(t)$, $(k,l) \in E$, changes with time according to a predefined pattern. We assume a time slotted operation, $t = 1, 2, \ldots, T$, with slot duration $T_0$. Link capacities remain constant within each time slot. First, we assume no storage capacity at all nodes. We assume that the traversal time is identical for all links and equal to the slot duration. In order to study network $G$ we employ the technique of time-expanded graphs, [29]. Time-expanded graphs are used to map a dynamic network $G$ for a given time period of $T$ slots, to an equivalent static network $G_T$. The transformation is accomplished as follows. For every node $k \in V$ of the original network $G$ we create $T$ nodes in the $G_T$, $k^{(t)}$, $t = 1, \ldots, T$. Moreover, for every arc $(k,l) \in E$ of $G$, we add a set of corresponding arcs $(k^{(t)}, l^{(t+1)})$, $t = 1, \ldots, T-1$. Network $G_T = (V_T, E_T)$ contains $N_T = |V_T|$ nodes. Finally, we substitute the time instances of the source and the destination nodes with 2 supernodes for ease of presentation. Now, we are able to study the properties and the performance of $G$ for $T$ slots, by applying to $G_T$ the well known methods and theorems that have been derived for static networks.

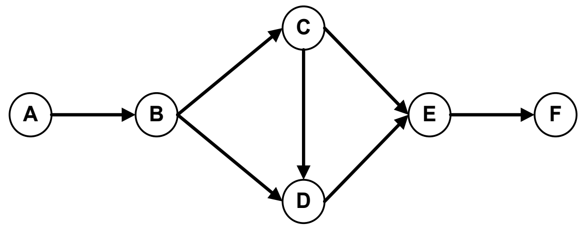
The graph $G_T$ incorporates the notion of time and is used in order to analyze the properties of $G$ for the time period $T$. Namely, the amount of data that can be transferred in network $G$ within horizon $T$, is upper bounded by the minimum cut of $G_T$. This is stated in the following lemma:

**Lemma 6.4.1.** *The maximum amount of data transferred from source to destination of a network $G$ within a time horizon $T$ is equal to the max-flow at an appropriately expanded graph $G_T$.*

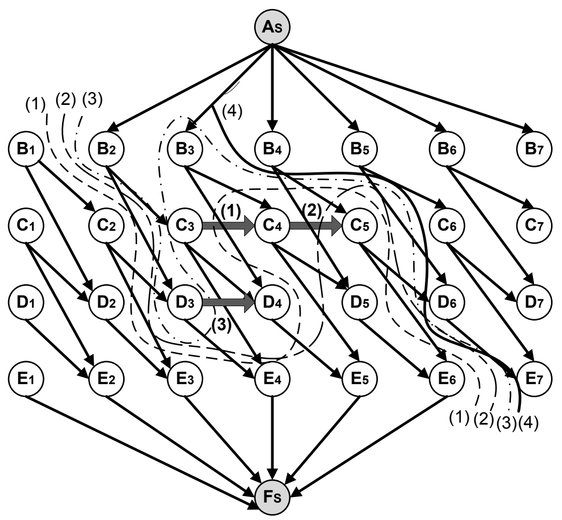
**Proof:** The proof follows directly from the definition of the time-expanded graph. ■

Here, we propose the increase of the min-cut capacity of $G_T$ by the addition of specific links, the *storage links*. The rationale of the method is visualized in Figure 6·4. Assume that $Q_T = [W_T, V_T \setminus W_T]$ is the initial min-cut of $G_T$ with capacity $C(Q_T)$, where $W_T$ is the set of nodes in which the source node belongs and $(V_T \setminus W_T)$ is the set containing the sink node. The critical observation is the following. If there exists a node $k \in G$ such that $k^{(t)} \in W_T$ and $k^{(t+1)} \in (V_T \setminus W_T)$ then we can increase the network capacity by connecting $k^{(t)}$ and $k^{(t+1)}$ with a virtual link of capacity $S_k(t)$. This link represents the capability of node $k$ for storing data during time slot $t$ up to an amount of $S_k(t)$ packets. With the addition of this virtual link the capacity of the network is increased up to $(C(Q_T) + S_k(t))$ units. Adding enough storage to node $k$ at time $t$, renders $Q_T$ a non-minimum cut. If the new min-cut $Q_T^{'}$ contains a node for which the above condition also holds, then we add again storage so as to make $Q_T^{'}$ a non minimum cut.

Specifically, Algorithm 2, describes the proposed methodology for Storage capacity allocation, (SCA), which is applied to graph $G_T$ and creates a new, storage-enhanced graph $G_T^s$. In each step, the min-cut can be found either through a flow-based technique, i.e. based on max-flow algorithms [7], or by using non-flow techniques [99],[52]. **Step 5** is required in order to ensure that there is no excessive storage usage. The maximum number of added storage links is bounded by the number of nodes. Additionally, the amount of




(a) Original graph.




(b) The Storage enhanced time-expanded graph.

**Figure 6·4:** A graph with time varying links and initial min-cut (max flow) of $C(Q_T) = 34$ packets ($p$). Link capacities for $T = 7$ slots: $AB(18, 16, 18, 20, 16, 18)$, $BC(4, 10, 10, 4, 6, 4)$, $BD(6, 16, 16, 4, 4, 6)$, $CD(6, 10, 12, 2, 12, 8)$, $CE(6, 8, 2, 10, 12, 8)$, $DE(4, 6, 2, 12, 12, 8)$, $EF(10, 12, 14, 10, 20, 22)$. In step (1) we add the storage link $S_C(3) = 8$, in step (2) we add link $S_C(4) = 4$ and in step (3) the link $S_D(4) = 6$. The final capacity is $C(Q_f) = 42$ data packets.

**Algorithm 3** (Storage Capacity Allocation -SCA)

**Input:** The time expanded graph $G_T = (V_T, E_T)$ where initially nodes do not have storage capacity.
**Output:** The storage-enhanced time expanded graph $G_T^s = (V_T, E_T^s)$ where the set of links $E_T^s \supseteq E_T$ contains the initial links of $E_T$ and the added virtual storage links.
**Step 0:** Find the current min-cut $Q_T = [W_T, V_T \setminus W_T]$ and calculate its capacity $C(Q_T)$. Go to Step 1
**Step 1:** If there exists a node $k$ of $G$ for which $k^{(t)} \in W_T$ and $k^{(t+1)} \in (V_T \setminus W_T)$, then add a link of minimum capacity $S_k(t) = S_0 > 0$ (*storage link*) connecting these two nodes and go to Step 2. If there exist more than one nodes satisfying this condition, select one of them randomly. If there is not such node, go to Step 4.
**Step 2:** Increase the new storage link capacity $S_k(t)$ as much as required so as to render $Q_T$ a non minimum cut. Go to Step 3.
**Step 3:** Find the new min-cut $Q_T' = [W_T', V_T \setminus W_T']$. Set $Q_T = Q_T'$ and go to Step 1.
**Step 4:** Set $Q_f = Q_T$. This is the final min-cut of the storage enhanced network $G_T^s$. Go to Step 5.
**Step 5:** Find all storage links that do not belong to the final min-cut $Q_f$ and decrease their capacity as long as $Q_f$ remains unchanged. The algorithm terminates.

storage is confined by the capacity of links. Storage addition will eventually result in a minimum cut consisting only of communication links. After this point, using more storage cannot further improve the min-cut of the network.

The SCA algorithm guarantees the maximum possible benefit from node storage use, in terms of the amount of end-to-end transferred data. The capacity of the new cut $C(Q_f)$ is larger than the capacity of the initial cut $C(Q_T)$ which means that the storage-enhanced network $G_T^s$ can transfer larger amount of data within time period $T$. However, SCA algorithm does not provide a method for achieving this bound. The main reason for introducing this algorithm is to give the intuition behind the idea of adding in-network storage. Also, through SCA it can be explained why and when storage capacity addition does not increase the amount of transferred end-to-end data after a certain point. In order to exploit the potential of storage we need to consider it in conjunction with routing. In the next section we provide a method for deriving the joint storage control and routing policy for a single data commodity that needs to be transferred from source to destination.

## 6.5 Joint Storage Control and Routing Optimization

Consider again the network $G = (V, E)$. The storage allocation policy that maximizes the amount of data that can be transferred from source to destination within the time interval $T$, can be derived by the methodology presented above (SCA algorithm). This algorithm takes as input the initial time-expanded graph $G_T$ and gives the storage-enhanced expanded graph, $G_T^s$. In this section we define and solve the joint storage capacity control and routing (JSR) optimization problem as a maximum flow problem on the time-expanded graph $G_T^s = (V_T, E_T^s)$. The solution of this problem yields the store and routing decisions that maximize the amount of data transferred within time $T$, by exploiting in-network storage.

