UNIVERSITY OF THESSALY

SCHOOL OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

Modal Characteristics Identification based on the Classically Damped Model

by

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A Thesis

Submitted in Partial Fulfillment of the Requirements for the Degree of Diploma

in Mechanical Engineering at the University of Thessaly

Volos, 2024

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Acknowledgments

I would like to thank my supervisor Prof. Costas Papadimitriou for supporting me through the strenuous period of writing the thesis both for providing me with knowledge on structural dynamics as well as inspiring me to get involved in this field of engineering.

I would also like to show my gratitude to all the members of the academic staff of the department of mechanical engineering for their providing foundations and guidance for becoming a professional in the field of Mechanical Engineering.

Last but not least, I would like to thank my parents for the constant support and trust they have shown over the years.

Abstract

The present thesis discusses the identification of modal characteristics (modal frequencies, damping ratios, modeshapes) of mechanical structures using acceleration sensors for measuring the external load and acceleration responses of a structure. The classically damped mathematical model was used to formulate a Single-Input Multiple-Output oscillating structure. The modal analysis for the multi-degree of freedom system is conducted in the frequency domain. The estimation of the modal characteristics is based on minimizing the mean-square error between the measured transfer function and the analytical transfer function obtained from modal analysis. This objective is a function of the modal frequencies, modal damping ratio and the mode shape components at the measured locations. A non-linear unconstrained optimization problem is used to minimize the objective with respect to the parameters. Efficient methods have been used to write the objective function with respect to the modal frequencies and modal damping ratios. A MATLAB software is developed to process the data collected from the sensors and perform the optimization. Both the mathematical analysis and the software are presented in the thesis for better understanding of structural dynamics analysis basics. Two simulated "experiments" were conducted so as to validate the software's capabilities and the accuracy of the optimization algorithms. The simulated data are generated from two models of shear buildings (3DOFs and 10DOFs) and the results from the study on the buildings are shown through tables and figures for the effectiveness of the methodology and the software to be confirmed.

Περίληψη

Στην παρούσα διπλωματική εργασία παρουσιάζεται μια μεθοδολογία προσδιορισμού των ιδιομορφικών χαρακτηριστικών (ιδιοσυχνότητες, συντελεστές απόσβεσης, ιδιομορφές) μιας κατασκευής στην οποία έχουν τοποθετηθεί επιταχυνσιόμετρα για την μέτρηση της διέγερσης και της απόκρησης της κατασκευής. Ο προσδιορισμός των ιδιομορφικών χαρακτηριστικών της κατασκευής γίνεται χρησιμοποιώντας το μοντέλο 'classically damped model' και η περίπτωση η οποία μελετάται είναι ένα σύστημα μονής-εισόδου πολλαπλών-εξόδων (SIMO). Η ιδιομορφική ανάλυση πραγματοποιήθηκε στο πεδίο συχνοτήτων. Επίσης για τον προσδιορισμό των ιδιομορφικών χαρακτηριστικών αναπτύχθηκε αντικειμενική συνάρτηση απαραίτητη για την λύση ενός μη-γραμμικού προβλήματος βελτιστοποίησης δίχως περιορισμούς. Σχέσεις που προσδιορίζουν την αντικειμενική συνάρτηση ως προς τις ιδιοσυχνοτήτες και τους συντελεστές απόσβεσης είναι αυτές οι οποίες βελτιστοποιούνται αναζητώντας το σημείο που ελαχιστοποιεί την διαφορά μεταξύ της μετρούμενης συνάρτησης μεταφοράς και της υπολογισμένης συνάρτησης μεταφοράς. Έγινε ανάπτυξη λογισμικού στην πλατφόρμα της MATLAB για την επεξεργασία δεδομένων από αισθητήρες και για την πραγματοποίηση υπολογισμών της βελτιστοποίησης. Προκειμένου να επιβεβαιωθεί η λειτουργικότητα και η αξιοπιστία του λογισμικού δύο παραδείγματα κτηρίων, με προσομοιωμένα δεδομένα εισόδου και εξόδου, χρησιμοποιήθηκαν. Στα παραδείγματα αυτά προσδιορίστηκαν τα ιδιομορφικά χαρακτηριστικά των κτηρίων καθώς και παρουσιάστηκαν τα αποτελέσματα από την μελέτη με Πίνακες και Διαγράμματα προκειμένου να αποδειχθεί η ακρίβεια της μεθοδολογίας και του λογισμικού.

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Chapter 1

1.1. Research Context

During the past decades modal analysis has become one of the most important processes for understanding the dynamic behaviour of mechanical structures in various fields of research like civil engineering, industrial machinery and energy industry, automotive and aerospace industries. Modal analysis is of vital importance because it helps analyze the response of complex mechanical systems, improve the design, detect damage in already built structures, and eventually render them safe and reliable [1].

As stated before, a lot of industries require certain specifications for their structures to be safe and reliable. These specifications tend to be the most important in the design process of a structure and later during the maintenance of the structure. In order to ensure the performance of a system is reliable and complies with its design criteria, the knowledge of its dynamic behavior is essential in case of extreme loads. Engineers build models to predict the dynamic behavior of a structure and therefore predict the performance of a structure subjected to dynamic loads. The dynamic behavior of a mechanical system relies on its modal characteristics (modal frequencies, damping ratios and modeshapes).

Modal analysis is the study of mechanical systems in order to identify their modal characteristics. These characteristics are usually modal frequencies, modal damping ratios and modeshapes. Modal frequencies are frequencies that depend on how the mass and stiffness are distributed on the structure. The need to find where these frequencies lie is very important to avoid resonance. The phenomenon called resonance is when dynamic excitation inputs to the system seem to match the modal frequencies of the structure. When a structure is in resonance even a small excitation force on the structure can produce large vibration response. When dealing with large structures like buildings or industrial machinery, avoidance of extreme vibration responses is very important to avoid damage which may be expensive or even fatal [2].

Of course, experimental modal analysis and operational modal analysis are the processes of identifying modal characteristics, which are a function of the structures' physical properties (mass, stiffness and damping). Therefore, changes in the physical properties of a structure can lead to altering the modal characteristics of the structure. Modifications in the physical properties of a structure happen regularly through deterioration of the materials, broken fasteners or linkages, or additions on the already made structure. All these minor changes on a mechanical structure can cause changes in the modal properties and therefore create unexpected dynamic behavior, proving the importance of measuring a structure regularly and updating or calibrating its dynamic model. Furthermore, indicating changes in modal characteristics can be helpful in detecting and identifying the location of potential damage within the structure [3].

There are two major modal identification techniques used in structural dynamics as mentioned before, Experimental Modal Analysis (EMA) and Operational Modal Analysis (OMA).

Experimental Modal Analysis (EMA), is an identification technique that calculates the modal parameters through input and output data acquired from measuring devices on a structure. Classically, excitation to the structure (input) happens by applying measurable forces on the structure's DOFs and measuring the response (usually acceleration) through sensors. EMA usually happens in a laboratory which is requires stopping the structure's operation. Experimental Modal Analysis has two main restrictions. First, applying artificial forces to large structures like buildings or bridges needs expensive equipment and may not be possible at all which is a major restriction for measuring the input signal on the structure. Moreover, stopping the operation of a structure for doing an EMA is not efficient, may not be possible for large structures and is definitely expensive.

Operational Modal Analysis (OMA), is a modal parameters identification technique also known as output only modal analysis. This technique is mainly used when structures are under operating conditions and utilizes input signals from ambient forces, forces coming from cyclic loads or earthquake loads as inputs to the system. Therefore, as excitation signals are unknown only response data at structures degrees of freedom are taken into account in the modal identification process. OMA method is mainly used in structures like bridges or buildings which are subjected to random loads generated in their environment. Furthermore, due to the fact that the structure remains to its normal operating conditions OMA is also very effective in cases like vehicles in road testing or aircrafts during flights for obtaining accurate dynamic models [4].

This thesis approaches the modal identification problem through manipulation of vibration data from sensors placed on the structure. This is an approach to Experimental Modal Analysis mentioned before and used by engineers through the years because it gives the opportunity to apply sensors on a structure (usually accelerometers) for identifying the modal characteristics as well as calibrating a digital twin model of the structure. Handling sensor data can be challenging and has certain limitations. To begin with, shortage of data due to a limited number of sensors on the structure leads to not having enough information about the full modeshape spectrum of the system. Moreover, usually experimenting with different frequencies of system excitation contributes to the shortage of quality data especially in large structures. Furthermore, sensor placement is important and it can be a trial and error process leading to acquiring useful data that don't represent the given system correctly. Last but not least, measurement errors that happen during the acquisition of data like excessive noise that is difficult to be filtered out, changes in the environment that produce unwanted data or even faulty equipment that leads to incorrect data can be problematic or catastrophic for the modal analysis [5].

The main goal of this thesis is to develop software that can identify dynamic modal characteristics of structures like modal frequencies, modal damping ratios and modeshapes at certain degrees of freedom (DOFs) of a structure using acceleration data from sensors installed on a structure to measure the excitations and acceleration responses. An identification algorithm has been developed using the classically damped model in the frequency domain. The modal identification happens through a least-squares optimization problem that minimizes the error between measured data and model predicted acceleration response using classically damped modal model .

Once the modal model is obtained, there are several applications that modal analysis can be utilized. To begin with, understanding the dynamic behavior of mechanical structures is one of the reasons why modal analysis is crucial. Getting to understand the dynamic behavior of structures helps with designing and implementing modern vibration control strategies. Moreover, finite element model updating or updating other simulation models depends on modal analysis results and measured modal data. This improves model's accuracy and reliability. In addition, tracking changes in modal parameters can be used, as mentioned before, in health monitoring of structures as well as detecting and locating damage [6].

There are a lot of cases where modal analysis could have prevented structure failure and subsequently saved lives and properties. At the Tacoma Narrows Bridge Disaster in 1940 winds blowing happened to match the resonant frequency of the bridge causing it to collapse instantaneously. Another catastrophic example is the Mexico City earthquake in 1985 which resulted in most 6 to 15 storey buildings collapsing because of the lack of proper design. Most modern high storey buildings in regions vulnerable to earthquake are designed to have controlled response in the event of a high scale earthquake [7]. Some other famous resonance incidents are the Broughton suspension bridge (1831) in the UK [8] and the Angers bridge (1850) in France [9] which collapsed from marching troops walking in with same frequency as the resonant frequency of the bridge. Both incidents resulted in over two hundred deaths. Last but not least, the most recent resonance incident was the London's Millenium bridge that was shut down in 2000 due to excessive oscillations created from pedestrians walking across the deck of the bridge.

1.2. Outline of the thesis

In the present thesis, a mathematical model of the estimation of modal characteristics is developed for a mechanical structure subjected to external loads. Modal analysis was conducted in the frequency domain based on the classically damped model in order to develop an expression for the frequency response functions. An output-error approach was used for the modal characteristics identification by minimizing the difference between the Frequency Response Functions and the model calculated Transfer Functions for each resonance mode. Moreover, a MATLAB software was developed in order to analyze data provided for the structure response, calculate the dynamic characteristics (frequency response functions), estimate the modal characteristics and present the results of the analysis. For better understanding of the thought process as well as the results from the software the thesis is separated in five chapters.

The second chapter of the thesis is dedicated to explaining the equations of motion for a multi-degree of freedom system approached with the classically damped oscillator model as well as presenting equations needed for the modal analysis problem of the system. The analysis is done both in time and frequency domain and the final equations show the response of the system degrees of freedom as a function of the eigenfrequencies and the damping ratios.

Chapter three covers the formulation of the modal analysis problem as a least squares optimization problem. An objective function is constructed from the error between the measured and the calculated transfer function (from the classically damped model), on the measured degrees of freedom, so as to find the minimum difference among the two arrays. The optimization process is formulated to seek the modal characteristics for each resonance mode and does not include identifying overlapping modes.

Chapter four explains and presents the working principles of the software developed in MATLAB to identify the modal characteristics of a structure. In this chapter, the software structure is separated into three stages for better understanding of the algorithm. Moreover, two examples are presented in order to make clear for the possibilities of the software and the way its results are presented. The two examples use simulated data generated from MATLAB but the software could be used in real world scenarios/structures.

Last chapter summarizes the conclusions of the thesis as well as the knowledge gained through the research. In addition, future work is proposed in order to enrich this research and approach the modal characteristics identification with alternative methods.

Appendix contains the MATLAB software written in the form of the Main Script with all its accompanying functions as well as explanations on its working principal.

Chapter 2

2.1. Introduction

Modal Analysis is the study of an object's/structure's vibration modes. Modal analysis is important for finding the frequencies, the damping ratios and the modeshapes of a structure which are built-in properties of it [10].