We need to emphasize here that the solution of this problem does not presume the execution of Algorithm 1 so as to find $G_T^s$. Instead, we can construct and use a network graph that stems from $G_T$ by adding at each node $i \in V_T$ storage capacity that is bounded by an upper limit $S_i^{max}$. The amount of node storage that is actually required in order to maximize the amount of end-to-end transferred data will be found by the solution of the max-flow problem that we present in the sequel. In other words, there exist two cases for which we can define and solve the JSR problem: **(i)** when the storage capacity of nodes is given and we want to derive the max-flow joint storage and routing control policy, and **(ii)** when initially the nodes do not have storage capacity and we want to find the optimal storage allocation and the joint max-flow storage and routing control policy. In both cases, we denote with $G_T^s$ the graph that we use to define and solve the JSR problem.

In detail, we define for each node $k \in G$ the storage control vector $S_k = (S_k(t) : t = 1, 2, \ldots, T)$, where $S_k(t)$ is the amount of data that is stored at node $k$ at time $t$. Also, we define for every link $(k, l) \in E$, the routing control vector as $R_{kl} = (R_{kl}(t) : t = 1, 2, \ldots, T)$ where $R_{kl}(t)$ is the amount of data that is sent over link $(k, l)$ at time $t$. Finally, we define the network storage control policy $\mathbf{S} = (S_k : k \in V)$ and the network routing control policy as $\mathbf{R} = (R_{kl} : (k, l) \in E)$. Our goal is to find the optimal joint storage and routing control (JSR) policies, $\mathbf{R}^*$, $\mathbf{S}^*$, that maximize the data transferred within time $T$. Specifically, we define the following problem:

**Definition 1.** ***Joint Storage Control and Routing Max-Flow Problem:*** *Given a dynamic network $G = (V, E)$ with a single source and a single destination, and with nodes with certain storage capacity, find how much data should be stored in each node ($\mathbf{S}^*$) and how much data should be routed over each link ($\mathbf{R}^*$), in every time slot, in order to maximize the amount of transferred data within a certain time period of $T$ slots.*

Similar approaches have been also proposed in [80], and recently in [18]. However, in this chapter we focus on a solution method that is highly suitable for distrituted implementation.

The JSR problem is a constrained optimization problem and can be solved by dual ascend methods. These methods solve the respective dual problem by iteratively updating the dual variables so as to improve the dual objective function. There are two classes of such algorithms: the primal-dual method and the relaxation method. These methods use different ascent directions but admit fairly similar implementation. The primal-dual ascent method tries at each iteration to use the steepest ascent direction and can be implemented by means of a shortest path computation. The relaxation method is usually faster in practice. It tries to use directions that are not necessarily steepest, but can be computed more quickly than the steepest ascend direction. It is also called a coordinate ascent method, since in each iteration only one price is updated.

Notice now that the objective of the JSR problem is a linear function and hence the respective dual problem has a non-differentiable objective function. This means that typical relaxation methods (or similar primal-dual methods) may not converge to the optimal

| Variables and Notation | |
|---|---|
| $k, l$ | Nodes of the original network graph $G=(V,E)$, $k,\, l \in V$ |
| $i, j$ | Nodes of $G_T=(V_T,E_T)$, $i,\, j \in V_T$ |
| $(i,j)$ | Link of $G_T=(V_T,E_T)$, $(i,j)\in E_T$, with $i \triangleq k^{(t)}$, $j \triangleq l^{(t+1)}$, $(k,l)\in E$ |
| $x_{ij}$ | Bytes sent over link $(i,j)\in E_T$ |
| $y_{in}$ | Bytes stored at node $k \in V$ in slot $t$, where $i \triangleq k^{(t)}$, $n \triangleq k^{(t+1)}$ |
| $x_{ds}$ | Data transferred from source to destination node over the time horizon $T$ |
| $F_i$ | Forward *communication* (child) nodes of node $i \in V_T$ |
| $B_i$ | Backward *communication* (parent) nodes of node $i \in V_T$ |
| $D$ | Amount of data to be transferred end-to-end |

problem solution. Therefore, we use the $\epsilon$-relaxation method which has improved convergence properties and is amenable to distributed implementation [6], [7]. The underlying idea of this method is that dual variables updates are allowed even if these worsen the dual cost function. The produced pairs of primal-dual variables satisfy the $\epsilon$-Complementary slackness (CS) which is a perturbed version of the traditional CS conditions.

### 6.5.1 Joint Storage Control and Routing Problem Formulation

First we add to $G_T^s$ an artificial link $(d,s)$ connecting the sink $d$ with the source $s$. Each node $i \in V_T$ that represents a node $k$ of graph $G$ for a certain time slot $t$, $i \triangleq k^{(t)}$, is connected with a node $m \in V_T$ that represents the previous instance of the same node, i.e. $m \triangleq k^{(t-1)}$. Similarly, each node $i \in V_T$ is connected with a node $n \in V_T$ that represents the subsequent instance of the same node, i.e. $n \triangleq k^{(t+1)}$. The capacity of these links models the available storage at node $i$ which is bounded. For each node $i \in V_T$ we define the set of forward (child) communication nodes $F_i = \{j : (i,j) \in E_T\} \setminus \{n\}$, and the set of backward communication nodes $B_i = \{j : (j,i) \in E_T\} \setminus \{m\}$. Therefore, there exist two classes of links in the expanded graph: (i) the *communication links* that connect two different nodes $i$ and $j$ at a specific time slot $t$ with capacity $\mathbf{C} = \{C_{ij} : i \in V_T, j \in F_i\}$, and (ii) the *storage links* that connect different time instances of the same node with (maximum) storage capacity $\mathbf{S}^{max} = \{S_i^{max} : i \in V_T,\, i \triangleq k^{(t)},\, k \in V\}$. Notice that the notion of time is incorporated in the time-expanded graph. Hence, the capacity of communication and storage links is measured in data packets and not in data packets per slot.

Let us define the vector $\mathbf{x} = \{x_{ij} : i \in V_T, j \in F_i\}$ where $x_{ij}$ denotes the amount of data that is sent over link $(i,j)$ with $i \triangleq k^{(t)}$ and $j \triangleq l^{(t+1)}$, $(k,l) \in E$. Similarly, we define the vector of storage variables $\mathbf{y} = \{y_{in} : i \in V_T,\, i \triangleq k^{(t)}, n \triangleq k^{(t+1)}\}$ where $y_{in}$ denotes the amount of data that is stored at node $k \in V$, in time slot $t$. The optimal storage-routing policy $(\mathbf{x}^*, \mathbf{y}^*)$ for the time period $T$ is derived from the solution of the max flow problem defined over the corresponding time-expanded graph, JSR Max-Flow Problem:

$$\min(-x_{ds}) \tag{6.8}$$

subject to

$$\sum_{j\in F_i} x_{ij} + y_{in} = \sum_{j\in B_i} x_{ji} + y_{mi},\ i \in V_T \setminus \{s,d\} \tag{6.9}$$

$$0 \le x_{ij} \le C_{ij},\ i \in V_T, j \in F_i \tag{6.10}$$

$$0 \le y_{in} \le S_i^{max},\ i,n \in V_T \tag{6.11}$$

Where it is $y_{in}$, $x_{ij}\ \ge 0$. Equation (6.9) is the data conservation constrain, in analogy with the flow conservation constraint, and $x_{ds}$ is the amount of data that is transferred from source to destination node. The solution of the JSR problem determines the optimal routing $\mathbf{x}^*$, and storage decisions $\mathbf{y}^*$ that maximize the amount of transferred data during the time interval $T$.

Notice that $\mathbf{x}^* = \{x_{ij}^* : i \in V_T, j \in F_i\}$ and $\mathbf{y}^* = \{y_{in}^* : i \in V_T,\ i \triangleq k^{(t)}, n \triangleq k^{(t+1)}\}$ are defined for the time-expanded network $G_T$. Obviously, these variables can be easily mapped to the respective routing and storage variables $R_{kl}^*(t)$ and $S_k^*(t)$ of the dynamic network $G$ and yield the respective optimal control policy $\mathbf{R}^*$ and $\mathbf{S}^*$. Finally, it is interesting to notice that this formulation can be used to find the incurred delay for the transfer of a certain amount of data $D$. Actually this is the minimum time $T^*$ for which the solution of the respective JSR problem satisfies $x_{ds}^* \ge D$. We can find $T^*$ by using a binary or another search method.

### 6.5.2 Distributed Algorithm for the JSR Problem

We are interested to solve the JSR problem in a distributed fashion. First, we define the Lagrangian by relaxing the constraint (6.9) and introduce the vector of dual variables $\mathbf{p} = \{p_i : i \in V_T\}$:

$$L(\mathbf{x},\mathbf{y},\mathbf{p}) = -x_{ds} + \sum_{i\in V_T}\sum_{j\in F_i}(p_j - p_i)x_{ij} + \sum_{i\in V_T}(p_n - p_i)y_{in} \tag{6.12}$$

The dual problem is

$$\max_p q(\mathbf{p}) \tag{6.13}$$

where

$$q(\mathbf{p}) = \min_{[x_{ij}\le C_{ij}, y_{in}\le S_{in}]} L(\mathbf{x},\mathbf{y},\mathbf{p}) \tag{6.14}$$

The objective function of the primal problem is linear and therefore the dual function is non-differentiable. The $\epsilon$-relaxation method ensures convergence to the optimal solution if $\epsilon < \frac{1}{NT}$, in polynomial time $O(N^3T^3)$. We omit the detailed description of the algorithm and refer the reader to [6, Chap.5.3].

We cast the $\epsilon$-relaxation algorithm presented in [6] to fit our problem. In a distributed setting the variables are circulated among nodes and therefore they need to be time-stamped. Notice that these time stamps refer to the algorithm execution time and they should not be confused with actual time $t$ that represents the slots, and which we denote below by $t_G$. The basic idea as in standard dual ascents methods is to exploit the separa-

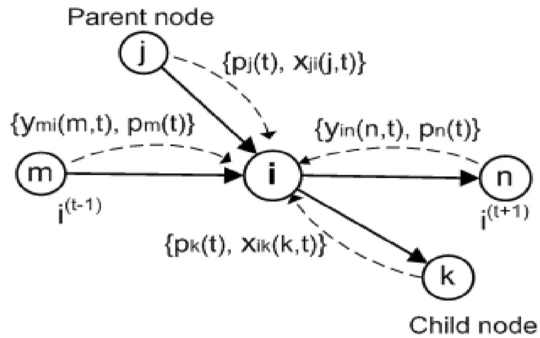



**Figure 6·5:** Distributed execution of the $\epsilon$-relaxation algorithm. Each node $i$ receives coordination messages from its neighbors, $\{x_{ji}(j,t), p_j(t)\}$ and $\{x_{ki}(k,t), p_k(t)\}$; and updates its storage decisions by considering its forward $n$ and backward $m$ instances.

bility property of the dual problem and group the decision variables per node. Specifically, every node $i \in V_T$ maintains the following variables:

- $p_i(t)$: dual variable of node $i$ at time $t$
- $p_j(i,t)$: dual variable of node $j$, which is a neighbor of node $i$, i.e. $j \in F_i \cup B_i \cup \{n\} \cup \{m\}$, communicated to node $i$ at time $t$. This is the local copy of the dual variable of node $j$, stored in node $i$.
- $x_{ij}(i,t)$: amount of data that node $i$ forwards to node $j$, $j \in F_i$ at time $t$.
- $x_{ji}(i,t)$: amount of data that node $i$ decides to admit from node $j \in B_i$ at time $t$.
- $y_{in}(i,t)$, $y_{mi}(i,t)$ : amount of data that node $i$ stores during the time slots $(t_G - 1)$ and $t_G$, where $i \triangleq k^{t_G}$, $m \triangleq k^{(t_G-1)}$ and $n \triangleq k^{(t_G+1)}$.
- $g_i(t)$: data and storage surplus at node $i$, i.e. $g_i(t) = \sum_{j \in B_i} x_{ji}(i,t) - \sum_{j \in F_i} x_{ij}(i,t) + y_{mi}(i,t) + y_{in}(i,t)$

The nodes circulate messages with their variables in order to achieve coordination and collectively solve the JSR Max Flow problem. Notice that adjacent nodes (neighbors) calculate the same variables and therefore it is required to reach consensus. For example, the final value of the data that node $i$ pushes to node $j$ should be equal to the data that node $j$ decides to admits, i.e. $x^*_{ij} = x^*_{ji}$. In detail, the distributed algorithm for the solution of the JSR Max Flow problem is presented in Algorithm 4.

This algorithm solves the JSR problem even when there does not exist a central network controller with global knowledge. Instead it is only required every node to be aware of its own link capacity variation patterns. This scenario is very important since it models a large set of networking examples. However, future knowledge about the network state is still a prerequisite. In the following section we explain why it is important to know the patterns of the link capacities and we discuss some cases where it is possible to derive suboptimal solutions even when there is lack of information.

## 6.6 Instances of Limited Knowledge About Link State

When there is no information about the future state of the network, a subset of the constraint set of the JSR problem is not known nor can it be determined through message passing. In this case, the joint storage control and routing policy becomes an online problem where nodes must decide using only the currently available information. In general, these problems are solved through dynamic programming techniques and optimal solutions are difficult to characterize and derive, [56]. Letting storage aside, distributed dynamic routing has been studied both for wireless [102], and wire-line networks, [5]. The underlying idea is the same in both algorithms. Namely, each node independently takes routing decisions so as to balance network load by forwarding its data packets to its neighbors with the less backlog, i.e. the smaller queues. If all of its neighbors are congested, the node refrains from sending its packets and the detected congestion is gradually signaled back to the source which temporary pauses data transmission. This scheme constitutes a proactive end-to-end flow control mechanism which may degrade the data delivery capability of the network. Therefore, these algorithms are not delay-aware and there is much ongoing research aiming at their improvement [110], [13].

In this context, we can consider storage utilization as a method for modifying the above congestion detection mechanism in order to reduce data transfer delay. To make this clear, consider a congested node which is aware that its outgoing link capacity will significantly increase in the near future. In this case, this node would be able to decide not to signal back to its parent nodes the congestion so as to keep receiving data from them. The excess data would be stored in the storage area of the node and returned to the queues when the backlog is reduced as described in Figure 6·6. This way, the node would prevent time consuming temporary pausing of flow and hence eventually enable faster data delivery to the sink. In other words, node storage could be used to transform the end-to-end flow control to a hop-by-hop operation. The challenge in this setting is to detect the conditions that render storage utilization beneficial for the network performance. Without information about the future state of the network links the described congestion-biasing technique may deteriorate the network performance. Clearly, when a node decides to store some data it must know that this cannot be routed at that time from alternative shorter non-congested paths.

The fundamental difficulties encountered in the online version of the JSR problem motivate the exploration of specific network operation scenarios where suboptimal solutions are possible. For example, consider a network where the administrator (or the nodes) can predict the future values of link capacities and node available storage with bounded error. In this case, we can solve a variation of the JSR problem, name it EJSR, which stems from the original problem if we substitute the actual with the estimated parameters at the constraint set. Namely, instead of $C_{ij}$, and $S_{ij}$, we can use the worst-case predictions $\hat{C_{ij}} = (C_{ij} - e^c_{ij})$ and $\hat{S_{ij}} = (S_{ij} - e^s_{ij})$. The quantities $e^c_{ij}$ and $e^s_{ij}$ represent the maximum prediction error in the link capacities and node storage respectively. Obviously, the optimal

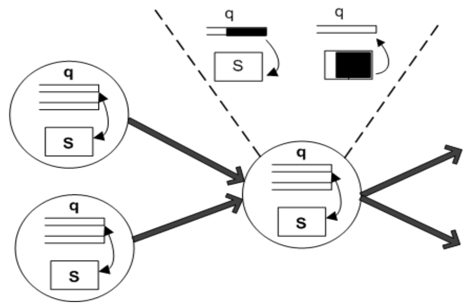



**Figure 6·6:** Storage and queue control for enabling hop-by-hop flow control. When queue backlog increases, excess data is moved to storage area to avoid congestion signaling. Then, when backlog is reduced, data is restored in respective queues.

solution of the EJSR problem is feasible for the respective JSR while its optimality depends on the accuracy of predictions.

Another interesting scenario is when the nodes are aware of their links and storage capacities only for the near future. In this case we can find the short-term storage policy using a similar algorithm with the one presented in section 6.4. Namely, we can use distributed algorithms such as those in [87], for finding the current min-cut $Q^{(t)}$ and also the min-cut of the next few slots, $Q^{(t+1)}, \ldots, Q^{(t+M)}$. This is accomplished through message passing among nodes with information about their current and future state. Now assume that a certain node $i$ belongs both to the set $W^{(t)}$ of the $Q^{(t)}$ and to the set $V \setminus W^{(t+1)}$ of the $Q^{(t+1)}$ cut. In this case, node $i$ can infer that it must use its storage capacity and admit the excess data that is routed to it. This policy enables the network performance improvement through the use of storage, although this is not the maximum possible, as was the case with the SCA algorithm.