A single degree of freedom system (SDOF) is a system that can be described with a single coordinate system and can be used to describe the motion of a single concentrated mass [11]. The SDOF systems are expressed using a single differential equation and its solution gives us the equation of motion of the system. In most cases an SDOF is not useful for real problems because it has limited potential in describing the dynamics of most mechanical structures.

A continuous system is a system that has an infinite number of degrees of freedom (DOFs). A beam can be described using a continuous system because it has an infinite number of points on it and each point can be corresponded to a DOF. These types of systems require complex mathematics and a lot of computational power to simulate them.

Finally, a multi degree of freedom system (MDOF) is assumed to be a system that describes a mechanical structure with a finite number of degrees of freedom. An MDOF system is a result of discretizing a continuous system into a system with a finite number of components. A discretization method used for problems that require calculations on a continuous mean is usually Finite Element Analysis (FEA). Using the MDOF and making certain assumptions for the continuous system to be converted into a discrete one, the mathematical model needed for explaining the dynamic behavior of a structure becomes less complex and less time-consuming.

When dealing with vibrating structures modeled as multi degree of freedom systems, there are two approaches to characterizing the damping forces. The most common approach is that the damping matrix is a diagonal matrix linearly connected to the mass and stiffness matrixes. This approach is known as Rayleigh damping or classically damped model. Many modern structures are comprised of various substructures usually built with different materials. These complex structures may not be sufficiently explained using the classically damped model and there are other approximations like the non-classically damped model that can be used for such cases [12].

In the analysis below the equations of motion as well as the solution of the eigenvalue problem are presented so that the identification of the modal parameters problem can be comprehended and explained. The approach to the vibrations of a structure is done with the classically damped oscillator model.

2.2. Equations of Motion (Time Domain)

The equation of motion of a structure with multiple DOFs is a system of differential equations that contain the displacement $\underline{u}(t) \in R^m$ with m being the amount of DOFs existing in the system, as presented in Natsiavas' "Vibrations of Mechanical Structures" [13]. The following analysis assumes having a structure with its mass being concentrated on the degrees of freedom. Figure 1 shows a depiction of the MDOF system whose equations of motion will be analyzed. This idealized system

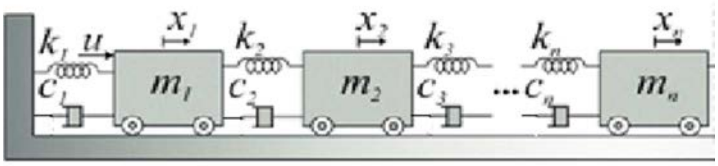

Figure 1: Depiction of an idealized MDOF system

The example examined in this analysis is also characterized as "shear building problem" and analyzed in various structural dynamics journals [14]. This well known structure uses a multi-storey vibrating building with its mass being distributed on the buildings floors. The usage of the shear building assumes no rotation of the horizontal floors leading to the deformed shape being similar to the shape of a deformed beam subjected to shear forces only. Moreover, the resistance of the structure to the shear forces applied is translated into stiffness of the structure and therefore a stiffness matrix can be constructed for the structure. Furthermore the damping of the shear forces on the structure is assumed to happen using linear viscous dampers.

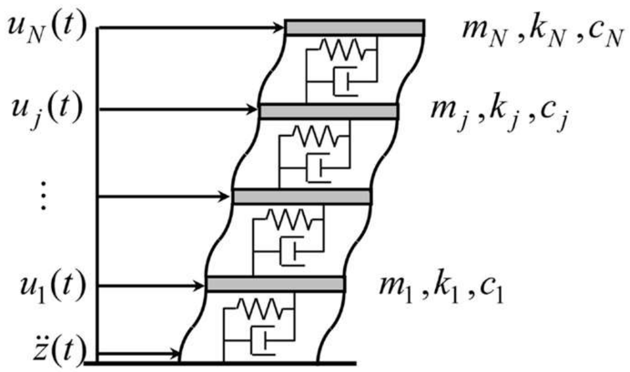

Figure 2: Shear building depiction

The response of a linear structure subjected to a load can be described by the follow system of m differential equations.

$$M * \{\ddot{u}\} + C * \{\dot{u}\} + K * \{u\} = \{F(t)\} \qquad (2.1)$$

Where $M \in R^{mxm}$ is the mass matrix, $C \in R^{mxm}$ is the damping matrix and $K \in R^{mxm}$ is the stiffness matrix. Also $\{\ddot{u}\}$ is the acceleration vector, $\{\dot{u}\}$ is the velocity vector and $\{u\}$ is the displacement vector. The matrixes and vectors above have the form:

$$M = \begin{bmatrix} m_1 & 0 & \cdots & 0 & 0 \\ 0 & m_2 & \cdots & \cdots & 0 \\ \vdots & \cdots & \ddots & \cdots & \vdots \\ 0 & \cdots & \cdots & m_{m-1} & 0 \\ 0 & 0 & \cdots & 0 & m_m \end{bmatrix},$$

$$C = \begin{bmatrix} c_1 + c_2 & -c_2 & \cdots & 0 & 0 \\ -c_2 & c_2 + c_3 & -c_3 & \cdots & 0 \\ \vdots & -c_3 & \ddots & \cdots & \vdots \\ 0 & \cdots & \cdots & c_{m-1} + c_m & -c_{m-1} \\ 0 & 0 & \cdots & -c_{m-1} & c_m \end{bmatrix},$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & \cdots & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & \cdots & 0 \\ \vdots & -k_3 & \ddots & \cdots & \vdots \\ 0 & \cdots & \cdots & k_{m-1} + k_m & -k_{m-1} \\ 0 & 0 & \cdots & -k_{m-1} & k_m \end{bmatrix},$$

Also,

$$\{\ddot{u}\} = \begin{Bmatrix} \ddot{u}_1 \\ \vdots \\ \ddot{u}_m \end{Bmatrix}, \{\dot{u}\} = \begin{Bmatrix} \dot{u}_1 \\ \vdots \\ \dot{u}_m \end{Bmatrix}, \{u\} = \begin{Bmatrix} u_1 \\ \vdots \\ u_m \end{Bmatrix}, \{F(t)\} = \begin{Bmatrix} F_1(t) \\ \vdots \\ F_m(t) \end{Bmatrix}$$

The acceleration, velocity and displacement vectors have dimensions $mx1$. Moreover, the $\{F(t)\}$ vector represents the forces asked on each of the degrees of freedom.

It is apparent that the mass matrix, damping matrix and stiffness matrix are symmetrical with

$$M = M^T, C = C^T, K = K^T$$

2.3. Modal Analysis

Using the classically damped model presented before, which assumes M, C and $K$are symmetric and $\{M, C, K\} \in R^{mxm}$, the solution to the equations of motion for the displacement are $\underline{u}(t) \in R^{mx}$. Through modal analysis the solution of the modal problem can be written in the form:

$$\underline{u}(t) = \sum_{r=1}^{m} \underline{\Phi_r} * \xi_r(t) \tag{2.2}$$

Where $\xi_r(t)$ are given by the equations.

$$\ddot{\xi}_r(t) + 2\zeta_r\omega_r\,\dot{\xi}_r(t) + \omega_r{}^2\,\xi_r = \underline{\Phi_r}^T * \underline{F}(t) \tag{2.3}$$

On equation (2.3) ω_r represents the modal frequency, ζ_r is the damping ratio and $\underline{\Phi_r}$ represents the modeshape vector. Using Fast Fourier Transformation, the equation above becomes

$$\underline{\hat{u}}(\omega) = \sum_{r=1}^{m} \underline{\Phi_r} * \widehat{\xi_r}(\omega) \tag{2.4}$$

and

$$(i\omega)^2\hat{\xi}_r(\omega) + 2\zeta_r\omega_r(i\omega)\hat{\xi}_r(\omega) + \omega_r^{\,2}\hat{\xi}_r(\omega) = \underline{\Phi_r}^T * \hat{\underline{F}}(\omega) \Rightarrow$$

$$\hat{\xi}_r(\omega) = \frac{\underline{\Phi_r}^T * \hat{\underline{F}}(\omega)}{(i\omega)^2 + 2\zeta_r\omega_r(i\omega) + \omega_r^{\,2}} \tag{2.5}$$

Where ω is the frequency range of the excitation and the response of the structure.

Using the frequency domain instead of the time domain for this analysis helps better understand and visualize the results of a vibrating structure. A signal presented in the frequency domain displays the variation of the signal within a frequency band which includes a range of frequencies [15].

The combination of equations (2.4) and (2.5) leads to the following equation that describes the displacement in the frequency domain:

$$\hat{\underline{u}}(\omega) = \left[\sum_{r=1}^{m} \frac{\underline{\Phi_r} \cdot \underline{\Phi_r}^T}{-\omega^2 + 2\zeta_r\omega_r(i\omega) + \omega_r^{\,2}}\right] * \hat{\underline{F}}(\omega) \tag{2.6}$$

$\underline{H}(i\omega)$ can now be defined as the transfer function of the model. A transfer function in general is a mathematical function that represents the relationship between a system's output signal based on each different input signal. For the modal model above the transfer function is:

$$\underline{H}(i\omega) = \sum_{r=1}^{m} \frac{\underline{\Phi_r} \cdot \underline{\Phi_r}^T}{-\omega^2 + 2\zeta_r\omega_r(i\omega) + \omega_r^{\,2}} \text{ },\underline{H}(i\omega) \in C^{mxm} \tag{2.7}$$

In the special case when the excitation on the structure $\hat{\underline{F}}(t)$ acts on some of the DOFs, then $\hat{\underline{F}}(t)$ can be expressed as:

$$\hat{\underline{F}}(t) = \underline{L}_p * \hat{\underline{f}}(t) \tag{2.8}$$

Therefore transferring $\hat{\underline{F}}(t)$ in the frequency domain through Fourier Transformation, equation (2.8) becomes:

$$\hat{\underline{F}}(\omega) = \underline{L}_p * \hat{\underline{f}}(\omega), \qquad \text{where} \quad \hat{\underline{f}}(\omega) \in C^{mx1} \tag{2.9}$$

It should be noted that the case examined and coded in MATLAB assumes that the structure analyzed has an excitation only on one of the degrees of freedom (in case of the shear building the excitation is assumed to be on the base of the structure). This makes the system a Single Input Multiple Output system and the participation factor turns out to be

$$\underline{L}_p = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \underline{L}_p \in Z^{mx1}$$

and

$$\hat{f}(\omega) \in C^{1xN}$$

where N is the number of measurements in the frequency domain.

With the use of this special case in the response equation (2.6) derived before, it becomes:

$$\underline{\hat{u}}(\omega) = \left[\sum_{r=1}^{m} \frac{\underline{\Phi}_r \cdot \underline{\Phi}_r{}^T \cdot \underline{L}_p}{-\omega^2 + 2\zeta_r \omega_r (i\omega) + \omega_r{}^2} \right] * \hat{f}(\omega) \tag{2.10}$$

Defining the participation factor as $\underline{P}_r = \underline{\Phi}_r{}^T \cdot \underline{L}_p$ equation (2.10) becomes

$$\underline{\hat{u}}(\omega) = \left[\sum_{r=1}^{m} \frac{\underline{\Phi}_r \cdot \underline{P}_r}{-\omega^2 + 2\zeta_r \omega_r (i\omega) + \omega_r{}^2} \right] * \hat{f}(\omega) \tag{2.11}$$

Making the assumption that the frequency range can be separated into 'k' equal spaces with size $\Delta\omega$, we can prove that

$$\omega = k * \Delta\omega \tag{2.12}$$

which helps relate every frequency value 'ω' with the parameter 'k'.

It is clear that $\underline{u}(k), \underline{H}(k)$ is a function of the parameter set $\theta = \{\omega_r, \zeta_r, \underline{\Phi}_r,\ for\ r = 1,2, \dots, m\}$. As a result, we can rephrase the equation above as :

$$\underline{\hat{u}}(k;\theta) = \left[\sum_{r=1}^{m} \frac{\underline{\Phi}_r \cdot \underline{P}_r}{-(k\Delta\omega)^2 + 2\zeta_r \omega_r \big(i(k\Delta\omega)\big) + \omega_r{}^2} \right] * \hat{f}(k\Delta\omega) \Rightarrow$$

$$\underline{\hat{u}}(k;\theta) = \underline{H}(k;\theta) * \hat{f}(k\Delta\omega) , \underline{\hat{u}}(k;\theta) \in C^{mxN} \tag{2.13}$$

where m is the number of the k intervals used in discretizing the frequency range.

The modal analysis in the frequency domain done in this chapter is of vital importance in finding the response of the system's degrees of freedom and being able to form an algorithm for approaching the modal identification problem. In the following chapter an optimization problem is used in order to compare the classically damped model and the measured output of a structure and estimate its modal characteristics.