## 6.7 Numerical Results

In order to verify the validity of our approach we simulated the operation of 3 networks with storage-capable nodes. Two of them are linear networks with 3 and 5 nodes respectively, while the third one is the original graph of Figure 6·4. The objective was to demonstrate the impact of intermediate storage to the performance of the network for various capacity evolution scenarios. These scenarios are modeled through the link capacities dissimilarity index $L$. Recall that higher values of $L$ imply more diverse capacity patterns. The performance metric is either the amount of data that can be transmitted within a given time period or equivalently, the incurred delay for the transfer of a certain amount of data from source to sink. The later is visualized through *Delay - Storage curves* where we depict the delay versus the aggregate storage of nodes for the transfer of various amounts of data. Clearly, the benefit of using storage varies for different networks and different values of $L$ from zero to substantial improvement on performance.

We begin with the 3-nodes linear network of Figure 6·2 where node $B$ has storage capability of $S_B$ units. We consider a time slotted operation for $T = 40$ slots and assume that link capacities $C_{AB}$ and $C_{BC}$ vary with time. The network operation is described by equations (6.1) - (6.5). In Figure 6·7 we depict the delay for the transfer of $D = 450$ units of data from node $A$ to node $C$ for different values of $L$. We see that as storage $S_B$ increases, the incurred delay reduces down to a minimum value. Further usage of storage does not improve the performance of the network. Similarly, in Figure 6·8 we depict the delay for the transfer of $D = 450$ units of data in a linear network of 5 nodes. We see again that the benefit from storage use to the network performance is almost proportional to the dissimilarity index $L$. Namely, notice that the distance of the maximum to the minimum delay value for every plot increases with $L$.

In Figure 6·9 we fix the value of $L$ and plot the delay for different amounts of transferred data for the 3-nodes network. Notice that the lower bounds of incurred delay are different for different amounts of data. Finally, in Figure 6·10 we depict the maximum amount of transferred data $D$, for a time period of $T = 20$ time slots from source to sink in the network of Figure 6·4. We see that this amount increases as a function of aggregate available storage at intermediate nodes $B, C, D$, and $E$ up to a maximum value. Further increase in storage capacity does not improve the performance of the network. This upper limit depends both on the network graph and on the dissimilarity index $L$ of the links for the time period $T$.

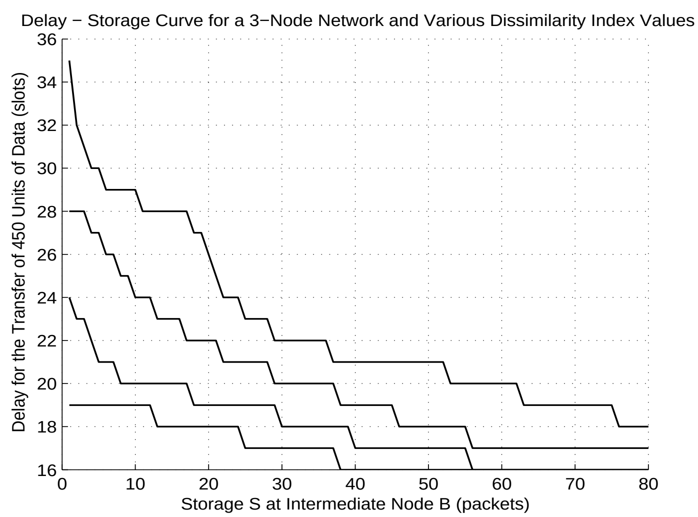



**Figure 6·7:** Delay - Storage curves for a 3-node network with intermediate storage, for the transfer of $D = 450$ units of data and various values of $L$. From the lower to the upper curve, it is $L = 867$, $L = 884$, $L = 909$, and $L = 934$.

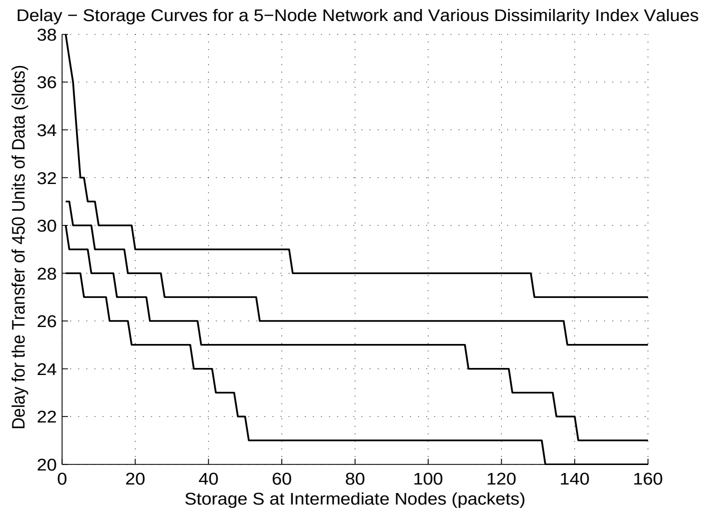



**Figure 6·8:** Delay - Storage curves for a 5-node linear network with intermediate storage, for the transfer of $D = 450$ units of data and various values of $L$. From the lower to the upper curve it is $L = 1120$, $L = 1204$, $L = 1306$, and $L = 1421$. Storage is equally distributed to nodes.

## 6.8 Conclusions

In this chapter we showed that storage under certain conditions can improve the network performance of dynamic networks. This improvement is realized either as increase of the amount of data that can be transported from the source to the destination within a finite time horizon or, equivalently, as reduction of the incurred delay for the delivery of certain amounts of data. The optimal storage control policy is the one that guarantees maximum benefit from storage use and can be derived for every network using the presented SCA algorithm. In order to realize this benefit, storage must be considered in conjunction with routing. The joint storage control - routing (JSR) policy can be derived through the solution of a max flow problem defined over a time-expanded graph. This policy determines how much data should be routed over each link and how much data should be stored in each node, in each time slot.

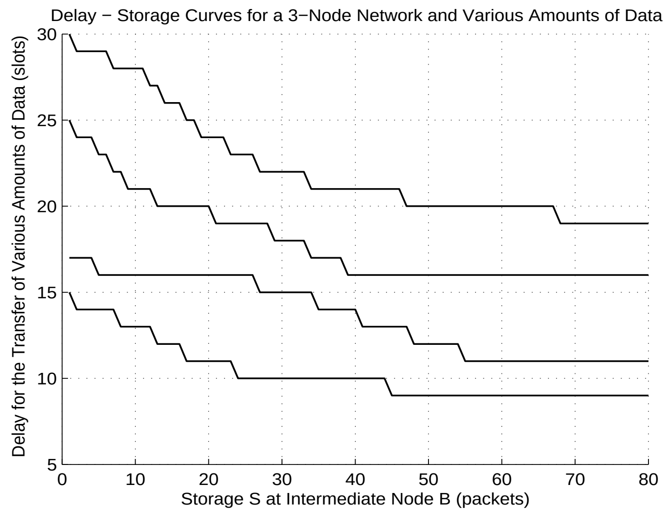



**Figure 6·9:** Delay - Storage curves for different amounts of transferred data in a 3-node linear network with fixed intermediate storage $S_B$, and dissimilarity index $L = 909$. From the lower to the upper curve it is $D = 200$, $D = 300$, $D = 400$, and $D = 500$.

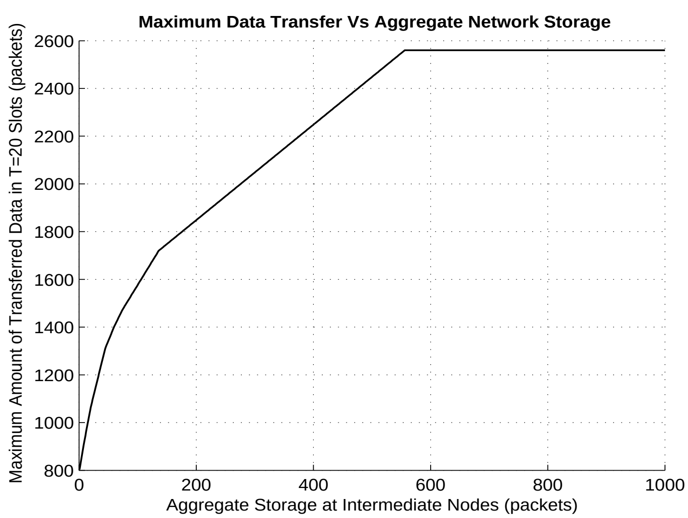



**Figure 6·10:** Maximum amount of data transferred in $T = 20$ time slots for the network of Figure 3(a), as a function of the total available storage at the intermediate nodes.