Chapter 3

3.1. Least Squares Problem

In this chapter, a methodology for identifying dynamic modal parameters in the frequency domain is presented. The modal parameters are derived from input and output signals of a structure and the analysis below can be used in Single Input Single Output (SISO) systems or Single Input Multiple Output (SIMO) systems.

The main goal of identifying the modal parameters that are contained in the parameter set $\underline{\theta} = \{\omega_r, \zeta_r, \underline{\Phi_r},\ for\ r = 1,2, \dots, m\}$ is to minimize the difference between the Transfer Function $\left(\underline{H}(k;\theta)\right)$ of the classically damped model and the Frequency Response Function $(\underline{\hat{H}}(k))$ measured on the structure.

Frequency Response Function (FRF) is calculated from the response and excitation signal on any system. It is sometimes referred to as a Transfer Function, a term used in the mathematical explanation of control systems, and it describes the correlation between two signals. In modal analysis the Frequency Response Function uses the output signal on a measured degree of freedom (usually the acceleration response) and input signal on the system (usually a force excitation). The FRF is a complex value signal in the frequency domain with either "Magnitude" and "Phase", of "Imaginary" and "Real" information [16]. A typical FRF used in this thesis is illustrated in Figure 1 below, where it is obvious that the peaks in both the real and the imaginary part of the FRF indicate the modes searched.

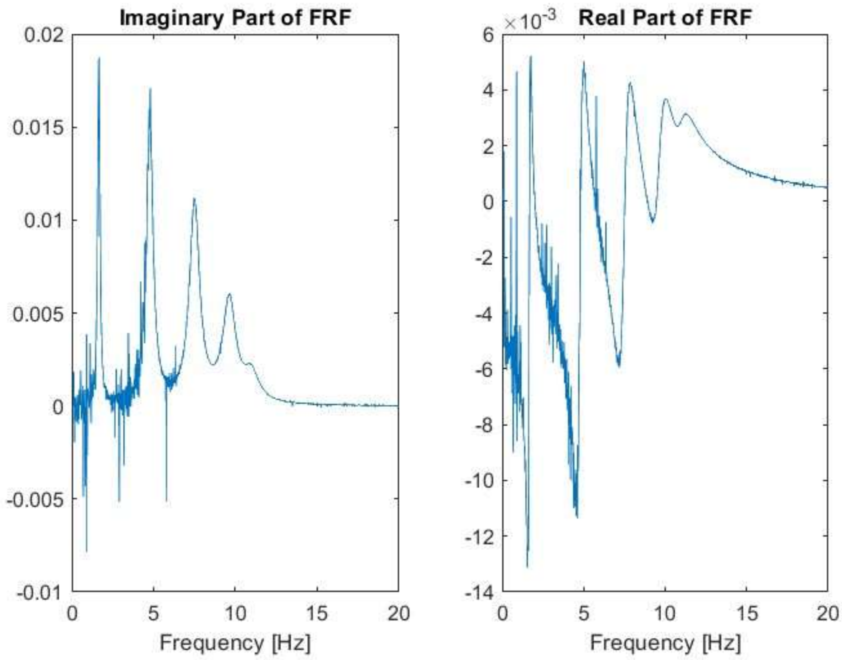

Figure 3: Frequency Response Function Imaginary and Real Part

An error function approach seeks the optimal values for the parameter set $\underline{\theta}$ so that the measured and the calculated data fit together on the frequency domain. As explained in Nikolaou [17] this modal model output-error identification approach uses an objective function minimized with different unconstrained optimization algorithms in order to find the corresponding values for the variables in the parameter set $\underline{\theta}$. Thus, this objective function $J(\underline{\theta})$ can be formulated to express the absolute error between these two data sets.

So, the measurement versus the model output objective function can be formulated to be

$$J(\underline{\theta}) = \sum_{k=1}^{N} \left\| \underline{u}(k;\underline{\theta}) - \underline{\hat{u}}(k;\underline{\theta}) \right\|^{2} \tag{3.1}$$

N being the full frequency spectrum where the minimization is executed.

Consequently, because both the model and the measured structure have the same excitation signal $\hat{f}(k\Delta\omega)$, it appears that:

$$\underline{H}\big(k;\underline{\theta}\big) = \frac{\underline{u}\big(k;\underline{\theta}\big)}{\hat{f}(k\Delta\omega)} \quad \text{and} \quad \underline{\widehat{H}}\big(k,\underline{\theta}\big) = \frac{\underline{\hat{u}}\big(k,\underline{\theta}\big)}{\hat{f}(k\Delta\omega)} \tag{3.2}$$

where $\underline{H}\big(k;\underline{\theta}\big)$ is the Transfer Function predicted by the model whereas $\underline{\widehat{H}}\big(k,\underline{\theta}\big)$ is the Transfer Function measured from the structure. The introduction of equations (3.2) in the objective function:

$$J\big(\underline{\theta}\big) = \frac{1}{V}\sum_{k=1}^{N}\left\|\underline{H}\big(k;\underline{\theta}\big) - \underline{\widehat{H}}\big(k,\underline{\theta}\big)\right\|^{2} \tag{3.3}$$

and the expansion of the values inside the norm transform the equation (3.3) to:

$$J\big(\underline{\theta}\big) = \frac{1}{V}\sum_{k=1}^{N}[\underline{\overline{H}}\big(k;\underline{\theta}\big) - \underline{\overline{\widehat{H}}}(k)]^{T}[\underline{H}\big(k;\underline{\theta}\big) - \underline{\widehat{H}}(k)] \tag{3.4}$$

where

$\underline{H}\big(k;\underline{\theta}\big)$: Model predicted transfer function

$\underline{\widehat{H}}(k)$: Measured transfer function (FRF) at the DOFs of the structure

$\underline{\overline{H}}\big(k;\underline{\theta}\big)$: Conjugate values of the model predicted transfer function

$\underline{\overline{\widehat{H}}}(k)$: Conjugate values of the measured transfer function (FRF) at the DOFs of the structure

$\underline{\theta}$: Parameter set that contains $\{\omega_r, \zeta_r, \underline{\Phi_r},\ for\ r = 1,2,\dots,m\}$

k: Frequency step for intervals $\Delta\omega$

N: Total number of frequency range data

$V = \sum_{k=1}^{N}\underline{\widehat{H}}(k)^{T} * \underline{\widehat{H}}(k)$: Normalization factor needed for the frequency domain

It should be noted that $J\big(\underline{\theta}\big)$ is expected to be a scalar value and be near to 0 at the optimal point.

3.2. Modified Objective Function

The objective function shown in equation (3.4) is a function of the parameter set $\underline{\theta} = \{\omega_r, \zeta_r, \underline{\Phi}_r,\ for\ r = 1,2, \dots, N\}$. The solution of the minimization problem is expected to have $(2 * r + m * r)$ solutions (in case of 3 DOFs we need to approximate the value of 15 parameters). There is an analytical expression that shows that the modeshapes $\underline{\Phi}_r$ are dependent on $\{\omega_r, \zeta_r\}$ for each mode r. This leads to the conclusion that the objective function can be reformed in

$$J(\underline{\theta}) = J(\omega_r, \zeta_r, \underline{\Phi}_r) = J\left(\{\omega_r, \zeta_r\}, \underline{\Phi}_r(\omega_r, \zeta_r)\right) = J(\underline{\varphi})$$

where $\underline{\varphi}$ is a new parameter set containing $\{\omega_r, \zeta_r\}$.

The existence of the boundary condition (3.5) near the actual point (ω_r, ζ_r) :

$$\frac{\partial J\left(\underline{\varphi}\right)}{\partial \underline{\Phi}_{jr}} = 0, j = 1,2, \dots, m \tag{3.5}$$

yields the derivative of $J(\underline{\varphi})$ shown in equation (3.6).

$$\frac{\partial J(\underline{\varphi})}{\partial \underline{\Phi}_{jr}} = \sum_{k=1}^{N} [\underline{H}(k; \underline{\theta}) - \underline{\widehat{H}}(k)]^T * \frac{\partial \underline{H}(k, \underline{\varphi})}{\partial \underline{\Phi}_{jr}} + conj\left(\sum_{k=1}^{N} \left[\underline{H}(k; \underline{\theta}) - \underline{\widehat{H}}(k)\right]^T * \frac{\partial \underline{H}\left(k, \underline{\varphi}\right)}{\partial \underline{\Phi}_{jr}}\right)$$

$$\Rightarrow \frac{\partial J(\underline{\varphi})}{\partial \underline{\Phi}_{jr}} = 2 * Re\left\{\sum_{k=1}^{N} [\underline{H}(k; \underline{\theta}) - \underline{\widehat{H}}(k)]^T * \frac{\partial \underline{H}(k, \underline{\varphi})}{\partial \underline{\Phi}_{jr}}\right\} \tag{3.6}$$

Inserting Kronecker Delta "δ_j" to identify a specific mode j

$$\frac{\partial \underline{H}\left(k, \underline{\varphi}\right)}{\partial \underline{\Phi}_{jr}} = \frac{1}{-(k\Delta\omega)^2 + 2\zeta_r \omega_r \left(i(k\Delta\omega)\right) + {\omega_r}^2} * \ \delta_j, \tag{3.7}$$

where ,$\delta_j^{\,mx} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \end{bmatrix}$

and defining the denominator as

$$\underline{\rho}(k, \underline{\varphi}) = -(k\Delta\omega)^2 + 2\zeta_r\omega_r\big(i(k\Delta\omega)\big) + {\omega_r}^2 \tag{3.8}$$

converts equation (3.6) into:

$$\frac{\partial J(\underline{\varphi})}{\partial \underline{\Phi}_{jr}} = 2 \cdot Re\left\{\sum_{k=1}^{N} \overline{\underline{\rho}}\left(k, \underline{\varphi}\right) \cdot \underline{\rho}\left(k, \underline{\varphi}\right) \underline{\Phi}_{jr}{}^{T} \delta_j\right\} - 2 \cdot Re\left\{\sum_{k=1}^{N} \underline{\widehat{H}}(k) \cdot \underline{\rho}\left(k, \underline{\varphi}\right) \delta_j\right\} = 0$$

Solving the equation above for $\underline{\Phi}_{jr}$ leads to the conclusion that

$$\underline{\Phi}_r(k, \underline{\varphi}) = \frac{Re\left\{\sum_{k=1}^{N} \underline{\widehat{H}}(k) \cdot \underline{\rho}\left(k, \underline{\varphi}\right)\right\}}{Re\left\{\sum_{k=1}^{N} \overline{\underline{\rho}}\left(k, \underline{\varphi}\right) \cdot \underline{\rho}\left(k, \underline{\varphi}\right)\right\}} \tag{3.9}$$

Equation (3.9) is valid when searching for one resonant frequency in the frequency range specified and cannot be used having overlapping modes. For multiple modes the analysis would be different and Kronecker Delta "δ_j" wouldn't identify a single mode.

Having defined the $\underline{\Phi}_r(k, \underline{\varphi})$ it is now possible to rephrase the objective function equation (3.4) so that it depends only on the parameter set $\underline{\varphi} = \{\omega_r, \zeta_r, r = 1,2, \dots, m\}$.

$$J\left(\underline{\varphi}\right) = \frac{1}{V}\sum_{k=1}^{N}\left[\underline{H}\big(k; \underline{\theta}\big) - \underline{\widehat{H}}(k)\right]^{T}\left[\underline{H}\big(k; \underline{\theta}\big) - \underline{\widehat{H}}(k)\right] \Rightarrow$$

$$J\left(\underline{\varphi}\right) = \frac{1}{V}\sum_{k=1}^{N}\left[\frac{\underline{\Phi}_r(k, \underline{\varphi})}{\overline{\underline{\rho}}\left(k, \underline{\varphi}\right)} - \overline{\underline{\widehat{H}}}(k)\right]^{T}\left[\frac{\underline{\Phi}_r(k, \underline{\varphi})}{\underline{\rho}\left(k, \underline{\varphi}\right)} - \underline{\widehat{H}}(k)\right] \tag{3.10}$$

The last expression (3.10) of the objective function is easier to minimize because the number of parameters estimated are now $2 * m$, where m is the number of DOF of the system.

3.3. Objective Function Gradient

Solving an unconstrained nonlinear optimization problem, like the modal parameters identification, requires an Objective Function like the one shown in equation (3.10). To find extrema's of a function, a lot of optimization algorithms require supplying the gradients of the Objective Function. In this section of the thesis the analytical gradients for the least squares objective function will be computed.

Based on the analysis by Nikolaou [17] and using equation (3.10), the analytical gradients of the objective function with respect to each parameter in the parameter set $\underline{\varphi}$ are defined as:

$$\frac{\partial J(\underline{\varphi})}{\partial \varphi} = \frac{1}{V}\sum_{k=1}^{N}\left[\frac{\underline{\Phi_r}(k,\underline{\varphi})}{\overline{\underline{\rho}}\left(k,\underline{\varphi}\right)} - \overline{\underline{\widehat{H}}}(k)\right]^T \left[\frac{1}{\underline{\rho}\left(k,\underline{\varphi}\right)}\frac{\partial \underline{\Phi_r}}{\partial \varphi} - \frac{\underline{\Phi_r}(k,\underline{\varphi})}{\underline{\rho}^2\left(k,\underline{\varphi}\right)}\frac{\partial \underline{\rho}\left(k,\underline{\varphi}\right)}{\partial \varphi}\right] + CT \quad (3.11)$$

where CT stands for "conjugate transpose" of the term

$$\left[\frac{\underline{\Phi_r}(k,\underline{\varphi})}{\overline{\underline{\rho}}\left(k,\underline{\varphi}\right)} - \overline{\underline{\widehat{H}}}(k)\right]^T \left[\frac{1}{\underline{\rho}\left(k,\underline{\varphi}\right)}\frac{\partial \underline{\Phi_r}}{\partial \varphi} - \frac{\underline{\Phi_r}(k,\underline{\varphi})}{\underline{\rho}^2\left(k,\underline{\varphi}\right)}\frac{\partial \underline{\rho}\left(k,\underline{\varphi}\right)}{\partial \varphi}\right].$$

Given the fact that the problem only requires the real part of the complex values handled above, equation (3.11) becomes

$$\frac{\partial J\left(\underline{\varphi}\right)}{\partial \varphi} = \frac{2}{V} Re\left\{\sum_{k=1}^{N}\left[\frac{\underline{\Phi}_r\left(k,\underline{\varphi}\right)}{\overline{\underline{\rho}}\left(k,\underline{\varphi}\right)} - \overline{\underline{\widehat{H}}}(k)\right]^T \left[\frac{1}{\underline{\rho}\left(k,\underline{\varphi}\right)}\frac{\partial \underline{\Phi}_r}{\partial \varphi} - \frac{\underline{\Phi}_r\left(k,\underline{\varphi}\right)}{\underline{\rho}^2\left(k,\underline{\varphi}\right)}\frac{\partial \underline{\rho}\left(k,\underline{\varphi}\right)}{\partial \varphi}\right]\right\} \quad (3.12)$$

$$\frac{\partial J\left(\underline{\varphi}\right)}{\partial \varphi} = \frac{2}{V}\begin{bmatrix} Re\left\{\sum_{k=1}^{N}\left[\frac{\underline{\Phi_r}\left(k,\underline{\varphi}\right)}{\overline{\underline{\rho}}\left(k,\underline{\varphi}\right)} - \overline{\underline{\widehat{H}}}(k)\right]^T \left[\frac{1}{\underline{\rho}\left(k,\underline{\varphi}\right)}\frac{\partial \underline{\Phi_r}}{\partial \omega_r} - \frac{\underline{\Phi_r}\left(k,\underline{\varphi}\right)}{\underline{\rho}^2\left(k,\underline{\varphi}\right)}\frac{\partial \underline{\rho}\left(k,\underline{\varphi}\right)}{\partial \omega_r}\right]\right\} \\ Re\left\{\sum_{k=1}^{N}\left[\frac{\underline{\Phi_r}\left(k,\underline{\varphi}\right)}{\overline{\underline{\rho}}\left(k,\underline{\varphi}\right)} - \overline{\underline{\widehat{H}}}(k)\right]^T \left[\frac{1}{\underline{\rho}\left(k,\underline{\varphi}\right)}\frac{\partial \underline{\Phi_r}}{\partial \zeta_r} - \frac{\underline{\Phi_r}\left(k,\underline{\varphi}\right)}{\underline{\rho}^2\left(k,\underline{\varphi}\right)}\frac{\partial \underline{\rho}\left(k,\underline{\varphi}\right)}{\partial \zeta_r}\right]\right\} \end{bmatrix} \quad (3.13)$$

The first line of the matrix indicates that parameter $\varphi == \omega_r$ and the second line of the matrix indicates that parameter $\varphi == \zeta_r$.

Moreover the derivatives of $\underline{\Phi_r}$ with respect to ω_r and ζ_r are defined as

$$\frac{\partial \underline{\Phi_r}}{\partial \varphi} = -{a_r}^3 Re\left\{\sum_{k=1}^{N}\left[\frac{\overline{\underline{\widehat{H}}}(k)}{\underline{\rho}^2\left(k,\underline{\varphi}\right)} * \frac{\partial \underline{\rho}\left(k,\underline{\varphi}\right)}{\partial \varphi}\right]\right\} + {a_r}^2 Re\left\{\sum_{k=1}^{N}\left[\frac{1}{\left|\underline{\rho}\left(k,\underline{\varphi}\right)\right|^2}\frac{\partial\left|\underline{\rho}\left(k,\underline{\varphi}\right)\right|^2}{\partial \varphi}\right] * \frac{\overline{\underline{\widehat{H}}}(k)}{\underline{\rho}\left(k,\underline{\varphi}\right)}\right\} \tag{3.14}$$

where

- $\underline{\rho}\left(k,\underline{\varphi}\right) = -(k\Delta\omega)^2 + 2\zeta_r\omega_r\big(i(k\Delta\omega)\big) + {\omega_r}^2$
- $\frac{\partial \underline{\rho}\left(k,\underline{\varphi}\right)}{\partial \varphi} = \begin{bmatrix} 2\omega_r + 2\zeta_r\big(i(k\Delta\omega)\big) \\ 2\omega_r\big(i(k\Delta\omega)\big) \end{bmatrix}$

First line of the matrix indicates derivating over ω_r ($\varphi == \omega_r$) and the second line of the matrix indicates derivating over ζ_r ($\varphi == \zeta_r$).

- $\left|\underline{\rho}\left(k,\underline{\varphi}\right)\right|^2 = [{\omega_r}^2 - (k\Delta\omega)^2]^2 + 4{\zeta_r}^2{\omega_r}^2(k\Delta\omega)^2$
- $\frac{\partial\left|\underline{\rho}\left(k,\underline{\varphi}\right)\right|^2}{\partial \varphi} = \begin{bmatrix} 2[{\omega_r}^2 - (k\Delta\omega)^2](2\omega_r) + 8{\zeta_r}^2\omega_r(k\Delta\omega)^2 \\ 8\zeta_r{\omega_r}^2(k\Delta\omega)^2 \end{bmatrix}$

First line of the matrix indicates derivating over ω_r ($\varphi == \omega_r$) and the second line of the matrix indicates derivating over ζ_r ($\varphi == \zeta_r$).

- $a_r = \sum_{k=1}^{N}\frac{1}{\left|\underline{\rho}\left(k,\underline{\varphi}\right)\right|^2}$

Having proven the Objective Function Gradient analytically, it is now possible to solve the minimization problem in order to obtain the optimal parameters ω_r *and* ζ_r. The mathematical analysis above is applied in a MATLAB software to estimate the modal characteristics of different structures. The software's operation is presented in the following chapter.

Chapter 4

4.1. Software Structure

In this chapter the structure of the MATLAB script, developed during the period of the thesis, is explained. The goal of the software was to identify the modal characteristics of a structure using input and output signals from sensors on the structure. The mathematical analysis shown in Chapters 2 and 3 was used for the identification process and also different optimization algorithms were checked so that a comparison between them would be made. The software developed is able to identify modal frequencies, modal damping ratios and modeshapes of any MDOF system which is a Single Input Multiple Output (SIMO) control system using the classically damped dynamic model. The MATLAB software approaches the problem of modal characteristics estimation, as stated before, in three stages that will be presented extensively below.

4.1.1. Signals and FRFs (Stage 1)

The first function of the MATLAB software is to analyze the data input from the structure, supplied by the user. These data inputs are acceleration measurements of the excitation and the response of the structure on each of the DOFs that has a measuring device (accelerometer). The time domain data provided by the user are converted into the frequency domain data using Fast Fourier Transform and the Frequency Response Functions (FRFs) are calculated. The FRFs, as shown below, are a result of the following equation:

$$FRF_m^{Nx1} = \frac{\underline{\hat{u}}_m}{\underline{\hat{f}}_{sys\_input}} \in C^{Nx1}$$

where, m is a particular DOF whose FRF is calculated, $\underline{\hat{u}}_m$ is the measured response at the particular DOF and $\underline{\hat{f}}_{sys\_input}$ is the input excitation at a particular DOF of the system, usually the base of the structure.

The Frequency Response Functions of the structure are of vital importance to make the initial estimation for the modal frequencies and damping ratios (ω_r, ζ_r).

Showcasing the absolute values of the FRFs for all the DOFs available follows, and an initial estimation for (ω_r, ζ_r) for $r = 1,2,\dots,m$ is made by the user. The initial estimation renders the software less time consuming and the estimated values for the modal properties more accurate.

4.1.2. Solving the Optimization Problem (Stage 2)

In Stage 2 the software uses the FRFs calculated in Stage 1 and the initial estimations of the modal properties that the user registered, and using a MATLAB function file that contains the Weighted Least-Squares Objective Function (equation 3.1.4), introduced in Chapter 3, tries to solve the non-linear unconstrained optimization problem. The optimization problem that should be solved is a minimization problem with $J(\underline{\theta})$ being the function minimized. The minimization of the Objective Function is solved with two different optimization algorithms whose results are compared below.

The first algorithm that is capable of solving this non-linear, unconstrained optimization problem is the "Quasi-Newton" algorithm which is a common optimization method used because of the minimal computational effort needed, while being accurate, compared to other arithmetic algorithms like "Full-Newton" method or "Steepest Decent" method [18]. Quasi-Newton method can be applied to finding extrema's of scalar-valued functions searching for zeros in the gradient of the function. These arithmetic calculations of gradients are all done through the built in 'fminunc' function that MATLAB has in its Optimization Toolbox [19].

The other algorithm used to solve the unconstrained optimization problem is the "Trust Region" algorithm. The main difference here is that the gradient is user provided and in the modal identification problem the gradients provided $\left(\frac{\partial J(\theta)}{\partial \omega_r} \; and \; \frac{\partial J(\theta)}{\partial \zeta_r}\right)$ were presented in the previous Chapter. The working principle of the "Trust Region" algorithm is that instead of finding a solution to the original objective function, a neighborhood around the current solution is built aiming to find a new best solution in this neighborhood. This neighborhood represents the original function decently in order to specify a new trust region and find a new best solution in each iteration. User provided analytical gradients help with determining the direction of seeking the optimal point [20].

The search of modes takes place by splitting the full frequency range in smaller blocks and running the optimization algorithm in these small frequency blocks. This happens because the mathematical analysis used in the software helps to identify only one mode per searching block, which is very restricting if two different modes exist in nearby frequencies. The optimization algorithms are iterative processes which stop when optimality conditions, like Step Size or First Optimality Order, have reached the pre-specified limit. Those parameters limits have some default values assigned by MATLAB, but the user can change them to make the solution of the optimization more accurate [21]. The results from the minimization process are saved into matrices so as to be used in further stages.

4.1.3. Constructing the classically damped model transfer function, curve fitting and displaying the results (Stage 3)

Having solved the optimization problem in Stage 2 the software has now obtained the calculated values of modal frequencies and damping ratios, $\omega_r \; and \; \zeta_r$ for every $r = 1,2,\dots,Ndof$, and also calculated the modeshapes from the equation (3.9) introduced in Chapter 3. All these calculated values are saved into a matrix and used to construct the classically damped model transfer function which is the main function needed for the comprehension of the dynamic behavior of the structure. Equation (2.7) in chapter 2 shows the proof of the transfer function for a classically damped system. Having constructed the model transfer function, the software plots the results as well as the actual measurements, showing if the two arrays are compatible. Moreover, the results of the modal frequencies and damping ratios are printed in the Command Window for the user to see.

4.2. Validation of the Identification Algorithm using Simulated Data

In the present section simulated data are used with the aim of validating the identification methodologies introduced in the previous chapters. A 3 DOF and a 10 DOF system are used to validate the software's abilities to estimate the modal characteristics of a spring-damper mass model which has predetermined Mass, Damper and Stiffness matrices. Figure 2 illustrates the configuration of the structure.