**Algorithm 4** (Joint Storage Routing - JSR - Max Flow)

**Variables**: Each node $i \in G_T$ maintains the variables: $p_i(t)$, $p_j(i,t)$, $x_{ij}(i,t)$, $x_{ji}(i,t)$, $y_{in}(i,t)$, $y_{mi}(i,t)$, $g_i(t)$, $\forall j \in F_i \cup B_i$, $n = i^{(t+1)}$, $m = i^{(t-1)}$.

**Execution**: The algorithm is executed continuously in a time sequence $t = (t_0, t_1, \ldots)$. Each specific time, every node $i$ executes one of the Actions below (**Action 1**-**Action 2**) and then checks the **Termination Condition**.

(**Action 1** - *Variables Update*): Node $i$ calculates its data and storage surplus $g_i(t)$ and:

**1.1 If** $[g_i(t) > 0]$ **Then** updates its local variables $[p_i(t),\ p_j(i,t),\ x_{ij}(i,t),\ x_{ji}(i,t),\ y_{ij}(i,t),\ y_{ji}(i,t)]$ by executing Steps 2 - 5 of the $\epsilon$-relaxation algorithm, [6, Chap.5.3].

(**Action 2** - *Notification*): Node $i$ sends messages with its primal-dual vars to its neighbors:

**2.1** Send $[p_i(t), x_{ij}(i,t)]$ to every child node $j \in F_i$

**2.2** Send $[p_i(t), x_{ji}(i,t)]$ to every parent node $j \in B_i$

(**Action 3** -*Coordination*): Node $i$ receives messages from its neighbors and updates his variables:

**3.1** For every message received at $t' < t$ from a child node $j \in F_i$, update the local variables:

**3.1.1 If** $[\,p_j(i,t) \leq p_j(t')\,]$ **Then** set $p_j(i,t) = p_j(t')$

**3.1.2 If** $[(\,p_i(t) \leq p_j(t') + \alpha)\,\&\,(x_{ij}(i,t) > x_{ij}(j,t'))]$ **Then** set $x_{ij}(i,t) = x_{ij}(j,t')$

**3.2** For every message received at $t' < t$ from a parent node $j \in B_i$, update the local variables:

**3.2.1 If** $[p_j(i,t) \leq p_j(t')]$ **Then** set $p_j(i,t) = p_j(t')$

**3.2.2 If** $[(p_i(t) \leq p_j(t') - \alpha)\,\&\,(x_{ji}(i,t) < x_{ji}(j,t'))]$ **Then** set $x_{ji}(i,t) = x_{ji}(j,t')$

where $\alpha = -1$ if $(i,j) = (d,s)$ and $\alpha = 0$, otherwise.

(**Action 4** - *Store Decisions Update*): Node $i$ considers the information from its instances $m = i^{(t_G-1)}$ and $n = i^{(t_G+1)}$ for $t' < t$ and updates its storage decisions as follows:

**4.1 If** $[p_n(i,t) \leq p_n(t')]$ **Then** set $p_n(i,t) = p_n(t')$.

**4.2 If** $[p_i(t) \leq p_n(t')\ \&\ y_{in}(i,t) > y_{in}(n,t')]$ **Then** set $y_{in}(i,t) = y_{in}(n,t')$.

**4.3 If** $[p_m(i,t) \leq p_m(t')]$ **Then** set $p_m(i,t) = p_m(t')$.

**4. If** $[p_i(t) \leq p_m(t')\ \&\ y_{mi}(i,t) < y_{mi}(m,t')]$ **Then** set $y_{mi}(i,t) = y_{mi}(m,t')$.

**Termination Condition**: The algorithm terminates when the data and storage surplus for all nodes becomes zero: $g_i(t) = 0\ \forall i \in V_T$.

# Chapter 7

# Conclusions and Future work

## 7.1 Summary of contributions

The goal of this dissertation was to analyze the challenges in emerging dynamic spectrum markets and propose mechanisms that will contribute to their efficient operation. The deployment of these markets is expected to improve spectrum utilization and enable the satisfaction of the ever increasing user demand for wireless services. However, market-based solutions are not a panacea and if they are not properly designed they will fail to yield the anticipated results. Our analysis was based on game theory and network economics. Whenever existing market mechanisms were not suitable for the problems we studied we proposed novel methods such as the $\beta$-optimal auction in Chapter 3 and the dynamic pricing mechanism in Chapter 6.

Additionally, we studied the impact of in-network storage on the capability of a network to convey data. We showed that, under certain conditions, node storage can be used as a low-cost alternative to the expensive link capacity. This technique will be particularly important the coming years when operators will need to manage the huge volume of users data traffic.

### Hierarchical Spectrum Markets

First we studied *hierarchical spectrum allocation schemes* that are expected to proliferate in emerging dynamic spectrum (DS) markets. We considered the scenario where a governmental agency (CO) sells channels to Primary Operators (POs) who subsequently resell them to Secondary Operators (SOs). We showed that this hierarchical scheme results in inefficient channel allocation because of the revenue-maximizing strategy of the POs. Each PO reassigns his channels so as to increase his revenue and not the welfare of his underneath secondary market. This strategy induces efficiency loss. Moreover, it creates a coordination problem. That is, the CO fails to allocate the channels efficiently to the POs in the first stage of the hierarchical allocation. We proposed an incentive mechanism that aligns the objective of the POs with the objective of the state agency and alleviates these issues.

The basic idea of the mechanism is that the CO exploits feedback information provided by the secondary operators and reimburses each PO in proportion to the welfare of his secondary market. This pricing induces the POs to reconsider their channel management policy. In other words, this mechanism creates a coupling between the channel reallocation decisions of the POs and their cost for buying the spectrum from the CO. Technically, the proposed scheme is a combination of an auction and a pricing mechanism. The auction is used in order to elicit the hidden information about spectrum demand while the pricing component enables the alignment of the objective of the POs with the goal of the agency.

We also discussed how this mechanism can be applied to spectrum markets where the CO-POs and the PO-SOs interactions are realized in different time scale. In this case, the coordination problem is inherently unsolvable. The CO assigns the channels in the beginning of each time period while the needs of the SOs under each PO change randomly on a per-slot basis (each period consists of many slots). Despite this issue, the mechanism increases the secondary market welfare since it induces the POs to adopt a more efficient channel allocation strategy.

Understanding the machinery and analyzing the efficiency of hierarchical spectrum allocation schemes is of crucial importance for emerging spectrum markets. Such hierarchies are expected to proliferate in DS markets in many different cases. For example, a similar hierarchy arises when spectrum is allocated from the controller to the POs and from the latter to their primary users.

**Competition in Wireless Services Markets**

Accordingly, we analyzed the *competition of operators in a wireless services market* for a common pool of users (clients) and discussed the necessity of regulation. The particular characteristic of our model is the assumption that users have an alternative out-of-the-market option to satisfy their communication needs. This is a very likely scenario today where users have concurrently access to multiple different networks. For example, a user may either access the Internet through a 3G (or 4G) wireless connection or through a WiFi network. We modeled the alternative option by introducing the concept of the *neutral operator* $P_0$ who offers a service of value $U_0$ to users. The existence of $P_0$ affects both the operator selection strategy of the users and the price competition among the operators. For example, in this setting, unlike other competition markets, operators cannot collude and fix very high prices because this will induce users to leave the market and select the alternative communication method ($P_0$).

We considered a very large population of users and employed an evolutionary game theoretic model in order to capture the users interaction and analyze their operator selection strategy. In evolutionary games the players update (revise) their strategy according to a certain revision protocol. For this problem, we assumed that users employ a hybrid revision protocol that is based both on imitation of better strategies and direct selection of the neutral operator. We derived the differential equations that describe the evolution of the market and found its stationary points. The advantage of using evolutionary game theory is that we were able to capture the realistic aspect of the users' limited information about the market, and at the same time to describe the dynamics of the users interaction with a good precision.

Due to the existence of the neutral operator, the price competition game of operators differs significantly from other similar games. We studied an oligopolistic market, with $I > 2$ operators. Each operator selects the price that believes it will yield the highest revenue. We assumed that the pricing game among operators is a non-cooperative game, with simultaneous moves and complete information. However, we allowed the operators to update their pricing strategy based on the previous prices other operators have adopted. This gives rise to a repeated interaction where in every stage the same static pricing game is played. We assume that operators are myopic and use best-response strategies on a per-stage basis. Our model is very similar to *Bertrand* price competition model. We proved that this is a Potential game which means that it has pure Nash equilibriums (NE) and moreover they are reachable under any finite improvement path. That is, any best response strategy of operators will finally drive the game to (one of) its equilibriums.