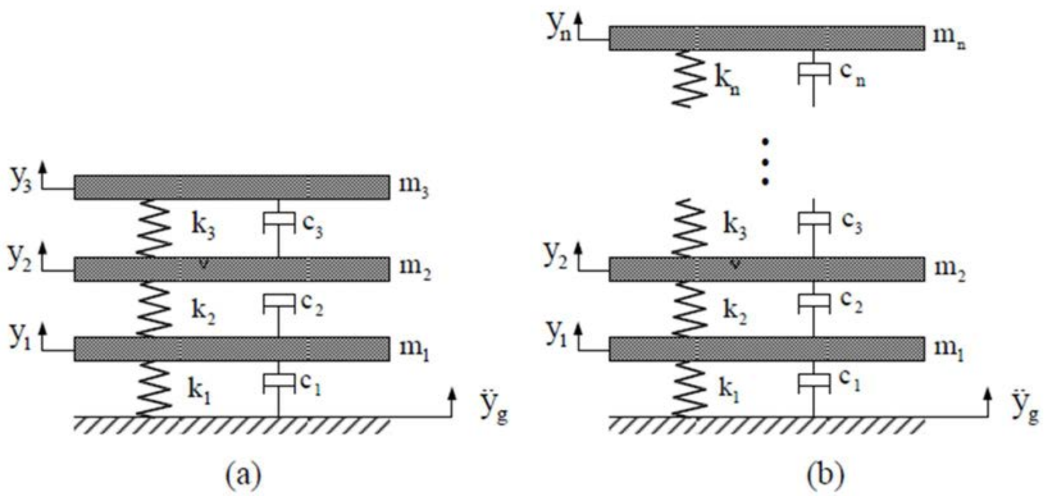

Figure 4: Configuration of the N-DoF system

To begin with, the simulated data had to be generated by MATLAB. The response of each of the measured DOFs is calculated by the built-in MATLAB function "lsim". It should be noted that conversion of the equations of motion of the structure into a State Space system and entering the excitation data on the base of the structure were necessary steps to run "lsim" function. The state space formulation of the equations of motion was done based on Natsiavas [13].

In Figure 5 (a) the excitation signal lies in the time domain. This excitation signal represents an earthquake at the base of the structure. In Figure 3 (b) the same signal is presented after applying the Fast Fourier Transformation and transferring it on the frequency domain. A signal converted from time to frequency domain through Fourier Transformation gives a complex valued array. In Figure 3 (b) the absolute value of this complex array is depicted.

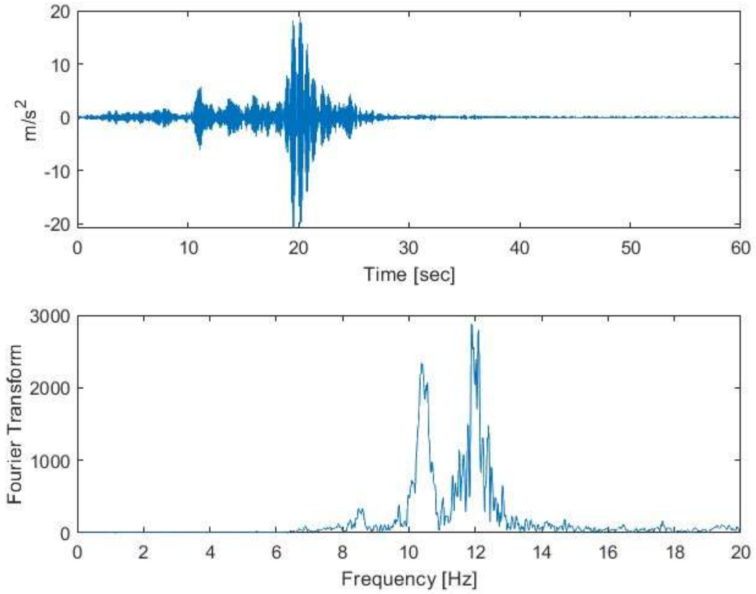

Figure 5: Earthquake Excitation Signal of the System in a) Time Domain, b)Frequency Domain

An excitation on the first floor of the structure is expected to give certain response on the rest of the floors of the building. The response of the measured degrees of freedom is predicted through the "lsim" MATLAB function and presented in Figure 4. All the DOFs' responses are shown in the frequency domain for better visualization of the natural frequencies. Figure 6 shows the response for each of the degrees of freedom of the 3DOF system and Figure 7 shows the response of the 10DOF system.

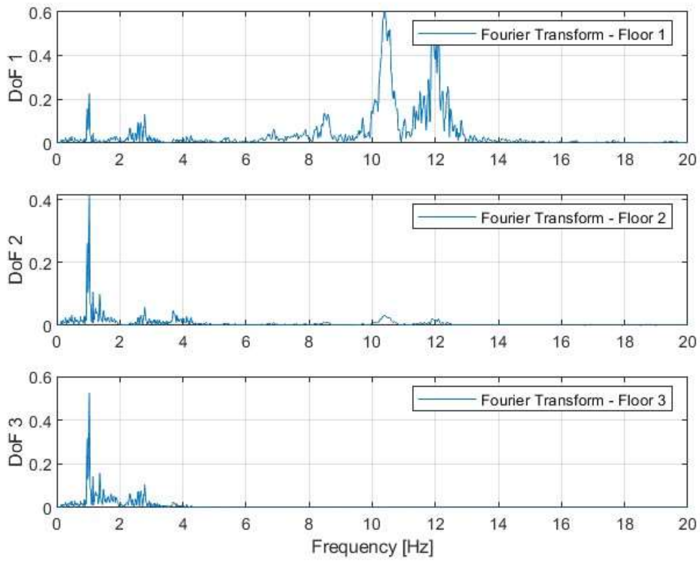

Figure 6: Simulated Response for the 1st, 2nd and 3rd floor of 0 the 3DOF system

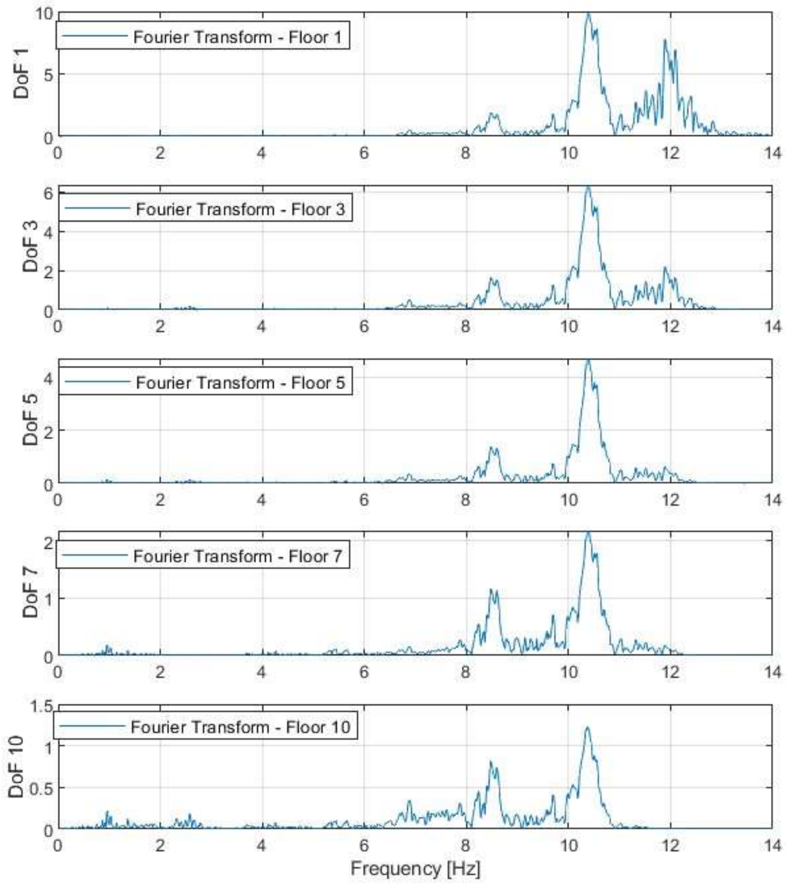

Figure 7: Simulated Response for the 1st, 3rd, 5th, 7th and 10th floor of the 10DOF system

The use of the excitation on the base of the system as well as the simulated responses of each floor (input – output signals) produces the Frequency Response Functions which are shown so that the user can make an initial estimation of the modal parameters $\omega_r\ and\ \zeta_r$ for $r = 1,2,\dots,Ndof$. These Frequency Response Function arrays that are plotted in Figures 8 and 9 are the ones used in the objective function (equation 3.10) for the measured transfer function $\underline{\widehat{H}}(k)$.

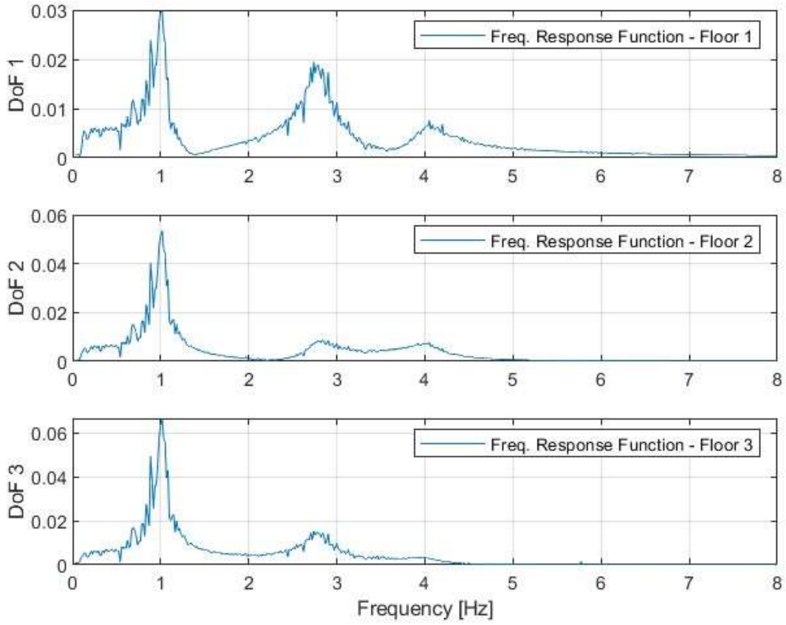

Figure 8: Frequency Response Functions for 1st, 2nd and 3rd floor of the 3DOF model

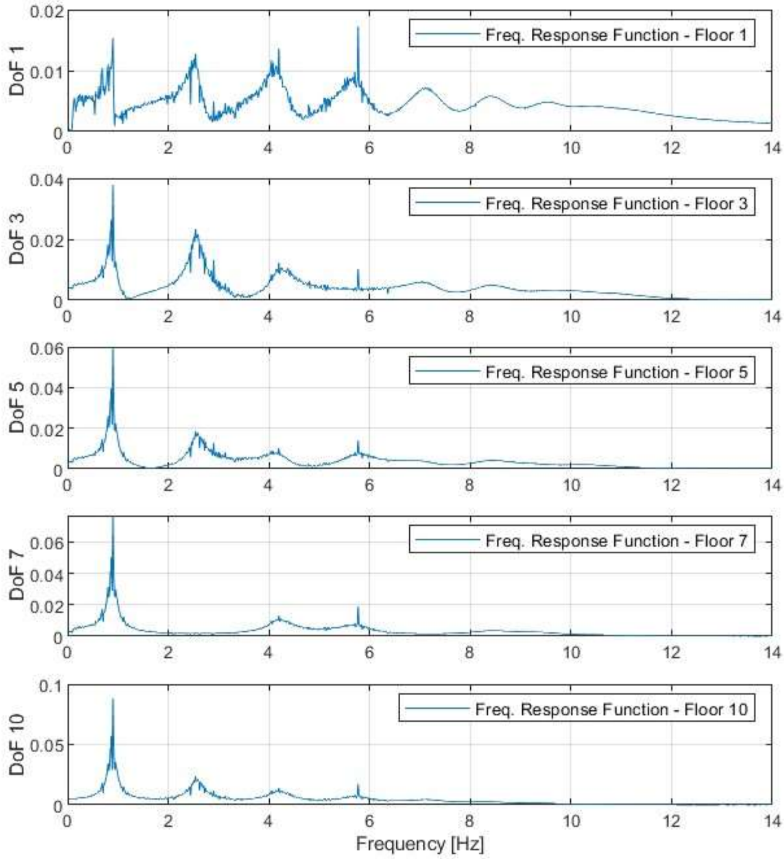

Figure 9: Frequency Response Functions for 1st, 3rd, 5th, 7th and 10th floor of the 10DOF model

Frequency Response Functions shown in Figure 8 and Figure 9 indicate certain frequency points where the structure has significantly large response. These peaks indicate possible existence of modes. The larger the peak the more significant the particular frequency for the building. The frequencies where the peaks appear are the modal frequencies the software aims to compute.

Shown the Frequency Response Functions the user estimates the modal frequency values and the modal damping ratios of the structure (typically for cement structures $\zeta_r = 0.05$ and for steel structures $\zeta_r = 0.1$).

Running the optimization script with the objective function defined in Chapter 3 and providing all the input and output data in the system the results of the Modal Parameter Estimation software are shown in Table 1 and Table 2. For each of the two examples presented in this thesis (3DOF and 10DOF) the minimization problem was solved with both the Quasi-Newton and the Trust-Region algorithm.

4.2.1. <u>3 DOF Example Results</u>

Table 1: Comparison of Calculated and Real Modal Parameters for 3DOF model.

3 DOF Example						
Mode Number	Simulated Model Parameters		Calculated Modal Parameters			
			Quasi-Newton Algorithm		*Trust Region Algorithm*	
	ω_r $[Hz]$	$\zeta_r[\%]$	ω_r $[Hz]$	$\zeta_r[\%]$	ω_r $[Hz]$	$\zeta_r[\%]$
1	0.984	0.05	0.983	0.044	0.984	0.051
2	2.757	0.05	2.752	0.056	2.748	0.049
3	3.984	0.05	3.989	0.048	3.987	0.048

Notice the different algorithms end up in almost equal results with the Trust Region algorithm, which handles analytical derivatives of the objective function, being slightly more accurate.

Using the calculated values for ω_r and ζ_r from the optimization process it is now possible to implement them in a classically damped model Transfer Function and do a fitting between the calculated Transfer Function and the Frequency Response Functions of the measured DOFs presented in Figure 4. The expected result from the data fitting is validating that the FRFs are near to identical with the Transfer Functions calculated. The better the fit the more accurate the modal parameter estimation from the software. Figure 10 illustrates a great fit between the FRFs and the Transfer Functions calculated for each of the floors of the building. Obtaining the correct transfer function for the 3DOF building, it is possible to calculate the response for each of the floors of the building from the equation

$$\underline{H}(k;\underline{\theta}) = \frac{\underline{u}(k;\underline{\theta})}{\hat{f}(k\Delta\omega)} \Longrightarrow \underline{u}(k;\underline{\theta}) = \underline{H}(k;\underline{\theta}) * \hat{f}(k\Delta\omega)$$

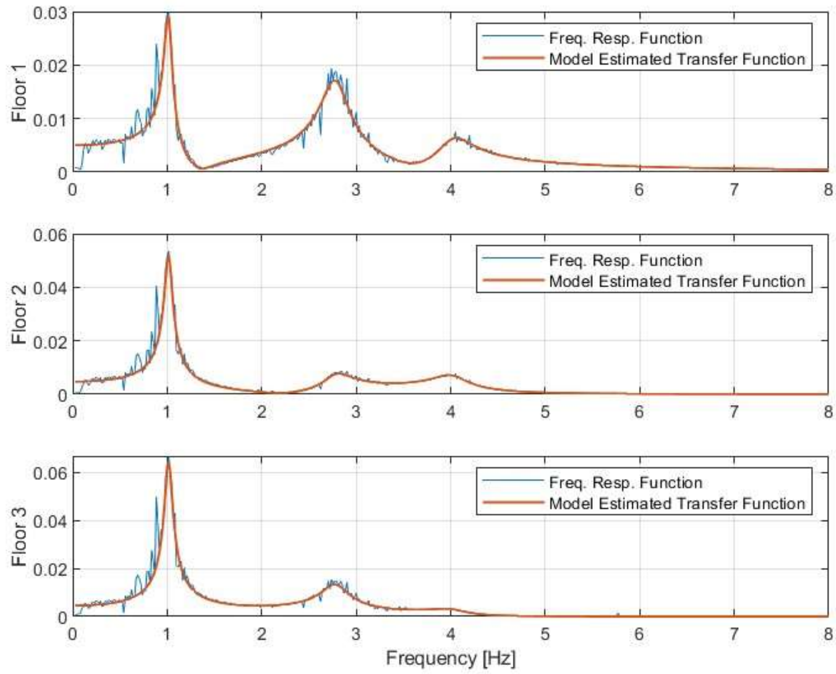

Figure 10: Fitting between Calculated and Real data for the 3DOF system.

It is apparent that the estimated modal frequencies and damping ratios have been computed with great accuracy which validates that the identification algorithm and the minimization script work decently. The modeshapes from the estimated modal characteristics are shown in Figure 11.

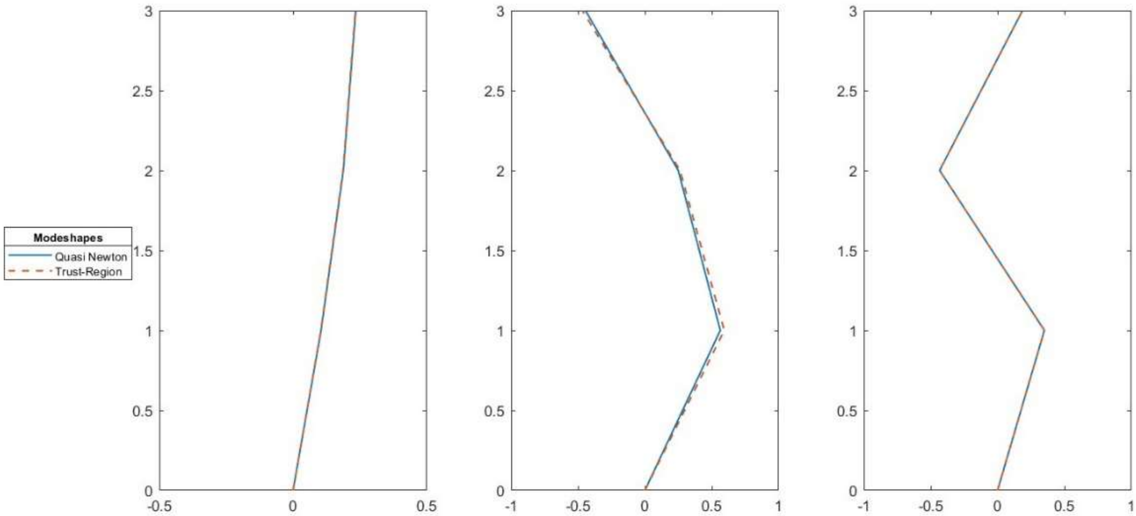

Figure 11: Modeshapes for the 3DOF example

The modeshapes that arise for the 3DOF example show how the structure deforms at the different modal frequencies. In general modeshapes are also useful for locating which regions experience higher stresses in extreme load scenarios or in case of resonance [22].

4.2.2. 10 DOF Example Results

Table 2: Comparison of Calculated and Real Modal Parameters for the 10DOF model

10 DOF Example						
Mode Number	Simulated Model Parameters		Calculated Modal Parameters			
			Quasi-Newton Algorithm		*Trust Region Algorithm*	
	ω_r $[Hz]$	$\zeta_r[\%]$	ω_r $[Hz]$	$\zeta_r[\%]$	ω_r $[Hz]$	$\zeta_r[\%]$
1	0.853	0.05	0.852	0.059	0.853	0.051
2	2.541	0.05	2.544	0.049	2.540	0.049
3	4.171	0.05	4.158	0.049	4.162	0.048
4	5.709	0.05	5.777	0.057	5.733	0.048
5	7.119	0.05	7.117	0.048	7.118	0.048
6	8.369	0.05	8.421	0.047	8.411	0.040
7	9.434	0.05	9.433	0.042	9.432	0.040
8	10.287	0.05	10.338	0.036	10.280	0.035
9	10.911	0.05	11.078	0.032	10.946	0.023
10	11.290	0.05	11.171	0.022	10.902	0.049

The two algorithms appear to have the almost the same results. It is clear that the first modes that are clearly visible in the Frequency Response Functions plots are the ones calculated more accurately from the minimization algorithm. Even though bigger modes are not calculated with the same accuracy the results are not disappointing as the relative error between the real and the calculated values are small. The resonant frequencies are obviously calculated with higher accuracy that the damping ratios. Damping ratios happen to be difficult to identify with higher accuracy even when approached with other models like the non-classically damped model.

The data fitting for the two different methods is shown below in Figures 9 and 10. Figure 9 depicts the Quasi-Newton algorithm results and Figure 10 the Trust Region algorithm results.

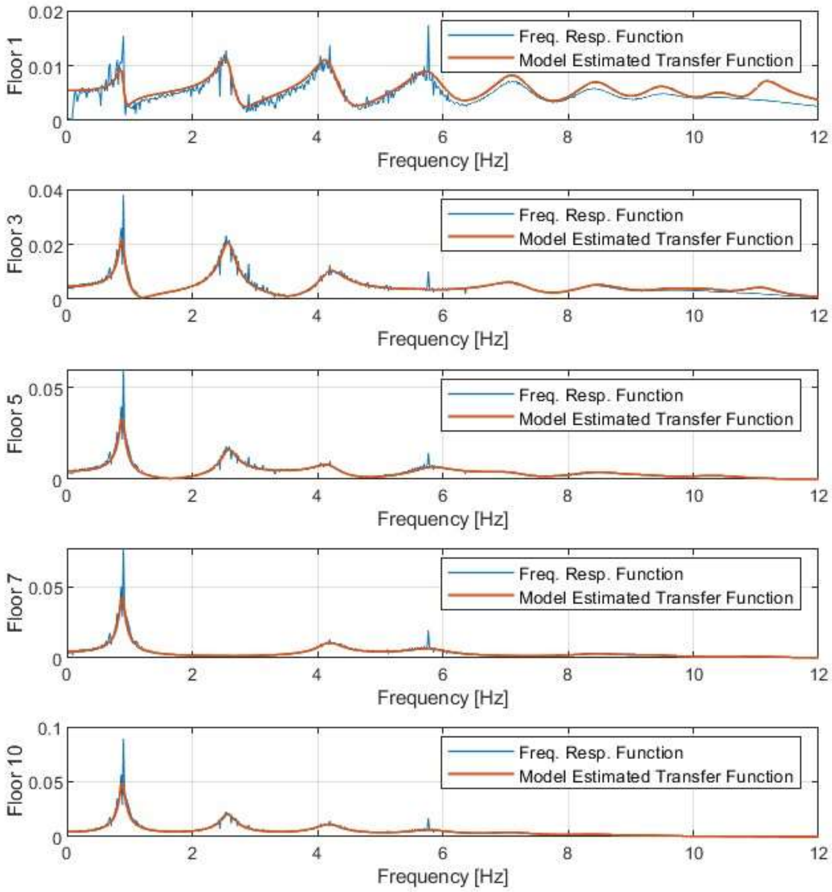

Figure 12: Data Fitting Between the Calculated and the Real Data for the 10DOF example with Numerical Gradients

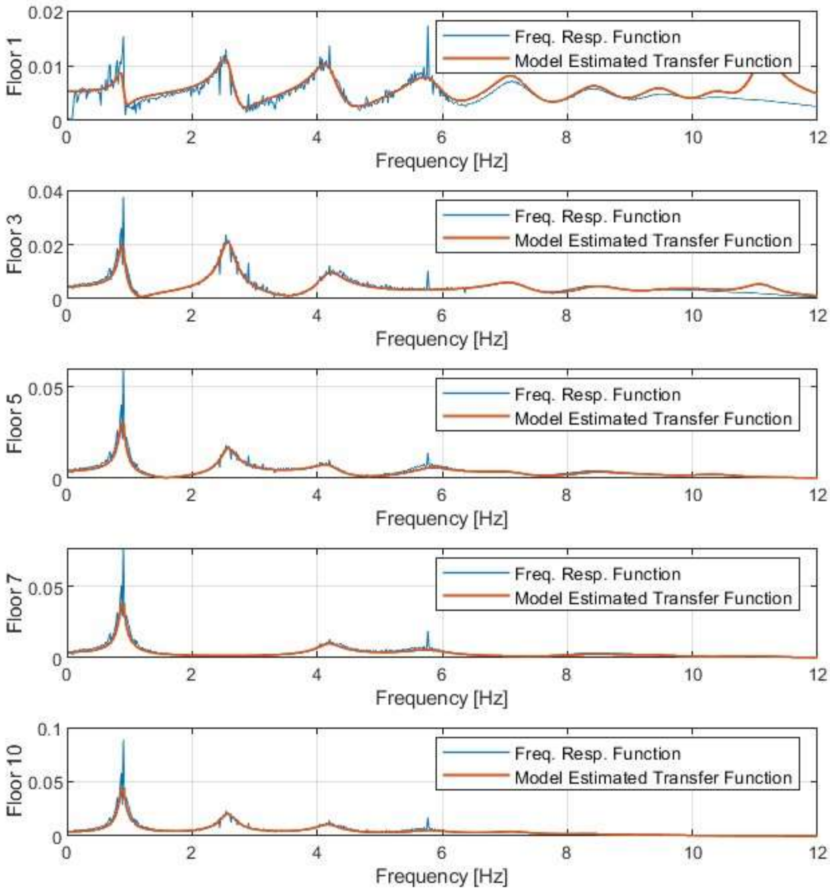

Figure 13: Data Fitting Between the Calculated and the Real Data for the 10DOF example with Analytical Gradients

In the previous figures the fitting between the calculated and the real Transfer Functions seems to be very good especially for the lower frequency modes. In higher frequencies the results from both the Quasi-Newton and the Trust Region methods deviate from the real data. Such behavior is common because there is no significant signal in higher frequencies and the estimation algorithm fails to have high accuracy. Nevertheless, both Table 2 and the curve fitting shown in Figures 11 and 12 validate that modal characteristics of the structure have been accurately calculated and depicted. In Figure 14 the modeshapes of the 10DOF structure are presented.