We found the Nash equilibriums of the pricing game for the particular case that operators have the same amount of spectrum $W$. Interestingly, the NE depend on $W$ and on the value $U_0$ of the neutral operator service. This allows us to consider how a regulator can intervene in the market and steer the equilibrium according to his objective. The regulator may either tune $U_0$ or change the total spectrum of each operator. For example, if the regulator is interested in increasing the efficiency of the market, i.e. improve the services that users enjoy, he can increase $U_0$. This will induce the market operators to lower their prices in order to offer more attractive services than $P_0$. On the other hand, if the regulator wants to increase the revenue of the operators, he may decrease $U_0$ or allow them to acquire more spectrum (e.g. by lowering its price). Different regulation methods have different results on the market efficiency and on the revenue of operators.

### Dynamic Pricing Mechanisms for Spectrum Markets

Next, we focused on secondary markets where peer entities such as Secondary operators (SOs) or secondary users will interact directly with each other in order to satisfy their dynamic communication needs. For example, each SO will be able to temporary lease his idle spectrum channels to other SOs and request channels from them when he has increased spectrum needs. Similarly, users will exchange bandwidth by routing each other traffic and satisfy their communication needs in an ad hoc fashion. These scenarios are expected to proliferate in dynamic spectrum markets.

We assumed a perfect competition market. That is, there exist many market entities (SOs or users) and hence no one of them can independently estimate the impact of his strategy on the price the spectrum (or bandwidth) is traded. In other words, the players are assumed to be price-takers. Each entity has to decide how much of his resource to sell, at what price, and how much he is willing to pay for the resource other entities offer to him. The basic characteristic of the spectrum markets we studied is that each entity is at the same time a resource provider and a resource consumer. This distinguishes this market from other similar resource trading markets and calls for novel market clearing mechanisms that will ensure the social welfare maximization.

We introduced a dynamic pricing mechanism that captures the double role of the network entities (SO or SU) and we proved that there exist bidding and charging strategies that maximize social welfare and we explicitly computed them. The mechanism determines the resource allocation, resource request and pricing strategies of each player. We designed also a decentralized realization of this scheme that relies only on lightweight feedback from the market, through which the entities coordinate in a distributed fashion. This is an important property of the mechanism since it allows the market to operate without a central coordinator or broker. Finally, we explained that the mechanism can be used also for the optimization of a generic network objective, other than the social welfare. For example, consider a set of users trying to serve each others traffic and at the same time achieve load balancing.

#### Storage Capacity Control Policies for Dynamic Networks

Finally, we discussed methods for exploiting in-network storage. We identified the benefits that node storage capacity has for a network and the conditions under which these benefits are realizable. We showed that for networks with time-varying link capacities it is possible to use properly designed store and forward policies in order increase the amount of data that can be transferred from source to destination within a certain time interval. This result is of high interest for network operators because storage has very low cost compared to bandwidth and is available in large scale.

We began our study with linear networks where the routing policy is simple. Accordingly, we extended our analysis in general networks and provided a formulation based on time-expanded graphs. We showed that by adding storage in certain nodes, one can increase the minimum cut of the graph and hence improve its data transfer capability. Additionally, we proposed the conjunction of storage control with routing and defined the joint storage control and routing max-flow problem. We solved this problem using a relaxation method which is amenable to parallel execution. The solution reveals how much data should be stored in each node in every time slot and how much data should be routed over each link.

### 7.2 Future Work

#### Hierarchical Spectrum Markets

First, it is challenging to consider the scenario where the SOs anticipate the impact of their bidding to the mechanism and strategize against it in order to increase their perceived utility. One can also consider the scenario of POs colluding with their SOs clients so as to deceive the controller. Another intriguing direction is to consider the more realistic setting where there is no prior knowledge about the SOs types or the family of POs and SOs valuation functions, and apply learning schemes to elicit this hidden information. Finally, it is important to quantify the cost of regulation for the controller. That is, how much money has the CO to inject into the market so as to improve its efficiency? The cost of regulation should be compared with the improvement in the welfare of the secondary markets.

Our study can also contribute towards understanding hierarchical resource allocation mechanisms. These schemes are expected to arise in many different settings in the near future. Dynamic spectrum markets is only one of the many examples. Cloud services markets or even power electricity market are fields where such hierarchies will proliferate. The common characteristic of all these instances is that more than two different classes of entities interact concurrently, while traditional resource allocation problems involve usually two parties (e.g. client-server architectures).

### Competition and Regulation in Wireless Services Markets

The liberalization of the spectrum market will increase competition among spectrum sellers and among providers of associated wireless services. It is important to analyze all the aspects of this new market environment and understand when and how one should intervene so as to ensure its efficient operation. A basic challenge is to devise realistic and detailed models of the market which at the same time will be tractable.

In this context, it is interesting to extend our study by considering the possibility that operators cooperate and make peering agreements for jointly serving the users or collude and set their prices without competing. Strategies like these will have an impact on the market equilibrium and affect the welfare of the market. More interestingly, we can analyze the price competition game not only for the stationary point of the market but even before the users dynamics reach a stable point. This will allows us to understand how the operators should select their optimal pricing policy in a real time fashion.

### Dynamic Pricing Mechanisms for Spectrum Markets

We introduced a dynamic pricing framework which is generic and can be used for devising resource allocation algorithms for various different settings such as peer-to-peer networks, disruption-tolerant storage clusters and wireless ad-hoc networks with energy constrained nodes. In all these instances, each network entity possesses some resource and can engage in transactions with others to achieve its needs. Nodes face the dilemma of devoting their limited resource to their own benefit and thereby directly gaining utility, versus acting altruistically, with the anticipation to be aided themselves when needed. Specifically, scenarios that our proposed mechanism can be applied, are the following:

- Wireless ad hoc networks, where nodes may use their limited battery energy to transmit their own traffic to the next hop en route to the destination, or forward other nodes' traffic to their respective next hop. The underlying scarce resource is energy. A node clearly benefits only if it uses its energy to transmit its own traffic.

- Peer-to-peer networks, where peers may use their access link bandwidth either to download content from other peers or to let other peers upload content from them. That is, a peer may act both as client and a server. The resource here is link bandwidth or equivalently service time. Clearly, a node obtains utility that captures node satisfaction only as client, namely through downloads.

- Disruption-tolerant networks, where nodes store content in case of intermittent connectivity and transmit it when link conditions allow it. In that case, the resource may be the cache memory that is used for short-term storage, or the disk space for more permanent storage. Nodes need to decide whether to allocate their storage space to their own content to facilitate their own transmission or reserve some amount for received content by others.

Finally, on of the most fascinating directions for future work is to relax the assumption of price-taking users and study the impact of strategic, price anticipating behavior of nodes on the system overall performance.

**Storage Capacity Control Policies for Dynamic Networks**

In-network storage is a resource that has not be exploited adequately until now. Under certain conditions it can improve the performance of a dynamic network. The methodology we proposed has many interesting applications. For example, it can be used to analyze and improve inter-data center communication where the cost of bulk data transfer is extremely high and time varying, [62]. Intra-data center networking is another area that we believe it can benefit from this analysis. Designing the architecture of a data center is a very challenging task and must take into account both the performance and the cost of the equipment, [39]. Hence, it is very important to use efficiently both the available storage and links capacity resources. Moreover, node storage can be used to enhance the operation of peer-to-peer systems where the performance bottleneck is the uplink capacity, [60]. It is important to analyze carefully all these cases and devise proper algorithms tailored to the needs of each specific networking scenario.

This was a first attempt to understand the impact of storage capacity in networks with full knowledge over the link capacity state and its evolution. The next big step that can be pursued is towards understanding the online version of the problem. In this case, link capacities obey a known discrete or continuous probability distribution, but the controller knows only the current value of link capacities just before taking a decision. Again, it is imperative to consider algorithms which are amenable to distributed implementation.

# Bibliography

[1] V. Abhishek and B. Hajek. Efficiency loss in revenue optimal auctions. In *IEEE CDC*, Atlanta, Georgia, 2010.

[2] D. Acemoglou and A. Ozdaglar. Price competition in communication networks. In *Infocom*, 2006.

[3] M. Armbrust, A. Fox, and et. al. *An Annotated Overview of Dynamic Network Flows.* INRIA, Technical Report No 4936, 2003.