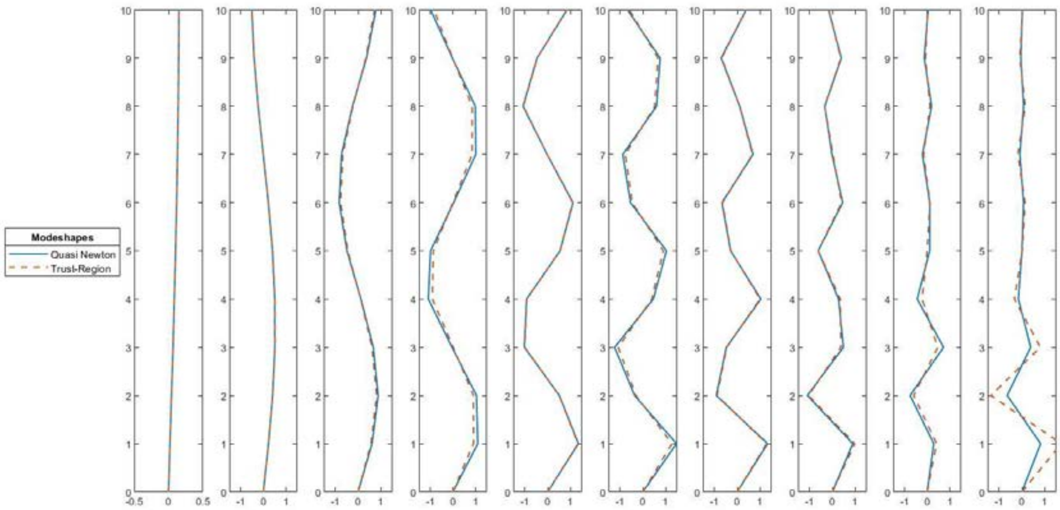

Figure 14: Modeshapes for the 10DOF example

Both algorithms ended up calculating almost indistinguishable modeshapes of the 10 DOF structure which leads to the conclusion that the sensitivity of both algorithms is similar.

Examining the Frequency Response Functions of the structure plotted by the software in the beginning of the script it is clear that on higher frequencies both the data are poor and the structure response tends to zero out. It should be noted that the user can choose to seek for some of the modes not all of them. The user can give an initial estimate of how many modal frequencies and damping ratios and therefore search for less modes. For example, the user may be seeking for the first 5 modes, which happen to be really dominant. User's initial estimations after seeing the FRFs in Figure 13 from the structure would probably be the ones on Table 3.

Table 2: Initial Estimates based on the FRFs

Initial Estimates		
Mode Number	$\omega_r\ [Hz]$	$\zeta_r [\%]$
1	0.9	0.05
2	2.5	0.05
3	4.1	0.05
4	5.7	0.05
5	7.0	0.05

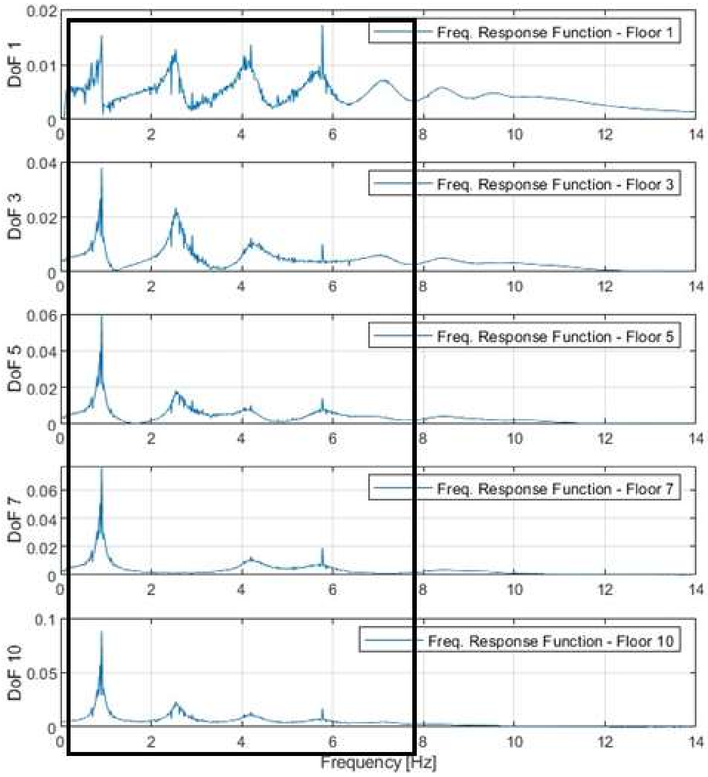

Figure 15: Estimation of 5 dominant modes

Giving the software five estimates of modes is expected to result in 5 search intervals and therefore the software aims to find the five modal frequencies and damping ratios near the estimates the user gave. On Figure 14 the data fitting for the case of 5 modes estimation is shown.

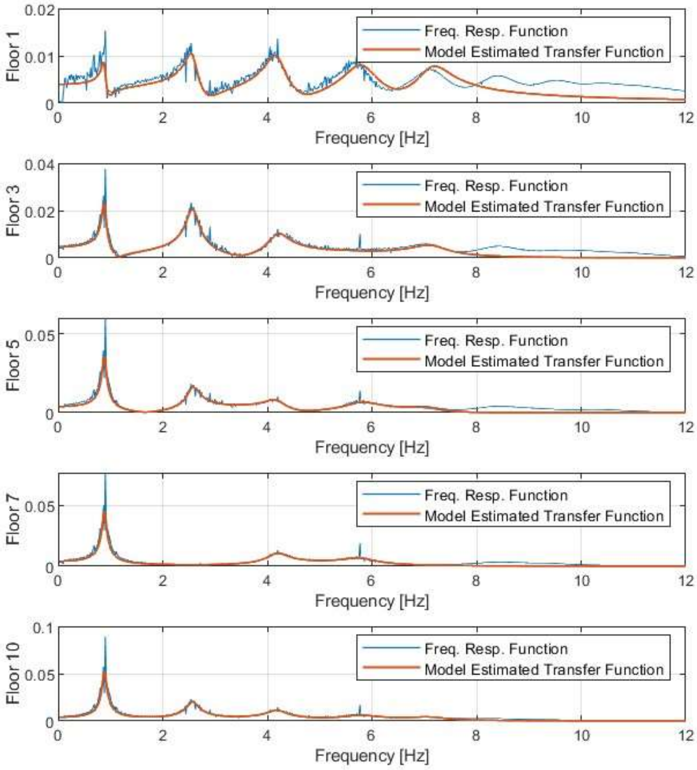

Figure 16: Data Fitting for 5 dominant modes

The plots show the software's ability to be adaptive and that even with less data the results are accurate for both the resonant frequencies and the damping ratios. Moreover searching for some of the dominant modes is less time consuming and needs less computational power.

Chapter 5

Conclusions and Future Work

In this thesis a mathematical methodology was used to approach the problem of identification of modal characteristics of linear structures subjected to loads. The classically damped modal model was used for approaching multi-degree of freedom structures and the modal analysis was carried out in the frequency domain. Specifically the analysis was done in the case of having a single excitation on the structure while examining the response of all the DOFs like a Single Input Multiple Output system. Additionally, a MATLAB software was developed to implement the theoretical analysis done and be able to visualize the results from the identification process. Due to the fact that the identification process is based on a minimization problem two different unconstrained optimization algorithms were examined for calculating the modal characteristics. Both of the optimization algorithms proved to be able to solve the minimization problem having great accuracy. In Chapter 4 of the thesis two experiments were conducted and showcased in order to validate both the mathematical analysis and the execution of the MATLAB software. Tables and Figures in Chapter 4 exhibit the results of the experiments and prove the high accuracy of the methodology as well as the sensitivity to data imperfections. Furthermore, it is proven that the identification algorithm examined requires minimal computational power while being able of achieving great accuracy. In the appendix of the thesis the MATLAB software developed is shown and explained in detail for further usage or enriching with more capabilities.

Modal Identification is of vital importance and has a lot of possibilities for future work despite this thesis. First of all, on the mathematical analysis basis more oscillation models could be used to calculate the theoretical response of a mechanical system like the Non-Classically Damped model presented in Nikolaou Msc Thesis (2006) [17]. Comparison between the results of different vibration models is necessary and usage of more complex structures despite the one dimensional MDOF shear building studied in the present thesis. Moreover, as stated in Chapter 4, the mathematical analysis and the algorithm used to solve the identification problem uses small frequency ranges to find each modal frequency and damping ratio separately, because the mathematical methodology doesn't include the overlapping modes identification. It is important to study the case where two different modes are in nearby frequencies or else referred as overlapping modes and be able to identify them in the same frequency block. In addition, different optimization algorithms could be checked for the nonlinear unconstrained optimization problem introduced in Chapter 3. Algorithms like "Linesearch" methods or "Steepest Descent" methods could be checked for obtaining quicker and possibly more accurate results [23].

In addition, as mentioned in Chapter 1, Operational Modal Analysis can be executed using different types of data like ambient data measured at the structure from wind forces, negligible forces from people or vehicles etc. The development of a modal characteristics identification algorithm avoids the extreme earthquake scenario and gives the opportunity to obtain larger amount of data, thus have a constantly updated model.

Regarding the software perspective of the thesis, additions could be made like forming a user interface in order to create a friendlier environment. Furthermore, the software could be applied in already constructed structures in order to use it as a part of the experimental modal analysis or health monitoring. This may require troubleshooting regarding optimal sensor placement or data management for the software to be accurate and reliable. In addition, modal characteristics identification proposed in this study could be compared or correlated with other structural dynamics methods like Finite Element Analysis so as to create dependable models that accurately represent modern structures.

In general, sensors are proven to be an integral part in modal parameter identification algorithms and accelerometers are one of the most important devices used for structural dynamics. Thus advancements in sensors can reduce noise and faulty data while improving modal analysis results and make structural dynamics progress. In addition, big data management is a difficult task that can be made easy with the use of machine learning algorithms. Doing data analysis through machine learning could help identify patterns or trends in the data that humans couldn’t recognize as well as reduce noise or faulty data [24].

In conclusion, experimental modal analysis is proven to be a very useful and important tool nowadays for designing, building, inspecting and maintenance of structures.