[4] M. Armbrust, A. Fox, and et. al. *Above the Clouds: A Berkeley View of Cloud Computing.* Berkeley, Technical Report,(EECS-2009-28), 2009.

[5] B. Awerbuch and T. Leighton. Improved approximation algorithms for the multi-commodity flow problem and local competitive routing in dynamic networks. In *ACM STOC*, 1994.

[6] D. P. Bertsekas and J. N. Tsitsiklis. *Parallel and Distributed Computation: Numerical Methods.* Athena Scientific, 1997.

[7] Dimitri Bertsekas. *Network Optimization: Continuous and Discrete Models.* Athena Scientific, Belmont, Massachusetts, 1998.

[8] G. Bianchi. Performance analysis of the ieee 802.11 distributed coordination function. *IEEE Journal on Selected Areas in Communication*, 18(3), 2000.

[9] M. Bitsaki, G. D. Stamoulis, and C. Courcoubetis. An efficient auction-based mechanism for hierarchical structured bandwidth markets. *Elsevier Computer Communications*, pages 911–921, 2006.

[10] M. Bitsaki, G.D. Stamoulis, and C. Courcoubetis. A new strategy for bidding in the network-wide progressive second price auction for bandwidth. In *CoNEXT*, 2005.

[11] S. Boyd and L. Vandenberghe. *Convex Optimization.* Cambridge University Press, 2004.

[12] F. Branco. Multiple unit auctions of an indivisible good. *Economic Theory*, 8:77–101, 1996.

[13] L. Bui, R. Srikant, and A. Stolyar. Novel architectures and algorithms for delay reduction in back pressure scheduling and routing. *CoRR*, 2009.

[14] J. Bulow and J. Roberts. The simple economics of optimal auctions. *Journal of Political Economy*, 97:1060–1090, 1989.

[15] M. Cagalj, S. Ganeriwal, and I. Aad. On selfish behavior in csma/ca networks. In *IEEE INFOCOM*, 2005.

[16] G. Carofiglio, M. Gallo, and L. Muscariello. Bandwidth and storage sharing performance in information centric networking. In *ACM SIGCOMM Workshop on Information-Centric Networking*, 2011.

[17] X. Chen and J. Huang. Evolutionarily stable spectrum access in a many-users regime. In *IEEE Globecom*, 2011.

[18] P. Chhabra, V. Erramilli, N. Laoutaris, R. Sundaram, and P. Rodriguez. Algorithms for constrained bulk-transfer of delay-tolerant data. In *IEEE ICC*, 2010.

[19] S.H. Chun and R.J. La. Auction mechanism for spectrum allocation and profit sharing. In *GameNets*, 2009.

[20] E. Clark. Multipart pricing of public goods. *Public Choice*, 2:19–33, 1973.

[21] C. Courcoubetis and R. R. Weber. *Pricing Communication Networks: Economics, Technology and Modelling*. Willey, 2003.

[22] M. Diallo, S. Fdida, V. Sourlas, P. Flegkas, and L. Tassiulas. Leveraging caching for internet-scale content-based publish/subscribe networks. In *ICC*, Japan, 2011.

[23] A. Dimakis, R. Jain, and J. Walrand. Mechanisms for efficient allocation in divisible capacity networks. In *45th IEEE Conf. on Decision Control*, pages 1264–1269, 2006.

[24] L. Duan, J. Huang, and B. Shou. Competition with dynamic spectrum leasing. In *IEEE DySPAN*, 2010.

[25] B. Edelman, M. Ostrovsky, and M. Schwarz. Internet advertising and the generalized second price auction: Selling billions of dollars worth of keywords. In *2nd Workshop Sponsored Search Auctions*, 2006.

[26] T. Erpek, M. Lofquist, and K. Patton. Spectrum occupancy measurements loring commerce centre limestone, maine september. *The Shared Spectrum Company*, pages 18–20, 2007.

[27] T. Erpek, K. Steadman, and D. Jones. Dublin ireland spectrum occupancy measurements collected on april 16-18, 2007. *The Shared Spectrum Company*, 2007.

[28] L. Fleischer and M. Skutella. Minimum cost flows over time without intermediate storage. In *ACM-SIAM Symposium on Discrete Algorithms*, pages 66–75, 2003.

[29] L. R. Ford and D. R. Fulkerson. *Flows in Networks*. Princeton University Press, 1962.

[30] D. Fudenberg and D. K. Levine. *Learning in Games*. MIT Press, 1998.

[31] L. Gao, X. Wang, and Y. Xu. Multiradio channel allocation in multihop wireless networks. *IEEE Transactions on Mobile Computing*, 2009.

[32] L. Gao, X. Wang, Y. Xu, and Q. Zhang. Spectrum trading in cognitive radio networks: A contract-theoretic modeling approach. *IEEE Journal on Selected Areas in Communications*, pages 843–855, 2011.

[33] S. Gitzenis, G. Konidaris, and S. Toumpis. Flow optimization in delay tolerant networks using dual decomposition. In *WiOpt/Rawnet*, Germany, 2012.

[34] N. Golrezaei, K. Shanmugam, and A. G. Dimakis. Femtocaching: Wireless video content delivery through distributed caching helpers. In *IEEE Infocom*, Orlando, US, 2012.

[35] Google. Ad word auctions. `http://www.adwords.google.com/`, 2006.

[36] A. Gopinathan and Z. Li. A prior-free revenue maximizing auction for secondary spectrum access. In *IEEE Infocom*, 2011.

[37] J. Gray. Long term storage trends and you. `http://research.microsoft.com/~Gray/talks/IO_talk_2006.ppt`, 2006.

[38] T. Groves. Incentives in teams. *Econometrica*, 41(4):617–631, 1973.

[39] C. Guo, H. Wu, K. Tan, L. Shiy, Y. Zhang, and S. Lu. Dcell: A scalable and fault-tolerant network structure for data centers. In *ACM SIGCOMM*, 2008.

[40] J. D. Hartline and T. Roughgarden. Optimal mechanism design and money burning. In 40*th ACM Symp. Theory of Comp.*, 2008.

[41] L. He and J. C. Walrand. Pricing and revenue sharing strategies for internet service providers. *IEEE Journal on Selected Areas in Communications*, 2006.

[42] Internet Engineering Task Force (IETF). A survey of in-network storage systems. `http://tools.ietf.org/html/rfc6392`, 2011.

[43] G. Iosifidis and I. Koutsopoulos. Challenges in auction theory driven spectrum management. *IEEE Communications Magazine*, 49(8), 2011.

[44] V. Jacobson, D. K. Smetters, J. D. Thornton, M. F. Plass, N. H. Briggs, and R. L. Braynard. Networking named content. In *ACM CoNEXT*, Rome, Italy, 2009.

[45] R. Jain and P. Varaiya. Efficient market mechanisms for network resource allocation. In *44th IEEE Conf. on Decision Control*, pages 1056–1061, London, 2005.

[46] S. Jain, K. Fall, and R. Patra. Routing in a delay tolerant network. In *ACM SIGCOMM*, 2004.

[47] J. Jia, Q. Zhang, Q. Zhang, and M. Liu. Revenue generation for truthful spectrum auction in dynamic spectrum access. In *ACM Mobihoc*, 2009.

[48] L. Johansen. Price-taking behavior. *Econometrica*, 4(7):1651–1656, 1977.

[49] R. Johari and J. Tsitsiklis. Efficiency loss in a network resource allocation game. *Math. of Oper. Res.*, 29(3):407–435, 2004.

[50] R. Johari and J. Tsitsiklis. Communication requirements of vcg-like mechanisms in convex environments. In 43*rd Allerton Conference on Communication, Control and Computing*, 2005.

[51] E. P. C. Jones, L. Li, J. K. Schmidtke, and P. Ward. Practical routing in delay-tolerant networks. *IEEE Trans. on Mob. Computing*, 6:943–959, 2007.

[52] D. Karger and C. Stein. An o(n2) algorithm for minimum cuts. In *25th ACM Symposium on the Theory of Computing*, New York, 1993.

[53] G. Kasbekar, E. Altman, and S. Sarkar. A hierarchical spatial game over licensed resources. In *GameNets*, 2009.

[54] F.P. Kelly. Charging and rate control for elastic traffic. *European Trans. Telecom.*, 8(3):33–37, 1997.

[55] F.P. Kelly, A. Maulloo, and D. Tan. Rate control for communication networks: Shadow prices, proportional fairness and stability. *J. Oper. Res. Soc.*, 49(3):237–252, 1998.

[56] J. Kleinberg and E. Tardos. *Algorithm Design*. Addison-Wesley, 2005.

[57] P. Klemperer. *Auctions: Theory and practice*. Princeton University Press, 2004.

[58] V. Krishna. *Auction theory 2nd ed*. Academic Press, 2010.

[59] N. Laoutaris and P. Rodriguez. Good things come to those who (can) wait or how to handle delay tolerant traffic and make peace on the internet. In *ACM HotNets-VII*, 2008.

[60] N. Laoutaris, P. Rodriguez, and L. Massoulie. Echos: Edge capacity hosting overlays of nano data centers. *ACM SIGCOMM Computer Communication Review*, 38(1):51–54, 2008.

[61] N. Laoutaris, M. Sirivianos, X. Yang, and P. Rodriguez. Inter-datacenter bulk transfers with netstitcher. In *ACM SIGCOMM*, 2011.

[62] N. Laoutaris, G. Smaragdakis, P. Rodriguez, and R. Sundaram. Delay tolerant bulk data transfers on the internet. In *ACM Sigmetrics*, 2009.

[63] A. Lazar and N. Semret. The progressive second price auction mechanism for network resource sharing. In *Int. Symp. on Dynamic Games and Applications*, 1997.

[64] K. Leung and J. Huang. Regulating wireless access pricing. In *IEEE ICC*, 2011.

[65] A. Likhodedov and Tuomas Sandholm. Auction mechanism for optimally trading off revenue and efficiency in multi-unit auctions. In *ACM Electronic Commerce*, 2004.

[66] R. Maheswaran and T. Basar. Nash equilibrium and decentralized negotiation in auctioning divisible resources. *J. Group Decision and Negotiation*, 13(2), 2003.

[67] P. Maille and B. Tuffin. Why vcg auctions can hardly be applied to the pricing of inter-domain and ad hoc networks. In *3rd EURO-NGI Conference on Next Generation Internet Networks, Design and Engineering for Heterogeneity (NGI 2007)*, Trondheim, Norway, 2007.

[68] P. Maille and B. Tuffin. Price war in heterogeneous wireless networks. *Computer Networks*, 54(13):2281–2292, 2010.

[69] E. S. Maksin and J. G. Riley. *Optimal Multi-unit Auctions.* Frank Hahn (ed) The Economics of Missing Markets, Information and Games, 1989.

[70] R.P. McAfee. A dominant strategy double auction. *Journal of Economic Theory*, 56(2):434–450, 1992.

[71] D. Monderer and L. S. Shapley. Potential games. *Games and Economic Behavior*, 14, 1996.

[72] R. B. Myerson. Optimal auction design. *Mathematics of Operations Research*, 6:58–73, 1981.

[73] Y. Narahari, D. Garg, R. Narayanam, and H. Prakash. Game theoretic problems in network economics and mechanism design solutions. *Springer, Advanced Information and Knowledge Processing*, 2009.

[74] T. Nguyen, H. Zhou, R. Berry, M. Honig, and R. Vohra. The impact of additional unlicensed spectrum on wireless services competition. In *IEEE Dyspan*, 2011.

[75] N. Nisan, T. Roughgarden, E. Tardos, and V. V. Vazirani. *Algorithmic Game Theory.* Cambridge University Press, 2007.

[76] D. Niyato and E. Hossain. Modeling user churning behavior in wireless networks using evolutionary game theory. In *IEEE WCNC*, 2008.

[77] D. Niyato and E. Hossain. Dynamics of network selection inh eterogeneous wireless networks: An evolutionary game approach. *IEEE/ACM Transactions on Vehicular Technology*, 2009.

[78] D. Niyato and E. Hossain. A microeconomic model for hierarchical bandwidth sharing in dynamic spectrum access networks: Distributed implementation, stability analysis, and application. *EEE Transactions on Computers*, pages 865–877, 2010.

[79] D. Niyato, E. Hossain, and Z. Han. Dynamics of multiple-seller and multiple-buyer spectrum trading in cognitive radio networks: A game-theoretic modeling approach. *IEEE/ACM Transactions on Mobile Computing*, 8(8), 2009.

[80] R. G. Ogier. Minimum-delay routing in continuous-time dynamic networks with piecewise-constant capacities. *Networks*, 18:303–318, 1988.

[81] A. Orda, R. Rom, and M. Sidi. Minimum delay routing in stochastic networks. *IEEE/ACM Trans. Networking*, 1(2):187–198, 1993.

[82] A. Ozdaglar and R. Srikant. *Incentives and Pricing in Communication Networks*. Algorithmic Game Theory, Cambridge University Press, 2007.

[83] C. W. Paper. Cisco visual networking index: Global mobile data traffic forecast update 20102015. *tech. rep., Cisco Public Information*, 2010.

[84] J. Park and M. van der Schaar. The theory of intervention games for resource sharing in wireless communications. *IEEE Journal on Selected Areas in Communications*, 30(1):165–175, 2011.

[85] J. M. Peha. Emerging technology and spectrum policy reform. In *ITU MMSM Workshop*, 2007.

[86] J. M. Peha and S. Panichpapiboon. Real-time secondary markets for spectrum. In *Telecommunications Policy*, pages 603–618, 2004.

[87] T. L. Pham, I. Lavallee, M. Bui, and S. Hoang Do. A distributed algorithm for the maximum flow problem. In *ISPDC*, 2005.

[88] Spectrum policy task force report. Technical report 02-135. *Federal Communications Commission*, 2002.

[89] E. P. Release. Ericsson predicts mobile data traffic to grow 10-fold by 2016. *Technical Report*, 2011.

[90] D. J. Roberts and A. Postlewaite. The incentives for price-taking behavior in large exchange economies. *Econometrica*, 44(1):115–127, 1976.

[91] W. H. Sandholm. Pairwise comparison dynamics and evolutionary foundations for nash equilibrium. *Games*, 1(1):3–17, 2010.

[92] N. Semret. Market mechanisms for network resource sharing. PhD Thesis, Columbia University, 1999.

[93] S. Shakkottai and R. Srikant. Economics of network pricing with multiple isps. *IEEE/ACM Transactions on Networking*, 14(6), 2006.

[94] S. Shakkottai and R. Srikant. *Network Optimization and Control*. Foundations and Trends in Networking, NoW Publishers, 2007.

[95] M. J. Smith. *Evolution and the Theory of Games*. Cambridge University Press, 1982.

[96] M. J. Smith. The stability of a dynamic model of traffic assignment an application of a method of lyapunov. *Transactions of Science*, 18:245–252, 1984.

[97] Spectrum. Bridge. `http://www.spectrumbridge.com/`, 2010.

[98] T. M. Stoenescu and D. Teneketzis. *Decentralized Resource Allocation Mechanisms in Networks: Realization and Implementation*. Advances in Control, Communication networks, and Transportation Systems, 2005.

[99] M. Stoer and F. Wagner. A simple min-cut algorithm. *Journal of the ACM*, 44(4):585591, 1997.

[100] W. Tang and R. Jain. Hierarchical auctions for network resource allocation. In *Proc. of the IntlConference on Game Theory in Networks (GameNets)*, Shanghai, China, 2011.

[101] W. Tang and R. Jain. Hierarchical auction mechanisms for network resource allocation. *submitted to IEEE JSAC*, 2012.

[102] L. Tassiulas and A. Ephremides. Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks. *IEEE Trans. on Automatic Control*, 37(12):1936–1948, 1992.

[103] P. D. Taylor and L. Jonker. Evolutionarily stable strategies and game dynamics. *Mathematical Biosciences*, 40:145–156, 1978.

[104] H. Varian. Online ad auctions. *American Econ. Review*, 99(2):430–434, 2009.

[105] W. Vickrey. Counterspeculation, auctions and competitive sealed tenders. *Journal of Finance*, 16(1):8–37, 1961.

[106] G. Vojislav, J. Huang, and B. Rimoldi. Competition of wireless providers for atomic users: Equilibrium and social optimality. In *Allerton*, 2009.

[107] J. G. Wardrop. Some theoretical aspects of road traffic research. In *Proceedings of the Institution of Civil Engineers*, 1952.

[108] Z. Wu, Z. Chen, L. Xu, and F. Guo. Routing protocols based on double auction for ad hoc networks. In *IEEE Int. Conf. on Networking, Archit., Storage (NAS)*, pages 55–61, 2007.

[109] S. Yang and B. Hajek. Vcg-kelly mechanisms for allocation of divisible goods: Adapting vcg mechanisms to one-dimensional signals. *IEEE Journal on Selected Areas in Communications*, 25:1237–1243, 2007.

[110] L. Ying, S. Shakkottai, and A. Reddy. On combining shortest path and back pressure routing over multihop wireless networks. In *IEEE Infocom mini conference*, 2010.