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Chapter 6

Appendix

6.1. Main Script (*Main_estimation_script.m*)

The main script contains the full data analysis of the acceleration of the structure provided by the user in a certain format in the InputFile.mat, which is explained in the sub-chapter 6.2. In the beginning of the script the user inputs some important parameters in order to explain the structure studied and also set up the modal identification algorithm. The user should input the number of degrees of freedom of the MDOF system analysed and the DOF that is excited should also be specified. Moreover, after the user ‘runs’ the code with, optimize parameter set to 0, an initial estimation of modal frequencies and modal damping ratios should be made for the modal estimation algorithm to work.

```
clc;
clear;
close all;

% ------------------------
% |           ___                     /   ___
% |---////---|   |---////---|   |---  / --|   |
% |          |   |          |   |    /    |   |
% |----[|----|___|----[|----|___|---/   --|___|
% |          Mass1          Mass2  /      MassN
% ------------------------
% How many DoFs does your system have?
Ndof=10;

% Where is the input
exc_mass = 1; % what mass is excited

% If you just want to see the response of the Ndof
% system make "optimize"=0.
% If you want to search for the modal properties
% of the system set "optimize"=1.
optimize=0;%<----

% If you want to searh for the modal properties of the system set your
% initial estimation on the Matrix below ("omegar_estimation")
omegar_estimation=[0.853;2.54;4.171;5.708;7.1];%;6;6.8
%if you make 4 estimations for omega_r it means you are searching
%for the first 4 modes.
%The maximum estimations you can make equals the amound of Ndof.
%Give your estimation in [Hz].

% Search region indicates the frequency range where the search for each
% mode is conducted.
% search_region(:,1)=lower bound of frequency range
```

```
% search_region(:,2)=higher bound of frequency range
% (each line indicates a particular mode)
search_region=[0.6,1.0;
               2.3,2.7;
               4.0,4.4;
               5.5,5.9;
               6.9,7.3];


zetar_estimation=0.05; % Initial Estimation for damping ratio (zeta_r)

% ------------ Input File -------------------------------
InputFile = load("10dof_input.mat");
% - 1st column = Time (sec)
% - 2nd column = A_excitation of the structure (m/sec^2)
% - 3rd column = A_response of the 1st DOF (m/sec^2)
% -     ...
% - (N+2)th column = A_response of the Nth DOF (m/sec^2)

InputFile=InputFile.input_file; %struct to double

Time = InputFile(:,1);
F=InputFile(:,2);
Response=InputFile(:,3:size(InputFile,2));
time_step = Time(2)-Time(1);
% the sensor takes a measurement every 'time_step' seconds
fs = 1/time_step; %sampling frequency
N=length(F);


% -------------------------------------------

input = fft(F);
output=zeros(size(Response));
for i=1:Ndof
output(:,i) = fft(Response(:,i));
end

FRF=zeros(size(Response));
% Pre-allocation of the memory for Frequency Respose Functions
for i=1:Ndof
    FRF(:,i)=output(:,i)./input;
end

fx=fs*(0:N/2-1)/N;

% ----------Set the frequency range you want in hz-------------------------
-
freq_start = 0; %[hz]
freq_stop  = 20;%[hz]
for i=1:length(fx)
    if abs(freq_start - fx(i)) <= 0.0165
        start=i;
    end
    if abs(freq_stop - fx(i)) <= 0.0165
        stop = i;
    end
end
```

```
fx_new=fs*(start:stop-1)/N;

figure
for i=1:Ndof
    subplot(Ndof,1,i);plot(fx_new,abs(FRF(1:length(fx_new),i)));
    xlim([0 8])
    grid on
    text="DoF "+num2str(i);
    ylabel(text)
    text="Freq. Response Function - Floor "+num2str(i);
    legend(text)
end
xlabel('Frequency [Hz]')
sgtitle("Frequency Response Functions")

% -------------------------------------------------------------
if optimize==1
fx_rad=fx*2*pi; %Fit frequency in rad/s

tf_real=FRF;
tf_real=tf_real(1:length(fx_rad),:);
%make the tf_real length equal to fx_rad length

% ----- Here comes the Optimization Script -----------
global fx_rad_min tf_real_min
fx_rad_min=fx_rad(start:stop);

% OPTIMIZATION FUNCTION
fun3=@(x)ObjectiveSIMO_GENERAL(x(1),x(2));
%make the objective-function handle

SOLUTION=zeros(length(omegar_estimation),2);
%Preallocate the solution matrix memory
SOLUTION_TRUST_REGION=zeros(length(omegar_estimation),2);
%Preallocate the solution matrix memory (trust-region)

for i=1:length(omegar_estimation)
    freq_start=search_region(i,1);
    %Search region lower point
    %(specified by the user in the beginning of the script)
    freq_stop=search_region(i,2);
    %Search region higher point
    % (         >>         )

    for k=1:length(fx)
        if abs(freq_start - fx(k)) <= 0.0165
            start=k;
        end
        if abs(freq_stop - fx(k)) <= 0.0165
            stop = k;
        end
    end
    fx_rad_min=fx_rad(start:stop);

    %Initial Estimation for Mode k
    omega_r=omegar_estimation(i)*2*pi;
    zeta_r=zetar_estimation;
    x0=[omega_r,zeta_r];
```

```
    tf_real_min=tf_real(start:stop,:);

    options = optimoptions("fminunc","Display","iter");
    SOLUTION(i,:)=fminunc(fun3,x0,options);%,options
    Psi(:,i)=CalculatePSI_general(SOLUTION(i,1),SOLUTION(i,2));
    options = optimoptions('fminunc','Algorithm','trust-region',...
             'SpecifyObjectiveGradient',true,'Display','iter');
    SOLUTION_TRUST_REGION(i,:)=fminunc(fun3,x0,options);

Psi_trust_region(:,i)=CalculatePSI_general(SOLUTION_TRUST_REGION(i,1),...
                          SOLUTION_TRUST_REGION(i,2));
End

fx_rad=fx*2*pi;
omega_r=SOLUTION(:,1);
zeta_r=SOLUTION(:,2);
omega_r_trustr=SOLUTION_TRUST_REGION(:,1); %results from the trust region
algorithm
zeta_r_trustr=SOLUTION_TRUST_REGION(:,2); %results from the trust region
algorithm

for i=1:length(fx_rad)

H(i,:)=H_tf_general(fx_rad(i),omega_r,zeta_r,Psi,Ndof,length(omegar_estimat
ion));%

H2(i,:)=H_tf_general(fx_rad(i),omega_r,zeta_r,Psi_trust_region,Ndof,length(
omegar_estimation));
end
% H: Transfer Function from the Quasi Newton
% H2: Transfer Function from the Trust Region
% --------------------------------------------------------------
figure
for i=1:size(H,2)
    subplot(size(H,2),1,i);plot(fx_new,abs(FRF(1:length(fx_new),i)));
    hold; plot(fx_new,abs(H(1:length(fx_new),i)),'LineWidth',1.2);
    xlim([0,12])
    grid on
    text="Floor "+num2str(i);
    ylabel(text)
    legend("Freq. Resp. Function","Model Estimated Transfer Function")
end
xlabel("Frequency [Hz]")
sgtitle("FRF vs Transfer Funtions of Model")

% Display Results in the Command Window
for i=1:length(omegar_estimation)
    text(i)="Calculated Modal Freq. "+ num2str(i)+...
            " = "+num2str(SOLUTION(i,1)/2/pi)+" Hz";
    disp(text(i))
end
disp("------------------")
for i=1:length(omegar_estimation)
    text(i)="Calculated Damping Ratio "+...
            num2str(i)+" = "+num2str(SOLUTION(i,2));
    disp(text(i))
end
```

```
% Plot the Modeshapes
PSI=[zeros(1,size(Psi,2));Psi];
PSI_TRUST=[zeros(1,size(Psi_trust_region,2));Psi_trust_region];

figure
for i=1:size(Psi,2)
subplot(1,size(Psi,2),i);
plot(-PSI(:,i),0:Ndof,"LineWidth",1.1)%
hold
plot(-PSI_TRUST(:,i),0:Ndof,'--',"LineWidth",1.2)
xlim([-1.5,1.5])
if i==1
    xlim([-0.5,0.5])
end
end

% add legend
Lgnd = legend('Quasi Newton','Trust-Region');
Lgnd.FontSize = 8;
Lgnd.Title.String = 'Modeshapes';
Lgnd.Position(1) = 0.07;
Lgnd.Position(2) = 0.5;


end
```

6.2. Input File

As stated before the main script uses an Input File containing all the necessary input and output data from the accelerometers on the structure. These data are of vital importance because all the analysis is based on acceleration data. The Input File inserted should have a certain format for the code to read it correctly. It should be a .mat file with:

- 1st column: Time measurements (sec)
- 2nd column: $\widehat{f}(\frac{m}{sec^2})$ Excitation measurement of the structure input signal
- 3rd column: $\widehat{u}_1(\frac{m}{sec^2})$ Response measurement of the structure (on the 1st DOF)

  ...

- (N+2) column: $\widehat{u}_N(\frac{m}{sec^2})$ Response measurement of the structure (on the Nth DOF)

The format of the Input File is also explained within the main code before calling it.

6.3. Objective Function (*ObjectiveSIMO_GENERAL.m*)

In this function the main goal is to transfer the analysis done in Chapter 3 for the non-linear unconstrained optimization problem and write a function that can be minimized in order to estimate the modal characteristics like the equation (3.10). Furthermore, this MATLAB function file contains the equations for the analytical derivatives whose analysis was shown in Chapter 3.3. This function called in the main code from the optimization toolbox through fminunc which is the solver used for every unconstrained optimization problem.

```
function [Objective,Gradient] = ObjectiveSIMO_GENERAL(omega_r,zeta_r)
% This is the function that contains the Objective Function that solves the
% optimization problem.
global fx_rad_min tf_real_min


omega=fx_rad_min;
```

```
% -------------------------------------------------------------------------
for k = 1:length(omega)
rho_model(k)=(omega_r^2-omega(k)^2+2*zeta_r*omega_r*(1i*omega(k)));
end
% -------------------------------------------------------------------------
psi_up=tf_real_min(1:length(omega),:)'./((rho_model));
for i=1:size(psi_up,1)
psiup(i)=real(sum(psi_up(i,:)));
end
% -------------------------------------------------------------------------
for k = 1:length(omega)
psi_down(k)=1./((conj(rho_model(k)).*rho_model(k)));
end
psidown=real(sum(psi_down));
% -------------------------------------------------------------------------
Psi=psiup/psidown;

% -------------------------------------------------------------------------
for i=1:length(Psi)
    H(i,:)=Psi(i)./(-omega.^2+2*zeta_r*omega_r*(1i.*omega)+omega_r^2);
end

V= sum(conj(tf_real_min).*tf_real_min);
object = (conj((H.'-tf_real_min(1:length(H),:))).*...
         (H.'-tf_real_min(1:length(H),:)));
Objective = real(real((1./V))*sum(object,1)'); %

% ANALYTICAL DERIVATIVES
if nargout>1

    arr=sum(1./abs(rho_model).^2);
% -------------------------------------------------------------------------
    der_rho_omega = (2*omega_r)+(2*zeta_r)*(1i.*omega);
    der_rho_omega=der_rho_omega.';
    sec_der_rho_omega = 4*omega_r*(omega_r^2-omega.^2)+...
                        8*zeta_r^2*omega_r*(omega.^2);
    sec_der_rho_omega=sec_der_rho_omega.';

    der_rho_zeta = 2*omega_r*(1i*omega);
    der_rho_zeta=der_rho_zeta.';
    sec_der_rho_zeta = 8*zeta_r*omega_r^2*(omega.^2);
    sec_der_rho_zeta=sec_der_rho_zeta.';
% -------------------------------------------------------------------------
    A=zeros(size(tf_real_min'));
    for i=1:3
        A(i,:)=tf_real_min(:,i).'./(rho_model.^2);
    end
    B=zeros(size(tf_real_min'));
    for i=1:3
        B(i,:)=tf_real_min(:,i).'./(rho_model);
    end
    A=A.';
    B=B.';

    der_psi_omega= (-arr.^3)*sum(real((A).*der_rho_omega))...
                  +(arr.^2) * sum(real(((1./(abs(rho_model).^4)).'...
                  .*sec_der_rho_omega).*B)) ;
```

```
    der_psi_zeta= (-arr.^3)*sum(real((A).*der_rho_zeta))...
                  +(arr.^2) * sum(real(((1./abs(rho_model).^4).'...
                  .*sec_der_rho_zeta).*B));

% -------------------------------------------------------------------------
    der_J_omega= (2./V)*(sum(real((((Psi'./rho_model).' - tf_real_min)...
        .* ((1./rho_model).' .* der_psi_omega - (Psi'./(rho_model.^2)).'...
        .*der_rho_omega)))))';

    der_J_zeta= (2./V)*(sum(real((((Psi'./rho_model).' - tf_real_min)...
        .* ((1./rho_model).' .* der_psi_zeta - (Psi'./(rho_model.^2)).'...
        .*der_rho_zeta)))))';
    Gradient = [der_J_omega;...
                der_J_zeta];

end

end
```

6.4. Modeshapes Calculation (*CalculatePSI_general.m*)

The *CalculatePSI_general.m* function is used to calculate the modeshapes for a pair of ω_r and ζ_r. This function is called after the identification of the correct modal characteristics in order to acquire the modeshapes of the structure.

```
function Psi = CalculatePSI_general(omega_r,zeta_r)
global fx_rad_min tf_real_min

omega=fx_rad_min;
% -------------------------------------------------------------------------
for k = 1:length(omega)
    rho_model(k)=(omega_r^2-omega(k)^2+2*zeta_r*omega_r*(1i*omega(k)));
end
% -------------------------------------------------------------------------
psi_up=tf_real_min(1:length(omega),:)'./((rho_model));
for i=1:size(psi_up,1)
    psiup(i)=real(sum(psi_up(i,:)));
end
% -------------------------------------------------------------------------
for k = 1:length(omega)
    psi_down(k)=1./((conj(rho_model(k)).*rho_model(k)));
end
psidown=real(sum(psi_down));
% -------------------------------------------------------------------------
Psi=psiup/psidown;
End
```

6.5. Transfer Function Generation (*H_tf_general.m*)

The following function is the last step of approaching the structure with the classically damped model. *H_tf_general.m* function uses all the parameters estimated from the main code and creates the transfer function of the system using the classically damped model. The transfer function of the model written in this function is explained in equation 2.7.

```
function H_tf_return = H_tf_general(omega,omega_r,zeta_r,rho,Ndof,est_num)
% This function Calculates the Transfer Function of a Classically Damped
MODEL

H_tf_remember=0;
for i=1:Ndof
    for j=1:est_num
        H_tf_1=rho(i,j)./(-omega.^2+2*zeta_r(j)*omega_r(j)*...
              (1i.*omega)+omega_r(j)^2);
        H_tf_remember=H_tf_remember+H_tf_1;
    end
    H(i)=H_tf_remember;
    H_tf_remember=0;
end

H_tf_return=H;

end
```